Fermi gas in a magnetic field and related anisotropy in quark stars

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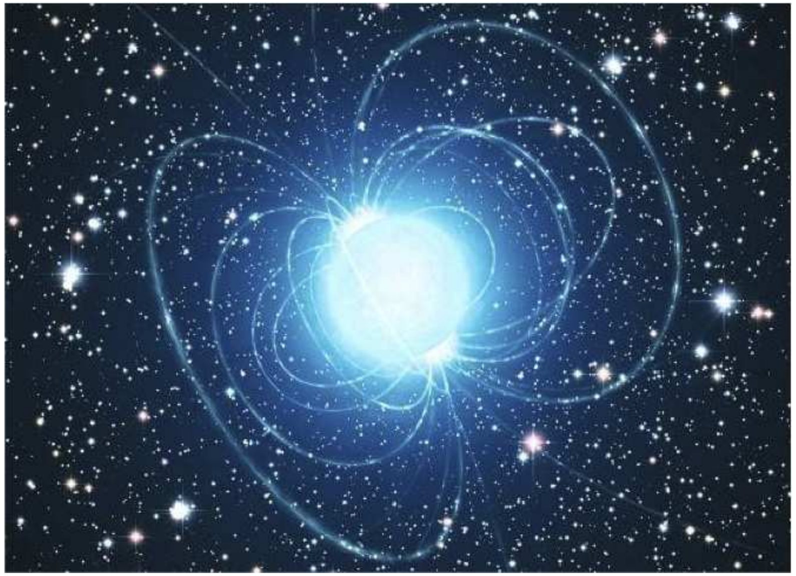
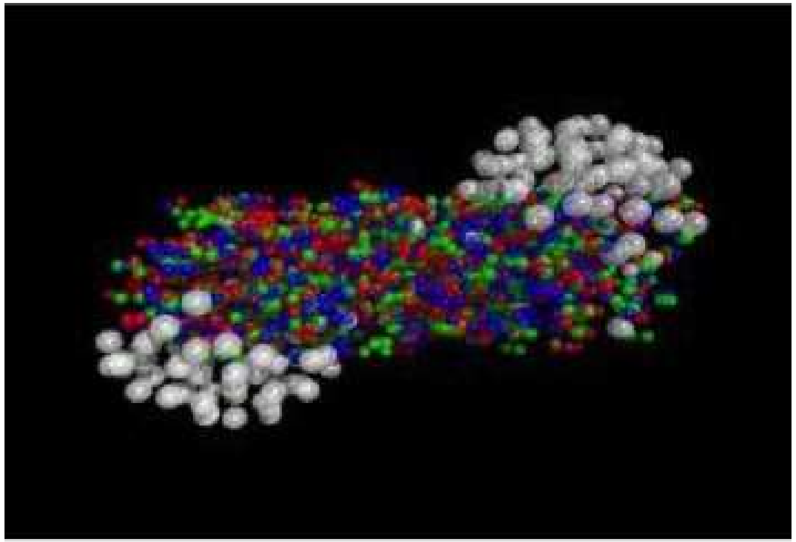
Motivation: why magnetic fields?



