Faraday Effect in 2D and 3D

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Motivation

It is well known that an electromagnetic wave propagating through a medium in an ambient magnetic field suffers Faraday rotation.

In astrophysiscs it may be important because radiation emitted from far stellar objects travels through very diluted concentrations of interstellar gases, passing through regions where weak magnetic fields exist.

In magneto-optics, Faraday rotation is also appreciated when ligth cross either a dielectric or a metal.

However, in most of papers the amount of rotation is derived in a non relativitic regime and for non degenerated system ofelectrons and nucleons.

Motivation

QHE and Faraday rotation have been studied in a two dimensional electron gas (2DEG). And recently both have been experimentally and theoretically studied in graphene.

From the point of view of high energy physics, graphene could be interesting to test some quantum field theories.

The aim of this work is to study Faraday effect in 3D and 2D.

Outline

We present the components of the polarization tensor in the case of $T \neq o$ and $\mu \neq o$ in the presence of a constant magnetic field.

The expression for the frequency of rotation of the polarization vector for an EM wave which travels in a relativistc electron-positron gas, along the direction of the magnetic field is obtained.

We address this problem in the general framework of QFT and we would be interested in its possible astrophysical applications!!!!

The particular case of 2D, massless and relativistic QED limit is taken.

An expression for the Faraday angle when an EM wave cross a graphene sheet is obtained.

Summary.

3D electron-positron gas

The propagation of photons in a relativistic electron-positron gas in the presence of a constant magnetic field was previously studied by Hugo Pérez and Shabad.

(H. Perez Rojas, A. E. Shabad, Ann. Phys. 121, 432-455 (1979))

3D electron-positron gas

To study the propagation of light in the presence of a constant magnetic field we are going to start with the components of the polarization operator in 3D in the one loop approximation.

$$\pi_{\mu\nu} = e^2 Tr \int \gamma_\mu G(x,z)\Gamma_\nu(z,y',y)G(y',x)d^3z\, d^3y'$$

Consider the case of photons propagating in a relativistic electron-positron gas in the presence of a constant magnetic field (B).

$\mu \neq 0$, $T \neq 0$

The particular case of propagation along the magnetic field is considered.

Polarization tensor

The diagonalization of the polarization tensor leads to the equation:

$$\prod_{\mu\nu} C_\nu^{(i)} = \kappa^{(i)} C_\mu^{(i)}$$

Having three non vanishing eigenvalues and three eigenvectors for i=1,2,3.

For the particular case of propagation along the external field B the second mode is pure longitudinal wave and the transverse modes are:

$$C_\mu^{\prime(1,3)} = R(C_\mu^{(3)} \pm iC_\mu^{(1)})$$

This describe a circularly polarized waves in the plane orthogonal to B having different eigenvalues.

The electric polarization vectors can then be written as: $e^{(1,3)} = (b_1^{(1)} \pm ib_2^{(3)})$

Corresponding to the eigenvalues : $\kappa_{1,3} = t \pm r$

Faraday effect

Faraday effect

If we assume that: $t, r << \omega$, then we can approximately write:

$$\omega_{\mp} = \sqrt{\mathrm{k}^2 - t} \mp \varepsilon' \quad \Longrightarrow \quad \varepsilon' = \frac{r}{2}\sqrt{\mathrm{k}^2 - t}$$

Then the superposition of both modes leads to the following wave, which shows that the polarization of the propagating wave rotates counterclockwise with frequency : ε'

$$\mathrm{E} = A[\mathrm{e}^1 e^{-i\varepsilon' t} + \mathrm{e}^3 e^{i\varepsilon' t}]\,\mathrm{Re}\, e^{i(\mathrm{k}\cdot\mathrm{x} - \omega t)}$$

H. Pérez Rojas and E. Roríguez Quertz, arXiv: 0708.2215v4

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In perspective

This is a general result, in the future we will be intersted in considering some particular situations which allow us to estimate the degree of Faraday rotation by calculating the values of the scalars ***r*** and ***t*** in some particular conditions.

