On the metric of the space of states in a modified QCD

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Overview

1. Review on possible mass generation effects under investigation within a modified version of QCD for massive quarks

2. The modified form of a local and gauge invariant QCD and its connection with the Lee-Wick theories

   It is discussed how the form of the resulting Feynman propagators in a proposed local and gauge invariant QCD for massive fermions suggests the existence of an indefinite metric associated to quark states, a property that might relate the model with the so called Lee-Wick theories.

3. The quantization of the free quark action

   The nature of the asymptotic free quark states in the theory is investigated by quantizing the quadratic part of the quark action. As opposite to the case in the standard QCD, the free theory does not show Hamiltonian constraints.

   3.1 The propagation modes

   The quark propagation modes are presented, they include a family of usual massless waves and a complementary set of also standard massive oscillations.

Overview

3.2 Quantization in the canonical procedure with anticommuting variables

The theory is quantized in a way that the massive modes show positive metric and the massless ones exhibit negative norms. After the quark field operator is defined, it is shown that this field satisfies the Heisenberg equations of motions of the system which coincide with the classical ones in this free problem.

4. Summary

It is remarked that, since QCD is expected to lack exact gluon or quark asymptotic states, the presence of negative metric massless modes does not constitute a definite drawback of the theory. In addition, the fact that the positive metric quark states are massive, seems to be a fine feature of the model, being consistent with the observed approximate existence of asymptotically free massive states in high energy processes.

1. Review on possible mass generation effects under investigation within a modified version of QCD for massive quarks

○ The **action** of the proposed modified QCD is illustrated below. Note the new two gluons-two quarks vertices which are included in it. These terms define masses for each of the six quark flavors and a one over *p* squared high momentum behavior of the quark propagator. If those terms are assumed as counterterms, it could be expected that they can induce additional four fermion counterterms, since they become allowed without loosing remormalizability, thanks to the new highly decreasing behavior of the quark propagator at large momenta.

○ Then, it is suggested one surprising possibility: that almost all the Lagrangean terms which define the Nambu-Jona–Lasinio (NJL) models are allowed to be considered in the resulting effective action of a renormalizable theory.

○ Therefore, the mass generation properties embodied in the usual non-renormalizable phenomenological NJL models, could happen to dynamically appear in the context of a specially renormalized massless QCD.

$$S_g = \int dx \ \{ -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2\alpha} \partial_\mu A^{a\mu} \partial_\nu A^{a\nu} + \overline{\Psi^i}_q \, i\gamma^\mu D^{ij}_\mu \Psi^j_q + \overline{c}^a \overleftarrow{\partial}_\mu D^{ab\mu} c^b +$$

$$+ \sum_{f=1,\ldots,6} \frac{C_f}{(2\pi)^D} \overline{\Psi^j}_f \gamma_\mu \overleftarrow{D}^{ji\mu} \gamma_\nu D^{ik\nu} \Psi^k_f + \qquad \xi, \xi', \varsigma, \varsigma' = (f, s, c)$$

$$+ \sum_{\xi,\xi',\varsigma,\varsigma'} \overline{\Psi}_\xi \, \overline{\Psi}_{\xi'} \, \Gamma^{\xi,\xi'}_{\varsigma,\varsigma'} \Psi^{\varsigma} \Psi^{\varsigma'} \}$$

The quark propagator of the new expansion took the form

$$S_f(p) = \frac{1}{-\gamma_\nu p^\nu - \frac{C_f}{(2\pi)^D} p^2} \equiv \frac{(-\gamma_\nu p^\nu - \frac{C_f}{(2\pi)^D} p^2)^{rr'} \delta^{ii'}}{p^2 (1 - (\frac{C_f}{(2\pi)^D})^2 p^2)}$$

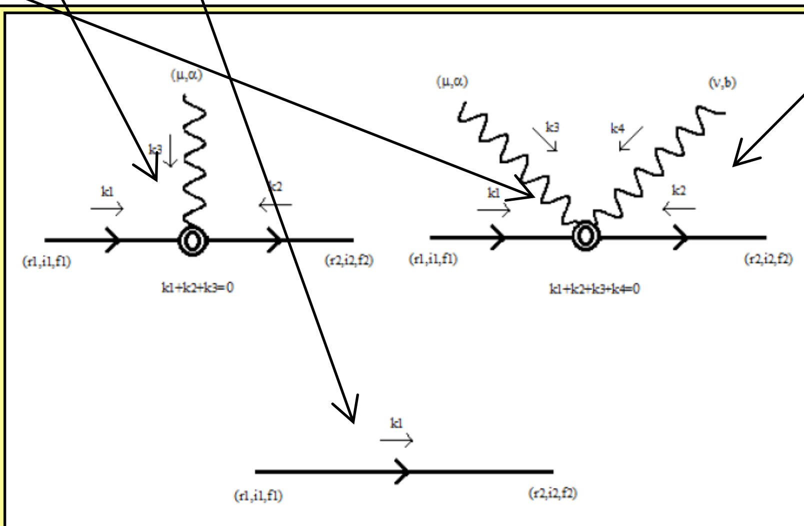
$$= \frac{m_f}{(m_f^2 - p^2)} - \frac{m_f^2}{(m_f^2 - p^2)} \frac{\gamma_\nu p^\nu}{p^2} = S_f^{(s)}(p) + S_f^{(f)}(p).$$

which clearly shows its decrease with the square of the momentum, and decomposes in the sum of a scalar like and a Dirac like components and determines masses for the quarks.