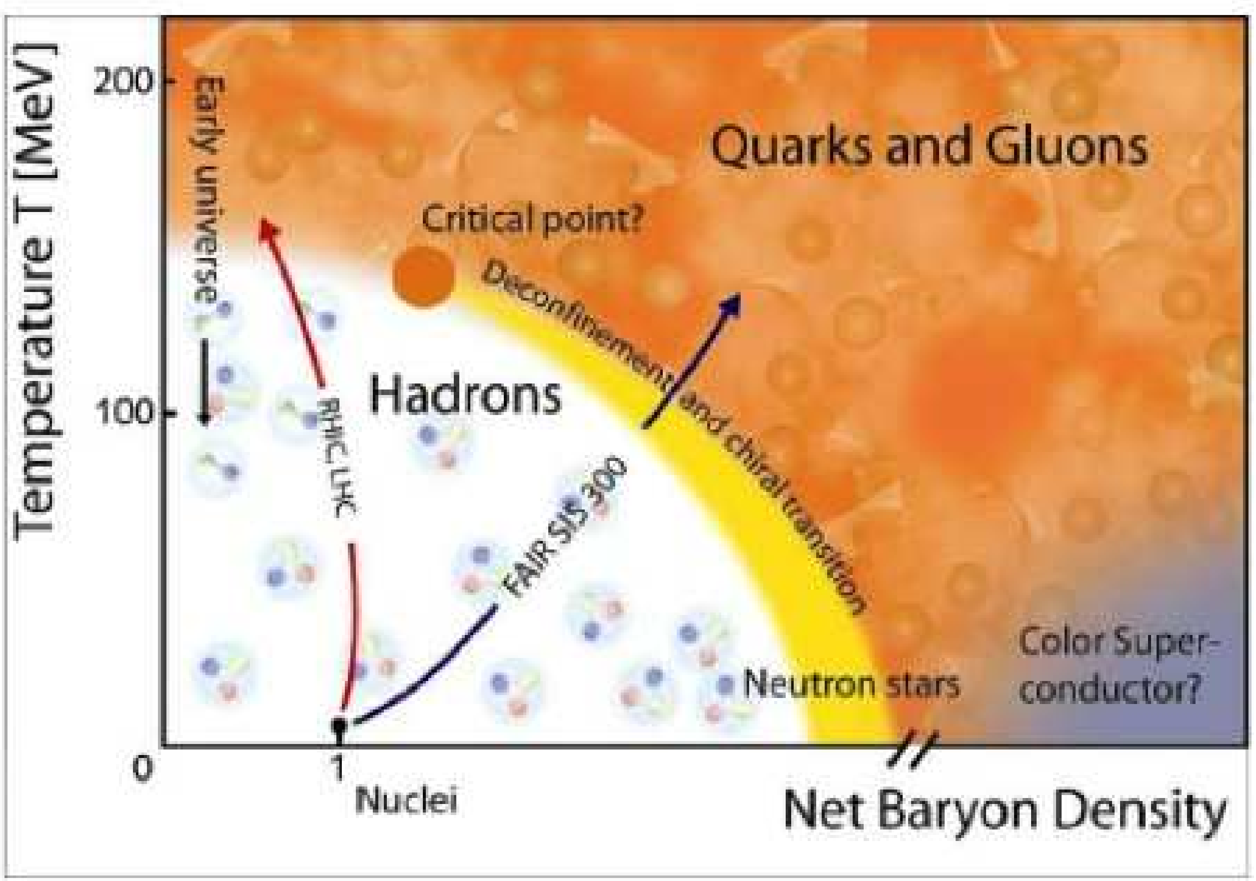


Magnetars - $eB \simeq 0.5 m_\pi^2$ $\quad m_\pi^2 \simeq 3.5 \times 10^{18}$ G

Non-central HIC - $eB \simeq 5 - 15 m_\pi^2$

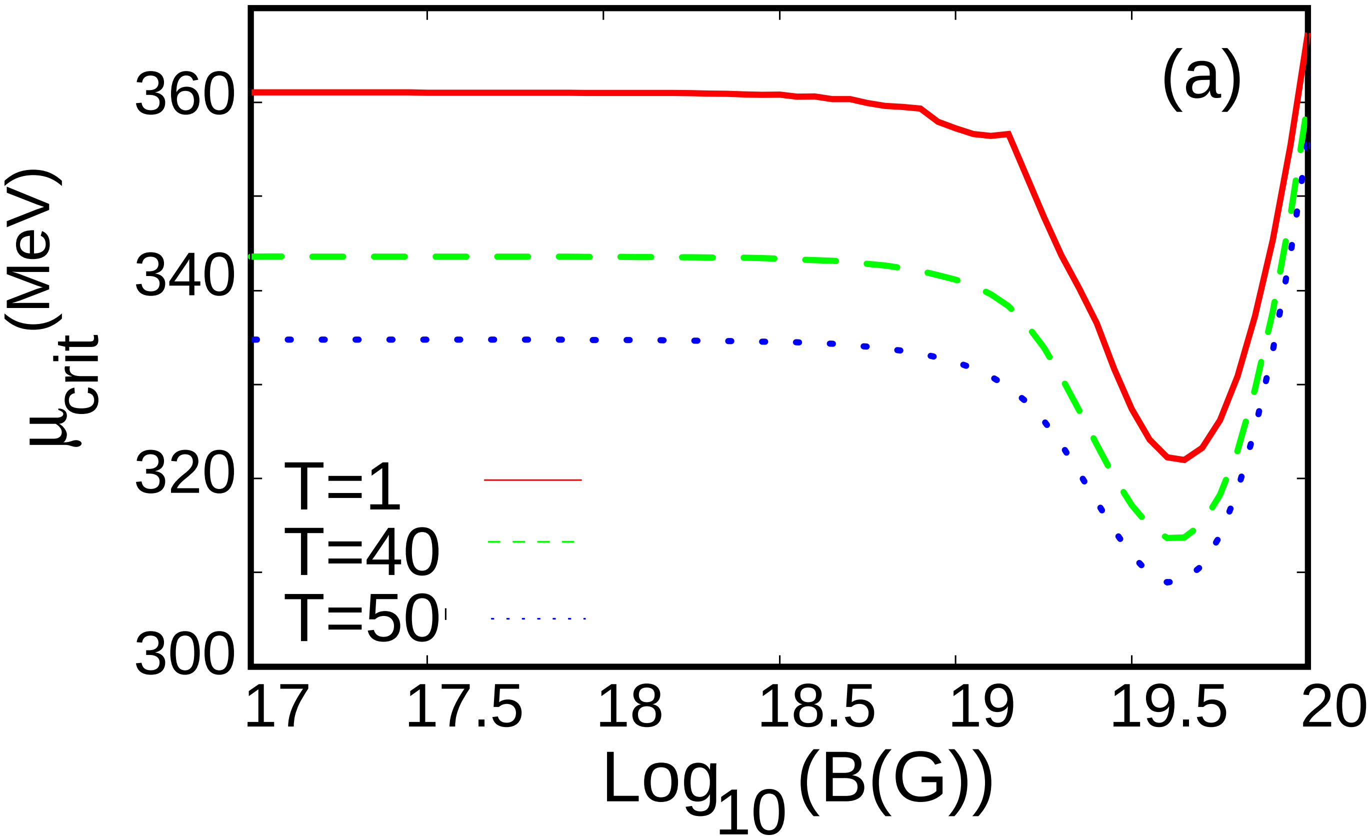
Early Universe - $eB \simeq 30 m_\pi^2$

QCD Phase Diagram



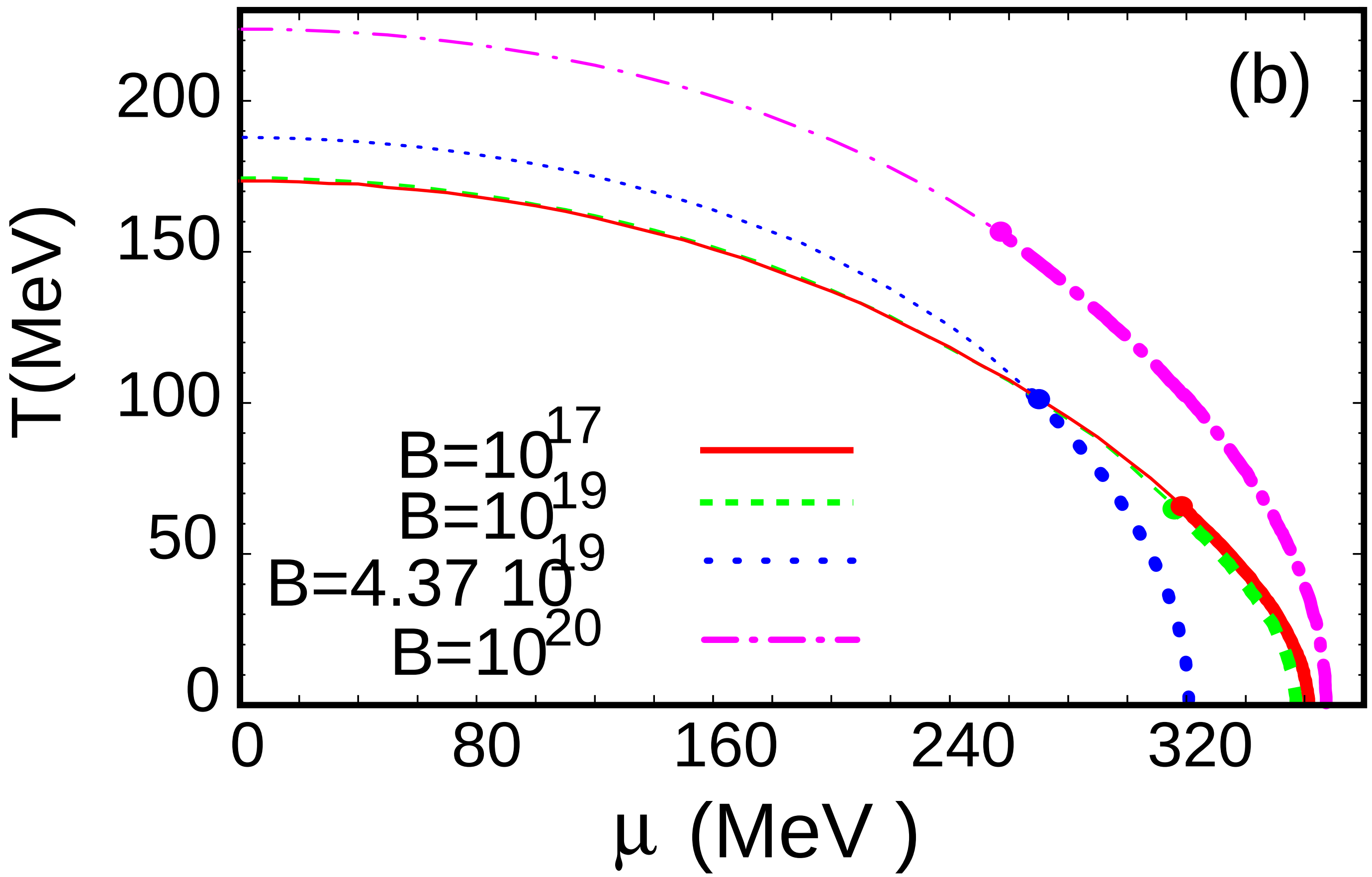
What would happen if matter were subject to strong magnetic fields?

