

Some comments on the full angular analysis of

$B \rightarrow K^* \ell^+ \ell^-$ (including CP violating observables)

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in collaboration with

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JHEP 0811:032,2008, arXiv:0807.2589;
JHEP 1010:056,2010, arXiv:1005.0571;
forthcoming paper

Implications of LHCb measurements and future prospects

16-18 April 2012, CERN

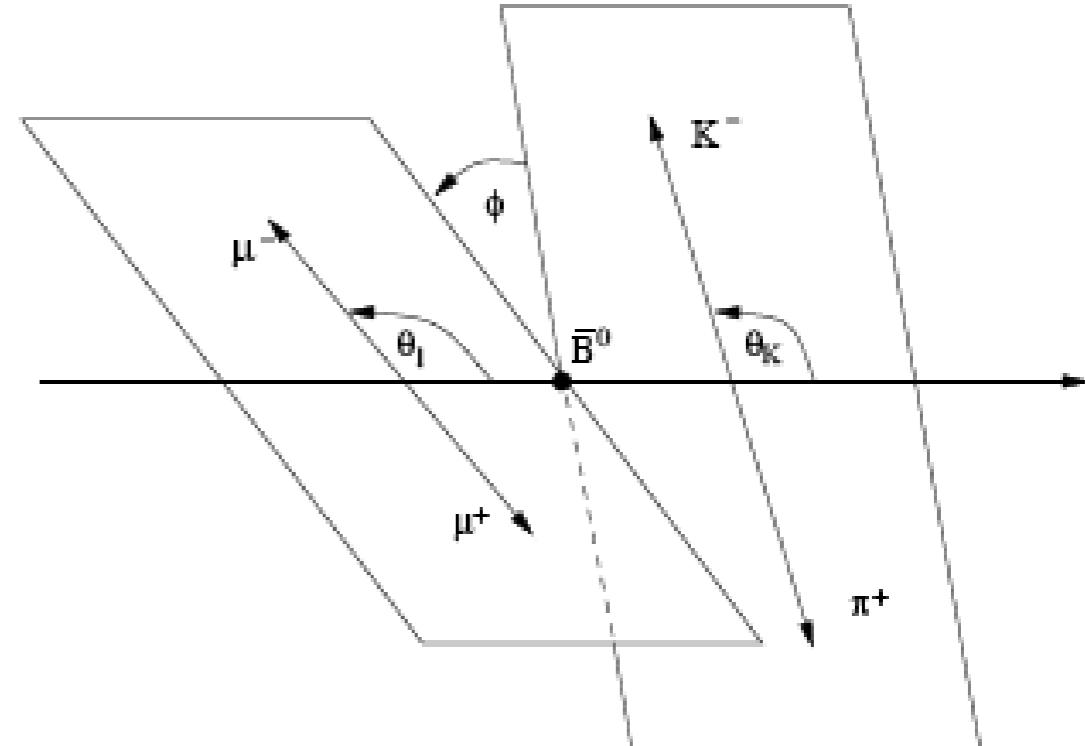
- Symmetries of angular distribution

Kinematics

- Assuming the \bar{K}^* to be on the mass shell, the decay $\bar{B}^0 \rightarrow \bar{K}^{*0}(\rightarrow K^-\pi^+) \ell^+ \ell^-$ described by the lepton-pair invariant mass, s , and the three angles θ_l , θ_{K^*} , ϕ .