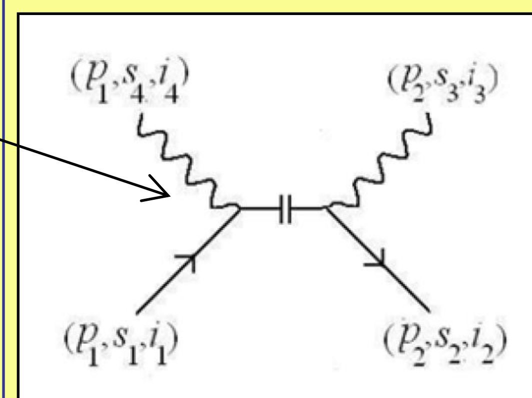
This action terms also create two new vertices in the Feynman expansion

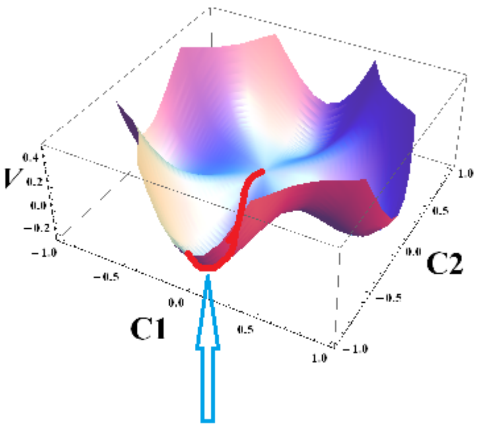
$$V^{(3)(\mu,a)}_{(r_1,i_1,f_1)((r_2,i_2,f_2)}(k_1,k_2,k_3) = g \frac{C_{f_1 f_2}}{(2\pi)^D} T_a^{i_1 i_2} (-(k_{1\alpha}\gamma^\alpha)^{r_1 s}(\gamma^\mu)^{s r_2} + (\gamma^\mu)^{r_1 s}(k_{2\alpha}\gamma^\alpha)^{s r_2}),$$

$$V^{(4)\,(\mu,a)(\nu,b)}_{(r_1,i_1,f_1)((r_2,i_2,f_2)}(k_1,k_2,k_3,k_4) = g^2 \frac{C_{f_1 f_2}}{(2\pi)^D} T_a^{i_1 i} T_b^{i i_2} (\gamma^\mu)^{r_1 s} (\gamma^\nu)^{s r_2}.$$

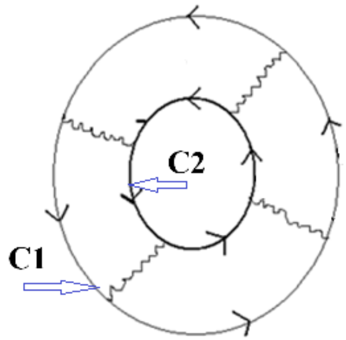


The four legs vertex is the local counterpart of the non local one appeared in the previous form of the modified expansion as a direct consequence of constructing the Wick expansion by including quark condensates in the free vacuum





The hierarchy: an approximate minimum for a single non vanishing condensate



It can only follow in a three or higher loop approximation where more than two types of quark flavours can "interfere" in a diagram.

○ The evaluated in a previous work two loop result for the effective potential, in the case of strong coupling were insufficient to decide about the possibility of appearance of a dynamical breaking of the flavor symmetry in the theory as signaled by non vanishing condensate dependent couplings.

○ This was a natural result, because in two loops, in a given diagram only one kind of quark propagator can appears. Thus, the results for the potential as a function of their condensate parameters are identical for all the quark types . Henceforth, , the minimal potential is simply the sum of six identical contributions, which does not show any flavor symmetry breaking.

○ However, in three and higher loop contributions, as the one illustrated in the above figure, two or more types of quark propagators can participate. Those diagrams are increasingly important at higher coupling values as the one associated to QCD .

○Therefore, these terms of the potential could be able to generate minima of the potential as functions of the six quark condensates (couplings), appearing around a finite value of a particular quark condensate, but for nearly null or smaller values of the other condensate types. Thus, the appearance of this effect could furnish an explanation for the top quark mass. Afterwards, if further analogous steps occur, the suspected hierarchical behavior of the quark mass spectrum could eventually appears.

2. The modified form of a local and gauge invariant QCD and its connection with the Lee-Wick theories

$$S=\int dx\ \ \Big(-\frac{1}{4}F^a_{\mu\nu}(x)F^{a\mu\nu}(x)-\frac{1}{2\alpha}\partial_\mu A^{a\mu}(x)\partial_\nu A^{a\nu}(x)+\overline{c}^a(x)\partial_\mu D^{ab\mu}c^b(x)+$$
$$\sum_f\overline{\Psi}^i_f(x)\ i\gamma^\mu D^{ij}_\mu\Psi^j_f(x)-\sum_f\varkappa\overline{\Psi}^j_f\ (x)\gamma_\mu\overleftarrow{D}^{ji\mu}\ \ \gamma_\nu D^{ik\nu}\Psi^k_f(x)\Big),$$

The action of the proposed form of QCD: The only new term with respect to the massless theory is this last one:

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu-g\ f^{abc}A^b_\mu A^c_\nu,$$

$$\Psi^k_f(x)\equiv\begin{pmatrix}\Psi^{k,1}_f(x)\\ \Psi^{k,2}_f(x)\\ \Psi^{k,3}_f(x)\\ \Psi^{k,4}_f(x)\end{pmatrix},$$

$$\Psi^{\dagger k}_f(x)\equiv(\Psi^k_f(x))^{T*}=\Big(\ (\Psi^{k,1}_f(x))^*\ \ (\Psi^{k,2}_f(x))^*\ \ (\Psi^{k,3}_f(x))^*\ \ (\Psi^{k,4}_f(x))^*\ \Big),$$

The employed notations for the gluon and quark fields. The index k is the color one k=1,2,3 and the spinor indices are hidden to simplify notation, f indicates the flavor of the quarks. The complex conjugation of the spinor , as usual is the transpose conjugate .

$$\overline{\Psi}^j_f\ (x)=\Psi^{\dagger k}_f(x)\gamma^0,$$
$$D^{ij}_\mu=\partial_\mu\delta^{ij}-i\ g\ A^a_\mu T^{ij}_a,\ \ \overleftarrow{D}^{ij}_\mu=-\overleftarrow{\partial}_\mu\delta^{ij}-i\ g\ A^a_\mu T^{ij}_a,$$
$$D^{ab}_\mu=\partial_\mu\delta^{ab}-g\ f^{abc}\ A^c_\mu.$$

The notations for the Dirac conjugation and the covariant derivatives in the the fundamental and adjoin representations.

