IoP joint HEPP and APP meeting 2012

Queen Mary
University of London

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Flavour and neutrino theory

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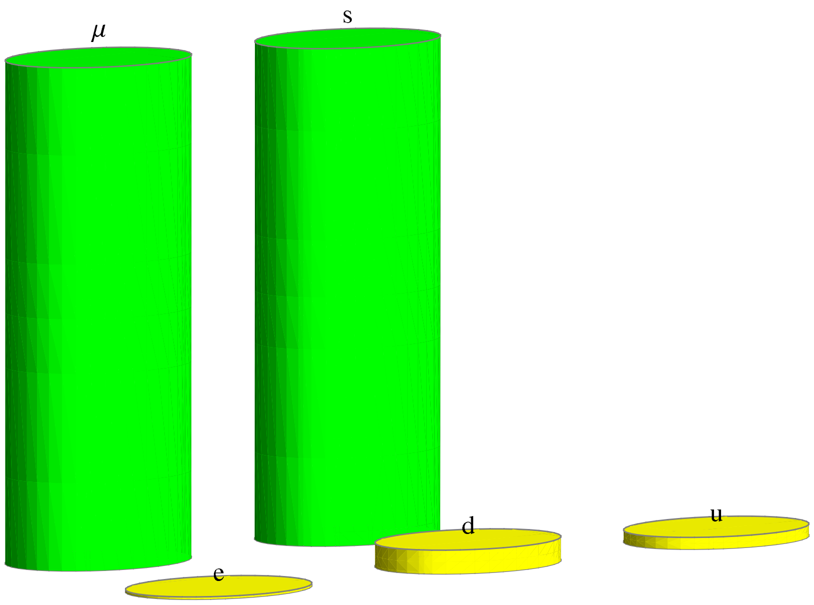
IP3

Outline

- basic facts about quarks and leptons
- what is the organising principle
  - hierarchy of masses
  - unification of quarks and leptons
  - neutrino masses and mixing pattern
- family symmetries – Abelian and non-Abelian
- tri-bimaximal neutrino mixing (until recently)
- latest twist: observation of $\theta_{13} \neq 0$
- strategies for implementing sizable reactor angle
- cosmological implications of family symmetries

