Neutrino emissivity under influence of strong magnetic field and its effects under cooling of neutron stars

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Introduction

- Study the effects of magnetic field in the interior of neutron starts (specific heat, thermal conductive, emissivity);
- Relativistic mean field theory of nuclear matter;
- Cooling of neutron stars by neutrino emission - Direct Urca Process

The relativistic mean field theory of nuclear matter (Walecka model)

The Lagrangian that describes this model is (Glendenning, 1997)

$$\begin{aligned} \mathscr{L} &= \sum_b \bar{\psi}_b \left[i\gamma_\mu D^\mu - m - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho \gamma_\mu \tau \cdot \rho^\mu \right] \psi_b \\ &+ \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ &- \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} - \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu + \sum_{l=e,\mu} \bar{\psi}_l (i\gamma_\mu \partial^\mu - m_l)\psi_l, \end{aligned}$$

where ψ_b is the Dirac spinor for the baryon octet $(n, p, \Lambda, \Sigma, \Xi)$, ψ_l is the field of leptons; σ, ω, ρ are the mesons; g_σ, g_ω and g_ρ are the coupling constants of the nucleons; $D^\mu = \partial^\mu + ieA^\mu$ and $A^0 = 0$, $\vec{A} \equiv (0, xB, 0)$; $U(\sigma) = \frac{1}{3} b m_n (g_{\sigma n}\sigma)^3 + \frac{1}{4} c (g_{\sigma n}\sigma)^4$ is the term of self-interactions.

Equations of motion

The Lagrange-Euler equation is given by

$$\frac{\partial \mathscr{L}}{\partial \phi(x)} - \partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} = 0,$$

where $\phi(x)$ is the field. Then for the Lagrangian above, the equations of motion are (Chakrabarty, 1997)

$$\begin{aligned}
\sum_b \gamma_\mu (i\partial^\mu - g_{\omega b}\omega^\mu) - (m_b - g_{\sigma b}\sigma)\psi_b &= 0, \\
(\partial_\mu \partial^\mu + m_{\sigma n}^2)\sigma(x) + b m_n g_{\sigma n}^3 \sigma^2(x) + c g_{\sigma n}^4 \sigma^3(x) &= \sum_b \bar{\psi}_b g_{\sigma b} \psi_b, \\
(\partial_\mu \partial^\mu + m_{\omega n}^2)\omega(x) - \partial_\nu \partial^\nu \omega_\nu &= \sum_b g_{wb} \bar{\psi}_b \gamma^\mu \psi_b, \\
\sum_l (i\gamma_\mu \partial^\mu - m_l)\psi_l &= 0
\end{aligned}$$

The meson field equations in mean field aproximation are

$$\omega_0 = \left(\frac{g_\omega}{m_\omega}\right)^2 \sum_b \chi_{\omega b}\rho_b,$$

$$\rho_{03} = \left(\frac{g_\rho m}{m_\rho}\right)^2 \sum_b \chi_{\rho b} I_{3b}\rho_b,$$

$$m_n^* = m_n + b m_n \left(\frac{g_{\sigma n}}{m_{\sigma n}}\right)^2 (m_n - m_n^*)^2 + c(m_n - m_n^*)^3 - \sum_b n_s,$$

where ω_0, ρ_{03} are the mean fields of ω-meson and ρ-meson; I_{3b} is the 3-component of the isospin; $m^* = m_n - \chi_{\sigma b} g_{\sigma n}\sigma$ and $\chi_{\sigma b} = \frac{g_{\sigma b}}{g_{\sigma n}}$, $\chi_{\omega b} = \frac{g_{\omega b}}{g_{\omega n}}$, $\chi_{\rho b} = \frac{g_{\rho b}}{g_{\rho n}}$.

Meson field equations

Scalar density

The scalar density is given by

$$n_s = n_s^{q=0} + n_s^{q\neq 0},$$

$$n_s^{q=0} = \frac{2}{(2\pi)^3}\left[\mu_b^* k_b - m_b^{*2} ln\left(\frac{\mu_b^* + k_b}{m_b^*}\right)\right],$$

$$n_s^{q\neq 0} = \frac{m_b^*|q_b|B}{(2\pi)^2}\sum_{\nu=0}^{\nu_{max(b)}} g_\nu ln\left[\frac{\mu_b^* + k_{b,\nu}}{(m_b^{*2} + 2\nu_b|q_b|B)^{1/2}}\right],$$

where q is the charge of the baryon b, μ_b^* is the effective chemical potential, k_b and $k_{b,\nu}$ are the Fermi momentum of the uncharged and charged baryons, $\nu_{max(b)}$ is the maximum Landau number