NJL model - Finite T



1^{st} order line oscillates with the increase of B

S.S. Avancini, D.P. Menezes, M.B. Pinto and C. Providência - Phys. Rev. D 85, 091901(R) (2012)



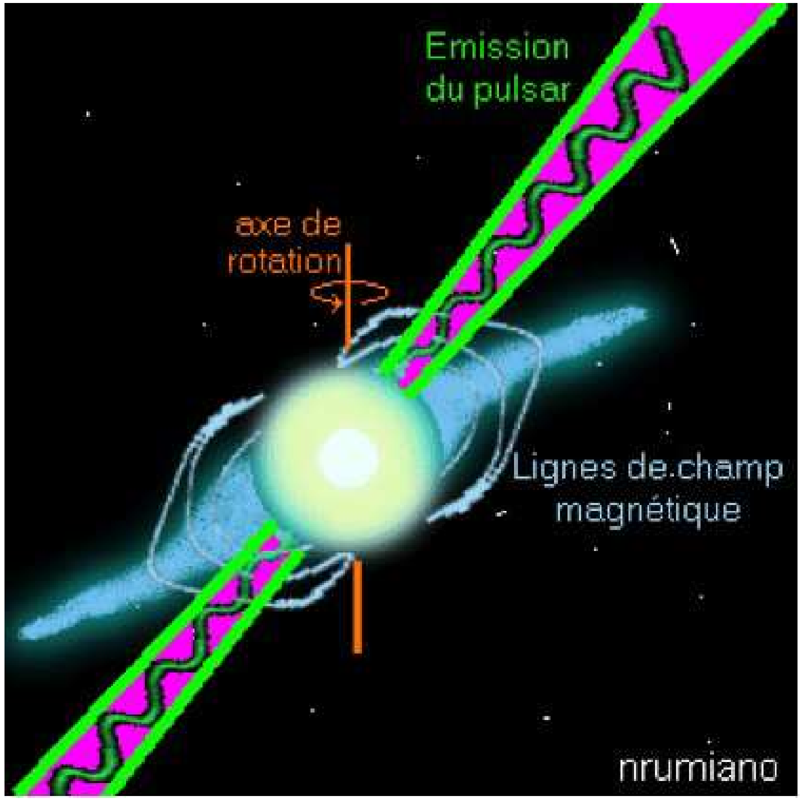
The thick curves represent first order transition lines which terminate at a critical end point identified with a full dot and the thin lines represent a crossover.

T_c decreases at high μ; size of 1^{st} order line increases with B; T_{pc} (cross-over) increases at $\mu = 0$ (the opposite happens in Lattice QCD calculations; more investigation required!)

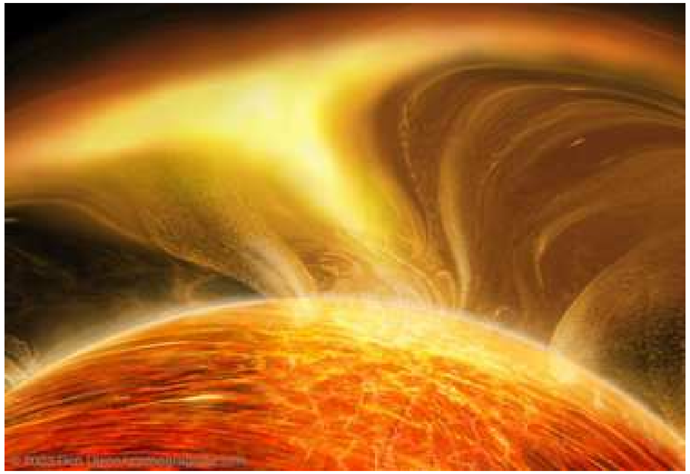
We have to understand magnetic field effects :

- At high densities and low temperatures $\rightarrow$ in neutron stars
- At low densities and hight temperatures $\rightarrow$ in heavy-ion collisions - (see, for instance, **M.G. Paoli and D.P. Menezes, arXiv: 1203.3175v1 [nucl-th]**)

Pulsares (NS) X Magnetares



$B = 10^{12}$ G na superfície



$B = 10^{15}$ G na superfície

Main NS manifestations:

- **Pulsars** - powered by rotation energy (1900 observed in radio-frequency)

- **Accreting X-Ray Binaries** - powered by gravitational energy (typical rotation periods 0.0015 - 1000 s)