~ 1970



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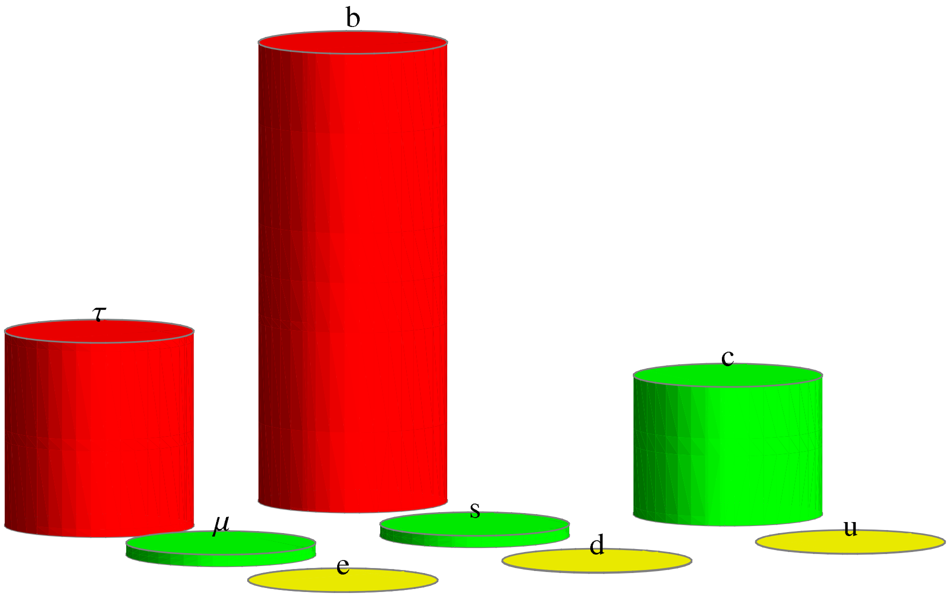
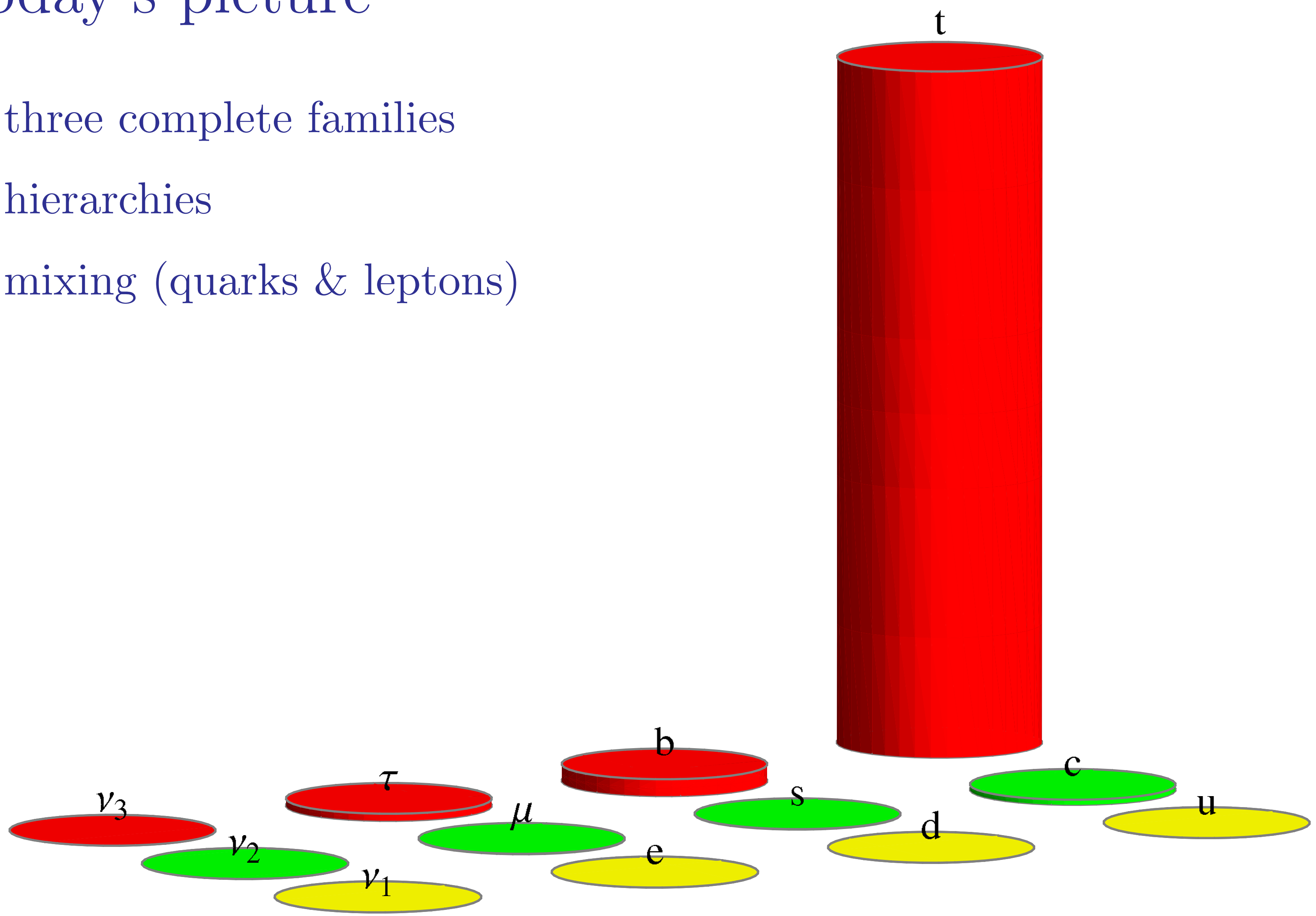


today's picture

three complete families

hierarchies

mixing (quarks & leptons)



Fermion masses and mixings

quarks

$$m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$$

$$m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1$$

$$\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim 0.22$$

$\Rightarrow$ hierarchical quark sector

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leptons

$$m_e : m_\mu : m_\tau \sim \lambda^{4\,\text{or}\,5} : \lambda^2 : 1$$

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \begin{cases} \lambda^{\geq 1} : \lambda : 1 \\ 1 : 1 : \lambda^{\geq 1} \\ 1 : 1 : 1 \end{cases}$$

$$\begin{pmatrix} 0.77-0.86 & 0.50-0.63 & 0.00-0.22 \\ 0.22-0.56 & 0.44-0.73 & 0.57-0.80 \\ 0.21-0.55 & 0.40-0.71 & 0.59-0.82 \end{pmatrix}$$