For example:

a degenerate gas

a non-relativistic gas

an ultra-relativistic gas

2D massless relativistic limit

Problem

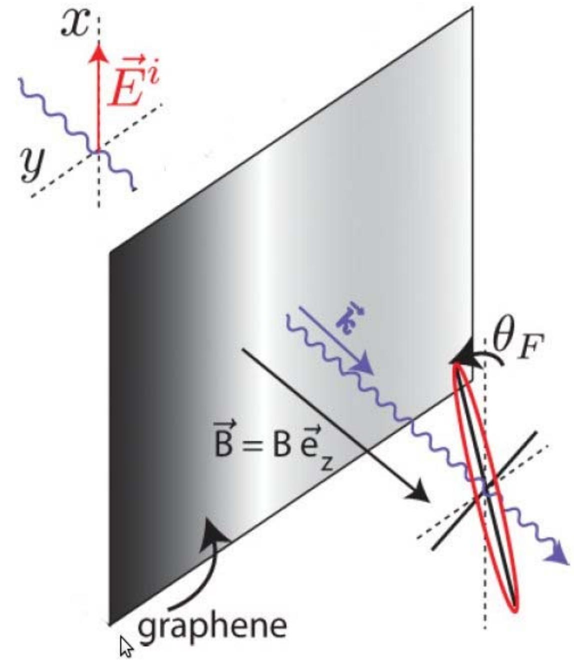
Propagation of an electromagnetic wave in the whole space:

$$\partial_\mu F^{\mu\nu} + \delta(z)\pi^{\nu\rho} A_\rho = 0$$

Subject to the mathcing conditions at the graphene surface:

$$A_\rho(z=0-) = A_\rho(z=0+)$$

$$\partial_3 A_\rho(z=0-) - \partial_3 A_\rho(z=0+) = \pi_\mu^\nu A_\nu(z=0)$$



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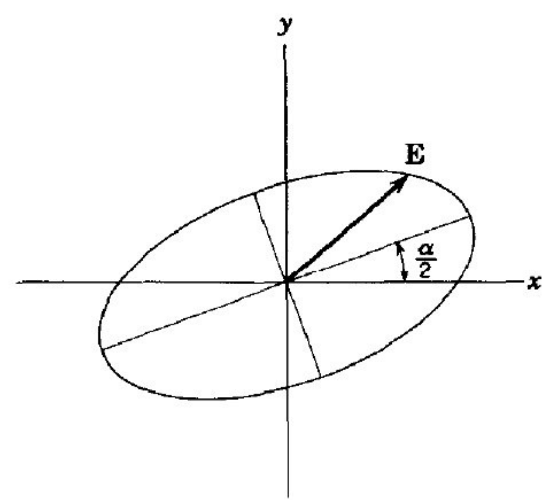
(1): PHYSICAL REVIEW B 84, 235410 (2011)

Problem

Considering a solution in the form of a plane wave propagating along the z-axis, with the initial polarization parallel to x:

$$A = e^{-i\omega t}\begin{cases}(\mathrm{e}_x e^{ik_3 z} + (r_{xx}\mathrm{e}_x + r_{xy}\mathrm{e}_y)e^{-ik_3 z}), & z < 0 \\ (t_{xx}\mathrm{e}_x + t_{xy}\mathrm{e}_y)e^{ik_3 z}, & z > 0\end{cases}$$

In the generic case the transmitted wave is elliptically polarized. The ellipse has its axes rotated by an angle:



$$\theta = \frac{\alpha}{2} = -\frac{1}{2}\arg\frac{t_{xx} - it_{xy}}{t_{xx} + it_{xy}} \approx -\frac{1}{2}\sigma_{xy}$$

Current density

The expression for the spatial part of the current density which is linear in a perturbative EM field A_μ is given in terms of the polarization tensor in the medium by: (**A. Perez, E. Rodríguez, H. Perez Rojas, R. Gaitan and S. Rodriguez-Romo, J. Phys. A: Math Theor. 44 (2011))**

$$j_i = \pi_{i\mu} A_\mu = Y_{ij} E_j$$

Current density

The expression for the spatial part of the current density which is linear in a perturbative EM field A_μ is given in terms of the polarization tensor in the medium by:

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Electric field

Current density

The expression for the spatial part of the current density which is linear in a perturbative EM field A_μ is given in terms of the polarization tensor in the medium by:

$$j_i = \pi_{i\mu} A_\mu = Y_{ij} E_j$$

Complex conductivity

$$Y_{ij} = \frac{\pi_{ij}}{i\omega}$$

For the case of a transverse wave propagating along the magnetic field we have:

$$\sigma_{ij} = \sigma^o \delta_{ij} + \varepsilon_{ij} \sigma^H \longrightarrow \sigma_{ij} = \mathrm{Re}(Y_{ij}) = \frac{\mathrm{Im}(\pi_{ij})}{\omega}$$

Previous results

The expression for the off diagonal part of the conductivity is given by:

$$\sigma_{ij} = \frac{\operatorname{Im} r}{\omega}$$

Where the expression for $\operatorname{Im} r$ have been previously obtained for a 3D relativistic electron -positron gas in the presence of a external magnetic field

$\mu \neq 0$ $T \neq 0$

$$\frac{\operatorname{Im} r}{\omega} = \frac{e^3 B}{2\pi^2} \sum_{n,n'} F^{(3)}_{n,n'}(0) \int_{-\infty}^{\infty} dp_3 \frac{(z_1 + 2eB(n+n'))}{|Q|^2} (n_e(\varepsilon_q) - n_p(\varepsilon_q))$$

H. Perez and A. E. Shabad 1982 Ann. Phys 138 1-35

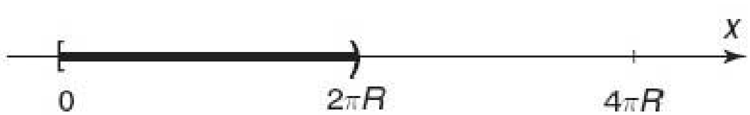
2D massless relativistic limit

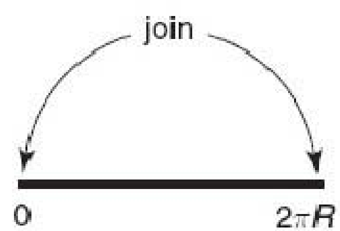
The 2D theory is obtained from the 3D one after a dimensional compactification:

Formally we obtain the 2D quantities from the 3D ones putting

$$k_3 = p_3 = 0$$

and removing from all the integrals $(1/2\pi\hbar)\int dp_3$





2D massless relativistic limit

In the following we were taken: $\nearrow T = 0$, $\searrow \mu > 0$, $m = 0$

$$\sigma_{xy} = \frac{e^3 B}{2\pi^2} \sum_{n,n'} F^{(3)}_{n,n'}(0) \int_{-\infty}^{\infty} dp_3 \frac{(z_1 + 2eB(n+n'))}{|Q|^2} (n_e(\varepsilon_q) - n_p(\varepsilon_q))$$

$\downarrow$

$$\sigma_{xy} = \frac{e^3 B}{\pi} \sum_{n,n'} F^{(3)}_{n,n'}(0) \frac{(z_1 + 2eB(n+n'))}{|Q|^2} \theta(n_e(\varepsilon_{n,o}))$$

2D massless relativistic limit

$$F^{(3)}_{n,n'}(0) = \delta_{n,n'-1} - \delta_{n-1,n'}$$

$$\sigma_{xy} = \frac{e^3 B}{\pi} \sum_{n,n'} F^{(3)}_{n,n'}(0) \frac{(z_1 + 2eB(n+n'))}{|Q|^2} \theta(n_e(\varepsilon_{n,o}))$$