$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}, \qquad [T_a,T_b] = i\ f^{abc}T_c.\ \gamma^0 = \beta,\ \gamma^j = \beta\ \alpha^j,\ j=1,2,3,$$

$$g^{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},\ \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},\ \alpha^j \equiv \begin{pmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{pmatrix},\ j=1,2,3,$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & -I \end{pmatrix},\ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The notations for the Dirac matrices, metric tensor and the SU(3) group generators . The σ 's are the Pauli matrices

$$x \equiv x^\mu = (x^0,\overrightarrow{x}) = (x^0,x^1,x^2,x^3), \quad x_\mu = g_{\mu\nu}x^\nu,\ x^0 = t.$$

The coordinates and time definitions

$$\mathcal{S}_{0,f} = \int dx\ \overline{\Psi}_f(x)(i\gamma^\mu\partial_\mu - \varkappa\,\partial^2)\Psi_f(x)$$
$$= -\int dx\ \overline{\Psi}_f\ (x)\Lambda_f(\partial)\Psi^j_{f'}(x)$$

The free part of the action associated to a given quark field of flavor *f* reduces to this expression. The κ constant has dimension of length and is equal to the coupling constant of the new vertex related with the particular flavor value *f* . It is the only dimensional quantity being associated to the theory . As it was remarked before, it appears in the place analogous to the one in which a quark condensate parameter was present, within the nonlocal vertex just motivating this QCD model.

The inverse of the kernel Λ_f is the propagator of the quark of flavor *f*

$$-(i\gamma^{\mu}\partial_{\mu} - \varkappa\, \partial^2)\; S_f(x-y) = \delta(x-y),$$

Then, the quark propagators obey

$$S_f(x) = \int dp\, S_f(p) \exp(-ip.x)),$$

After Fourier transforming them

$$-\,(\gamma^{\mu}p_{\mu} + \varkappa\, p^2)S_f(p) = I.$$

The propagators in the momentum representation satisfy

$$S_f(p) = \frac{1}{-\gamma_{\nu}p^{\nu} - \varkappa\, p^2} \equiv \frac{(-\gamma_{\nu}p^{\nu} - \lambda\, p^2)^{rr'}\delta^{ii'}}{p^2(1-\lambda^2p^2)}$$
$$= -\frac{1}{\gamma_{\nu}p^{\nu}} - (-\frac{1}{\gamma_{\nu}p^{\nu} + m_f}), \qquad m_f = \frac{1}{\varkappa}.$$

After the finding the inverse of the entering matrix, it follows that the propagator can be expressed as the difference between a usual massless Dirac propagator and one also usual but massive Dirac propagator, in which the mass is defined as the inverse of the coupling for the new corresponding quark vertex.

At this point it can be underlined, the massless propagator component has the appropriate sign to correspond to positive norm states and on another hand, the massive component has the sign related with negative norm states. For the QCD assumed to being describing Nature it is currently interpreted that nor gluons or quarks show asymptotic states. Thus, the negative metric of the massive free states seem not be a direct drawback of the model. However, the fact that in very high energy processes , a description based in massive quarks in short living asymptotic states, seems to describe the experiences, suggests that an approach in which the massive quarks would have positive norms would be more convenient.

$$x = -x', \qquad \partial_{\mu} \equiv \frac{\partial}{\partial\, x^{\mu}} = -\partial'^{\mu} \equiv \frac{\partial}{\partial\, x'^{\mu}},$$
$$\Psi(x) = \Psi_f^k(-x') = \Psi_f'^k(x'), \qquad \overline{\Psi}(x) = \overline{\Psi}_f^k(-x') = \overline{\Psi}_f'^k(x'),$$
$$A^{a\mu}(x) = A^{a\mu}(-x') = -A'^{a\mu}(x'),$$
$$c(x) = c(-x') = c'(x'), \quad \overline{c}(x) = \overline{c}(-x') = \overline{c}'(x').$$

Then, in this work we argue that quantizing the theory after the fields and coordinates are redefined, the action of the theory can be expressed in a form leading to positive norms for the massive quarks and a negative one for the massless quarks.. The transformation is:

After doing this change of variables and eliminating the prime over the field and coordinates , the action can be rewritten in a way in that, its only modification is the change in the sign of the usual Dirac term :

$$S = \int dx \ (\ -\frac{1}{4}F^{a}_{\mu\nu}(x)F^{a\mu\nu}(x) - \frac{1}{2\alpha}\partial_{\mu}A^{a\mu}(x)\partial_{\nu}A^{a\nu}(x) + \overline{c}^{a}(x)\partial_{\mu}D^{ab\mu}c^{b}(x) - $$
$$- \sum_{f} \overline{\Psi}^{i}_{f}(x) \ i\gamma^{\mu}D^{ij}_{\mu}\Psi^{j}_{f}(x) - \sum_{f} \varkappa \overline{\Psi}^{j}_{f} \ (x)\gamma_{\mu}\overleftarrow{D}^{ji\mu} \ \gamma_{\nu}D^{ik\nu}\Psi^{k}_{f}(x) \ ).$$

Therefore, the free action associate to the quarks, also simply changes its sign

$$\mathcal{S}_{0,f} = \int dx \ \overline{\Psi}_{f}(x)(-i\gamma^{\mu}\partial_{\mu} - \varkappa \ \partial^{2})\Psi_{f}(x) = -\int dx \ \overline{\Psi}_{f} \ (x)\Lambda_{f}(\partial)\Psi^{j}_{f'}(x),$$

Therefore, the free propagator defined as before by the inverse of the kernel Λ, becomes

$$S_{f}(p) = \frac{1}{\gamma_{\nu}p^{\nu} - \varkappa \ p^{2}} = \frac{1}{\gamma_{\nu}p^{\nu}} - \left(\frac{1}{\gamma_{\nu}p^{\nu} - m_{f}}\right),$$

In this form the massive component shows the sign leading to the normal positive norm states and on the contrary the massless term is related with negative norm ones.

