NNLO corrections to qqbar -> ttbar

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Based on:

P. Baernreuther, M. Czakon, A. Mitov `12

- ✓ I'll discuss the total inclusive cross-section.

- ✓ Relevant for this workshop in the following ways:

    - ✓ Gives normalizations,

    - ✓ An orthogonal, stringent constraint on SM/data.

Until now analyzed in approximate NNLO

Beneke, Czakon, Falgari, Mitov, Schwinn `09

… as an extension of the NLO

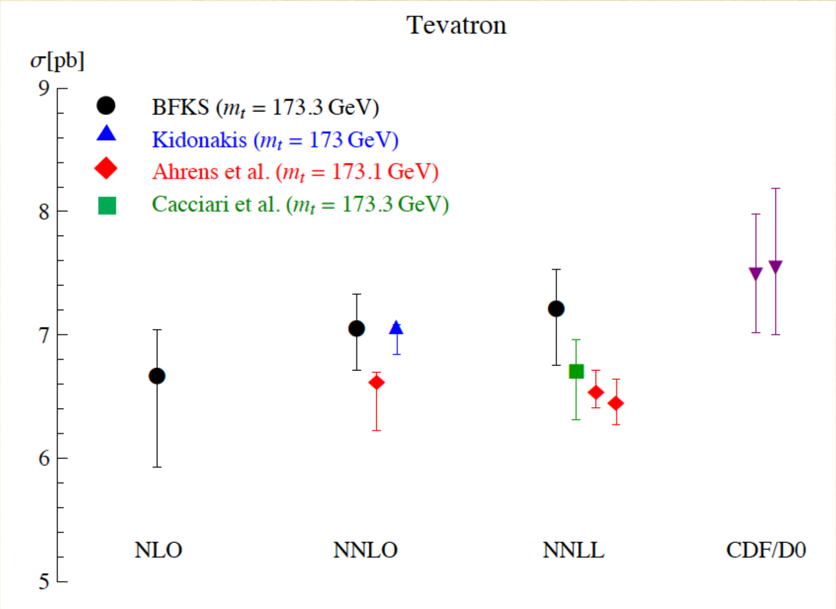
Nason, Dawson, Ellis '88
Beenakker et al '89

... resumed NLL

Sterman, Kidonakis '97
Bonciani, Catani, Mangano, Nason '98

and now NNLL resummation

Beneke, Falgari, Schwinn '10
Czakon, Mitov, Sterman '10
Ahrens et al '10-'11



Beneke, Falgari, Klein, Schwinn `11

Comparison between various groups shows:

- ✓ Significant differences between various predictions
- ✓ Suggests the true approximate NNLO uncertainty
- ✓ The realistic improvements over NLO+NLL are small (to be expected)

Cacciari, Czakon, Mangano, Mitov, Nason `11

Our answer: compute the full NNLO

Baernreuther, Czakon, Mitov `12

- So far published qq → tt +X
- Remaining reactions in the works

✓ First ever hadron collider calculation at NNLO with more than 2 colored partons.

✓ First ever NNLO hadron collider calculation with massive fermions.

Structure of the cross-section

$$\sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\max}} \mathcal{L}_{ij}(\beta)\hat{\sigma}(\beta)$$

Relative velocity of tT

$$\rho = \frac{4m_t^2}{s} \qquad \beta = \sqrt{1-\rho}$$

- ✓ The partonic cross-section computed numerically in 80 points. Then fitted.
- ✓ Many contributing partonic channels:

Computed. Dominant at Tevatron (~85%)

$$q\bar{q} \to t\bar{t}$$
$$q\bar{q} \to t\bar{t}g$$
$$q\bar{q} \to t\bar{t}gg$$
$$q\bar{q} \to t\bar{t}q'\bar{q}', \quad q \neq q'$$

$$gg \to t\bar{t}$$
$$gg \to t\bar{t}g$$
$$gg \to t\bar{t}gg$$
$$gg \to t\bar{t}q\bar{q}$$

$$qg \to t\bar{t}q$$
$$qg \to t\bar{t}qg$$
$$qq' \to t\bar{t}qq', \quad q \neq \bar{q}'$$
$$q\bar{q} \to t\bar{t}q\bar{q}$$

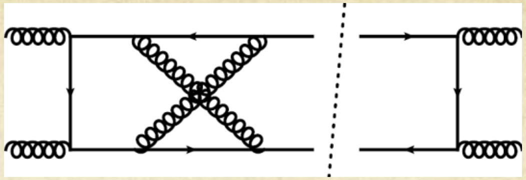
All of the same complexity. No more conceptual challenges expected (just lots of CPU)

What goes into the NNLO?