$\nu_{max(b)}$ and chemical potentials

The Fermi momentum of charged baryons is

$$k^2_{b,\nu_b} = u^{*2}_b + m^{*2}_b - 2\nu_b eB.$$

The upper limit $\nu_{max(b)}$ is defined by the condition $k^2_{b,\nu_b} \geq 0$, then

$$\nu_{max(b)} = \mathrm{int}\left[\frac{\mu^{*2}_b - m^{*2}_b}{2eB}\right].$$

The chemical potentials of uncharged and charged baryons

$$\begin{aligned} \mu_b^{q=0} &= \chi_{\omega b} g_{\omega n}\omega_0 + \chi_{\rho b} g_{\rho n} I_{3b}\rho_{03} + (k_b^2 + m_b^{*2})^{1/2}, \\ \mu_b^{q\neq 0} &= \chi_{\omega b} g_{\omega n}\omega_0 + \chi_{\rho b} g_{\rho n} I_{3b}\rho_{03} + (k_{b,\nu_b}^2 + m_b^{*2})^{1/2} \end{aligned}$$

β-equilibrium condition and baryon density

In the β-equilibrium condition

$$\mu_b = \mu_n + q_b \mu_e,$$

where μ_n and μ_e are the chemical potentials of neutron and electron, respectively. The baryon density is

$$\rho_b^{q=0} = \frac{k_b^3}{3\pi^2},$$

$$\rho_b^{q\neq 0} = \frac{eB}{2\pi^2} \sum_{\nu=0}^{\nu_{max(b)}} g_\nu k_{b,\nu(b)}$$

and to leptons (electrons and muons)

$$\rho_l = \frac{eB}{2\pi^2} \sum_{\nu=0}^{\nu_{max(l)}} g_\nu k_{l,\nu_l},$$

where g_ν =1 ($\nu = 0$) and g_ν =2 ($\nu > 0$) is the spin degeneracy.

$\nu_{max(l)}$ and conservation equations

The Fermi momentum of leptons is

$$k_{l,\nu_l}^2 = u_l^2 + m_l^2 - 2\nu_l |q_l| B$$

The upper limit $\nu_{max(l)}$ is defined by the condition $k_{l,\nu_l}^2 \geq 0$, then

$$\nu_{max(l)} = \mathrm{int}\left[\frac{\mu_l^2 - m_l^2}{2|q_l|B}\right],$$

Conservation of the quantity of baryons and neutrality of electrical charge

$$\rho = \sum_{b=1}^{8} \rho_b,$$

$$\sum_{b=1}^{8} q_b \rho_b - \sum_{l=e,\mu} \rho_l = 0$$

Energy density

The energy density total of the system is

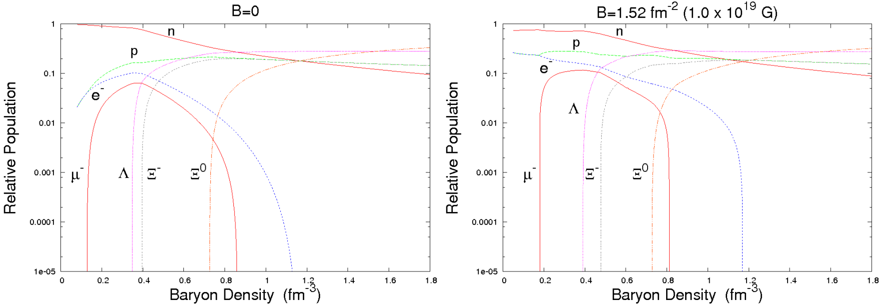
$$
\begin{aligned}
\varepsilon &= \frac{1}{3} b m_n (g_{\sigma n}\sigma)^3 + \frac{1}{4} c (g_{\sigma n}\sigma)^4 + \frac{1}{2}\left(\frac{g_{\sigma n}}{m_{\omega n}}\right)^2 (g_{\omega n}\omega_0)^2 \\
&+ \frac{1}{2}\left(\frac{g_{\rho n}}{m_{\rho n}}\right)^2 (g_{\rho_n}\rho_{03})^2 \\
&+ \sum_{b(q=0)}^{8} \frac{1}{8\pi^2}\left[2{\mu_b^*}^3 k_b - {m_b^*}^2 \mu_b^* k_b - {m_b^*}^4 \ln\left\{\frac{\mu_b^* + k_b^{1/2}}{m_b^*}\right\}\right] \\
&+ \frac{qB_m}{4\pi^2} \sum_{b(q\neq 0)}^{8} \sum_{\nu_b=0}^{\nu_{\max(b)}} g_\nu \left[\mu_b^* k_{b,\nu} + m^{*2}_{b,\nu_b} \ln\left\{\frac{\mu_b^* + k_{\nu_b}^{1/2}}{m^*_{\nu_b}}\right\}\right] \\
&+ \frac{qB_m}{4\pi^2} \sum_{l=e,l} \sum_{\nu_l=0}^{\nu_{\max(l)}} g_\nu \left[\mu_l k_{\nu_l} + m^2_{l,\nu_l} \ln\left\{\frac{\mu_l + k_{l,\nu_l}}{m_{l,\nu_l}}\right\}\right] + \frac{B^2}{8\pi}
\end{aligned}
$$

