3-flavour neutrino oscillations

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ν_μ disappearance

We know from several experiments, e.g. MINOS, T2K that some ν_μ change in such a way that they are no longer detected after travelling a few hundred kilometres - this is known as ν_μ disappearance.

There are good theoretical reasons to suppose that this is due to $\nu_\mu \rightarrow \nu_\tau$ oscillations, though this has not yet been confirmed experimentally.

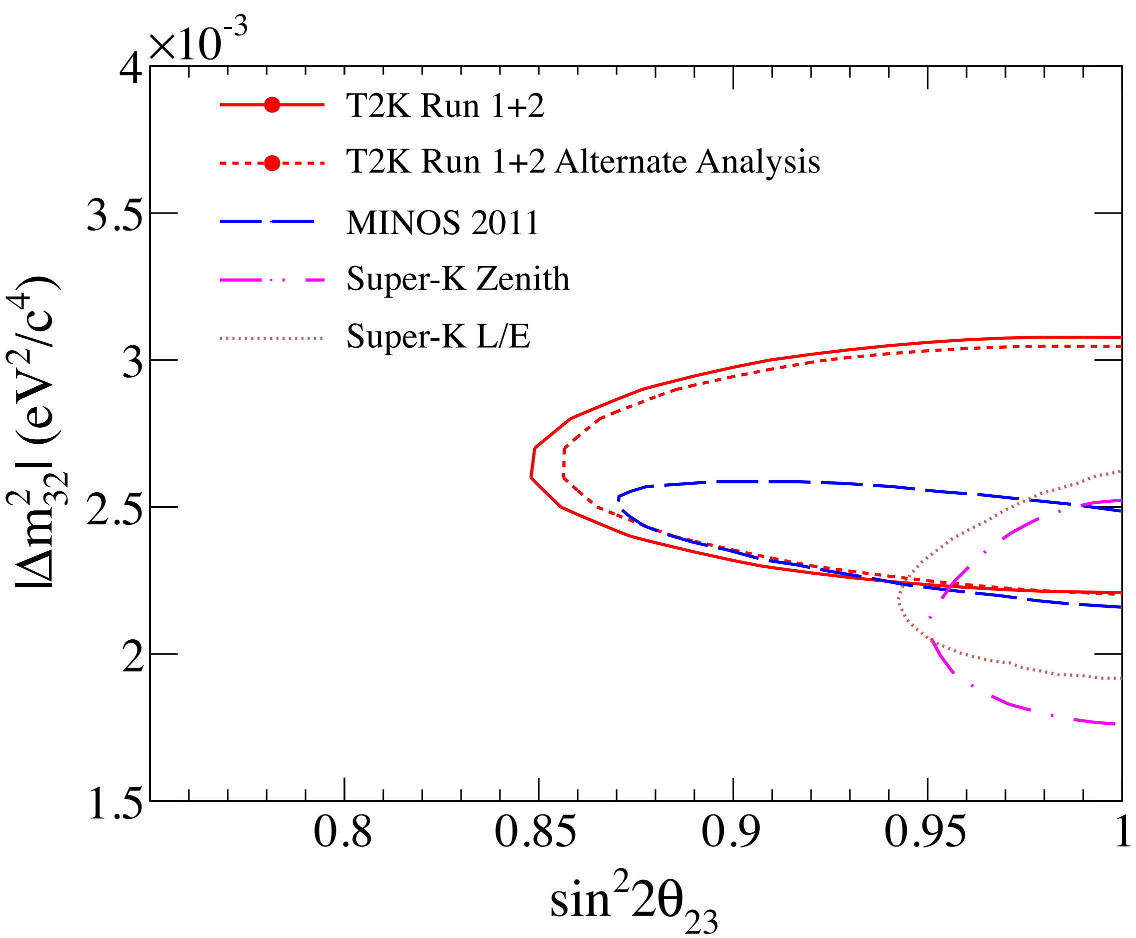
Neutrino experiments have mainly calculated the ν_μ-survival probability assuming that there are only two neutrino flavours (ν_μ and ν_τ):

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta_{23}) \sin^2(1.267\ \Delta m_{32}^2\ L / E)$$

T2K 2-flavour ν_μ-disappearance results

$$2\sum_{E_r}\left[N_{SK}^{data}\ln\left(\frac{N_{SK}^{data}}{N_{SK}^{exp}}\right)+(N_{SK}^{exp}-N_{SK}^{data})\right] \quad (1)$$

$$L = L_{\text{norm}}(\sin^2(2\theta_{23}),\Delta m^2_{32},\mathbf{f}) L_{\text{shape}}(\sin^2(2\theta_{23}),\Delta m^2_{32},\mathbf{f})L_{\text{syst}}(\mathbf{f}) \quad (2)$$



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T2K carried out two analyses to find the best-fit values of $\sin^2(2\theta_{23})$ and $|\Delta m_{32}^2|$. One analysis used a binned likelihood-ratio method and minimised equation 1.