- There are 3 principle contributions:

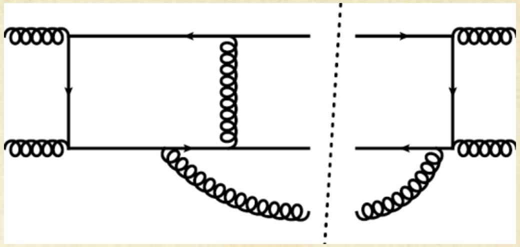
  - ✓ 2-loop virtual corrections (V-V)

    Czakon `08



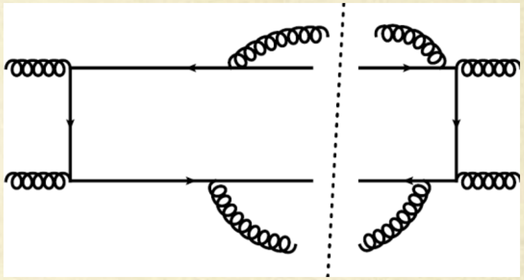
  - ✓ 1-loop virtual with one extra parton (R-V)

    1-loop amplitude: thanks to S. Dittmaier `07



  - ✓ 2 extra emitted partons at tree level (R-R)

    Czakon `10-`11



- And 2 secondary contributions:

  - ✓ Collinear subtraction for the initial state

    Known, in principle. Done numerically.

  - ✓ One-loop squared amplitudes (analytic)

    Korner, Merebashvili, Rogal `07

- Glued together in STRIPPER subtraction scheme

  Czakon `10

  (inspired by FKS and Sector Decomposition) Frixione, Kunszt, Signer `96

Results @ parton level

Notable features:

- ✓ Small numerical errors
- ✓ Agrees with limits

Partonic cross-section through NNLO:

$$\sigma_{ij}\left(\beta, \frac{\mu^2}{m^2}\right) = \frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)} + \alpha_S\left[\sigma_{ij}^{(1)} + L\,\sigma_{ij}^{(1,1)}\right] + \alpha_S^2\left[\sigma_{ij}^{(2)} + L\,\sigma_{ij}^{(2,1)} + L^2\sigma_{ij}^{(2,2)}\right] + \mathcal{O}(\alpha_S^3)\right\},$$

The NNLO term:

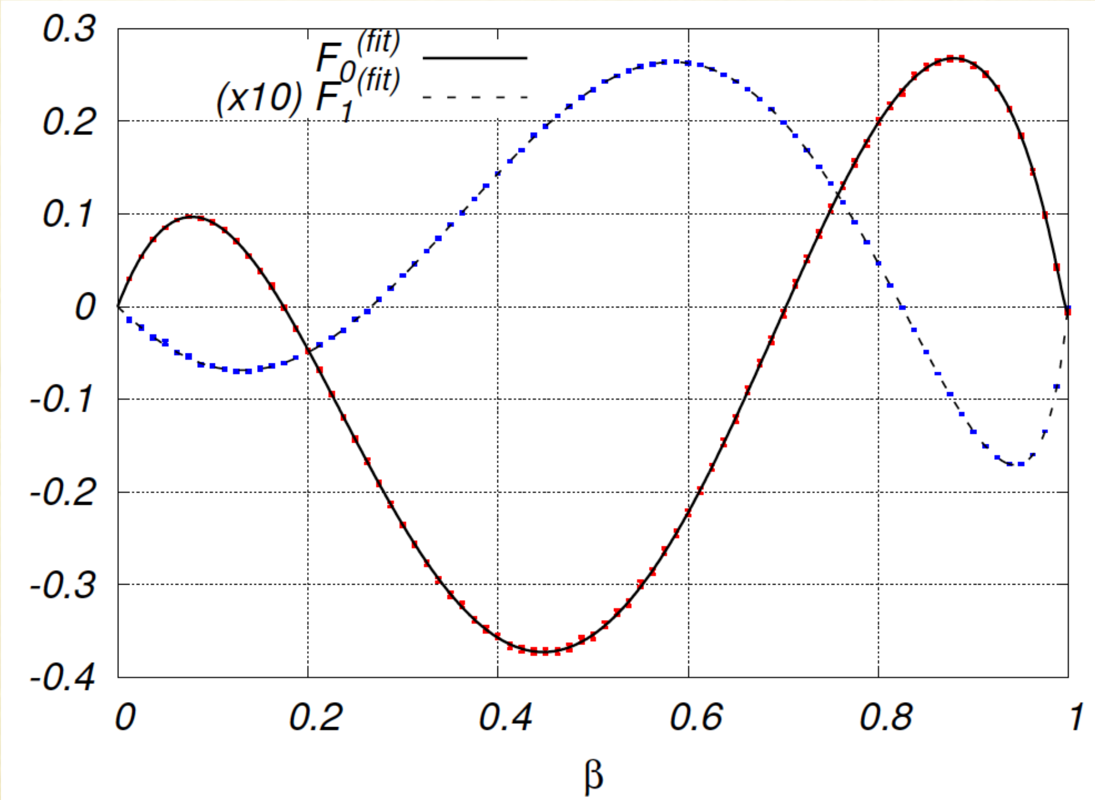
$$\sigma_{q\bar{q}}^{(2)}(\beta) = F_0(\beta) + F_1(\beta)N_L + F_2(\beta)N_L^2$$

Numeric (F_0, F_1) — Analytic (F_2)

$$F_i \equiv F_i^{(\beta)} + F_i^{(\mathrm{fit})}, i = 0, 1$$

The known threshold approximation ($F_i^{(\beta)}$)

