FIAS Frankfurt Institute for Advanced Studies

SMFNS2013

"Neutron stars (and more) within the pseudo-complex General Relativity"

Neutron stars: Isaac Rodriguez, Peter Otto Hess,
Walter Greiner, S. Schramm

pc-GR and experiment: T. Schönenbach, G. Caspar. M. Schäfer
T. Boller, A, Müller,

CONTENT

- Introduction: Motivation, what is pseudo-complex (pc) and justification.
- pc-GR and pc-Einstein equations.
- Possible origin of the dark energy.
- Some experimental predictions of pc-GR
- Ansatz for the energy-momentum tensor.
- TOV equations.
- Results
- Conclusions

Motivation

FIRST ATTEMPTS.

- A. Einstein, Ann.Math. **46** (1945), 518.
- A. Einstein, Rev. Mod. Phys. **20** (1948), 35.

(Unification of gravitation and electrodynamics)

- C. Mantz, T. Prokopec, (2008); arXiv:0804.0213

more recently: C. Mantz, T.Prokopec, Found. Phys. **41** (2011), 1597

(hermitian gravity and cosmology)

$$X^{\mu} = x^{\mu} + i\frac{l}{m}p^{\mu} \quad , \quad i^2 = -1$$

(Introduction of the Planck lenght, l)

Born's eqivalence principle:

$$\left[x^k, p^j\right] = i\hbar\delta_k \; , \left[x^k, x^j\right] = 0, \left[p^k, p^j\right] = 0$$

but

$$d^{2} = s\, g_{\mu} \, d_{\nu}{}^{\mu} dx^{\nu} \quad x$$

(M. Born, *Proc. Roy. Soc.* A **165** (1938), 291 and
M. Born, *Rev. Mod. Phys.* **21** (1949), 463.)

PROPOSAL
(M. BORN)

$$d\Omega^2 = dx_\mu dx^\mu + l^2 du_\mu du^\mu = dx_\mu dx^\mu \left(1 + l^2 \frac{du_\mu}{d\tau}\frac{du^\mu}{d\tau}\right)$$

$$\rightarrow dx_\mu dx^\mu \left(1 - l^2 a^2\right)$$

$$a^2 = -a_\mu a^\mu = \frac{du_\mu}{d\tau}\frac{du^\mu}{d\tau} \quad \square \quad l^2 \leq \frac{1}{a^2}$$

l is a minimal length

a = maximal acceleration!!!
E.R. Caianiello (1981), H.E. Brandt, R.G. Beil (1980’s)
S.G. Low (1990’s and more recently: representation theory)

Pseudo-complex

$$x^\mu \rightarrow X^\mu = x^\mu + I y^\mu \quad , \quad I^2 = 1$$

$$= X_+^\mu \sigma_+ + X_-^\mu \sigma_- \qquad \sigma_\pm = \frac{1}{2}(1 \pm I)$$

$$\sigma_\pm^2 = \sigma_\pm \quad , \quad \sigma_+ \sigma_- = 0$$

→ Zero divisor

PF Kelly and RB Mann, Class. Quant. Grav. **3** (1986), 705:
From all possible algebraic extensions only the pseudo-complex does not have ghost solutions in the limit of weak gravitational fields.

Final projection to the real 4-dimensional space: see talk of M. Schäfer at STARS2013.
Alternatively: modify variational principle such that the variation of the action is within the zero divisor.

The Theory

I. NEW VARIATIONAL PRINCIPAL:

$$S = \int L \, d\tau$$

$$\delta S = \delta S_{+} \sigma_{+} + \delta S_{-} \sigma_{-} \quad \in \quad z \quad e \quad d \quad r \quad i$$

$$\rightarrow \quad = \xi \quad _{-} \quad \sigma(c \quad o \quad) \; n \quad v$$

If we use =0, then: $\delta S_{\pm} = 0$ independently, thus two independent theories, i.e., no connection!

This results in the equations of motion

$$\frac{D}{D}\left(\frac{D}{D\dot{X}^{\mu}} s \right) - \left(\frac{D}{D \quad ^{\mu}} \right) \in \frac{L}{X} z \quad e \quad d \; r \quad i$$

(F. Schuller, PhD thesis, University of Cambridge (2003);
F. Schuller, *Ann. Phys. (N.Y.)* **299** (2002), 174,
F. S. has proposed this general variation principle in his thesis at Cambridge.)

Extension of the Theory of General Relativity:

- The metric is pseudo-complex, without torsion:

$$g_{\mu\nu} = \overset{+}{g}_{\mu\nu}\sigma_+ + \overset{-}{g}_{\mu\nu}\sigma_- \quad , \quad g_{\mu\nu} = g_{\nu\mu}$$

* pseudo-complex length element

$$d\omega^2 = g_{\mu\nu} DX^\mu DX^\nu$$

$$= \overset{+}{g}_{\mu\nu} DX_+^\mu DX_+^\nu \sigma_+ + \overset{-}{g}_{\mu\nu} DX_-^\mu DX_-^\nu \sigma_-$$

$$\rightarrow g_{\mu\nu}\left(dx^\mu dx^\nu + \left[\frac{l}{c}\right]^2 du^\mu du^\nu \right) \quad , \quad \left(\frac{l}{c} u^\mu << 1 \right)$$

$$and \qquad g_{\mu\nu} dx^\mu du^\nu = 0$$

← dispersion relation!