The alternate analysis used an unbinned maximum-likelihood method in which the likelihood was equation 2.

We expected 104 ± 14 single μ-like ring events in Super Kamiokande for the no-disappearance hypothesis, but observed 31 events. The best-fit values for 2-flavour $\nu_\mu \rightarrow \nu_\tau$ oscillations are

$\sin^2(2\theta_{23}) = 0.98$

$|\Delta m_{32}^2| = 2.65 \times 10^{-3}$ eV^2.

In both analyses, the 90% confidence regions are found using the Feldman-Cousins unified method.

ν_μ disappearance

However this 2-flavour analysis is only an approximation to the exact ν_μ-survival probability (in vacuum) if all three flavours are considered:

$$P(\nu_\mu \to \nu_\mu) = 1 - 4 \sum_{i(>j)} \sum_j |U_{\mu i}|^2 |U_{\mu j}|^2 \sin^2\left(\frac{1.27 \Delta m^2_{ij} L}{E}\right)$$

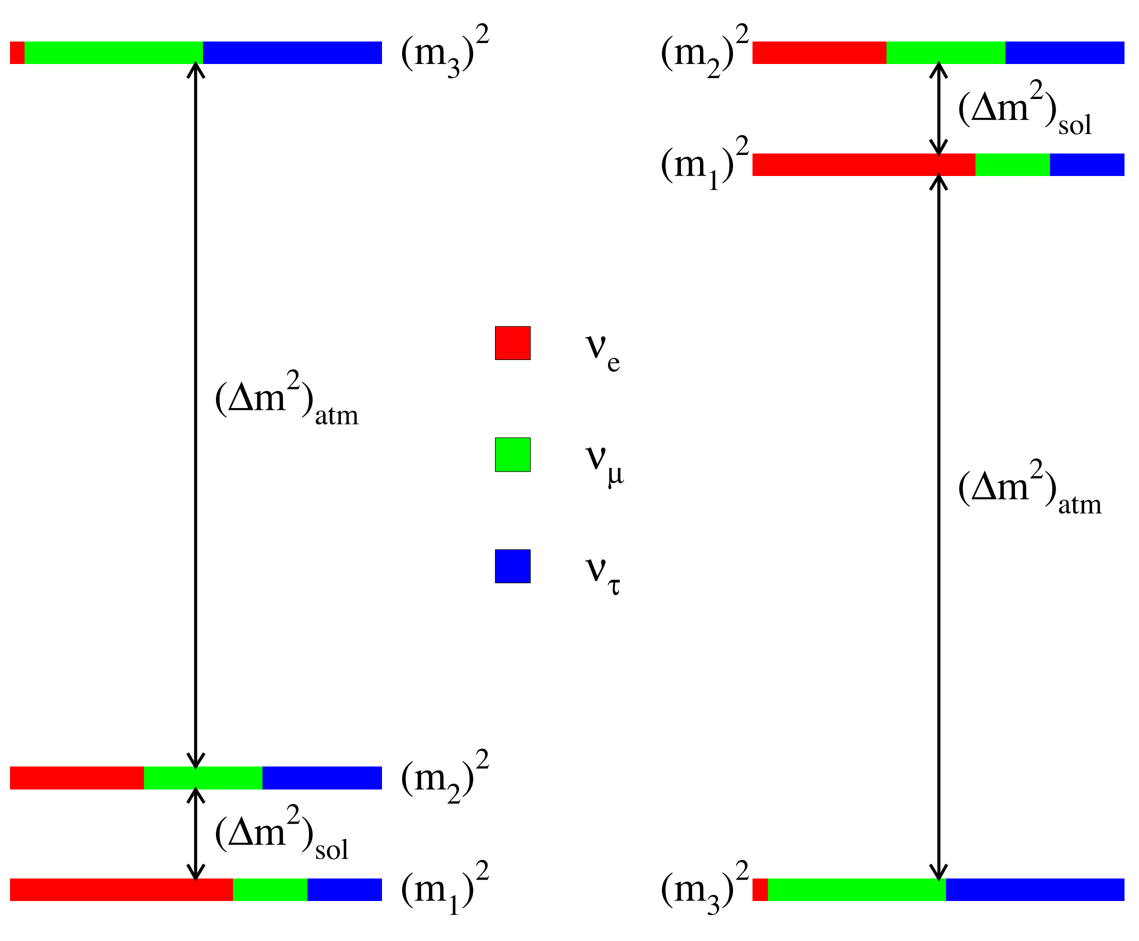
U is known as the PMNS matrix, and it relates neutrino mass eigenstates to flavour eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