Beneke et al `09

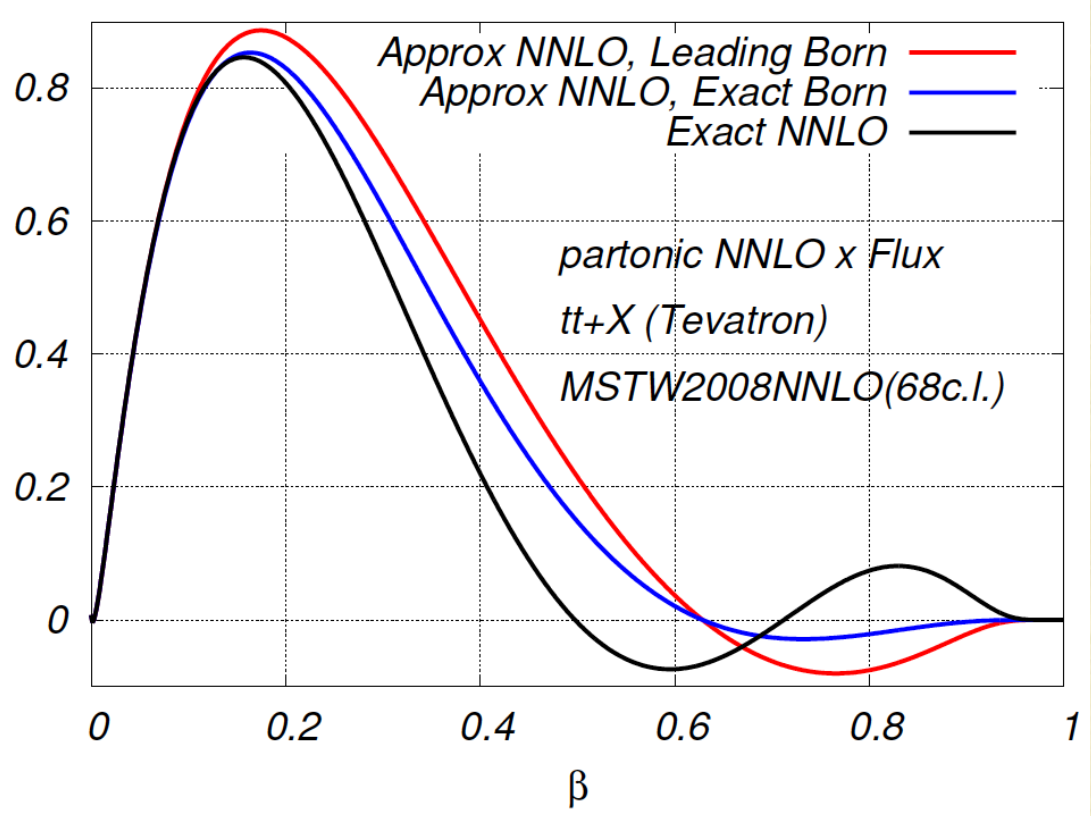


P. Baernreuther et al arXiv:1204.5201

What happens once we add the flux?

$$\sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\max}} \mathcal{L}_{ij}(\beta)\hat{\sigma}(\beta)$$

P. Baernreuther et al arXiv:1204.5201



- ✓ Approximate NNLO is a an OK approximation at parton level
- ✓ There are non-trivial cancellations; the integrated numbers are closer to the exact ones than one might anticipate
- ✓ The power corrections to the Leading Born term have important effect

Here are the numbers for the Tevatron:

P. Baernreuther et al arXiv:1204.5201

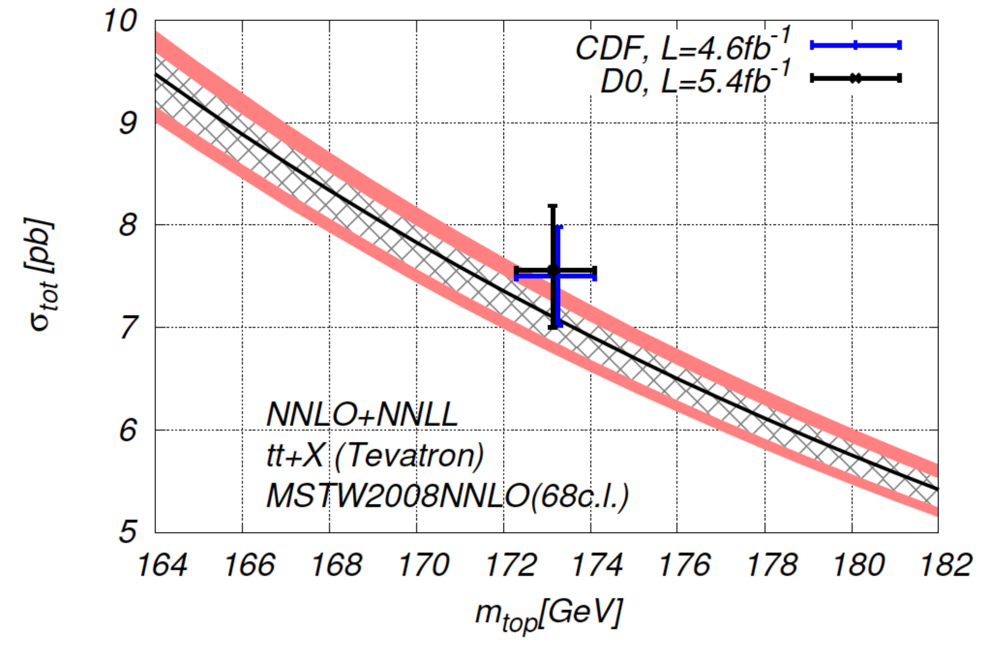
- ✓ Independent F/R scales
- ✓ MSTW2008NNLO
- ✓ mt=173.3

NNLO

$$\sigma_{\rm tot}^{\rm NNLO} = 7.005 \; {}^{+0.202\,(2.9\%)}_{-0.310\,(4.4\%)} \; [\text{scales}] \; {}^{+0.170\,(2.4\%)}_{-0.122\,(1.7\%)} \; [\text{pdf}]$$

$$\sigma_{\rm tot}^{\rm res} = 7.067 \; {}^{+0.143\,(2.0\%)}_{-0.232\,(3.3\%)} \; [\text{scales}] \; {}^{+0.186\,(2.6\%)}_{-0.122\,(1.7\%)} \; [\text{pdf}]$$