3. The quantization of the free quark action

3.1 The propagation modes

Let us present now the quantization of the free quark action

Firstly, let us write the propagation modes of this theory. Their general equations are

$$(-i\gamma^\mu \partial_\mu - \varkappa\, \partial^2)\, u(x) = 0, \quad u(x) = \int dq^4 u(q) \exp(-q_\mu x^\mu).$$

which in momentum representation become

$$(-\gamma^\mu q_\mu + \varkappa\, q^2)\, u(q) = 0.$$

A. Massive solutions

Selecting a rest frame in which the spatial momentum vanishes simplifies the equations

$$q_0(-\gamma^0 + \varkappa\, q_0)\, u(q) = 0 \equiv q_0 \begin{pmatrix} (-1+\varkappa\, q_0)I & 0 \\ 0 & (\varkappa\, q_0 + 1)I \end{pmatrix} \begin{pmatrix} u^{(1)} \\ u^{(2)} \end{pmatrix},$$

1. Positive energy waves

Then, for the positive q_0 solution, as in the usual Dirac equation, there are two spinor polarizations

$$u^r(\vec{0}) = \begin{pmatrix} \beta^r \\ 0 \end{pmatrix}, r = 1, 2$$

$$\beta^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \beta^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$$\varkappa = \frac{1}{m_f}.$$

After returning to the original reference frame, the two solutions for the four spinor get expressions

$$u^r(\vec{p}) = \sqrt{\frac{\epsilon_m(\vec{p}) + m_f}{2\epsilon_m(\vec{p})}} \begin{pmatrix} \beta^r \\ \frac{\vec{\sigma}\cdot\vec{p}}{\epsilon_m(\vec{p})+m_f}\beta^r \end{pmatrix}, r = 1, 2,$$

$$\vec{\sigma}.\vec{p} = \sigma^i p^i, \quad \epsilon_{m_f}(\vec{p}) = \sqrt{m_f^2 + \vec{p}^{\,2}}, q = (\epsilon_{m_f}(\vec{p}), \vec{p}),$$

in which the normalization was chosen in the special form

$$u^{r\dagger}(\vec{p})u^s(\vec{p}) = \delta^{rs}, \quad \overline{u}^r(\vec{p})u^s(\vec{p}) = \frac{m_f}{\epsilon_{m_f}(\vec{p})}\delta^{rs}.$$

2. *Negative energy waves*

In a very close way, the solutions associated to negative energy spinors may be written in the form

$$v^r(\vec{p}) = \sqrt{\frac{\epsilon_m(\vec{p}) + m_f}{2\epsilon_m(\vec{p})}} \begin{pmatrix} \frac{-\vec{\sigma}\cdot\vec{p}}{\epsilon_m(\vec{p})+m_f}\beta^r \\ \beta^r \end{pmatrix}, r = 1, 2,$$

$$\vec{\sigma}.\vec{p} = \sigma^i p^i, \quad \epsilon_{m_f}(\vec{p}) = \sqrt{m_f^2 + \vec{p}^{\,2}}, q = (-\epsilon_{m_f}(\vec{p}), \vec{p}),$$

with the normalization

$$v^{r+}(\vec{p})\, v^s(\vec{p}) = \delta^{rs}, \quad \overline{v}^r(\vec{p})\, v^s(\vec{p}) = -\frac{m_f}{\epsilon_{m_f}(\vec{p})}\delta^{rs}.$$

B. Massless solutions

The equations for the massless solutions simplify to coincide with the massless Dirac equation

$$i\,\gamma^\mu \partial_\mu\, u(x) = 0, \;\; u(x) = \int dq\; u(q) \exp(-q_\mu x^\mu),$$

which in momentum representation becomes

$$\gamma^\mu q_\mu\; u(q) = 0.$$

Now, since there is no a reference frame in which the momentum can vanish, a convenient way to write the equation is :

$$(q_0\gamma^0 - \gamma^i q^i)u(q) = \beta(q_0 - \alpha^i q^i)u(q)$$

$$\equiv \beta \begin{pmatrix} q_0 I & -\vec{\sigma}.\vec{q} \\ -\vec{\sigma}.\vec{q} & q_0 I \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = 0.$$

1. Positive energy massless waves

$$\beta^{+1}(\overrightarrow{p}) \equiv \frac{1}{\sqrt{2(n_3+1)}} \begin{pmatrix} n_3+1 \\ n_1+i\ n_2 \end{pmatrix}, \beta^{-1}(\overrightarrow{p}) = \frac{1}{\sqrt{2(n_3+1)}} \begin{pmatrix} -n_1+i\ n_2 \\ n_3+1 \end{pmatrix},$$

$$\overrightarrow{\sigma}.\overrightarrow{p}\ \beta^l(\overrightarrow{p}) = l\ \beta^l(\overrightarrow{p}), \quad l=+1,-1,$$

$$\overrightarrow{n}(\overrightarrow{p}) = \frac{\overrightarrow{p}}{|\overrightarrow{p}|} = (n_1,n_2,n_3),$$

It is clear that the spinor solutions can be found by employing the eigenfunctions of the two spinor matrix σ.p:

Thus the 4-spinors solving the Dirac equation associated to massless get the forms:

$$u_0^l(\overrightarrow{p}) = \sqrt{\frac{1}{2}} \begin{pmatrix} \beta^l(\overrightarrow{p}) \\ l\ \beta^l(\overrightarrow{p}) \end{pmatrix}, \ l=+1,-1,$$

$$\epsilon_0(\overrightarrow{p}) = |\overrightarrow{p}|, \qquad q = (\epsilon_0(\overrightarrow{p}), \overrightarrow{p}).$$