2-flavour approximation in ν_μ disappearance



normal hierarchy   inverted hierarchy

The 2-flavour ν_μ-survival probability is obtained by making two approximations:

1. $\theta_{13} = 0 \Rightarrow s_{13} = 0, c_{13} = 1$
2. $|\Delta m_{31}|^2 = |\Delta m_{32}|^2$, i.e. neglect Δm_{12}^2 which is $\approx$30 times smaller.

However in June 2011 T2K published indications for a non-zero θ_{13}, and in March 2012 Daya Bay found $\theta_{13} = 0.092 \pm 0.017$ with a non-zero θ_{13} at 5.2σ significance.

Also the error in the 2011 MINOS measurement of $|\Delta m_{32}^2|$ $\approx 4\text{x}10^{-4}$ eV^2 (compared with Δm_{12}^2 = $0.8\text{x}10^{-4}$ eV^2), and T2K expects to reduce this error within 1 year.

Matter effects

This means that the 2-flavour approximation is no longer valid, and we must use the 3-flavour formulation.

If we are to be more precise, we must also consider the effects on oscillations of interactions between neutrinos and the matter through which they travel between the near and far detectors.

All three neutrino flavours undergo neutral-current (NC) interactions with protons, neutrons and electrons in matter. These NC interactions have identical amplitudes for all three flavours, and they produce no observable effects on oscillation probabilities.

However the probabilities of oscillation between one neutrino flavour and another are changed by coherent forward scattering of ν_e in charged-current interactions with electrons in matter.

Calculation of 3-flavour oscillation probabilities

The calculation of 3-flavour oscillation probabilities in matter starts with $UMU^\dagger$

where $M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{12} & 0 \\ 0 & 0 & \Delta m^2_{13} \end{pmatrix}$

For neutrinos, matter effects are taken into account by adding the potential $V = 2E\sqrt{2}G_F N_e$ to $UMU^\dagger$:

$$\mathbf{U} \begin{pmatrix} 0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31} \end{pmatrix} \mathbf{U}^\dagger + \begin{pmatrix} V & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

where E is energy, G_F the Fermi coupling constant and N_e the number density of electrons in matter. For antineutrinos V is subtracted.

The resulting matrix is diagonalised. The eigenvalues are calculated by solving the (cubic) characteristic equation using Cardano's method (arXiv:physics/0610206). The differences between them are the effective Δm^2 in matter.

Calculation of 3-flavour oscillation probabilities

The eigenvectors are calculated algebraically. Set one component equal to 1.0 (real) and calculate the other two components using two of the equations $UMU^\dagger - \lambda I = 0$. Normalise the eigenvectors and they become the columns of the effective mixing matrix in matter (UMatter). Then carry out the following steps:

1. Define a complex column vector $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ to represent a ν_μ flavour eigenstate.