Best prediction at NNLO+NNLL



- ✓ Two loop hard matching coefficient extracted and included
- ✓ Very week dependence on unknown parameters (sub 1%): gg NNLO, A, etc.
- ✓ 50% scales reduction compared to the NLO+NNLL analysis of

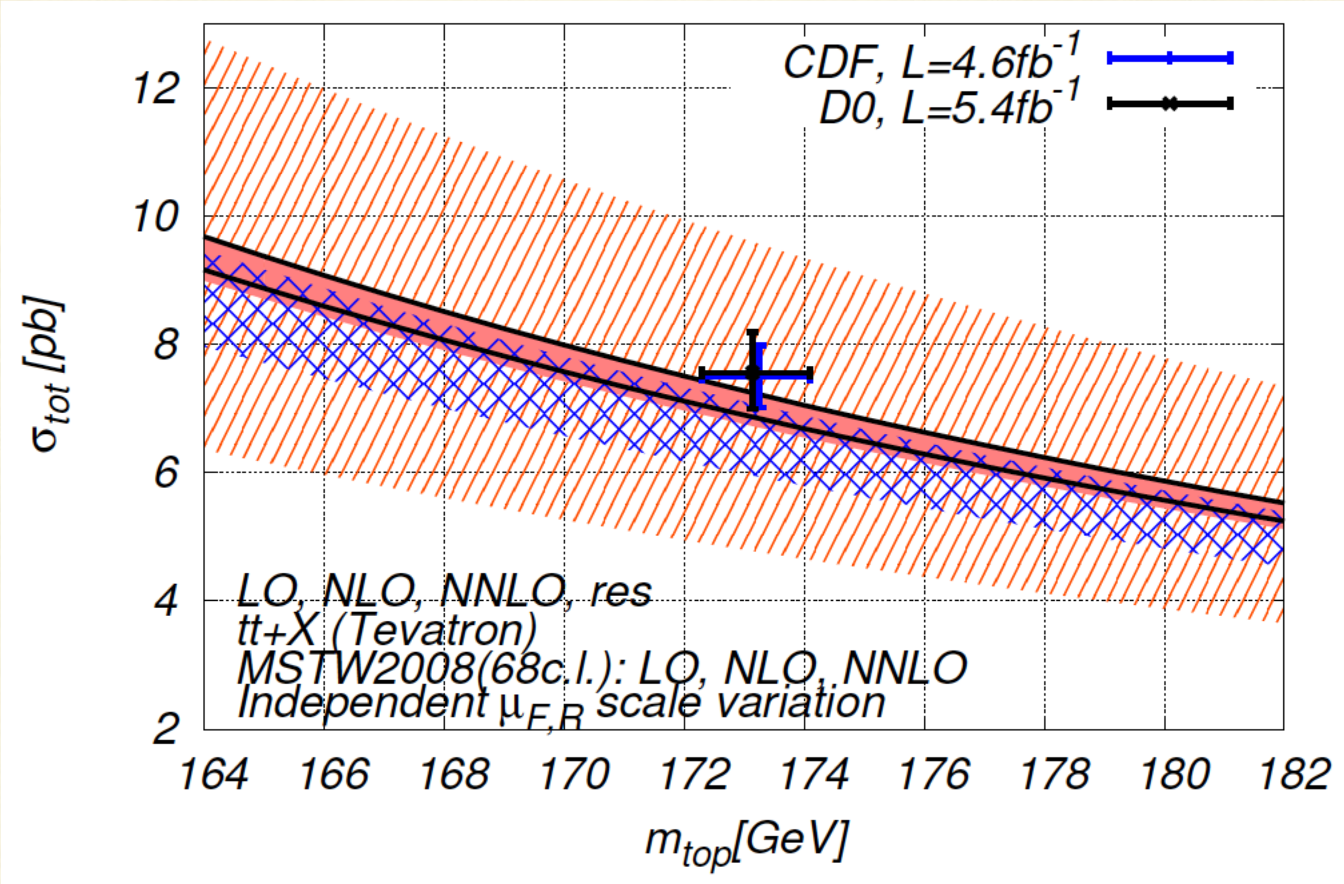
Cacciari, Czakon, Mangano, Mitov, Nason '11