Their normalization properties are

$$u_0^{\dagger\ l}(\overrightarrow{p})\ u_0^{l'}(\overrightarrow{p}) = \ \delta^{l\ l'}, \ \ \overline{u}_0^l(\overrightarrow{p})\ u_0^{l'}(\overrightarrow{p}) = 0.$$

2. Negative energy massless waves

$$q_0 = -|\overrightarrow{q}|$$

in a very close way, for the massless negative energy 4-spinor solutions, it follows

$$v_0^l(\overrightarrow{p}) = \sqrt{\frac{1}{2}} \begin{pmatrix} \beta^{-l}(\overrightarrow{p}) \\ l\ \beta^{-l}(\overrightarrow{p}) \end{pmatrix}, l=+1,-1,$$

$$\overrightarrow{n}(\overrightarrow{p}) = \frac{\overrightarrow{p}}{|\overrightarrow{p}|} = (n_1,n_2,n_3),$$

where the 2-spinors entering are the same eigenfunctions of the 2-spinor matrix σ.p

In this case the normalization properties are

$$v_0^{l\dagger}(\overrightarrow{p})\ v_0^{l'}(\overrightarrow{p}) = \ \delta^{l\ l'}, \ \ \overline{v}_0^l(\overrightarrow{p})\ v_0^{l'}(\overrightarrow{p}) = 0.$$

3.2 Quantization in the canonical procedure with anticommuting variables

$$L=\int dx^3\Psi^\dagger(x)(-\frac{i}{2}\alpha^\mu\overleftrightarrow{\partial}_\mu+\varkappa\,(\overleftarrow{\partial}_t\,\overrightarrow{\partial}_t-\overleftarrow{\nabla}.\overrightarrow{\nabla})\beta)\Psi(x),$$

$$\overleftrightarrow{\partial}_\mu=\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu,\alpha^\mu=(I,\alpha^1,\alpha^2,\alpha^3),\Psi^\dagger(x)=\Psi^{*T}(x),$$

From the action, after doing some integrations by parts, the Lagrangean L (the spatial integral of the Lagrangean density) of the free quarks of flavor *f* can be written in terms of the field and its first derivatives.

The momenta in the general classical formalism including anticommuting fermion variables are defined by the left functional derivatives of L over the velocities:

$$\Pi_\Psi(x)=L\frac{\overleftarrow{\delta}}{\delta(\partial_t\Psi(x))}=-\frac{i}{2}\Psi^\dagger(x)+\varkappa\,\partial_t\Psi^\dagger(x)\beta,$$

$$\Pi_{\Psi^\dagger}(x)=L\frac{\overleftarrow{\delta}}{\delta(\partial_t\Psi^\dagger(x))}=-\frac{i}{2}\Psi(x)-\varkappa\,\beta\,\partial_t\Psi(x),$$

One interesting point in the considered theory is that, at variance with the Dirac one, the mechanical system is regular. That is, the system does not show mechanical constraints. Then, all the velocities can be expressed in terms of the momenta:

$$\partial_t\Psi^\dagger(x)\beta=\frac{1}{\varkappa}(\frac{i}{2}\Psi^\dagger(x)+\ \Pi_\Psi(x)),$$

$$\beta\partial_t\Psi(x)=-\frac{1}{\varkappa}(\Pi_{\Psi^\dagger}(x)+\frac{i}{2}\Psi(x)).$$

Substituting the velocities as functions of the momenta, the Hamiltonian can be in terms of the fields and the momenta in the way shown at the left:

$$\begin{aligned}H&=\int dx^3(\Pi_\Psi(x)\partial_t\Psi(x)+\Pi_{\Psi^\dagger}(x)\partial_t\Psi^\dagger(x))-L\\&=\int dx^3\{\frac{1}{\varkappa}(-\Pi_\Psi(x)\ \beta\ \Pi_{\Psi^\dagger}(x)+\frac{1}{4}\Psi^\dagger(x)\ \beta\ \Psi(x)-\\&\frac{i}{2}\Psi^\dagger(x)\ \beta\ \Pi_{\Psi^\dagger}(x)-\frac{i}{2}\Pi_\Psi(x)\ \beta\ \Psi(x))+\\&\frac{i}{2}\Psi^\dagger(x)\alpha^i\overleftrightarrow{\partial}_i\Psi(x)+\varkappa\ \Psi^\dagger(x)\overleftarrow{\nabla}.\overrightarrow{\nabla}\beta\Psi(x)\}.\end{aligned}$$

$$\partial_t \Psi(x) = \frac{\delta}{\delta \Pi_\Psi(x)} H = -\frac{1}{\varkappa}\beta(\Pi_{\Psi^\dagger}(x) + \frac{i}{2}\Psi(x)),$$

$$\partial_t \Psi^\dagger(x) = \frac{\overrightarrow{\delta}}{\delta \Pi_{\Psi^\dagger}(x)} H = \frac{1}{\varkappa}(\Pi_\Psi(x) + \frac{i}{2}\Psi^\dagger(x))\beta$$

$$\partial_t \Pi_\Psi(x) = -H\frac{\overleftarrow{\delta}}{\delta \Psi(x)} = \frac{i}{2\varkappa}\Pi_\Psi(x)\beta - \frac{1}{4\varkappa}\Psi^\dagger(x)\beta + i\Psi^\dagger(x)\alpha^i \overleftarrow{\partial}_i + \varkappa\nabla^2\Psi^\dagger(x)\beta,$$

$$\partial_t \Pi_{\Psi^\dagger}(x) = -H\frac{\overleftarrow{\delta}}{\delta \Psi^\dagger(x)} = -\frac{i}{2\varkappa}\beta\ \Pi_{\Psi^\dagger}(x) + \frac{1}{4\varkappa}\beta\ \Psi(x) + i\ \alpha^i\partial_i\Psi(x) - \varkappa\beta\nabla^2\Psi(x),$$

By calculating the Poisson brackets of the fields and the momenta with the Hamiltonian, the equations of motion in the canonical procedure with anticommuting variables can be obtained in the form.

$$(-i\ \gamma^\mu\partial_\mu - \varkappa\ \partial^2)\Psi(x) = 0,$$

which after expressing the momenta in terms of the quark fields and the velocities, reproduces the original Lagrange equations.