Gonzalez-Garcia,Maltoni (2007)

$\Rightarrow$ neutrino sector is special

Aspects of the flavour puzzle

(a) hierarchy in the quark sector

(b) unification of quarks and leptons

(c) clues form the neutrino sector

- tiny mass $\rightarrow$ seesaw
- peculiar mixing $\rightarrow$ tri-bimaximal pattern
- non-Abelian family symmetry

(a) Hierarchy in the quark sector

Froggatt-Nielsen mechanism

- introduce an *Abelian* family symmetry
- assign generation dependent $U(1)$ charges to the quarks
- $U(1)$ symmetry forbids most of the Yukawa terms

- introduce flavon field ϕ which allows Yukawa-like terms at higher order
- flavon ϕ acquires a VEV $\rightarrow$ Yukawa terms are generated

	Q_1	Q_2	Q_3	D_1^c	D_2^c	D_3^c	H_u	ϕ
$U(1)$	3	2	0	1	0	0	0	-1

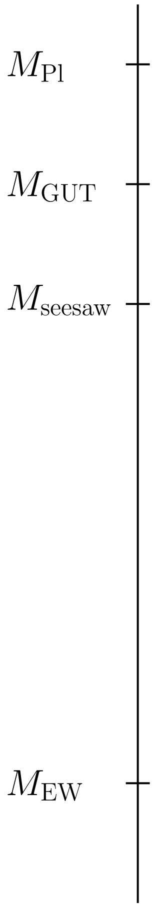
$$\frac{\langle\phi\rangle}{\Lambda} \sim \lambda$$

$$c_{ij}\, Q_i\, D_j^c\, H_u \left(\frac{\phi}{\Lambda}\right)^{x_{ij}} \rightarrow \quad c_{ij}\, Q_i\, D_j^c\, H_u\, \lambda^{x_{ij}} \quad \rightarrow \quad Y_d \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}$$

- hierarchies arise from spontaneous breakdown of $U(1)$ family symmetry
- drawback: $\mathcal{O}(1)$ coefficients c_{ij} not fixed

Scale of flavour physics

- models of flavour are typically formulated at high energies
- separate family symmetry from EW symmetry breaking
- renormalisation group running can modify high-energy prediction



flavon VEV breaks family symmetry
$\rightarrow$ Yukawas generated

Higgs VEV breaks electroweak symmetry
$\rightarrow$ fermion masses generated

(b) Unification of quarks and leptons

Relating Y_d and Y_ℓ in $SU(5)$

- fermionic $\mathbf{10}$ contains $Q\,, E^c, U^c$
- fermionic $\overline{\mathbf{5}}$ contains D^c, L
- down-type and charged lepton Yukawa terms originate in $\mathbf{10}_i \; Y_{ij} \; \overline{\mathbf{5}}_j \; \overline{\mathbf{5}}_{\mathbf{H}}$

$$Y_d \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ * & \lambda^2 & \lambda^2 \\ * & * & 1 \end{pmatrix} \quad \rightarrow \quad Y_\ell = Y_d^T \sim \begin{pmatrix} \lambda^4 & * & * \\ \lambda^3 & \lambda^2 & * \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

- bottom-tau unification at GUT scale
- mass ratios $m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1 \sim m_e : m_\mu : m_\tau$

- left-handed mixing determined by upper-right off-diagonals
- down and charged lepton mixing can be related for (partially) symmetric Y
- indeed Gatto-Sartori-Tonin relation $\theta_{12}^d \sim \sqrt{\frac{m_d}{m_s}} \sim \lambda$ suggests

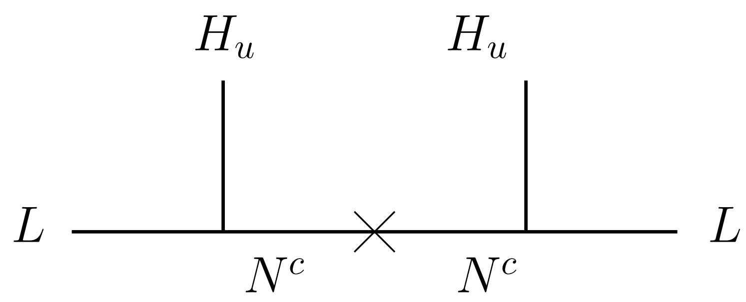
$$Y_{11} = 0 \quad \& \quad Y_{12} = Y_{21}$$