Numerical calculation

- The system of coupled nonlinear equations that describes the matter is solves numerically by iteration;
- Newton-Raphson method with global search of the solution;
- As output of the numerical calculations, we obtain the relative population of each species of particles as function of baryon density and energy density;
- We use the coupling constants given by NR model (Chiapparini et al. 2009)

Results

Relative Population



- The particle fractions in cold β-equilibrated neutron star with ($B = 1.0 \times 10^{19}$ G) (right panel) and without magnetic field (left panel)

- Direct Urca process

$$\begin{aligned} n &\rightarrow p + e^- + \bar{\nu}_e, \\ p + e^- &\rightarrow n + \nu_e \end{aligned}$$

- $k_{Fn} \leq k_{Fp} + k_{Fe}$, $k_{F\alpha} = (3\pi^2 n_\alpha)^{1/3}$;
- $p_n = p_p + p_e + p_\nu$;
- Strong magnetic fields lead to an increase of the proton fraction

- Weinberg-Salam theory for weak interactions, the interaction Lagrangian is given by

$$\mathscr{L}_{\text{weak}} = \frac{G_F}{\sqrt{2}} \cos\theta_c l_\mu j^\mu ,$$

where G_F is the Fermi Weak coupling constant, θ_c is the Cabibbo angle. Lepton and nucleon charged weak currents are

$$\begin{aligned} l_\mu &= \bar{\psi}_4 \gamma_\mu (1-\gamma_5) \psi_2 , \\ j^\mu &= \bar{\psi}_2 \gamma^\mu (g_V - g_A \gamma_5) \psi_1 , \end{aligned}$$

g_V and g_A are vector and axial-vector coupling constants and the indices $i = 1-4$ refer to the $n, p, e, \bar{\nu}_e$, respectively;

Emissivity of neutrinos
Direct Urca process

The emissivity due to the antineutrino emission process in presence of a uniform magnetic field B_m along z-axis

$$
\begin{aligned}
\varepsilon_\nu &= 2\int \frac{Vd^3p_1}{2\pi^3}\int \frac{Vd^3p_2}{2\pi^3}\int_{-qB_mL_x/2}^{qB_mL_x/2}\frac{L_y dp_{3y}}{2\pi}\int_{-PF_p}^{PF_p}\frac{L_z dp_{3z}}{2\pi} \\
&\times \int_{-qB_mL_x/2}^{qB_mL_x/2}\frac{L_y dp_{4y}}{2\pi^3}\int_{-PF_p}^{PF_p}\frac{L_z dp_{4z}}{2\pi} \\
&\times \sum_{\nu_e=0}^{\nu_{max}}\sum_{\nu_p=0}^{\nu_{max}} E_2 W_{fi} f(\vec{p}_1)[1-f(\vec{p_3})][1-f(\vec{p_4})]
\end{aligned}
$$

- The prefactor 2 takes into account the neutron spin degeneracy

The transition rate per unity volume W_{fi} is

$$W_{fi} = \frac{< |M_{fi}|^2 >}{tV},$$

The matrix element M_{fi} for the V-A interaction is given by

$$M_{fi} = \frac{G_F}{\sqrt{2}} \int d^4X \bar{\psi}_1(X)\gamma^{\mu}(g_V - g_A\gamma_5)\psi_3(X)\bar{\psi}_2(X)\gamma_5(1-\gamma_5)\psi_4(X)$$

where

- $< . >$ denotes an averaging over initial spin of n and a sum over spins of final particles (p, e);
- $\psi_3(X) = (1/\sqrt{L_y L_z}) exp(-iE_3 t + ip_{3y} y + ip_{3z} z)$ $\times f_{p_{3y}p_{3z}}(x)$;
- $f_{p_{3y}p_{3z}}(x)$ is the 4-component spinor solution