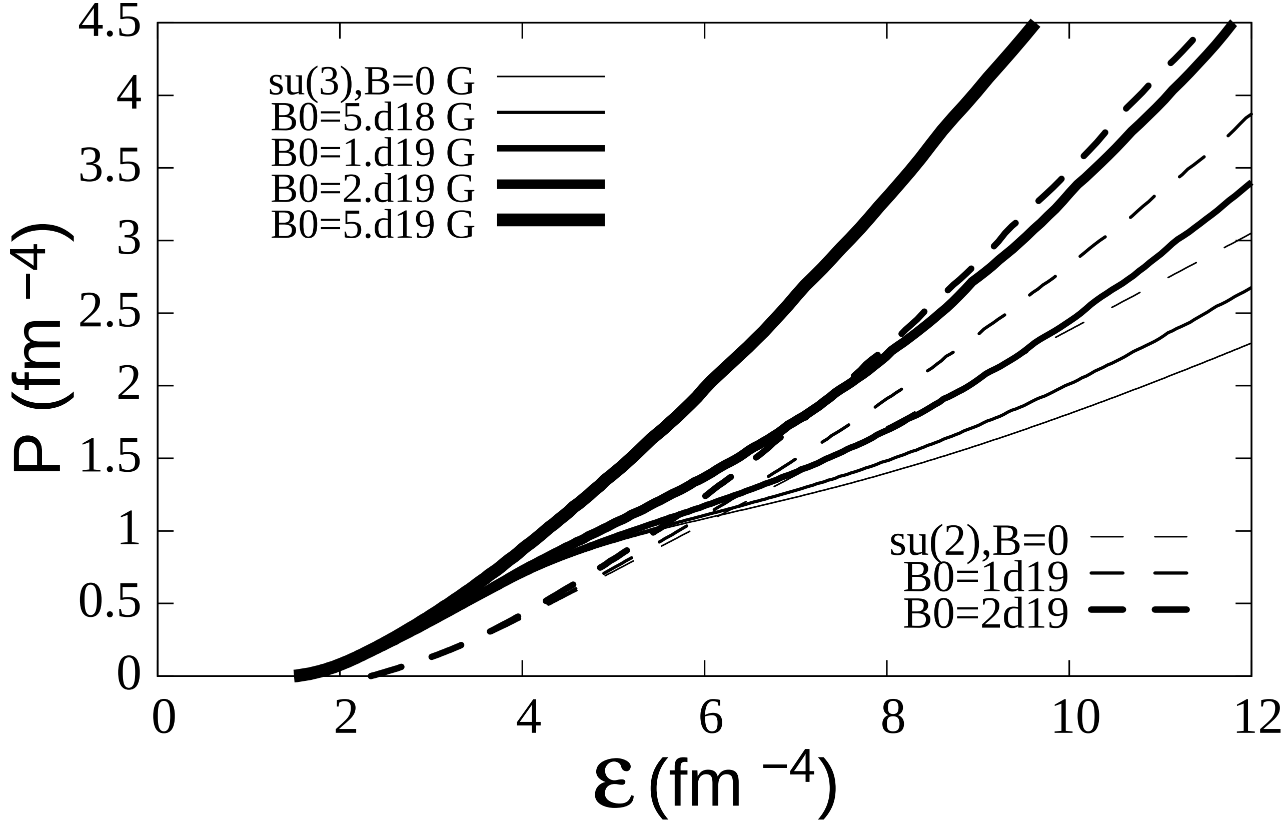
Magnetars don't fit into these categories! They are normally isolated NS whose main power source is the magnetic field.

There are 2 classes of magnetars:

- **Soft gamma-ray repeaters** (discovered in 1979 as transient X-ray sources and *giant flares*); 5 confirmed

- **Anomalous X-ray pulsars** (identified in 1990 as a class of persistent X-ray with no sign of a binary companion); 9 confirmed

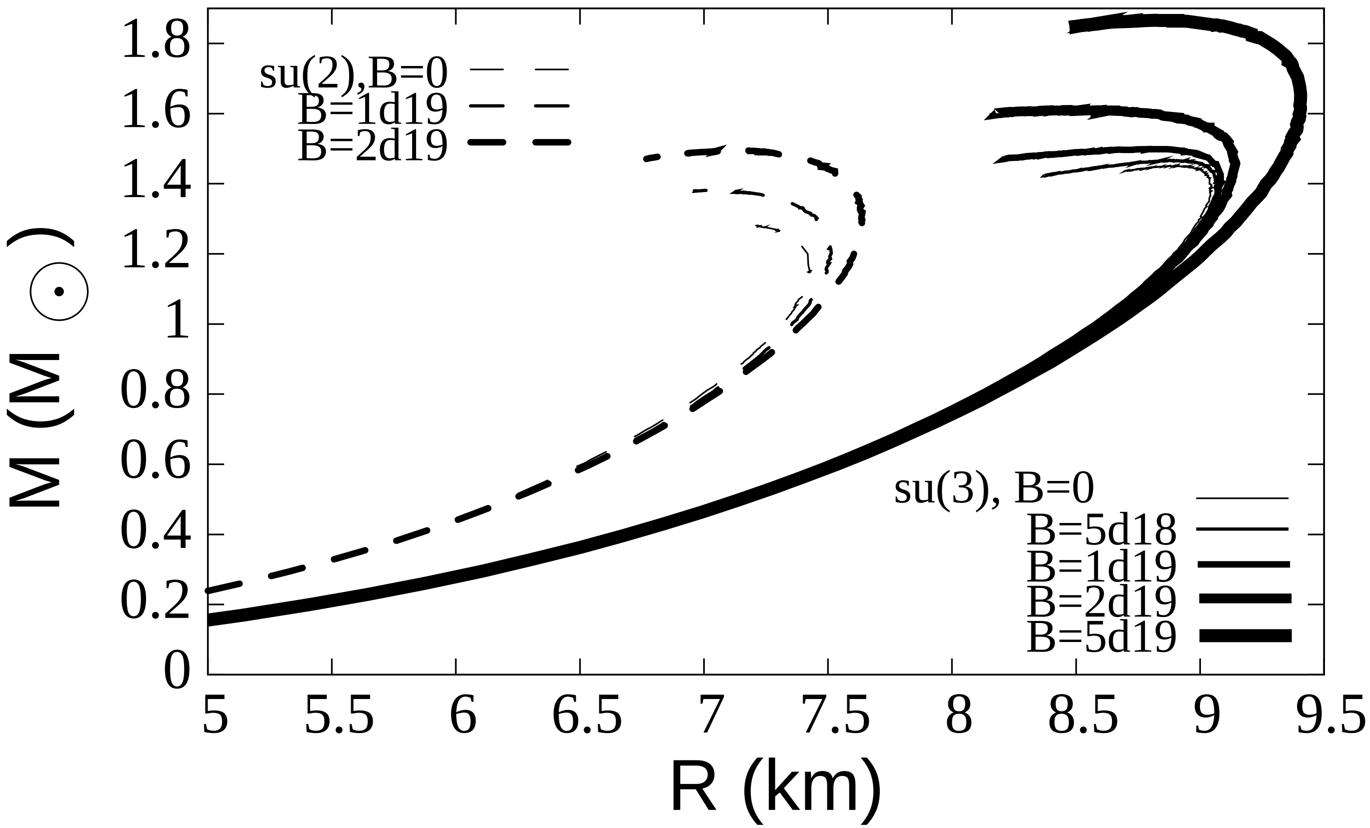
Magnetars - T=0 - NJL model



EoS becomes harder with increasing B; **isotropic EoS: many people here complained...**

Quark stars are bound by the nuclear force; for $B \leq 10^{18}$ G the curves coincide with the $B = 0$ results ($B \simeq 10^{15}$ G at the surface)

Higher stellar masses with increasing B



S. S. Avancini, D.P. Menezes, M.B. Pinto and C. Providência, PRC 80, 065805 (2009)

Anisotropy in a Fermi gas

$$T^{\mu\nu} = T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{fields}} \,. \tag{1}$$

We consider a background magnetic field B pointing along the z-direction

$T^{\mu\nu}_{\text{fields}} = \text{diag}(B^2/2, B^2/2, B^2/2, -B^2/2)$ - Heaviside-Lorentz units