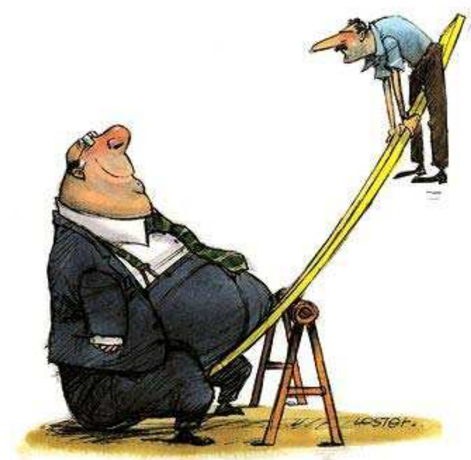
(c) Clues from the neutrino sector

The origin of neutrino mass

- neutrinos are much lighter than the other SM fermions
- suggestive of a different mass generation mechanism $\rightarrow$ seesaw
- introduce right-handed (Majorana) neutrinos N^c

Dirac term	Majorana term
$Y_\nu^{ij} L_i N_j^c H_u$	$M_R^{ij} N_i^c N_j^c$





- effective light neutrino mass matrix

$$\boxed{m_\nu^{\text{eff}} = Y_\nu \frac{v_u^2}{M_R} Y_\nu^T}$$

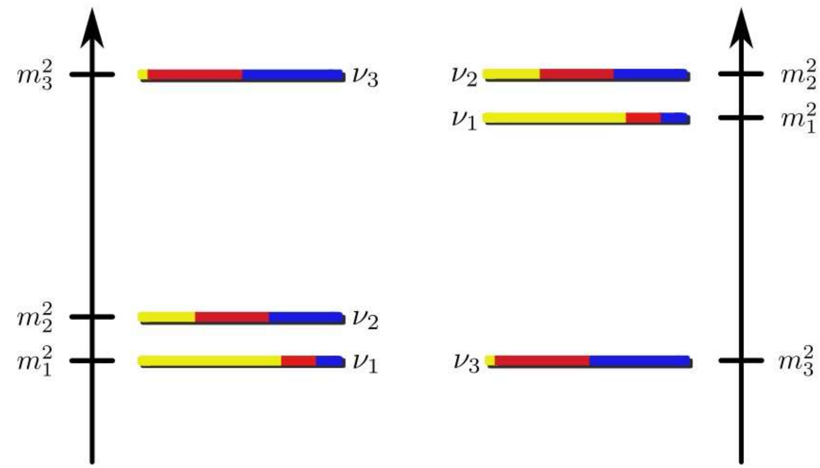
- $M_R \sim 10^{14}\,\text{GeV}$ gives viable light neutrino masses

Three neutrino flavour mixing

(in diagonal charged lepton basis)

flavour PMNS mixing mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



atmospheric reactor + Dirac solar Majorana

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{\frac{\alpha_3}{2}} \end{pmatrix}$$

Tri-bimaximal lepton mixing vs. global neutrino fits

$$U_{\rm PMNS} \approx U_{\rm TB} \equiv \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$