Einstein equation

$$L = \sqrt{-g}\,R$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = -\frac{8\pi\kappa}{c^2} T^{\mu\nu} \sigma_{-}$$

Origin of the dark energy?

M Visser, Phys. Rev. D **54** (1996) 5103 +…
C. Barceló et al., Phys. Rev. D **77** (2008), 044032

1) Semiclassical QM, with a Schwarzschild metric as back gound. Vacuum fluctuations are building up (dark energy!), which finally stop the collaps of a star (if the collapse is slow enough)
2) Advantage: They determine the density of the dark energy
   Disadvantage: They do not have a re-coupling of the dark energy density to the metric and thus have to take care of the event hiorizon

Our theory:
1) We assume a dark energy density and determine the re-coupling of the dark energy densi to the metric.
2) Advantage: We can determine the final metric
   Disadvantage: We can not determine the form of the dark energy density → assumed

Both ways are complementary! Both explain the stop of a collapse due to the *distorsion of space due to the presence of the dark energy distribution* such that *space itself retains the collapse of a star!*

Schwarzschild and Kerr metrics

(G. Caspar et al., Int. J. Mod. Phys. E 21 (2012), 1250015)

Schwarzschild:

$$g_{00}^{S} = \left(1 - \frac{2m}{r} + \frac{B}{2r^3}\right)$$

$$g_{11}^{S} = -\left(1 - \frac{2m}{r} + \frac{B}{2r^3}\right)^{-1}$$

$$g_{22}^{S} = -r^2$$

$$g_{33}^{S} = -r^2 \sin^2 \vartheta .$$

Kerr:

$$g_{00}^{K} = \frac{r^2 - 2mr + a^2 \cos^2 \vartheta + \frac{B}{2r}}{r^2 + a^2 \cos^2 \vartheta}$$

$$g_{11}^{K} = -\frac{r^2 + a^2 \cos^2 \vartheta}{r^2 - 2mr + a^2 + \frac{B}{2r}}$$

$$g_{22}^{K} = -r^2 - a^2 \cos^2 \vartheta$$

$$g_{33}^{K} = -(r^2 + a^2) \sin^2 \vartheta - \frac{a^2 \sin^4 \vartheta \left(2mr - \frac{B}{2r}\right)}{r^2 + a^2 \cos^2 \vartheta}$$

$$g_{03}^{K} = \frac{-a \sin^2 \vartheta \, 2mr + a \frac{B}{2r} \sin^2 \vartheta}{r^2 + a^2 \cos^2 \vartheta},$$

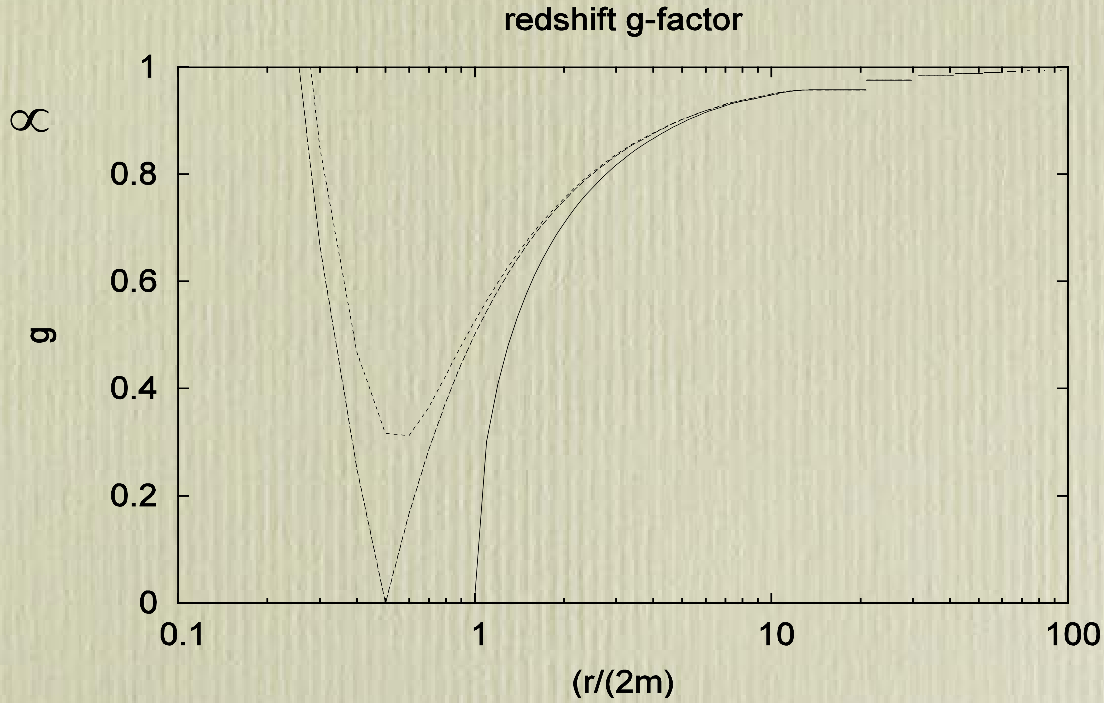
B is a measure of the dark energy Contribution:

$$B = bm^3$$

Schwarzschild:

$$g = \sqrt{g_{00}} \quad \propto V_{eff}(L=0)$$

$V_{eff}(L=0) \propto$



The radius of the dark disk is SMALLER in pc-GR!