The quantized fields and the norms of the states

$$H = \int dx^3 \{ \varkappa\, \partial_t \Psi^\dagger(x)\ \beta\ \partial_t \Psi(x) + i\ \Psi^\dagger(x) \alpha^i \partial_i \Psi(x) + \varkappa\, \Psi^\dagger(x)\ \overleftarrow{\nabla}.\overrightarrow{\nabla} \beta \Psi(x) \}$$
$$= \int dx^3 \Psi^\dagger(x) \{ \varkappa \overleftarrow{\partial}_t\ \beta\ \overrightarrow{\partial}_t + i\ \alpha^i \partial_i + \varkappa\ \overleftarrow{\nabla}.\overrightarrow{\nabla} \beta \} \Psi(x)$$
$$= \int dx^3 \Psi^\dagger(x)\, h(\partial)\, \Psi(x),$$

Let us now quantize this free model. For this purpose the Hamiltonian can be expressed in the form shown at the left

$$\Psi(x) = \Psi_m(x) + \Psi_0(x) = \sum_{\varkappa=0,m} \Psi_\kappa(x),$$

$$\Psi_0(x) = \frac{1}{\sqrt{V}} \sum_{\overrightarrow{p}} \sum_{l=\pm 1} (a_l(\overrightarrow{p}) u_0^l(\overrightarrow{p}) \exp(-i\ (\epsilon_0(\overrightarrow{p})t - \overrightarrow{p}.\overrightarrow{x})) +$$
$$c_l^\dagger(\overrightarrow{p}) v_0^l(\overrightarrow{p}) \exp(i\ (\epsilon_0(\overrightarrow{p})t + \overrightarrow{p}.\overrightarrow{x})))$$
$$= u_0(x) + v_0(x)$$

$$\Psi_m(x) = \frac{1}{\sqrt{V}} \sum_{\overrightarrow{p}} \sum_{r=1,2} (b_r(\overrightarrow{p}) u_{m_f}^r(\overrightarrow{p}) \exp(-i\ (\epsilon_{m_f}(\overrightarrow{p})t - \overrightarrow{p}.\overrightarrow{x})) +$$
$$d_r^\dagger(\overrightarrow{p}) v_{m_f}^r(\overrightarrow{p}) \exp(i\ (\epsilon_{m_f}(\overrightarrow{p})t + \overrightarrow{p}.\overrightarrow{x}))$$
$$= u_m(x) + v_m(x),$$

Now, as usual we will express the quark field as a linear combinations of all the just found propagation modes with coefficients defined by operators:

The spatial and time variations of the field operator are given by the same dependences specified by the modes solving the classical equations of motion.

$$\Psi(x) = \sum_{\kappa=0,m} u_\kappa(x) + \sum_{\kappa=0,m} v_\kappa(x).$$

The field also can be expressed in a compact form in terms of the positive and negative energy components of the massless and massive modes.

$$H=\int dx^3\Psi^\dagger(x)\ h(\partial)\ \Psi(x)$$
$$=\sum_{\kappa=0,m}\sum_{\kappa'=0,m}\Big(\int dx^3u_\kappa^\dagger(x)h(\partial)u_{\kappa'}(x)+\int dx^3v_\kappa^\dagger(x)h(\partial)v_{\kappa'}(x)+$$
$$\int dx^3u_\kappa^\dagger(x)h(\partial)v_{\kappa'}(x)+\int dx^3v_\kappa^\dagger(x)h(\partial)u_{\kappa'}(x)\Big).$$

The operator field expressions can be substituted in the Hamiltonian in order to express it as the usual sum of mode energies. In this process the nature of the operators introduced for each mode can be appropriately specified .

$$H=\int dx^3\Psi^\dagger(x)\ h(\partial)\ \Psi(x)$$
$$=\int dx^3u_m^\dagger(x)\ h(\partial)\ u_m(x)+\int dx^3v_m^\dagger(x)\ h(\partial)\ v_m(x)+$$
$$\int dx^3u_0^\dagger(x)\ h(\partial)\ u_0(x)+\int dx^3v_0^\dagger(x)\ h(\partial)\ v_0(x)$$
$$=\sum_{\overrightarrow{p}}\sum_{l=\pm1}(-\epsilon_0(\overrightarrow{p})\ a_l^\dagger(\overrightarrow{p})a_l(\overrightarrow{p})+\epsilon_0(\overrightarrow{p})\ c_l(\overrightarrow{p})c_l^\dagger(\overrightarrow{p}))+$$
$$\sum_{\overrightarrow{p}}\sum_{r=1,2}(\epsilon_{m_f}(\overrightarrow{p})\ b_r^\dagger(\overrightarrow{p})b_r(\overrightarrow{p})+\epsilon_{m_f}(\overrightarrow{p})\ d_r^\dagger(\overrightarrow{p})d_r(\overrightarrow{p})).$$

The previous definitions allow to express the Hamiltonian as a sum over terms being associated to each of the modes in the form illustrated at the left.

It can be noticed that the operators associated to the massive modes and their adjoins can be chosen to be interpreted as the usual creation and annihilation operators for massive quarks and antiquarks, respectively .

$$\{b_r^\dagger(\overrightarrow{p}_1),b_{r'}(\overrightarrow{p}_2)\}=s_m\ \delta_{r,r'}\delta^{(K)}(\overrightarrow{p}_1,\overrightarrow{p}_2),$$
$$\{d_r^\dagger(\overrightarrow{p}_1),d_{r'}(\overrightarrow{p}_2)\}=s_m\ \delta_{r,r'}\delta^{(K)}(\overrightarrow{p}_1,\overrightarrow{p}_2)$$
$$\{A,B\}=AB+BA,$$

$$s_m\ =\ \pm1$$

Then, the anticommutation relations for such operators will be chosen in the form

The sign factor s_m will not be yet selected . These operators will be assumed to anticommute with all the other already defined ones.

mode operators.

$$\widehat{a}_l(\overrightarrow{p}) = a_l^\dagger(\overrightarrow{p}), \quad \widehat{a}_l^\dagger(\overrightarrow{p}) = a_l(\overrightarrow{p}),$$
$$\widehat{c}_l(\overrightarrow{p}) = c_l^\dagger(\overrightarrow{p}), \quad \widehat{c}_l^\dagger(\overrightarrow{p}) = c_l(\overrightarrow{p}),$$

On another hand the massless operator and its adjoin should be assigned to creation and annihilation, ones respectively, in order to express the energy in the standard form.