$T^{\mu\nu}_{\text{fields}} = \text{diag}(B^2/8\pi, B^2/8\pi, B^2/8\pi, -B^2/8\pi)$ - Gaussian units

$$
\begin{aligned}
n &= \sum_s \int_k f \,, && (2)\\
\epsilon &= T^{00} = \sum_s \int_k E f \,, && (3)\\
P_{\|} &= T^{zz} = \sum_s \int_k \frac{k_z^2}{E} f \,, && (4)\\
P_{\perp} &= \frac{1}{2}(T^{xx} + T^{yy}) \\
&= \sum_s \int_k \frac{1}{E}\left[\frac{1}{2}\frac{k_\perp^2 \bar{m}(\nu)}{\sqrt{m^2 + k_\perp^2}} - s\kappa B \bar{m}(\nu)\right] f \,, && (5)
\end{aligned}
$$

$\bar{m}^2(\nu) \equiv (\sqrt{m^2 + k_\perp^2} - s\kappa B)^2$, κ - AMM

$k_\perp^2$ - (discretized) transverse momentum, $\sum_s$ over spin polarizations

Charged particles

$$\int_k \to \frac{|q|B}{(2\pi)^2} \sum_n \int_{-\infty}^{\infty} dk_z \,, \tag{6}$$

$$\nu = n + \frac{1}{2} - \frac{s}{2}\frac{q}{|q|} \,, s = \pm 1, \quad \text{spin1/2particles} \tag{7}$$

$$E = \sqrt{k_z^2 + \bar{m}^2(\nu)} \,, \bar{m}^2(\nu) \equiv (\sqrt{m^2 + 2\nu|q|B} - s\kappa B)^2 \tag{8}$$

Uncharged particles

$$\int_k \to \int \frac{d^3k}{(2\pi)^3} \,, \tag{9}$$

$$\bar{m} = (m - s\kappa B) \ . \tag{10}$$

Four possible cases:

- Finite temperature, with(out) AMM, $B = 5 \times 10^{18}$ G
- Zero temperature, with(out) AMM, $B = 5 \times 10^{18}$ G

For all of them we have proved that:

$$P_\perp = P_\| - MB$$

$\mu_N B$ is independent of the convention chosen, but:

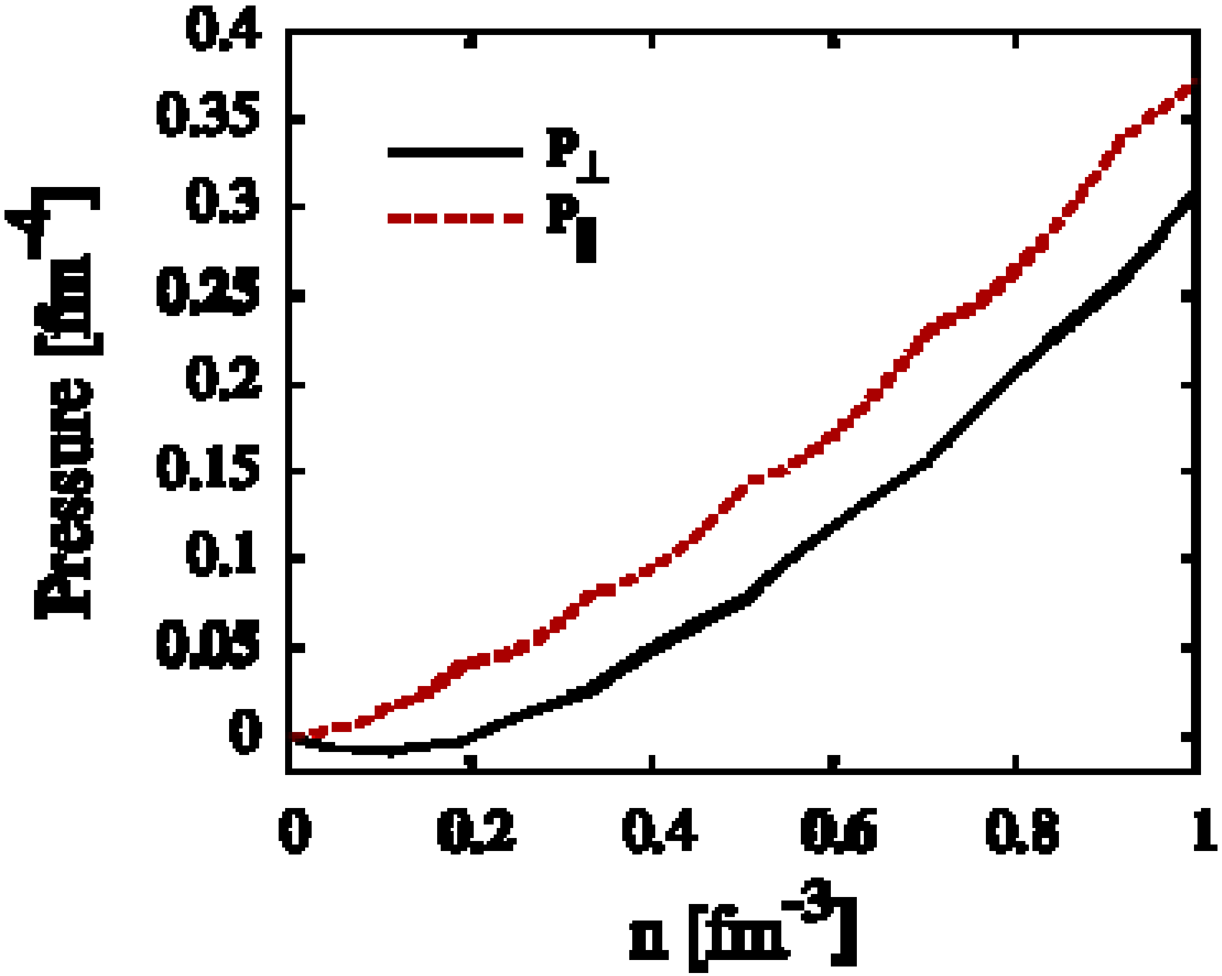
$e = \frac{1}{\sqrt{137}}$ - Heaviside-Lorentz units; $e = \sqrt{\frac{4\pi}{137}}$ - Gaussian units

In Heaviside-Lorentz units:

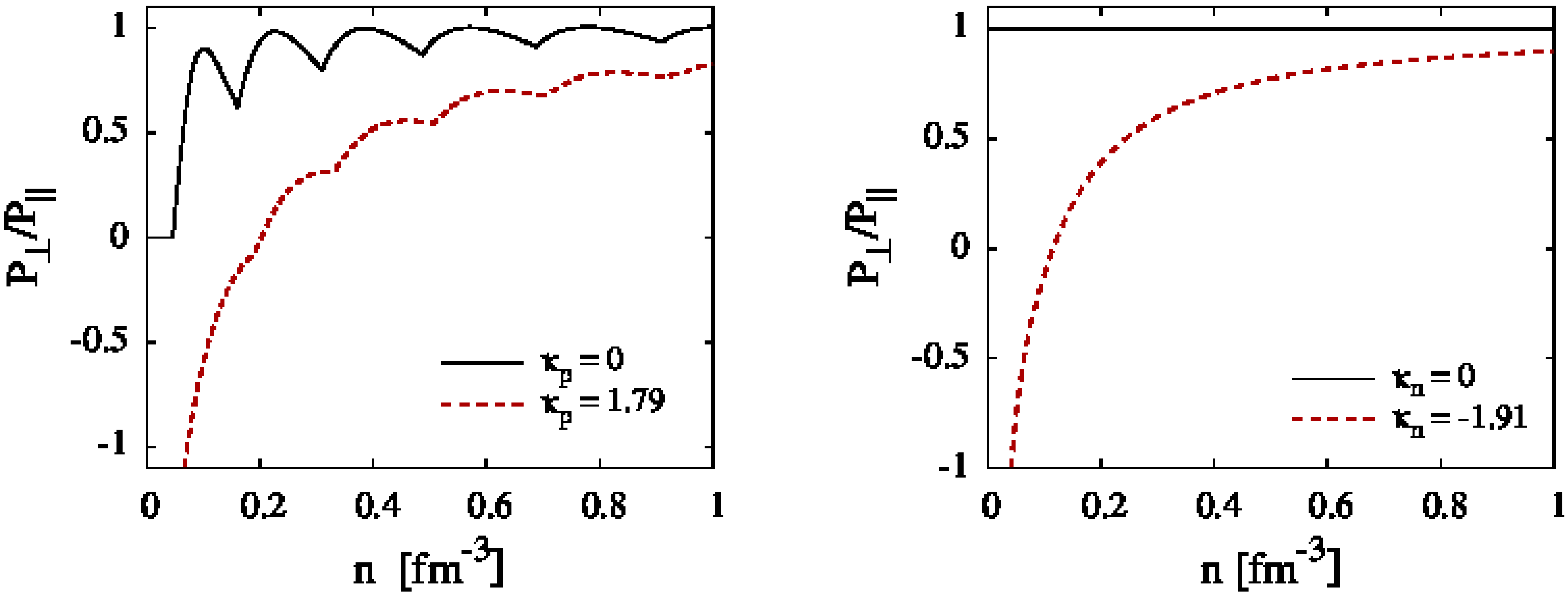
$\kappa_p \mu_N = 1.79 \cdot e/(2m_p) = 0.288633$ GeV^{-1}

$\kappa_n \mu_N = -1.91 \cdot e/(2m_n) = -0.307983$ GeV^{-1}

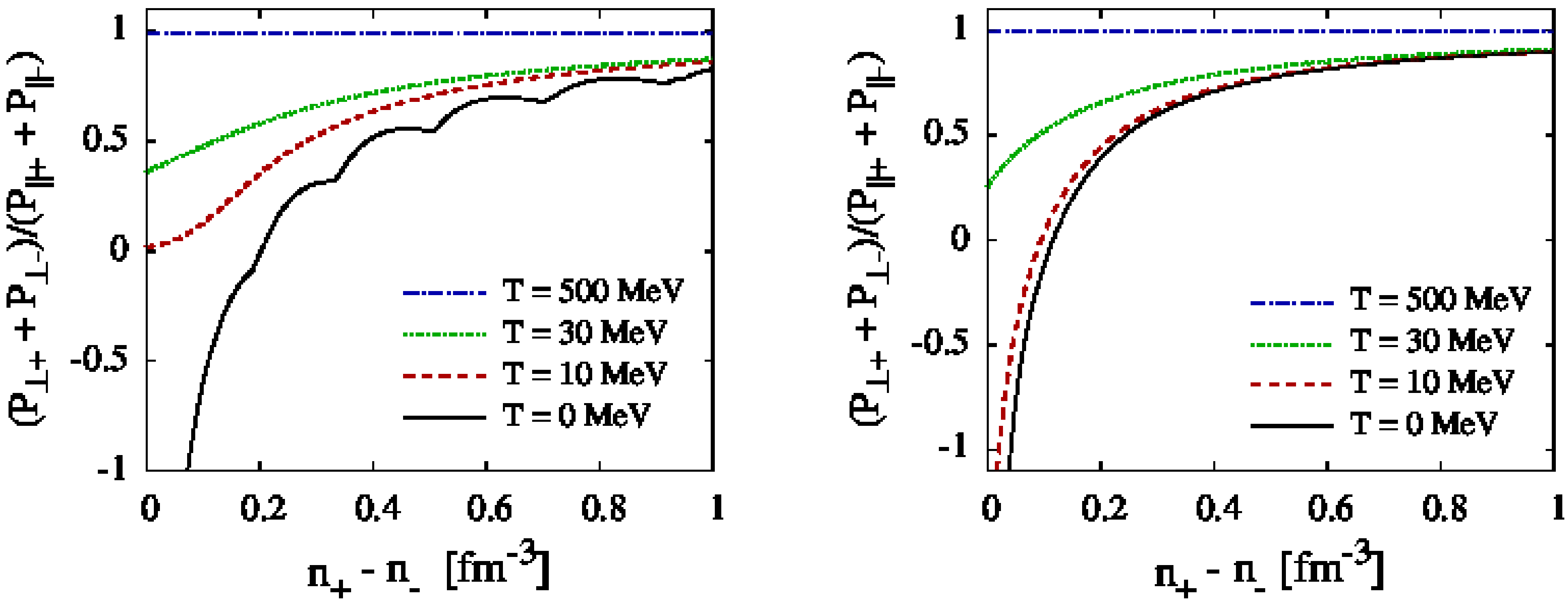
Proton and neutron gases



Transverse and longitudinal pressures of a gas of protons; T=0, with AMM



Ratio of transverse and longitudinal pressures T=0



Ratio of transverse to longitudinal pressure $T = \{0, 10, 30, 500\}$ MeV, with AMM

M. Strickland, V. Dexheimer and D.P. Menezes, Phys. Rev. D 86, 125032 (2012)

Protoquark stars - MIT + B

$$\mathcal{L} = \left[\bar{\Psi}_q\left(i\gamma^\mu\partial_\mu - e_q\gamma^\mu A_\mu - m_q\right)\Psi_q - \mathcal{B}\right]\Theta_V$$
$$- \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}_l\left(i\gamma^\mu\partial_\mu - e_l\gamma^\mu A_\mu - m_l\right)\Psi_l, \quad (11)$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\mathcal{B} = (154\ \text{MeV})^4$, chosen from the stability window analysis

$$\rho_i = \sum_\nu \frac{\gamma_i}{2\pi^2}|Q_{ei}|eB\int(f_{+i} - f_{-i})dk, \quad (12)$$