Circular orbital of a particle around a large mass

(T. Schönenbach et al., MNRAS **430** (2013), 2999

Lagrangian: $L = g_{00}c^2\dot{t}^2 + g_{11}\dot{r}^2 + g_{22}\dot{\vartheta}^2 + g_{33}\dot{\varphi}^2 + 2g_{03}c\dot{t}\dot{\varphi} = \frac{ds^2}{ds^2} = 1$

→ geodesic

$$\frac{d}{ds}\left(2g_{11}\dot{r}\right) = g'_{00}c^2\dot{t}^2 + g'_{11}\dot{r}^2 + g'_{22}\dot{\vartheta}^2 + g'_{33}\dot{\varphi}^2 + 2g'_{03}c\dot{t}\dot{\varphi}$$

Circular motion: $\dot{r} = 0 \quad , \quad \vartheta = \frac{\pi}{2}$ → $0 = g'_{00}(r_0)c^2\dot{t}^2 + g'_{33}(r_0)\omega^2\dot{t}^2 + 2g'_{03}(r_0)\omega c\dot{t}^2$

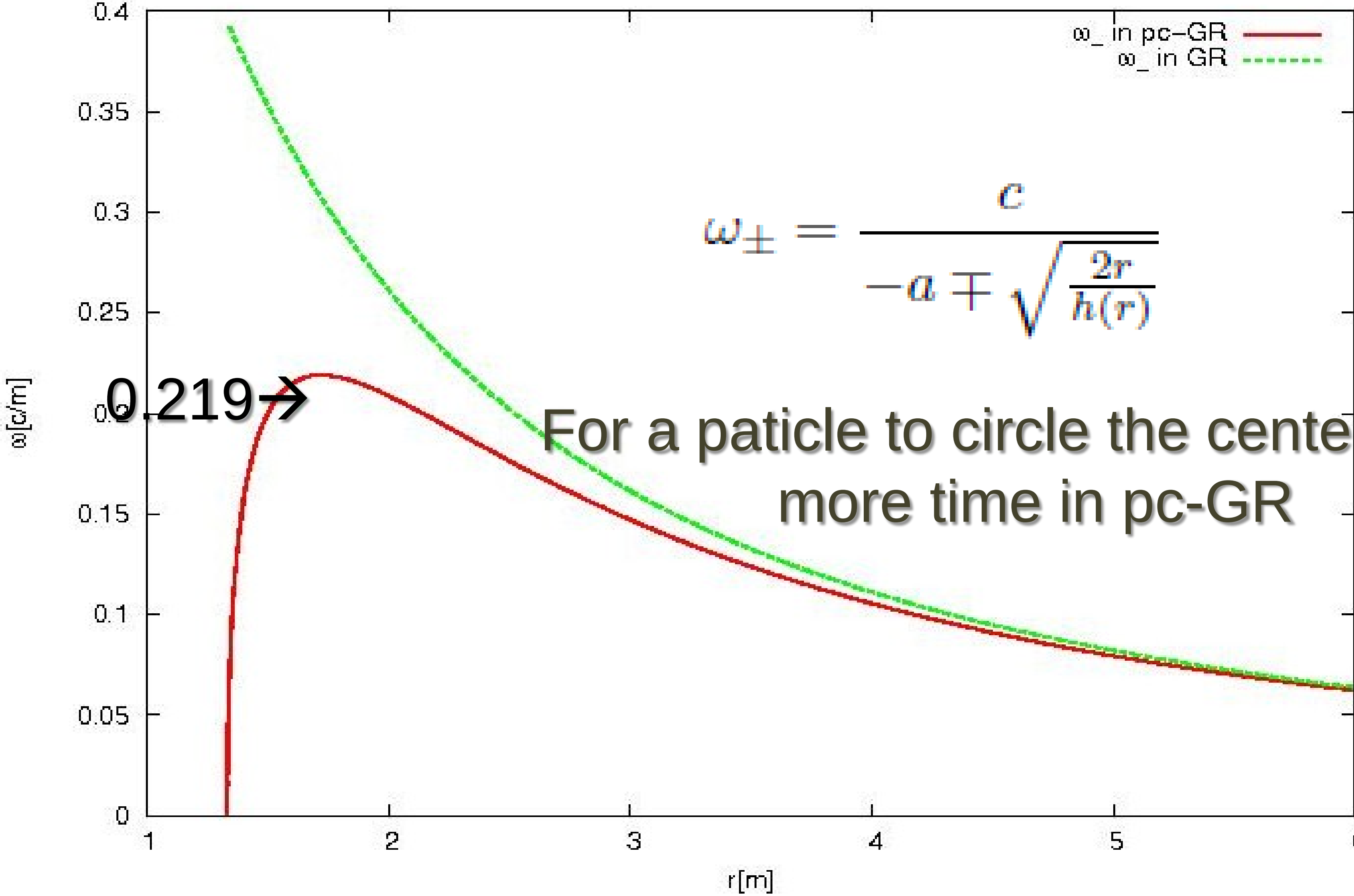
Resolve for the frequency:

$$\omega_\pm = c\frac{-g'_{03} \pm \sqrt{(g'_{03})^2 - g'_{00}g'_{33}}}{g'_{33}}$$