2. Multiply by $UMatter^\dagger$ to convert to mass eigenstates.

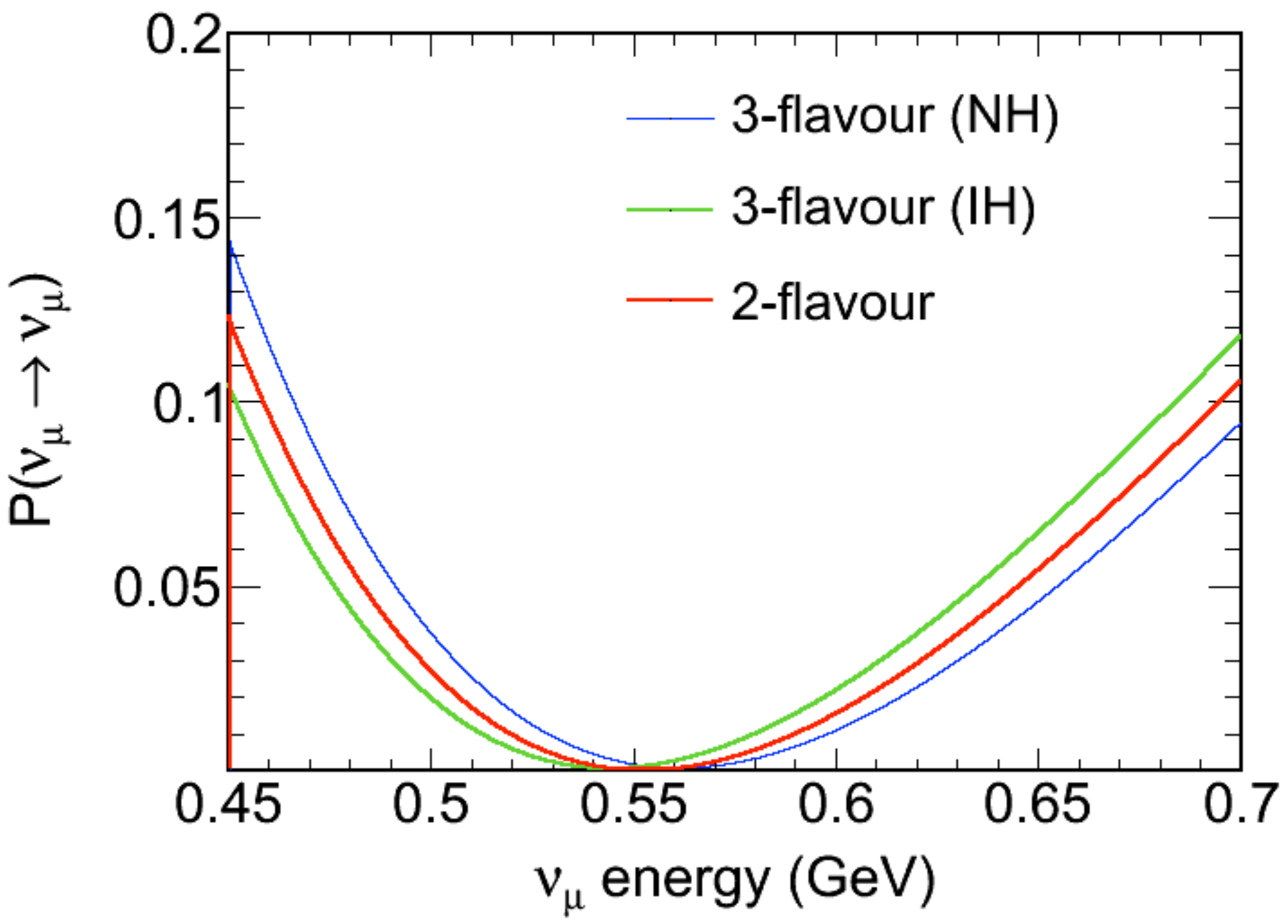
3. Propagate to Super Kamiokande by multiplying the jth component by $\exp(-i\ \Delta m_{1j}^2\ L\ /\ E)$.