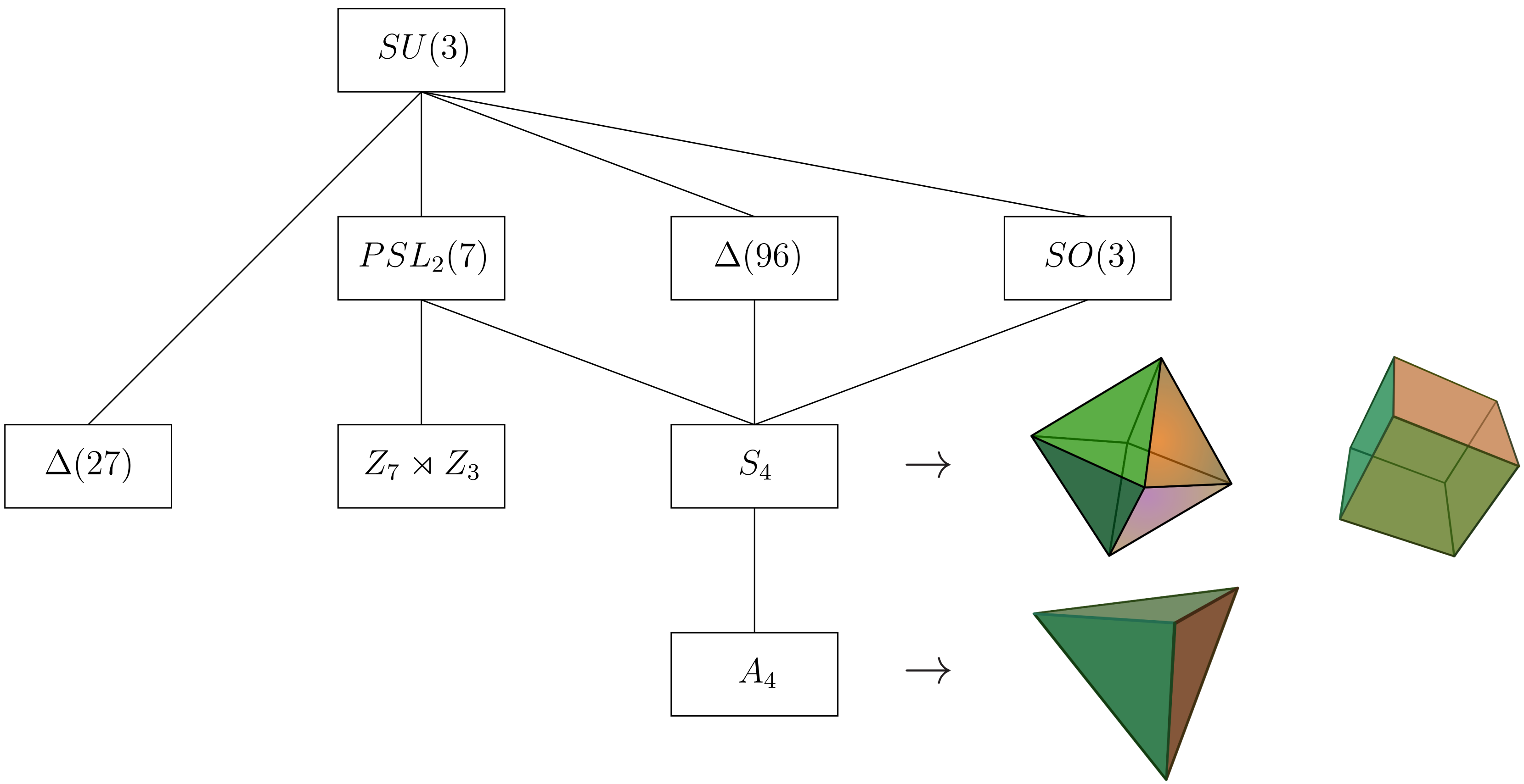
$\Rightarrow$

PMNS-angles	tri-bimax.	1σ exp.	1σ exp.
$\sin^2\theta_{12}$:	$\frac{1}{3}$	$0.297 - 0.329$	$0.296 - 0.329$
$\sin^2\theta_{23}$:	$\frac{1}{2}$	$0.45 - 0.58$	$0.39 - 0.50$
$\sin^2\theta_{13}$:	0	$0.008 - 0.020$	$0.018 - 0.032$
		Schwetz et al. (2011)	Fogli et al. (2011)

- tri-bimaximal mixing fits rather well
- motivates an underlying family symmetry, e.g. A_4, S_4

Non-Abelian family symmetries

- unify three families in multiplets of family symmetry
- group should have two- or three-dimensional representations



Symmetries of the mass matrices

charged leptons $M_\ell \;=\; \mathrm{diag}\,(m_e, m_\mu, m_\tau)$

symmetric under diagonal phase transformation h

$$\boxed{M_\ell \;=\; h^T M_\ell\, h^*} \qquad \text{e.g.} \quad h = \mathrm{diag}\,(1, e^{\frac{4\pi i}{3}}, e^{\frac{2\pi i}{3}})$$

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neutrinos $M_\nu \;=\; U_{\mathrm{PMNS}}\, \mathrm{diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\, U_{\mathrm{PMNS}}^T$

symmetry of M_ν depends on U_{PMNS}

$$\boxed{M_\nu \;=\; k^T M_\nu\, k} \qquad k = U_{\mathrm{PMNS}}^* \,\mathrm{diag}\,(\pm 1, \pm 1, \pm 1)\, U_{\mathrm{PMNS}}^T$$

require $\det k = 1$

four different k $\;\rightarrow\;$ generate $Z_2 \times Z_2$ symmetry group

Klein symmetry $\mathcal{K} = \{1, k_1, k_2, k_3\}$

for $U_{\mathrm{PMNS}} = U_{\mathrm{TB}}$:

$$k_1 \;=\; \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad k_2 \;=\; -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad k_3 \;=\; k_1 k_2$$