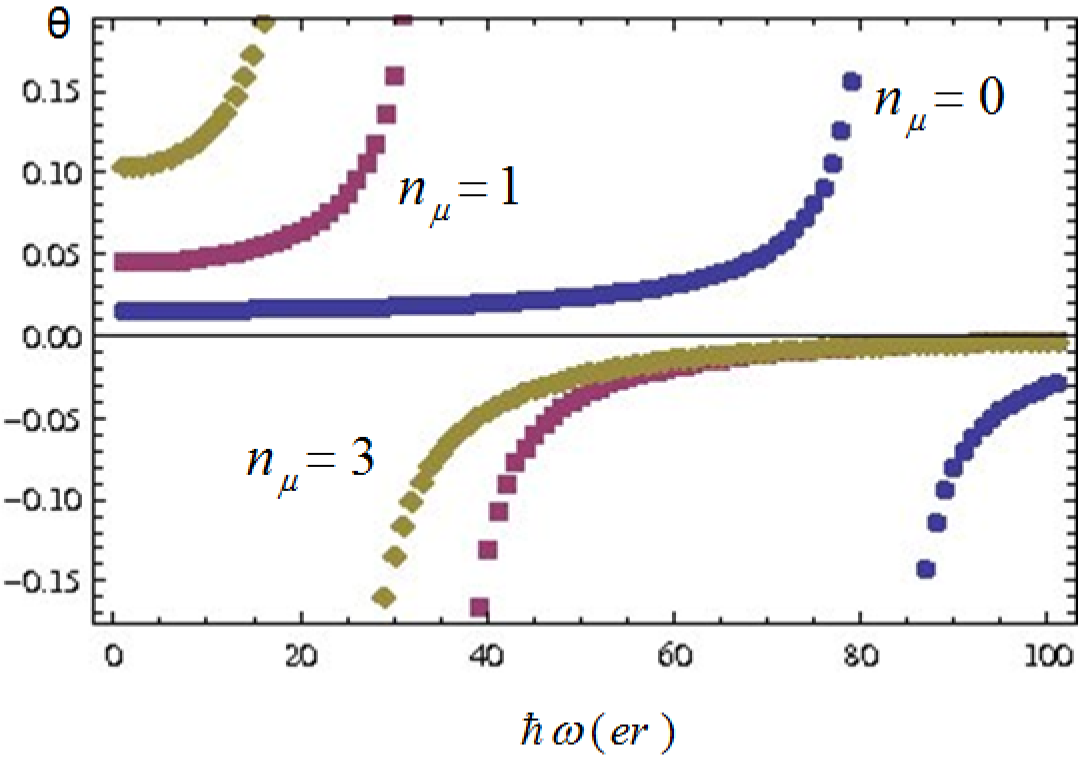
$$|Q|^2 = [-\omega^2 + 2eB(n'-n)]^2 - 4\omega^2 \varepsilon^2_{n,0}$$

$$\varepsilon_{n,0} = \sqrt{2enB}$$

$$\sigma_{xy} = \frac{e^3 B}{\pi} \left(\sum_{n=0}^{n_\mu} \frac{-\omega^2 + 2eB(2n+1)}{(2eB - \omega^2)^2 - 4\omega^2 \varepsilon^2_{n,0}} - \sum_{n=1}^{n_\mu} \frac{-\omega^2 + 2eB(2n-1)}{(2eB + \omega^2)^2 - 4\omega^2 \varepsilon^2_{n,0}} \right)$$

Results

$$\theta \approx -\frac{e^3 B}{\pi}\left(\frac{-\omega^2 + 2eB(2n_\mu + 1)}{(2eB-\omega^2)^2 - 4\omega^2\varepsilon^2_{n_\mu,0}}\right) \longrightarrow n_\mu = I\left[\frac{\mu^2}{2eB}\right]$$



$B = 7 * 10^4\ G$

Summary and future work

Faraday rotation is obtained in the case of an electromagnetic wave which travels along the direction of a constant magnetic field in a relativistic electron-positron gas in the general case of non zero temperature and non vanishing chemical potential.

The 2D relativistic massless limit is derived with the aim to describe graphene-like systems due to its possible applications to test some quantum field theories.

- Now we are looking for some possible astrophysical applications.

Thank you!!!

Gracias!!!