OR:

$$\omega_\pm = \frac{c}{-a \mp \sqrt{\frac{2r}{h(r)}}}$$

a=-0.995m



For a paticle to circle the center takes more time in pc-GR

0.219 for 4Millon sun masses = 9.4 minutes

Redshift

$d\tau^2 = g_{00} dt^2$, $\tau_0 = \sqrt{g_{00}} t_{obs}$ → $\nu_{obs} = \sqrt{g_{00}} \nu_0$

Redshift:

$$z := \frac{\nu_0 - \nu_{obs}}{\nu_{obs}} = \frac{1}{\sqrt{g_{00}}} - 1$$

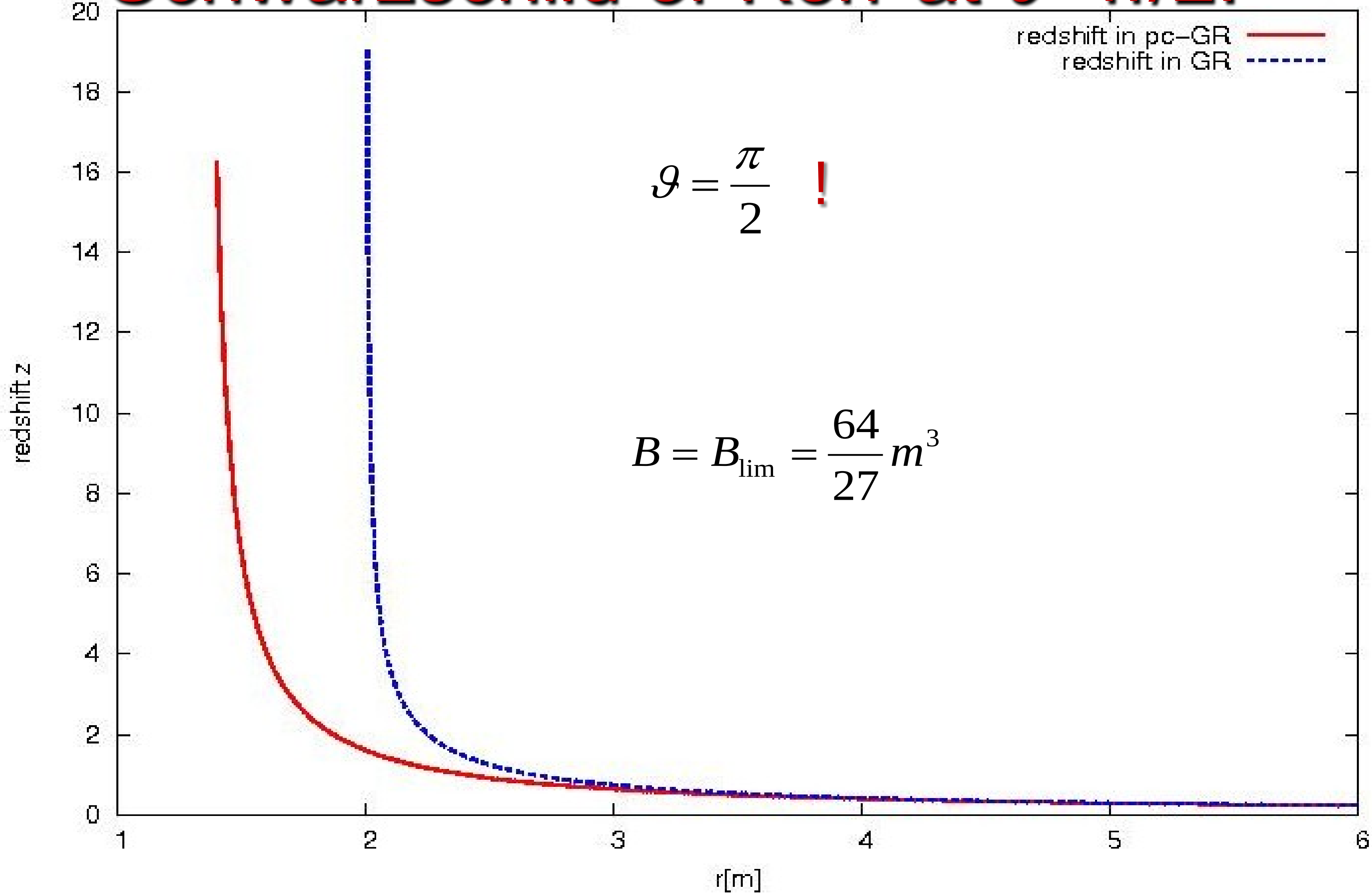
Kerr:

$$z = \frac{\sqrt{r^2 + a^2 \cos^2(\vartheta)}}{\sqrt{r^2 - 2mr + a^2 \cos^2(\vartheta) + \frac{B}{2r}}} - 1$$

Schwarzschild:

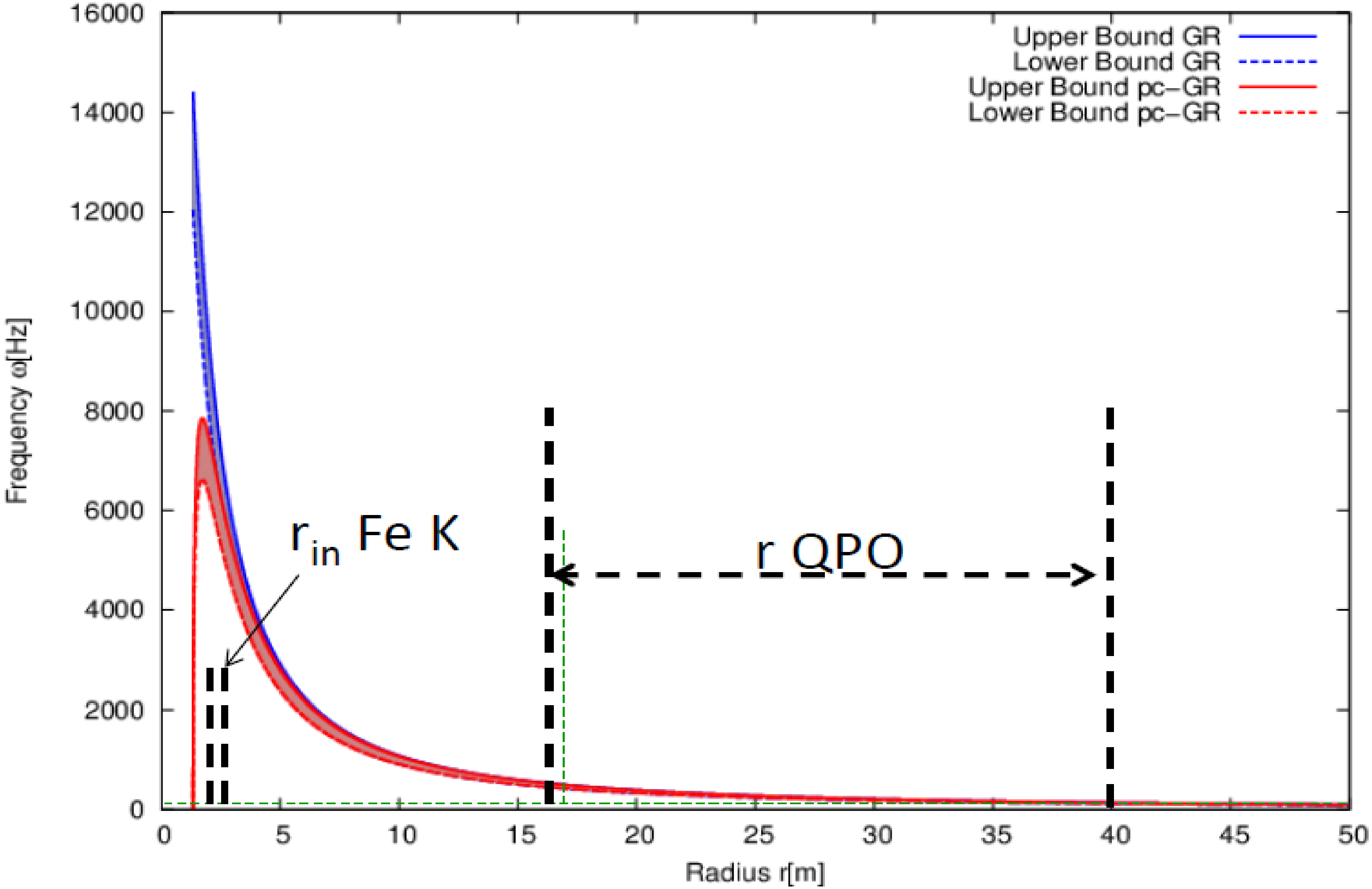
$$z = \frac{1}{\sqrt{1 - \frac{2m}{r} + \frac{B}{2r^3}}} - 1$$

Schwarzschild or Kerr at ϑ=π/2:



GRO J1655-40

r_in(2.0-2.4) a=(0.89..0.94), M=6.3+-0.5 68 per cent confidence



Pseudo-Complex General Relativity (pc-GR):

$$\left(\mathcal{R}^{\mu\,+}_{\nu\,i} - \frac{1}{2}g^{\mu\,+}_{\nu\,i}\mathcal{R}^{+}_{i}\right)\sigma_+ + \left(\mathcal{R}^{\mu\,-}_{\nu\,i} - \frac{1}{2}g^{\mu\,-}_{\nu\,i}\mathcal{R}^{-}_{i}\right)\sigma_- = -\frac{8\pi k}{c^4}T^{de\,\mu}_{i\;\;\nu}\sigma_- - \frac{8\pi k}{c^4}T^{m\,\mu}_{\;\;\nu}(\sigma_+ + \sigma_-)$$

$$\Downarrow$$

$$\mathcal{R}^{\mu\,+}_{\nu\,i} - \frac{1}{2}g^{\mu\,+}_{\nu\,i}\mathcal{R}^{+}_{i} = -\frac{8\pi k}{c^4}T^{m\,\mu}_{\;\;\nu}$$

$$\mathcal{R}^{\mu\,-}_{\nu\,i} - \frac{1}{2}g^{\mu\,-}_{\nu\,i}\mathcal{R}^{-}_{i} = -\frac{8\pi k}{c^4}T^{de\,\mu}_{i\;\;\nu} - \frac{8\pi k}{c^4}T^{m\,\mu}_{\;\;\nu}$$