Origin of the Klein symmetry

- direct models
  - Klein symmetry $\mathcal{K} \;\subset\;$ family symmetry $\mathcal{G}$
  - flavon fields ϕ break $\mathcal{G}$ down to $\mathcal{K}$ in neutrino sector
  - for TB mixing (k_1, k_2, h) generate S_4
- indirect models
  - Klein symmetry $\mathcal{K}$ not necessarily $\;\subset\;$ family symmetry $\mathcal{G}$
  - $\mathcal{G}$ responsible for generating particular flavon VEV configurations
  - for TB mixing – from e.g. $\Delta(27)$, $Z_7 \rtimes Z_3$

$$\langle\phi_1\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_2\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_3\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{L}_\nu \quad \sim \quad \nu\,(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\,\nu\; H\, H$$

What if tri-bimaximal mixing is ruled out?

2011/2012 indications of a non-zero θ_{13}

T2K [arXiv:1106.2822]

- $\theta_{13} \neq 0$ disfavoured at $\sim 2.5\sigma$
- $5° \lesssim \theta_{13} \lesssim 18°$ at 90% C.L.

MINOS [arXiv:1108.0015]

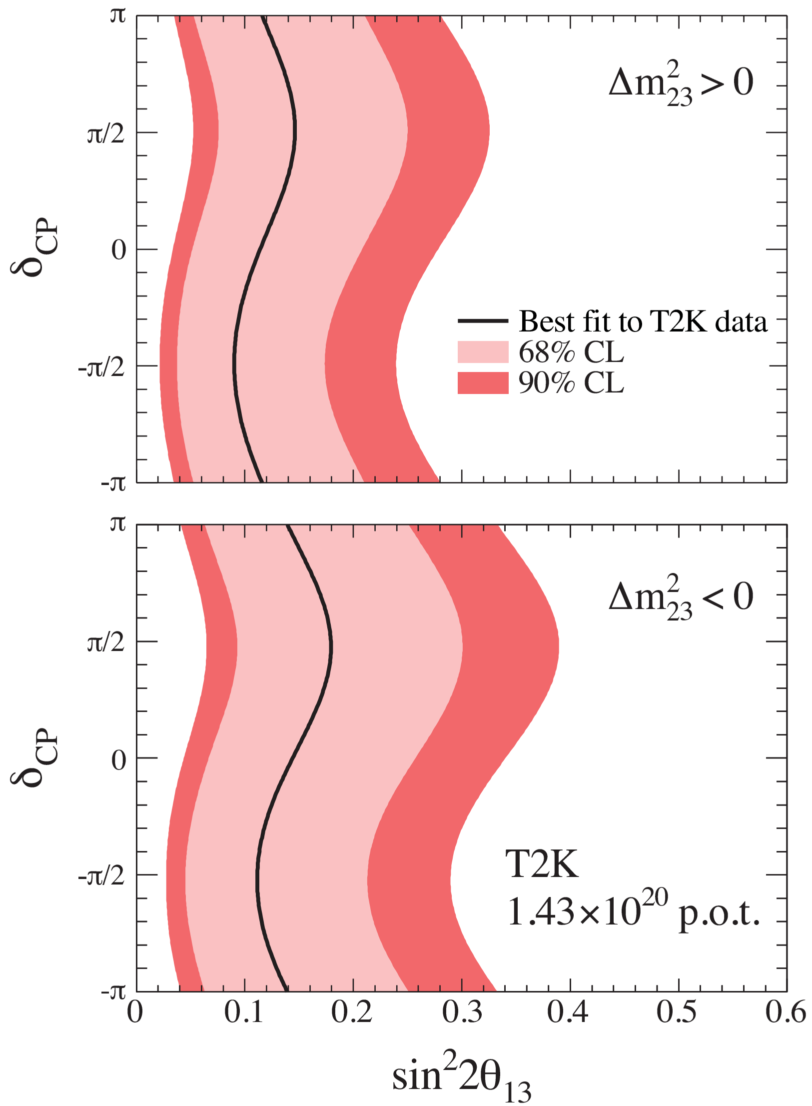
- $\theta_{13} \neq 0$ disfavoured at $\sim 1.6\sigma$
- $\theta_{13} \lesssim 13°$ at 90% C.L.