Then, their anticommutation relations are assumed in the forms

$$\{\widehat{a}_l^\dagger(\overrightarrow{p}_1), \widehat{a}_{l'}(\overrightarrow{p}_2)\} = s_0\ \delta_{l,l'}\delta^{(K)}(\overrightarrow{p}_1, \overrightarrow{p}_2), \{\widehat{c}_l^\dagger(\overrightarrow{p}_1), \widehat{c}_{l'}(\overrightarrow{p}_2)\} = s_0\ \delta_{l,l'}\delta^{(K)}(\overrightarrow{p}_1, \overrightarrow{p}_2),$$

$$s_0 = \pm 1$$

$\widehat{a}_l(\overrightarrow{p}),\ \widehat{c}_l(\overrightarrow{p}),\ b_r(\overrightarrow{p})$ and $d_r(\overrightarrow{p})$ — the adopted annihilation operators

The Hamiltonian has the form

$$H = \sum_{\overrightarrow{p}} \sum_{l=\pm 1} (\epsilon_0(\overrightarrow{p})\ \widehat{a}_l^\dagger(\overrightarrow{p})\widehat{a}_l(\overrightarrow{p}) + \epsilon_0(\overrightarrow{p})\ \widehat{c}_l^\dagger(\overrightarrow{p})\widehat{c}_l(\overrightarrow{p})) +$$
$$\sum_{\overrightarrow{p}} \sum_{r=1,2} (\epsilon_{m_f}(\overrightarrow{p})\ b_r^\dagger(\overrightarrow{p})b_r(\overrightarrow{p}) + \epsilon_{m_f}(\overrightarrow{p})\ d_r^\dagger(\overrightarrow{p})d_r(\overrightarrow{p})).$$

Then the expression for the quark field operator including its massless and massive component has the form

$$\Psi(x) = \Psi_m(x) + \Psi_0(x) = \sum_{\kappa=0,m} \Psi_\kappa(x),$$

$$\Psi_0(x) = \frac{1}{\sqrt{V}} \sum_{\overrightarrow{p}} \sum_{l=\pm 1} (\widehat{a}_l^\dagger(\overrightarrow{p})u_0^l(\overrightarrow{p}) \exp(-i\ (\epsilon_0(\overrightarrow{p})t - \overrightarrow{p}.\overrightarrow{x})) +$$
$$\widehat{c}_l(\overrightarrow{p})v_0^l(\overrightarrow{p}) \exp(i\ (\epsilon_0(\overrightarrow{p})t + \overrightarrow{p}.\overrightarrow{x})))$$
$$= u_0(x) + v_0(x),$$

$$\Psi_m(x) = \frac{1}{\sqrt{V}} \sum_{\overrightarrow{p}} \sum_{r=1,2} (b_r(\overrightarrow{p})u_{m_f}^r(\overrightarrow{p}) \exp(-i\ (\epsilon_{m_f}(\overrightarrow{p})t - \overrightarrow{p}.\overrightarrow{x})) +$$
$$d_r^\dagger(\overrightarrow{p})v_{m_f}^r(\overrightarrow{p}) \exp(i\ (\epsilon_{m_f}(\overrightarrow{p})t + \overrightarrow{p}.\overrightarrow{x})))$$
$$= u_m(x) + v_m(x).$$

$$
\begin{aligned}
\widehat{a}_l(\overrightarrow{p},t) &\equiv \exp(i\ H\ t)\ \widehat{a}_l(\overrightarrow{p})\exp(-iH\ t). \\
&= \exp(i\ \epsilon_0(\overrightarrow{p})\ \widehat{a}_l^{\dagger}(\overrightarrow{p})\widehat{a}_l(\overrightarrow{p})\ t)\ \widehat{a}_l(\overrightarrow{p})\exp(-i\ \epsilon_0(\overrightarrow{p})\ \widehat{a}_l^{\dagger}(\overrightarrow{p})\widehat{a}_l(\overrightarrow{p})\ t). \\
&= \exp(-s_0\ i\ \epsilon_0(\overrightarrow{p})\ t)\ \widehat{a}_l(\overrightarrow{p}) \to \exp(i\ \epsilon_0(\overrightarrow{p})\ t)\ \widehat{a}_l(\overrightarrow{p}) \\
\widehat{c}_l^{\dagger}(\overrightarrow{p},t) &= \exp(i\ H\ t)\ \widehat{c}_l^{\dagger}(\overrightarrow{p})\ \exp(-i\ H\ t). \\
& \exp(i\ \epsilon_0(\overrightarrow{p})\ \widehat{c}_l^{\dagger}(\overrightarrow{p})\widehat{c}_l(\overrightarrow{p})\ t)\ \widehat{c}_l^{\dagger}(\overrightarrow{p})\ \exp(-i\ \epsilon_0(\overrightarrow{p})\ \widehat{c}_l^{\dagger}(\overrightarrow{p})\widehat{c}_l(\overrightarrow{p})t). \\
&= \exp(i\ s_0\epsilon_0(\overrightarrow{p})\ t)\ \widehat{c}_l^{\dagger}(\overrightarrow{p}) \to \exp(-i\ \epsilon_0(\overrightarrow{p})\ t)\ \widehat{c}_l(\overrightarrow{p},0).
\end{aligned}
$$

Then, since the time evolution assumed for the modes was employed for evaluating the Hamiltonian in the previous discussion, it should be required that the quantum time evolution should identically reproduce the assumed classical time evolution of the modes.