$$f_{\pm i} = 1/[e^{(\sqrt{k_i^2+\bar{m}_i^2}\mp\mu_i)/T} + 1] \quad (13)$$

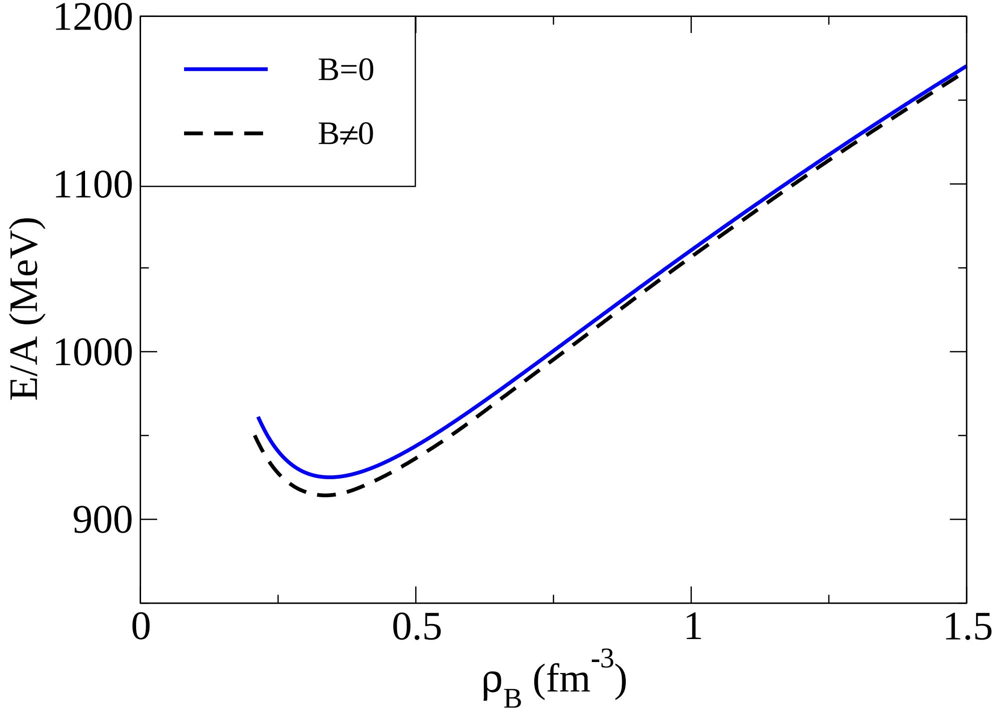
$$P_{m_\parallel} = \sum_{i,\nu} \frac{\gamma_i}{2\pi^2}|Q_{ei}|eB \int \frac{k_i^2}{\sqrt{k_i^2+\bar{m}_i^2}}(f_{+i}+f_{-i})dk - \mathcal{B} \tag{14}$$

$$\epsilon_m = \sum_{i,\nu} \frac{\gamma_i}{2\pi^2}|Q_{ei}|eB \int \sqrt{k_i^2+\bar{m}_i^2}(f_{+i}+f_{-i})dk + \mathcal{B}\,, \tag{15}$$

$$M = -\frac{\partial\Omega}{\partial B} = \frac{P_{m_\parallel}}{B} - \sum_{i,\nu} \frac{\gamma_i}{2\pi^2} Q_{ei}^2 e^2 B\nu \int \frac{1}{\sqrt{k_i^2+\bar{m}_i^2}} \times (f_{+i}+f_{-i})dk, \tag{16}$$

$$P_{m_\perp} = P_{m_\parallel} - MB, \quad \bar{m}_i = \sqrt{m_i^2 + 2|Q_{ei}|eB\nu} \tag{17}$$

Results



$B = 4.30 \times 10^{18}$ G

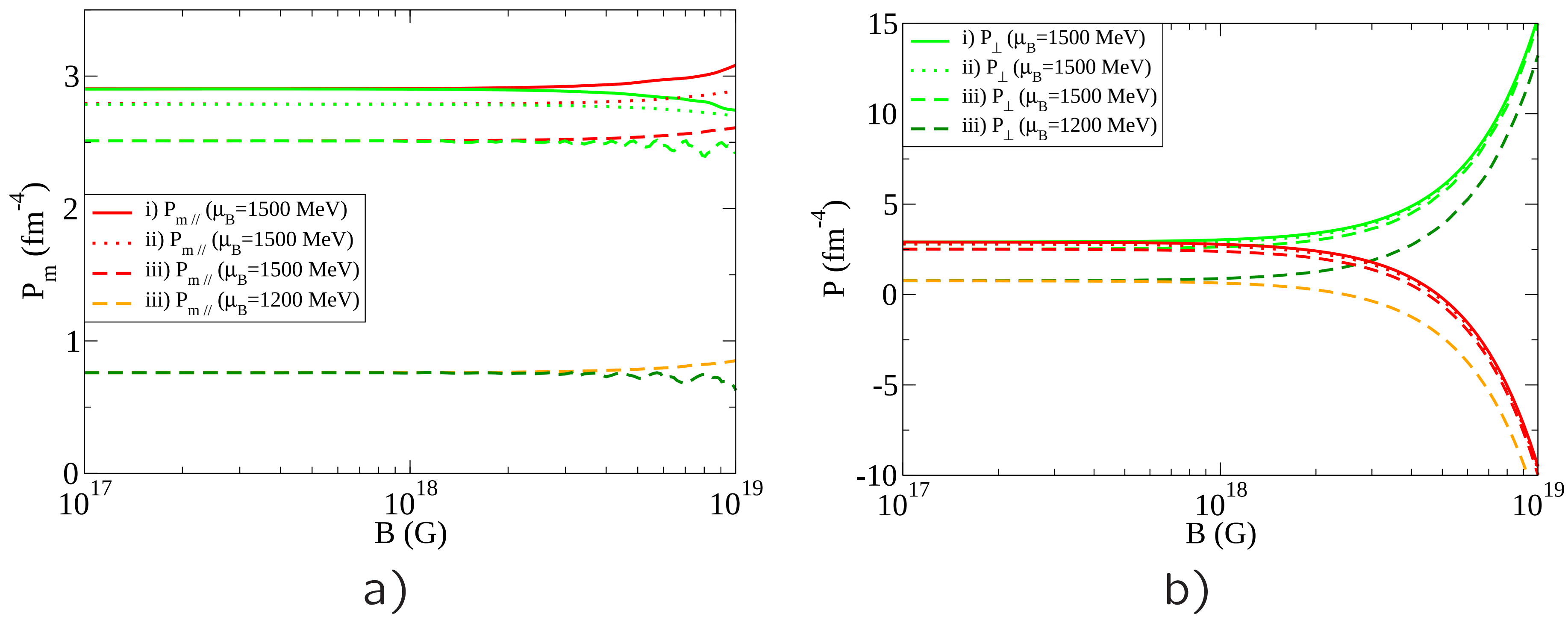
$$\epsilon = \epsilon_m + \frac{B^2}{8\pi}, \quad P_\perp = P_{m_\perp} + \frac{B^2}{8\pi}, \quad P_\parallel = P_{m_\parallel} - \frac{B^2}{8\pi}, \tag{18}$$

$$B(\mu_B) = 10^{15} + B_c\left[1 - e^{b\frac{(\mu_B - 938)^a}{938}}\right], \quad a = 2.5, b = -4.08 \times 10^{-4} \tag{19}$$