- The only positive energy spinor for protons in the chiral representation is then

$$f^{\nu=0}_{p_{3y};p_{3z}}(x) = N_{\nu=0} \begin{pmatrix} E^*_3 + p_{3z} \\ 0 \\ -m^* \\ 0 \end{pmatrix} I_{\nu=0;p_{3y}}(x)$$

where

- $N_{\nu=0} = 1/\sqrt{2E^*_3(E^*_3 + p_{3z})}$;
- $E^*_3 = (p^2_{3z} + m^{*2})^{1/2}$;
- $I_{\nu=0;p_{3y}} = (\frac{eH}{\pi})^{1/4} exp[-\frac{1}{2}eH(x - \frac{k_y}{eH})^2]\frac{1}{\sqrt{n!}}$ $\times H_n[\sqrt{2eH}(x - \frac{k_y}{eH})]$;

The transition rate per unity volume is

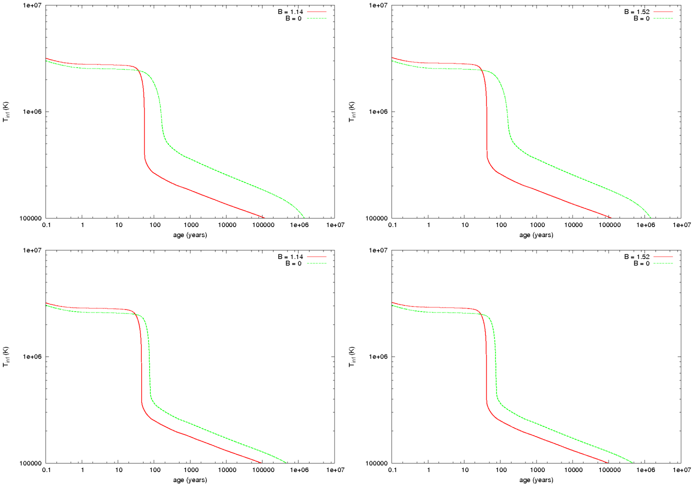
$$
\begin{aligned}
W_{fi} &= \frac{{G_F}^2}{{E_1}^* E_2 {E_3}^* E_4} \frac{1}{V^3 L_y Lz} \\
&\times\ exp\left(\frac{(p_{1x}-p_{2x})^2+(p_{3y}-p_{4y})^2}{2qB_m}\right) \\
&\times\ [(g_v+g_a)^2(p_1.p_2)(p_3.p_4)+(g_v-g_a)^2(p_1.p_4)(p_3.p_2) \\
&-\ ({g_V}^2-{g_A}^2)m^{*2}(p_4.p_2)](2\pi)^3\delta(E_1-E_2-E_3-E_4) \\
&\times\ \delta(p_{1y}-p_{2y}-p_{3y}-p_{4y})\delta(p_{1z}-p_{2z}-p_{3z}-p_{4z})
\end{aligned}
$$

- Then, the emissivity is

$$\begin{aligned}
\varepsilon_{\mathrm{URCA}} &= \frac{457\pi}{5040} G_F^2 \cos^2\theta_c (qB_m)[(g_v+g_a)^2(1-\frac{p_{F_p}}{\mu_p^*}) \\
&+ (g_v-g_a)^2(1-\frac{p_{F_n}}{\mu_n^*}\cos\theta_{14})-({g_v}^2-{g_A}^2)\frac{m^{*2}}{\mu_p^*\mu_n^*}] \\
&\times exp\left[\frac{(p_{F_p}+p_{F_e})^2-p_{F_n}^2}{2qB_m}\right]\frac{\mu_n^*\mu_p^*\mu_e}{p_{F_n}p_{F_p}} T^6\Theta,
\end{aligned}$$

where $\cos_{14} = (p_{F_n}^2 + p_{F_e}^2 - p_{F_n}^2)/2p_{F_n}p_{F_e}$,
$\Theta = \theta(p_{F_p} + p_{F_e} - p_{F_n})$ is the threshold factor, where $\theta(x) = 1$ for $x > 0$ and zero otherwise

Results
Cooling of neutron star by neutrino emission



Top panel: $M = 1.4M_\odot$ and bottom panel: $M = 1.6M_\odot$

Thank You!!

References

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