Firstly let us consider the time evolution of the operators associated to the massless modes. Then, the compatibility condition directly lead to the fixing the before defined sign parameter S_o to be equal to -1.

In a very close way, the sign for the massive modes S_m is defined to be equal to 1 , which completely fixes the commutation relations .

The defined field operator satisfies the quantum Hamiltonian equations

$$\{\Psi(x),\Psi(x')\}|_{x_0=x'_0}=\{\Psi^\dagger(x),\Psi^\dagger(x')\}|_{x_0=x'_0}=0,$$
$$\{\Psi(x),\Psi^\dagger(x')\}|_{x_0=x'_0}=\{\Psi_0(x),\Psi_0^\dagger(x')\}|_{x_0=x'_0}+\{\Psi_m(x),\Psi_m^\dagger(x')\}|_{x_0=x'_0}=0,$$

Firstly, the coincidence of the massive and massless field components with the ones associated to the Dirac equations allows to show the vanishing of the equal time commutation relations among all the field operators

$$\Pi_\Psi(x)=-\frac{i}{2}\Psi^\dagger(x)+\varkappa\,\partial_t\Psi^\dagger(x)\beta,$$
$$\Pi_{\Psi^\dagger}(x)=-\frac{i}{2}\Psi(x)-\varkappa\,\beta\,\partial_t\Psi(x),$$

Next, the momentum time operators must be expressed in terms of the field and its time derivative by

This allows to also derive for them the rest of the commutation relations

$$\{\Psi(x),\Pi_\Psi(x')\}|_{x_0=x'_0}=i\,\delta^{(D)}(\overrightarrow{x}-\overrightarrow{x}')I.$$

$$\{\Pi_\Psi(x),\Pi_\Psi(x')\}|_{x_0=x'_0}=\{\Pi_{\Psi^\dagger}(x),\Pi_{\Psi^\dagger}(x')=0,$$

$$\{\Psi^\dagger(x),\Pi_{\Psi^\dagger}(x')\}|_{x_0=x'_0}=i\,\delta^{(D)}(\overrightarrow{x}-\overrightarrow{x}')I,$$

$$\{\Pi_\Psi(x),\Pi_{\Psi^\dagger}(x')\}|_{x_0=x'_0}=0.$$

$$H=\int dx^3(\Pi_\Psi(x)\partial_t\Psi(x)+\Pi_{\Psi^\dagger}(x)\partial_t\Psi^\dagger(x))-L$$
$$=\int dx^3\{\frac{1}{x}(-\Pi_\Psi(x)\;\beta\;\Pi_{\Psi^\dagger}(x)+\frac{1}{4}\Psi^\dagger(x)\;\beta\;\Psi(x)-$$
$$\frac{i}{2}\Psi^\dagger(x)\;\beta\;\Pi_{\Psi^\dagger}(x)-\frac{i}{2}\Pi_\Psi(x)\;\beta\;\Psi(x))+$$
$$\frac{i}{2}\Psi^\dagger(x)\alpha^i\overleftrightarrow{\partial}_i\Psi(x)+\varkappa\;\Psi^\dagger(x)\overleftarrow{\nabla}.\overrightarrow{\nabla}\beta\Psi(x)\}.$$

These relations show that the fields and momenta obey the usual commutation relations determined by the quantization condition of substituting the classical Poisson brackets by the anticommutators.

Now, the Hamiltonian operator can be constructed in terms of the of the defined field and momenta operators in a form that coincides with the classical one.

$$\partial_t \Psi(x) = -\frac{1}{\varkappa}\beta\ (\Pi_{\Psi^\dagger}(x) + \frac{i}{2}\Psi(x)),$$

$$\partial_t \Psi^\dagger = \frac{1}{\varkappa}(\Pi_\Psi(x) + \frac{i}{2}\Psi^\dagger(x))\ \beta,$$

$$\partial_t \Pi_\Psi(x) = \frac{i}{2\varkappa}\Pi_\Psi(x)\ \beta - \frac{1}{4\varkappa}\Psi^\dagger(x)\ \beta + i\ \Psi^\dagger \alpha^i \overleftarrow{\partial}_i + \varkappa \nabla^2 \Psi^\dagger \beta,$$

$$\partial_t \Pi_{\Psi^\dagger}(x) = -\frac{i}{2\varkappa}\beta\ \Pi_{\Psi^\dagger}(x) + \frac{1}{4\varkappa}\beta\ \Psi(x) + i\ \alpha^i \partial_i \Psi^\dagger - \varkappa\ \beta\ \nabla^2 \Psi.$$

Therefore, the consistency of the construction follows after evaluating the Heisenberg evolution equations, that is, the commutator of the Hamiltonian operator with the fields and the momenta, which should define the time derivatives of these quantities. The result exactly gives a set of operator equations which coincide in form with the classical Hamiltonian equations .

$$(-i\ \gamma^\mu \partial_\mu - \varkappa\ \partial^2)\ \Psi(x) = 0,$$

$$\Psi^\dagger(x)\ (i\ \gamma^\mu \overleftarrow{\partial}_\mu - \varkappa\ \overleftarrow{\partial}^2) = 0.$$

After eliminating the momenta operators in the above equations the satisfaction of the operator form of the Lagrange equation also follows.

4. Summary

1. The nature of the space of states of an alternative of QCD for massive quarks is investigated.

2. The quantization is done in a form in which the massive creation and annihilation operators satisfy the usual anticommutation rules defining positive metric for the states, and on another hand, the massless states show negative norms.

3. Since the quarks do not show asymptotically free states, the conclusions does not seem to indicate a direct drawback of the theory.

4. However, the fact that the massive modes show usual positive norms, might be helpful in describing the Physics at high energies, where indirect scattering results indicate the approximate existence of quarks in massive states.

5. The work is planned to be extended in the sense of evaluating the higher contributions to the vacuum energy as a functional of the flavor dependent masses.