4. Convert back to flavour eigenstates by multiplying by UMatter.

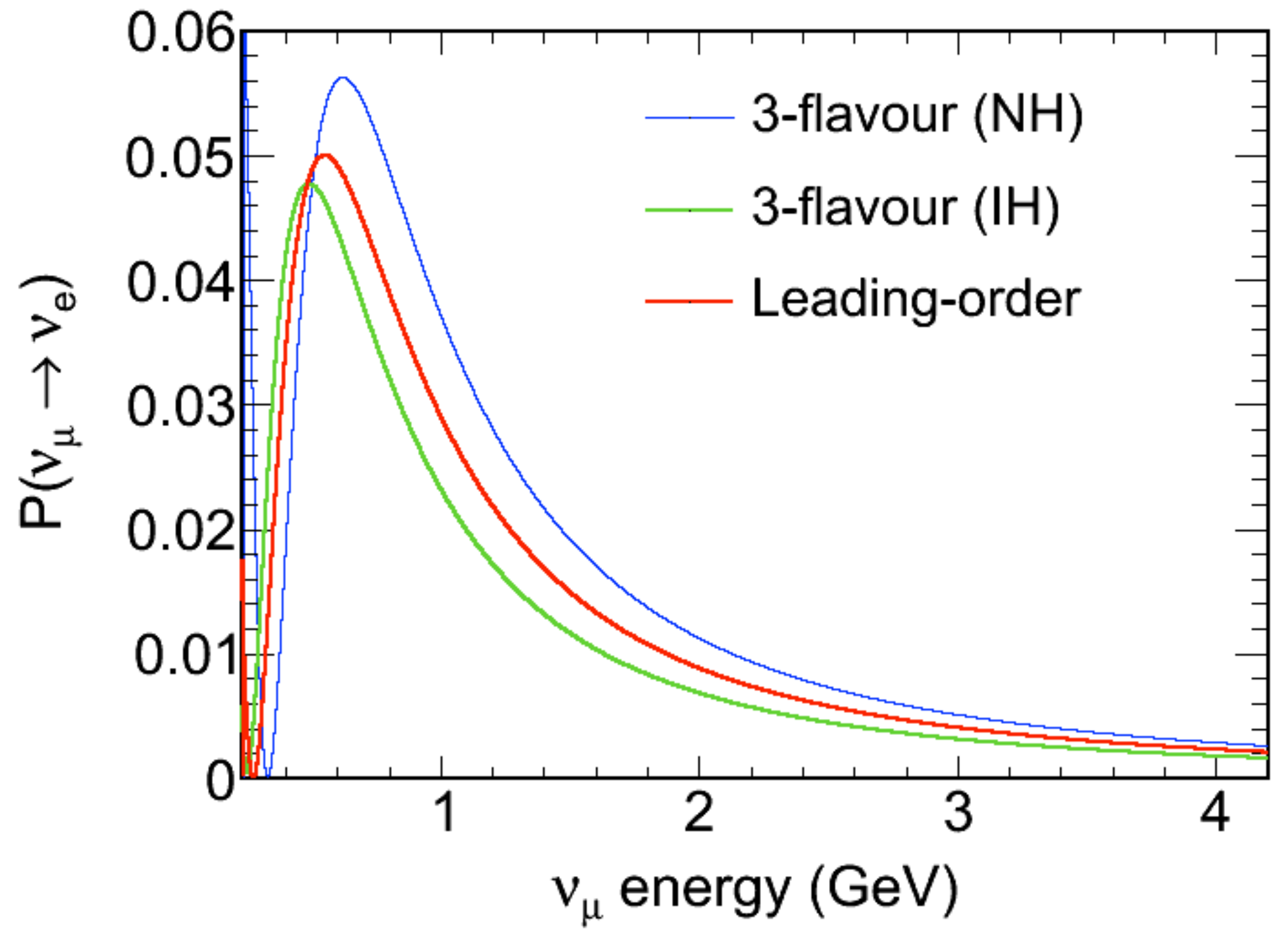
5. The probabilities of each neutrino flavour are given by the moduli squared of the components of the resulting 1x3 complex vector.

Comparison of 2- and 3-flavour oscillation probabilities

Comparison of 2-flavour and 3-flavour oscillation probabilities with matter effects for $P(\nu_\mu \rightarrow \nu_\mu)$



Comparison of leading-order and 3-flavour oscillation probabilities with matter effects for $P(\nu_\mu \rightarrow \nu_e)$



$\sin^2(2\theta_{12}) = 0.862$
$\sin^2(2\theta_{13}) = 0.1$
$\sin^2(2\theta_{23}) = 1.0$

$\Delta m_{12}^2 = 7.6 \times 10^{-5}\ eV^2$
$|\Delta m_{32}^2| = 2.32 \times 10^{-3}\ eV^2$
$\delta_{CP} = 0$

Earth crust density = 2.7 g/cm^3
NH = normal mass hierarchy
IH = inverted mass hierarchy

Summary

We have implemented a 3-flavour ν_μ-disappearance analysis that gives more precise measurements of $\sin^2(2\theta_{23})$ and $|\Delta m_{32}^2|$.

We are currently working on a 3-flavour joint ν_μ-disappearance/ν_e-appearance analysis. Since the leading-order term in $P(\nu_\mu \rightarrow \nu_e)$ is

$$\sin^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2(1.267\ \Delta m_{31}^2\ L / E),$$

we will report the measurement of θ_{23} in the form $\sin^2(\theta_{23})$ rather than the more traditional $\sin^2(2\theta_{23})$. If $\theta_{23} \neq 45°$, this will result in two best-fit points.