Pseudo-Complex General Relativity (pc-GR):

Perfect anisotropic fluid at rest

$$T^{\mu}_{\nu} = \begin{bmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & -p_r & 0 & 0 \\ 0 & 0 & -p_\theta & 0 \\ 0 & 0 & 0 & -p_\theta \end{bmatrix}$$

→ Isotropic assumption $p_r = p_\theta$

↓

Matter

$$T^{m\,\mu}_{\ \ \nu} = \begin{bmatrix} \varepsilon_m & 0 & 0 & 0 \\ 0 & -p_m & 0 & 0 \\ 0 & 0 & -p_m & 0 \\ 0 & 0 & 0 & -p_m \end{bmatrix}$$

pc-component

$$T_i^{de\,\mu}{}_{\nu} = \begin{bmatrix} \varepsilon_{\Lambda i} & 0 & 0 & 0 \\ 0 & -p_{\Lambda i} & 0 & 0 \\ 0 & 0 & -p_{\Lambda i} & 0 \\ 0 & 0 & 0 & -p_{\Lambda i} \end{bmatrix}$$

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Pseudo-Complex TOV system:

Anisotropic interior

$$\frac{dp_{rI}}{dr} = -\frac{(\varepsilon_I(r) + p_{rI}(r))}{r\left[r - 2m_m(r) + 2m_{\Lambda i}(r)\right]}\left[m_m(r) - m_{\Lambda i}(r) + \frac{4\pi k}{c^4} r^3 p_{rI}(r)\right]$$

$$\Delta p_I = p_{\theta I} - p_{rI}$$

$$p_{rI} = p_m + p_{\Lambda ri}$$

$$\varepsilon_I = \varepsilon_m + \varepsilon_{\Lambda i}$$

$$\frac{dp_m}{dr} = -\frac{(\varepsilon_m(r) + p_m(r))}{r\left[r - 2m_m(r) + 2m_{\Lambda i}(r)\right]}\left[m_m(r) - m_{\Lambda i}(r) + \frac{4\pi k}{c^4} r^3 (p_{\Lambda ri} + p_m)(r)\right]$$

$$\frac{dm_m}{dr} = 4\pi r^2 \varepsilon_m(r) \qquad p_m = p_m(\varepsilon_m)$$

$$\frac{dp_{\Lambda ri}}{dr} = -\frac{(\varepsilon_{\Lambda i}(r) + p_{\Lambda ri}(r))}{r\left[r - 2m_m(r) + 2m_{\Lambda i}(r)\right]}\left[m_m(r) - m_{\Lambda i}(r) + \frac{4\pi k}{c^4} r^3 (p_{\Lambda ri} + p_m)(r)\right]$$

$$\frac{dm_\Lambda}{dr} = -4\pi r^2 \varepsilon_\Lambda(r) \qquad p_\Lambda = p_\Lambda(\varepsilon_\Lambda)$$

→ Isotropic assumption $p_r = p_\theta$

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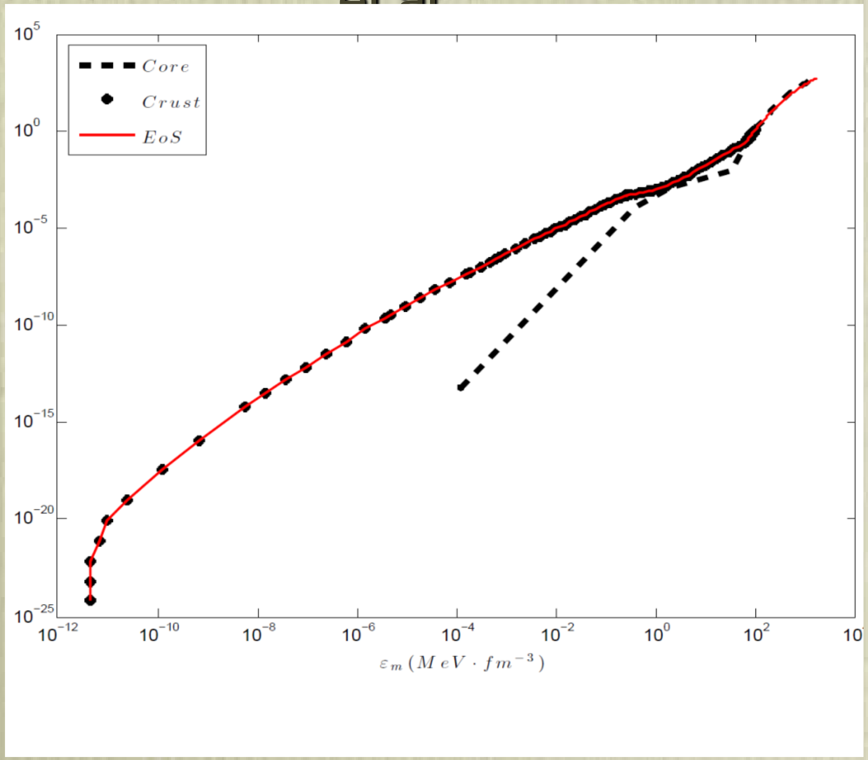
Equations of State:

Matter

chiral SU(3) Schramm et al



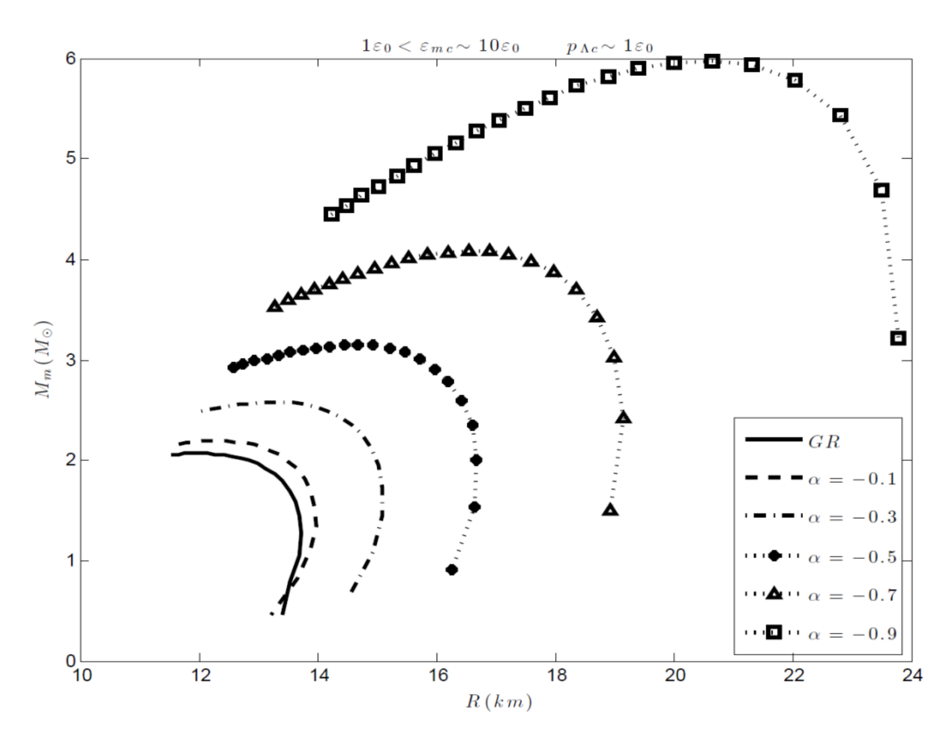
pc-component

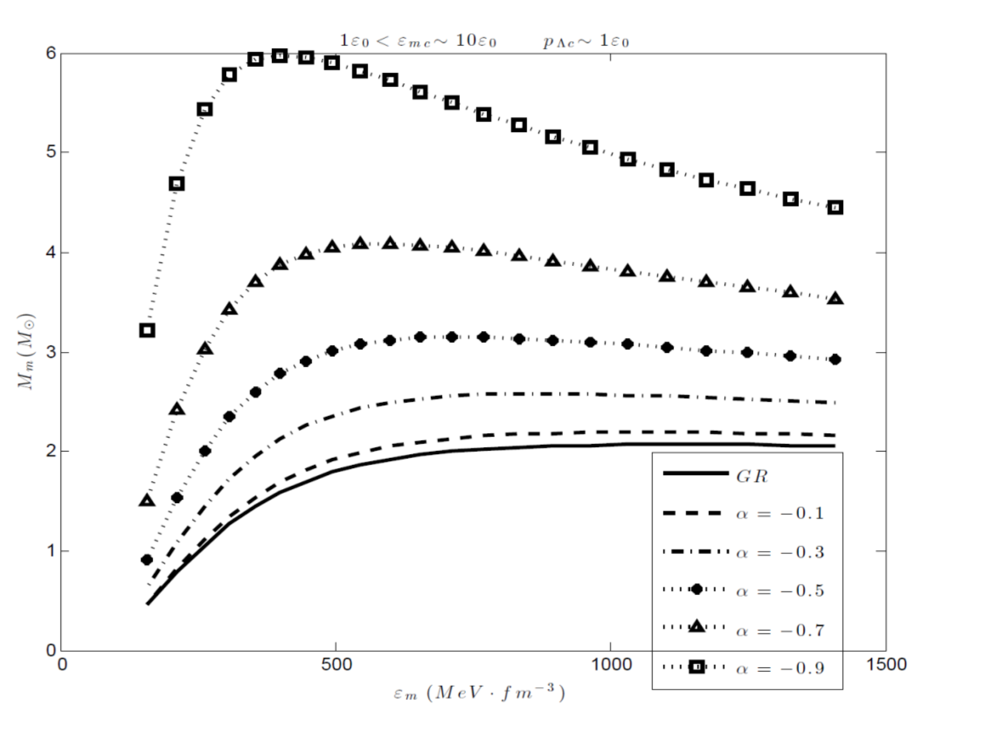
?