Double Chooz [arXiv:1112.6353]

- $\theta_{13} \neq 0$ disfavoured at $\sim 2\sigma$
- $4° \lesssim \theta_{13} \lesssim 12°$ at 90% C.L.

Daya Bay [arXiv:1203.1669]

- $\theta_{13} \neq 0$ disfavoured at $\sim 5.2\sigma$
- $7° \lesssim \theta_{13} \lesssim 10°$ at 90% C.L.



Family symmetry models and sizable θ_{13}

direct models

TB plus corrections

other family symmetries
with non-standard $\mathcal{K}$

indirect models

TB plus corrections

non-standard flavon
VEV configurations

A direct S_4 model of leptons

- S_4 representations: $\mathbf{1} \quad \mathbf{1'} \quad \mathbf{2} \quad \mathbf{3} \quad \mathbf{3'}$
- diagonal charged leptons
- in neutrino sector $\quad (L \sim N^c \sim \mathbf{3} \qquad H_u \sim \mathbf{1})$

$$\boxed{W_\nu \;\sim\; L N^c H_u + (\phi_{\mathbf{3'}} + \phi_{\mathbf{2}} + \phi_{\mathbf{1}}) N^c N^c + \frac{1}{M} \widetilde{\phi}_{\mathbf{1'}} \phi_{\mathbf{2}} N^c N^c}$$

- trivial Dirac neutrino Yukawa
- neutrino mixing governed by heavy right-handed neutrinos
- S_4 multiplication rule

$$\mathbf{3} \otimes \mathbf{3} = (\mathbf{3'} + \mathbf{2} + \mathbf{1})_s + \mathbf{3}_a$$

- three TB conserving flavons $\quad \phi_{\mathbf{3'}} \quad \phi_{\mathbf{2}} \quad \phi_{\mathbf{1}}$
- add new flavon $\widetilde{\phi}_{\mathbf{1'}} \sim \mathbf{1'}$ (otherwise neutral)
- resulting higher order term breaks TB structure

Breaking of the TB Klein symmetry $\mathcal{K}$

Dirac term LN^cH_u respects $\mathcal{K} \subset S_4$

Majorana terms $\left(\phi_{\mathbf{3'}} + \phi_{\mathbf{2}} + \phi_{\mathbf{1}} + \frac{1}{M}\widetilde{\phi}_{\mathbf{1'}}\phi_{\mathbf{2}}\right) N^cN^c$ respect k_1 but break k_2

S_4 irrep	k_1	k_2	VEV alignment
$\mathbf{3'}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\langle \phi_{\mathbf{3'}} \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\langle \phi_{\mathbf{2}} \rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{1}$	1	1	$\langle \phi_{\mathbf{1}} \rangle \propto 1$
$\mathbf{1'}$	1	-1	$\langle \widetilde{\phi}_{\mathbf{1'}} \rangle \propto 1$

Resulting mixing

$$M_R = \frac{M_1+M_3}{6}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \frac{2M_2+M_3-M_1}{6}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \frac{M_1+M_2-M_3}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$+ \Delta \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad \leftarrow \quad \text{small TB breaking term}$$

$$\Longrightarrow \quad \boxed{U_{\text{PMNS}} \approx \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \end{pmatrix}}$$

$$\text{Re}\,\alpha \approx -\sqrt{3}\cdot\left[\text{Re}\left(\frac{\Delta}{M_1-M_3}\right) + \text{Im}\left(\frac{\Delta}{M_1-M_3}\right)\frac{\text{Im}\left(\frac{M_1+M_3}{M_1-M_3}\right)}{\text{Re}\left(\frac{M_1+M_3}{M_1-M_3}\right)}\right]$$

$$\text{Im}\,\alpha \approx \sqrt{3}\cdot\frac{\text{Im}\left(\frac{\Delta}{M_1-M_3}\right)}{\text{Re}\left(\frac{M_1+M_3}{M_1-M_3}\right)}$$

Trimaximal neutrino mixing

- second column of $U_{\text{PMNS}} \;\propto\; (1,1,1)^T$
- one could have guessed this special structure