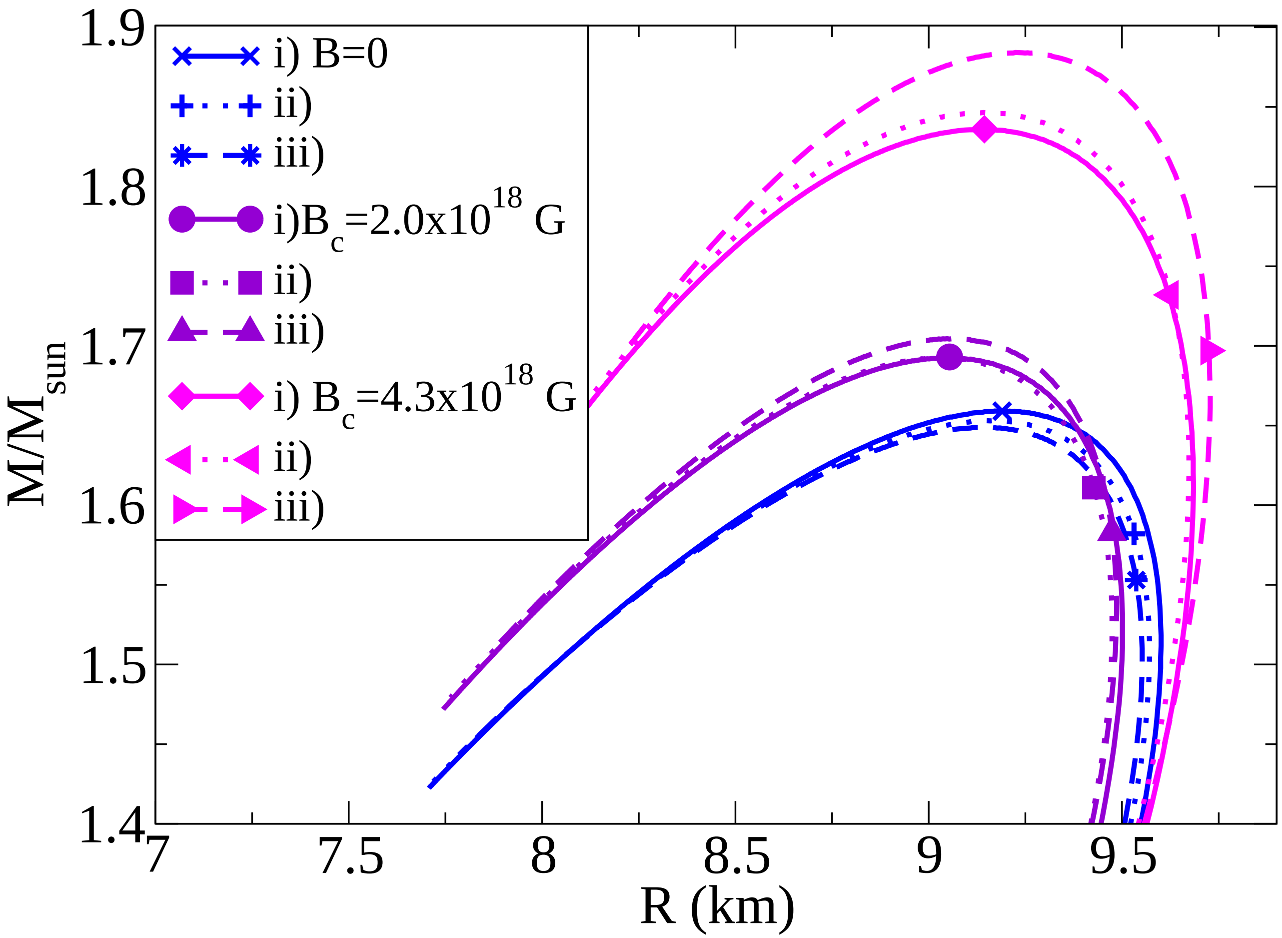
$$\frac{S}{A} = \frac{s}{\rho_B} = \frac{\epsilon + P_\parallel - \mu_B \rho_B}{T\rho_B}, \quad Y_l = \frac{\sum_i Q_{l_i}\rho_i}{\rho_B} \tag{20}$$

We consider three snapshots of the time evolution of a quark star in its first minutes of life:

- $s/\rho_B = 1$, $Y_l = 0.4$,
- $s/\rho_B = 2$, $\mu_{\nu_l} = 0$,
- $s/\rho_B = 0$, $\mu_{\nu_l} = 0$.



Parallel (open symbols on red/orange lines) and perpendicular (respective full symbols on green/dark green lines) pressures Pure magnetic field contribution $B^2/8\pi$ a) not included; b) included



i) B=0
ii)
iii)
i)B$_c$=2.0x10^{18} G
ii)
iii)
i) B$_c$=4.3x10^{18} G
ii)
iii)
M/M$_{sun}$
R (km)
1.4
1.5
1.6
1.7
1.8
1.9
7
7.5
8
8.5
9
9.5

stages	$B = 0$			$B_c = 2.0 \times 10^{18}$			$B_c = 4.3 \times 10^{18}$		
	i	ii	iii	i	ii	iii	i	ii	iii
$M_{max}(M_\odot)$	1.66	1.65	1.65	1.69	1.69	1.71	1.84	1.85	1.88
$M_{max}(M_\odot)$ fixed A	1.66	1.58	1.55	1.69	1.61	1.58	1.84	1.73	1.70
R (km) fixed A	9.19	9.53	9.54	9.06	9.43	9.47	9.14	9.62	9.73

V. Dexheimer, D.P. Menezes and M. Strickland, arXiv: 1210.4526 [nucl-th]

Conclusions

- Strong magnetic fields modify quark star masses

- The evolution of isolated stars needs to be constrained by fixed baryon number, which lowers the star masses.

- The level of pressure anisotropy at stage i) is relatively small $P_{\|}/P_{\perp} \simeq 0.85$ for the lower value of the magnetic field. We have then used the isotropic TOV equations which assume $P_{\perp} = P_{\|}$.

- For the larger value of the magnetic field studied, the level of pressure anisotropy is quite large with $P_{\|}/P_{\perp} \simeq 0.4$.

- The MIT bag model for a $\mathcal{B}^{1/4} = 154$ MeV obtained from an investigation of the adequate stability window cannot reproduce the very massive neutron stars recently detected, not even if very intense magnetic fields are considered.

- However, at such values of the magnetic field one should solve Einstein's equations in an axisymmetric metric which is determined self-consistently from the axisymmetric energy-momentum tensor for the star. Numerical solution for the axisymmetric case is needed and we are currently working towards this goal.

Collaboration with Veronica Dexheimer and Mike Strickland

Sponsors:

Thank you