6.72 + 3.6% - 6.1%

Good perturbative convergence:

P. Baernreuther et al arXiv:1204.5201

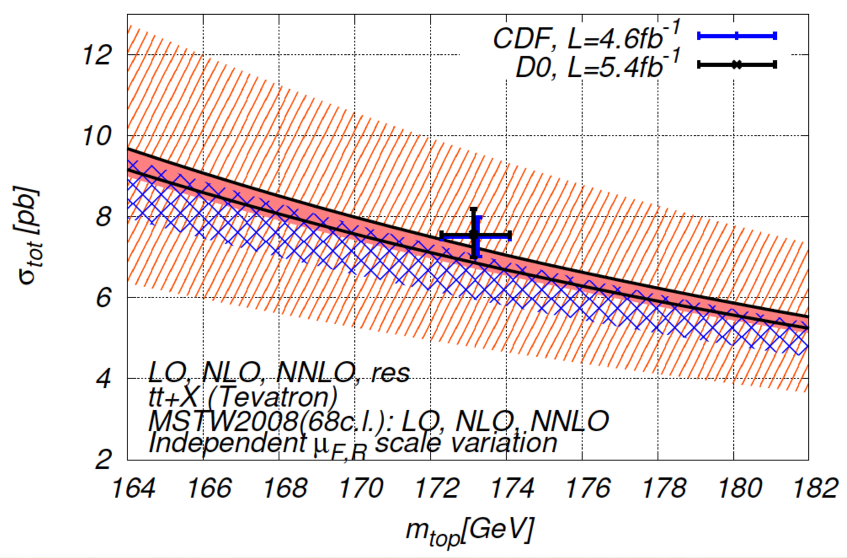
✓ Independent F/R scales
✓ mt=173.3



✓ Good overlap of various orders (LO, NLO, NNLO).
✓ Suggests our (restricted) independent scale variation is good

K-factors:

- Tevatron
- Fixed Order only



Same K-factors with NLO pdf

σ^{NNLO} (NNLO pdf) =

$$5.22059 + 1.23417 + 0.548064$$

$(\alpha_S)^2$; $(\alpha_S)^3$; $(\alpha_S)^4$

$K_{NLO/LO}=1.24$

$K_{NNLO/NLO}=1.08$ ($K_{NNLO+NNLL/NLO}=1.09$)

σ^{NNLO} (LO, NLO, NNLO pdf's) =

$$6.61926 \; ; \; 6.68123 \; ; \; 7.00531$$

$(\alpha_S)^2$; $(\alpha_S)^2+(\alpha_S)^3$; $(\alpha_S)^2 \ldots (\alpha_S)^4$

$K_{NLO/LO}=1.01$

$K_{NNLO/NLO}=1.05$

K-factors alone not totally adequate without taking uncertainties into account

Implementation and numbers

- ✓ We have also prepared the tools for top physics:

Top++ : a C++ program for the calculation of the total cross-section:

- ✓ Includes:

Czakon, Mitov `11

  - Fixed Order:
    - LO,NLO,
    - NNLO_approx (gg),
    - exact NNLO (qqbar)
  - Resummation (in Mellin space; full NNLL already there).

- ✓ Very user friendly.

- ✓ Developments:
  - ✓ ver. 1.1: Approx NNLO +NNLL (Released)
  - ✓ ver. 1.2: NNLO(qqbar) + NNLL. Complete Tevatron pheno. (Released)
  - ✓ ver. 2.0: Full NNLO + NNLL (Sometime this year)

Summary and Conclusions

✓ Computed the NNLO to qqbar -> ttbar

Significantly improved precision; right now O(½) from the experimental one at Tevatron

✓ Future work:

- ➢ Compute the remaining partonic reactions
- ➢ Compute the forward-backward asymmetry
- ➢ Compute differential distributions
- ➢ Add top decay

✓ Compute many more processes: dijets, W+jet, H+jet, etc @ NNLO

Facing the future, the stumbling block seems to be the availability of 2-loop amplitudes

✓ Our work is a strong motivation for new developments in this direction, too.