BACKUP SLIDES

Validation of 3-flavour oscillation probabilities (vacuum)

The 3-flavour probabilities in vacuum were validated by comparing them with an alternative formulation in the PDG review:

$$P(\nu_\alpha \to \nu_\beta) = \sum_j |U_{\beta j}|^2 |U_{\alpha j}|^2 + 2 \sum_{j>k} |U_{\beta j} U^*_{\alpha j} U_{\alpha k} U^*_{\beta k}| \cos\left(\frac{\Delta m^2_{jk} L}{2E} - \phi_{\beta\alpha;jk} \right)$$

where $\phi_{\beta\alpha;jk} = \arg(U_{\beta j} U^*_{\alpha j} U_{\alpha k} U^*_{\beta k})$

There was agreement to 14 significant figures between the two alternative formulations.

Validation of 3-flavour oscillation probabilities (matter)

Several validation checks are made of the 3-flavour probabilities in matter as they are calculated:

1. Sum of eigenvalues is compared with trace of $UMU^{\dagger}$.

2. $(UMU^{\dagger} - \lambda I)$ x eigenvector is compared with zero.

3. A check is made that the eigenvectors of $UMU^{\dagger}$ are orthogonal.

4. A check is made that $UMatter^{\dagger}$ is the inverse of UMatter - multiply them together and compare the result with an identity matrix.

The 3-flavour probabilities were also compared with those calculated by an independently-written Fortran program. There was good agreement between the two programs, and the fractional differences were $\approx 2 \times 10^{-6}$ (thanks to Terry Sloan for his help in making these comparisons).

Accuracy of 3-flavour oscillation probabilities (matter)

The accuracy of the 3-flavour oscillation probabilities was estimated in two separate ways:

1. The matter probabilities for zero density were compared with the vacuum probabilities. The fractional differences were $\approx 2\text{-}5 \times 10^{-6}$.

2. The sum of three probabilities e.g. $P(\nu_\mu \rightarrow \nu_e) + P(\nu_\mu \rightarrow \nu_\mu) + P(\nu_\mu \rightarrow \nu_\tau)$ was compared with 1.0. Again there was good agreement, and the differences were $\approx 2 \times 10^{-6}$.

This means that the 3-flavour probabilities should be considered to be accurate to 5 significant figures.

Comparison of 3-flavour oscillation probabilities in vacuum and matter

Energy (GeV)	$P(\nu_\mu \to \nu_e)$		$P(\nu_\mu \to \nu_\mu)$		$P(\nu_\mu \to \nu_\tau)$	
	Vacuum	Matter	Vacuum	Matter	Vacuum	Matter
0.3	0.00053673	0.00016523	0.95474	0.95689	0.044722	0.042946
0.4	0.024267	0.029263	0.34914	0.35017	0.62659	0.62057
0.5	0.046292	0.051604	0.036946	0.037281	0.91676	0.91111
0.6	0.051967	0.056260	0.010939	0.011023	0.93710	0.93272
0.7	0.049725	0.052978	0.094524	0.094528	0.85575	0.85249
0.8	0.044815	0.047255	0.20472	0.20470	0.75047	0.74804
0.9	0.039464	0.041309	0.31118	0.31116	0.64936	0.64753
1.0	0.034489	0.035903	0.40481	0.40479	0.56070	0.55930
1.1	0.030127	0.031227	0.48430	0.48429	0.48557	0.48448
1.2	0.026392	0.027261	0.55095	0.55094	0.42266	0.42180
1.3	0.023224	0.023920	0.60668	0.60668	0.37009	0.36940
1.4	0.020539	0.021104	0.65341	0.65341	0.32605	0.32549
1.5	0.018260	0.018723	0.69276	0.69276	0.28899	0.28852
1.6	0.016318	0.016702	0.72608	0.72609	0.25760	0.25721
1.7	0.014654	0.014976	0.75448	0.75449	0.23086	0.23054
1.8	0.013222	0.013494	0.77883	0.77884	0.20794	0.20767
1.9	0.011982	0.012214	0.79984	0.79985	0.18817	0.18794
2.0	0.010903	0.011102	0.81807	0.81808	0.17102	0.17082

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Normal mass hierarchy

Earth crust density $= 2.7\ \mathrm{g/cm}^2$