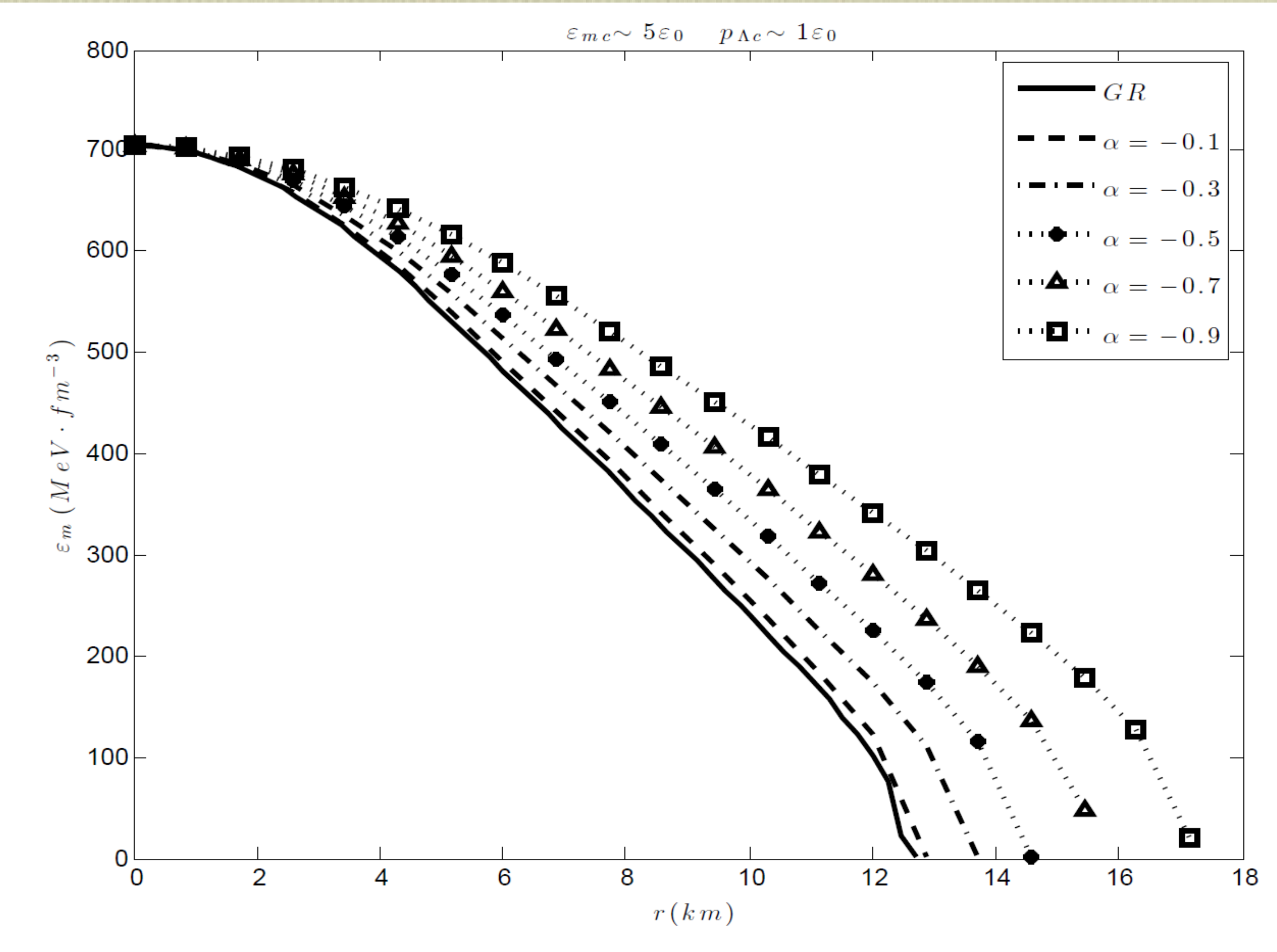
$$\varepsilon_\Lambda = \alpha \varepsilon_m \qquad \alpha < 0$$

C- R- Ghezzi, „Anisotropic Dark Energy Stars“, gr-qc:0908.0779

Results:


$1\varepsilon_0 < \varepsilon_{mc} \sim 10\varepsilon_0$ $p_{\Lambda c} \sim 1\varepsilon_0$
$M_m(M_\odot)$
$R\,(km)$
GR
$\alpha = -0.1$
$\alpha = -0.3$
$\alpha = -0.5$
$\alpha = -0.7$
$\alpha = -0.9$


$1\varepsilon_0 < \varepsilon_{mc} \sim 10\varepsilon_0$ $p_{\Lambda c} \sim 1\varepsilon_0$
$M_m(M_\odot)$
$\varepsilon_m\ (MeV \cdot fm^{-3})$
GR
$\alpha = -0.1$
$\alpha = -0.3$
$\alpha = -0.5$
$\alpha = -0.7$
$\alpha = -0.9$


$\varepsilon_{mc} \sim 5\varepsilon_0$ $p_{\Lambda c} \sim 1\varepsilon_0$
$\varepsilon_m \, (MeV \cdot fm^{-3})$
$r \, (km)$
GR
$\alpha = -0.1$
$\alpha = -0.3$
$\alpha = -0.5$
$\alpha = -0.7$
$\alpha = -0.9$
0
100
200
300
400
500
600
700
800
0
2
4
6
8
10
12
14
16
18

Conclusion & Future Work:

- It was shown that pc-GR can produce neutron stars with a significant larger mass as obtaind in standard GR.
- Problems of the correct equation of state of the mass and the coupling of the dark energy density to the mass density remain.
- In future we try to solve this problem.

Thanks!

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