After summing over the spins of the final particles:

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_l d\cos\theta_K d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$



$$J(q^2, \theta_l, \theta_K, \phi) =$$

$$\begin{aligned}
 &= J_{1s} \sin^2 \theta_K + J_{1c} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2c} \cos^2 \theta_K) \cos 2\theta_l + J_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi \\
 &\quad + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi + (J_{6s} \sin^2 \theta_K + J_{6c} \cos^2 \theta_K) \cos \theta_l \\
 &\quad + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi + J_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi
 \end{aligned}$$

However: Subtleties in measuring the 12 coefficients J_i

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- Angular distribution functions: depend on the 6 complex K^* spin amplitudes

$$J_i = J_i(A_{\perp L/R}, A_{\parallel L/R}, A_{0L/R}) \quad A_{\perp,\parallel} = (H_{+1} \mp H_{-1})/\sqrt{2}, \quad A_0 = H_0$$

- By inspection one finds: $J_{1s} = 3J_{2s}$, $J_{1c} = -J_{2c}$

Moreover, $J_{6c} = 0$ for $m_{lepton} = 0$

12 theoretical independent amplitudes A_j

?

\Leftrightarrow 9 independent coefficient functions J_i

Symmetries of $J_i = J_i(A_{\perp L/R}, A_{\parallel L/R}, A_{0L/R})$

Angular distribution spin averaged !

- Global phase transformation of the L amplitudes

$$A'_{\perp L} = e^{i\phi_L} A_{\perp L}, \quad A'_{\parallel L} = e^{i\phi_L} A_{\parallel L}, \quad A'_{0L} = e^{i\phi_L} A_{0L}$$

- Global phase transformations of the R amplitudes

$$A'_{\perp R} = e^{i\phi_R} A_{\perp R}, \quad A'_{\parallel R} = e^{i\phi_R} A_{\parallel R}, \quad A'_{0R} = e^{i\phi_R} A_{0R}$$

- Continuous $L-R$ rotation

$$\begin{aligned} A'_{\perp L} &= +\cos\theta A_{\perp L} + \sin\theta A_{\perp R}^* \\ A'_{\perp R} &= -\sin\theta A_{\perp L}^* + \cos\theta A_{\perp R} \\ A'_{0L} &= +\cos\theta A_{0L} - \sin\theta A_{0R}^* \\ A'_{0R} &= +\sin\theta A_{0L}^* + \cos\theta A_{0R} \\ A'_{\parallel L} &= +\cos\theta A_{\parallel L} - \sin\theta A_{\parallel R}^* \\ A'_{\parallel R} &= +\sin\theta A_{\parallel L}^* + \cos\theta A_{\parallel R}. \end{aligned}$$

Only 9 amplitudes A_j are independent in respect to the angular distribution

Observables as $F(I_i)$ are also invariant under the 3 symmetries !

Additional symmetry

Observation -correlations in the Monte-Carlo fit between different A_i -guided us to fourth symmetry:

$$n'_i = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cosh i\tilde{\theta} & -\sinh i\tilde{\theta} \\ -\sinh i\tilde{\theta} & \cosh i\tilde{\theta} \end{bmatrix} n_i, \quad \begin{aligned} n_1 &= (A_{\parallel}^L, A_{\parallel}^{R*}) \\ n_2 &= (A_{\perp}^L, -A_{\perp}^{R*}) \\ n_3 &= (A_0^L, A_0^{R*}) \end{aligned}$$

where θ and $\tilde{\theta}$ can be varied independently.

There is an additional non-trivial relationship between the angular distributions J_i

$$\begin{aligned} J_{1s} &= 3J_{2s} & J_{1c} &= -J_{2c} & J_{1c} &= 6 \frac{(2J_{1s} + 3J_3)(4J_4^2 + J_7^2) + (2J_{1s} - 3J_3)(J_5^2 + 4J_8^2)}{16J_1^{s2} - 9(4J_3^2 + J_6^{s2} + 4J_9^2)} \\ & & & & & - 36 \frac{J_{6s}(J_4J_5 + J_7J_8) + J_9(J_5J_7 - 4J_4J_8)}{16J_{1s}^2 - 9(4J_3^2 + J_{6s}^2 + 4J_9^2)}. \end{aligned}$$

Number of symmetries depend on assumptions:

Case	Coefficients	Dependencies	Amplitudes	Symmetries
$m_\ell = 0, A_S = 0$	11	3	6	4
$m_\ell = 0$	11	2	7	5
$m_\ell > 0, A_S = 0$	11	1	7	4
$m_\ell > 0$	12	0	8	4

Not all observables are independent :

$$S_i^{(a)} = \left(J_i^{(a)} + \bar{J}_i^{(a)} \right) \Bigg/ \frac{d(\Gamma + \bar{\Gamma})}{dq^2} \quad A_i^{(a)} = \left(J_i^{(a)} - \bar{J}_i^{(a)} \right) \Bigg/ \frac{d(\Gamma + \bar{\Gamma})}{dq^2}$$

Full angular fit has to be done with an independent set of K^* spin amplitudes or equivalently with an independent set of observables
(see Matias et al. 2012)

Additional problem in full angular fit

Binned analysis: $\langle O_1 \rangle_{q^2} < O_2 \rangle_{q^2}$ unequal $\langle O_1 O_2 \rangle_{q^2}$

- problem occurred already in latest $B \rightarrow K^* \ell \ell$ analysis of LHCb (A_T^2, F_L versus S_3)
- cannot be circumvented in full angular fit

Solutions:

- Use polynomials in q^2 to fit the q^2 dependence of the fit variables
Egede et al. 2008
 - * More fit parameters (more statistics needed)
 - * Complicated fourth symmetry ‘incompatible’ with polynomial of second order Egede et al. 2010
 - * However, if fourth symmetry is ignored, polynomials fix the corresponding ambiguity automatically!
- Sufficiently small q^2 bins (again more statistics needed)
- Direct fit of independent set of observables