  (i) $(1,1,1)^T$ is an eigenvector of M_R

  (ii) k_1 generator of TB Klein symmetry $\mathcal{K}$ unbroken
- such a TB breaking affects θ_{13} and θ_{23} – but not θ_{12}
- get correlations between TB deviation parameters r, a, s

$$\sin\theta_{13} \equiv \frac{1}{\sqrt{2}}\, r \qquad \sin\theta_{23} \equiv \frac{1}{\sqrt{2}}(1+a) \qquad \sin\theta_{12} \equiv \frac{1}{\sqrt{3}}(1+s)$$

$$r\cos\delta \;\approx\; -\frac{2}{\sqrt{3}}\,\mathrm{Re}\;\alpha \qquad a \;\approx\; \frac{1}{\sqrt{3}}\,\mathrm{Re}\;\alpha \qquad \delta \;\approx\; \pi + \arg\;\alpha$$

$\rightarrow$ testable sum rules $\boxed{a \approx -\frac{1}{2}\, r\cos\delta \qquad s \;\approx\; 0}$

Cosmological implication

Baryogenesis via leptogenesis

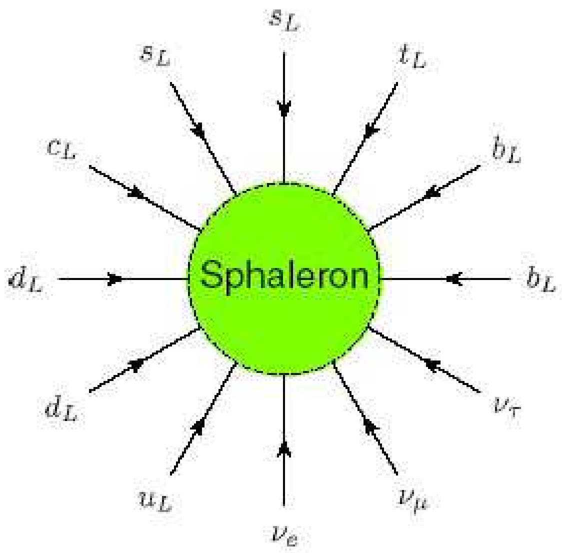
- origin of tiny matter-antimatter asymmetry $\frac{n_B}{n_\gamma} \approx 6.2 \times 10^{-10}$
- right-handed neutrinos N^c decay out of equilibrium

$$N_i^c \xrightarrow{\Gamma_i} L + H_u \qquad\qquad N_i^c \xrightarrow{\overline{\Gamma}_i} \overline{L} + \overline{H}_u$$

- generate net lepton number L
- non-perturbative sphaleron transitions

lepton number $L \rightarrow$ baryon number B

$$\prod_{k=1}^{3} Q_k Q_k Q_k L_k$$



Leptogenesis in family symmetry models

- generation of a net lepton number from N^c decay requires CP asymmetry

$$\epsilon_i = \frac{\Gamma_i - \overline{\Gamma}_i}{\Gamma_i + \overline{\Gamma}_i} \approx \frac{1}{8\pi (Y_\nu^\dagger Y_\nu)_{ii}} \sum_{j \neq i} \mathrm{Im}\left[(Y_\nu^\dagger Y_\nu)^2_{ij} \right] f\left(\frac{M_j^2}{M_i^2}\right)$$

- here Y_ν in basis of diagonal right-handed neutrinos
- often PMNS mixing purely due to M_R (trivial Dirac neutrino Yukawa)
- then $Y_\nu = U_{\mathrm{PMNS}}$ in basis with diagonalised M_R
- $(Y_\nu^\dagger Y_\nu)_{ij} = \delta_{ij} \;\rightarrow\;$ CP asymmetry ϵ_i vanishes
- general criterion: form dominance Chen,King (2009)

  columns of Y_ν proportional to columns of U_{PMNS}
- RG running of Y_ν gives rise to violation of form dominance
- successful leptogenesis possible Cooper,King,Luhn (2011)

Conclusion

- fermion masses and mixings suggest underlying family symmetry
  - Froggatt-Nielsen $U(1)$
  - non-Abelian symmetries like e.g. S_4
- experimental evidence for sizable θ_{13}
- review role of family symmetries
- necessary to discuss deviations from TB mixing
- testable mixing sum rules
- family symmetries compatible with leptogenesis

Thank you

(for staying until the very end of the meeting)