work in progress

- CP violating observables

CP violating observables

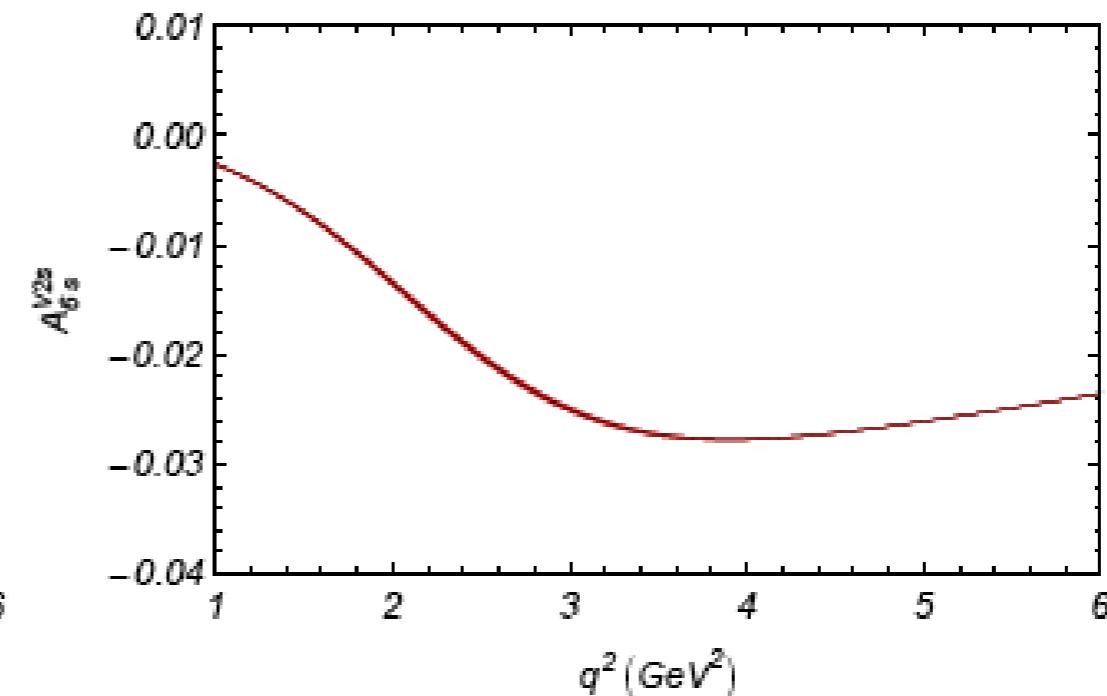
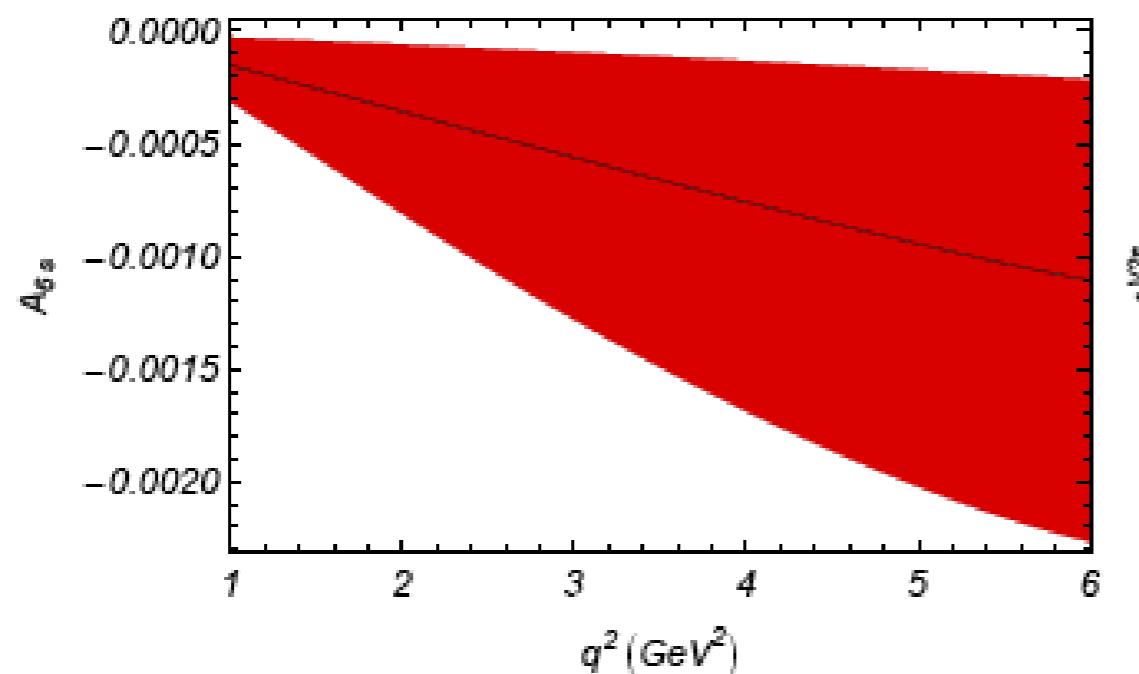
- Angular distributions allow for the measurement of 7 CP asymmetries
(Krüger,Seghal,Sinha² 2000,2005)
- NLO (α_s) corrections included: scale uncertainties reduced
(however, some CP asymmetries start at NLO only)
(Bobeth,Hiller,Piranishvili 2008)
- New CP-violating phases in C_{10}, C'_{10}, C_9 , and C'_9 are by now NOT very much constrained and enhance the CP-violating observables drastically
(Bobeth,Hiller,Piranishvili 2008; Buras et al. 2008)
- New physics reach of CP-violating observables of the angular distributions depends on the theoretical and experimental uncertainties:
 - soft/QCD formfactors
 - other input parameters
 - scale dependences
 - Λ/m_b corrections
 - experimental sensitivity in the full angular fit

Appropriate normalization eliminates the uncertainty due to form factors

Example

$$A^{6s} = \frac{I^{6s} - \bar{I}^{6s}}{d(\Gamma + \bar{\Gamma})/dq^2}$$

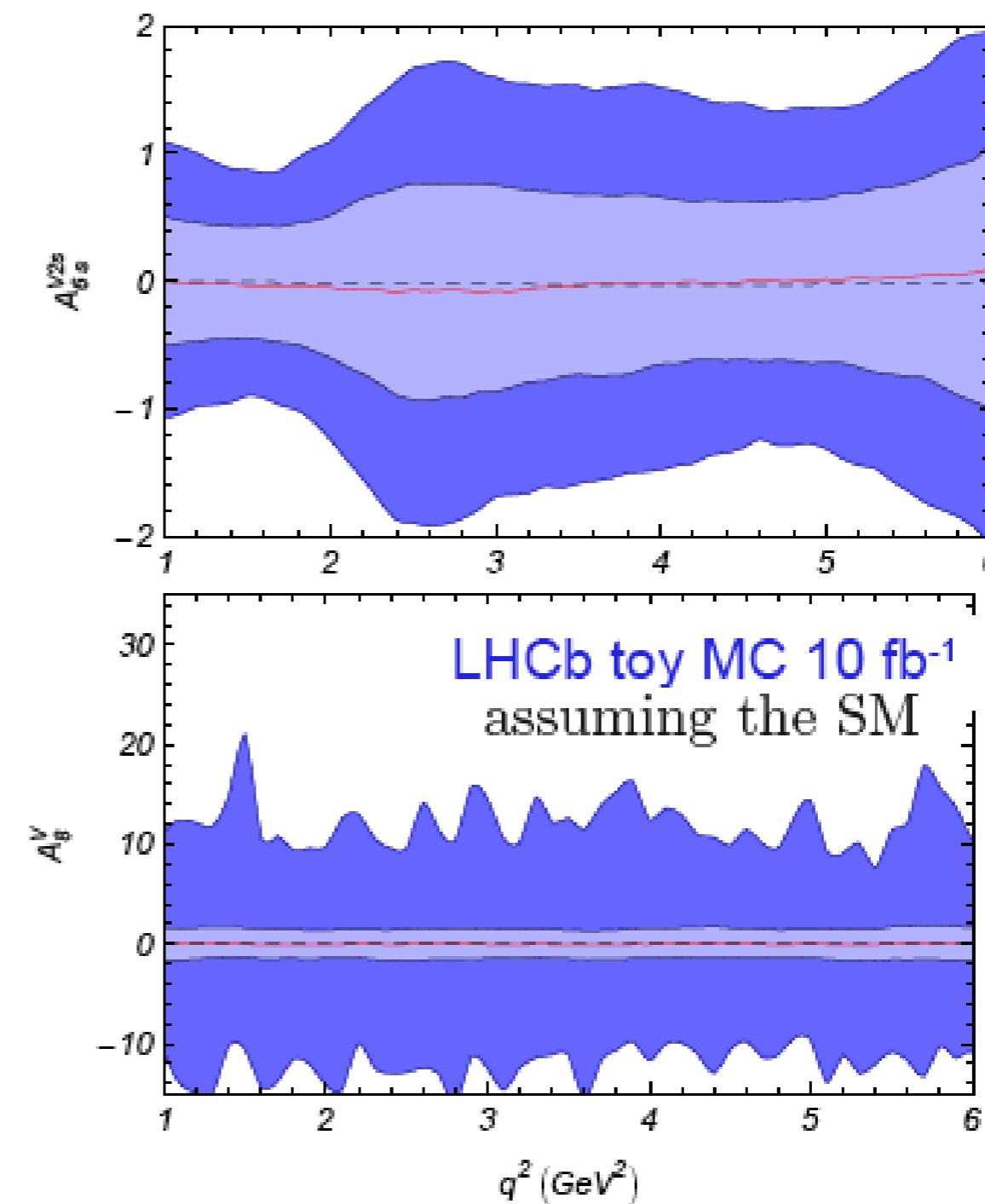
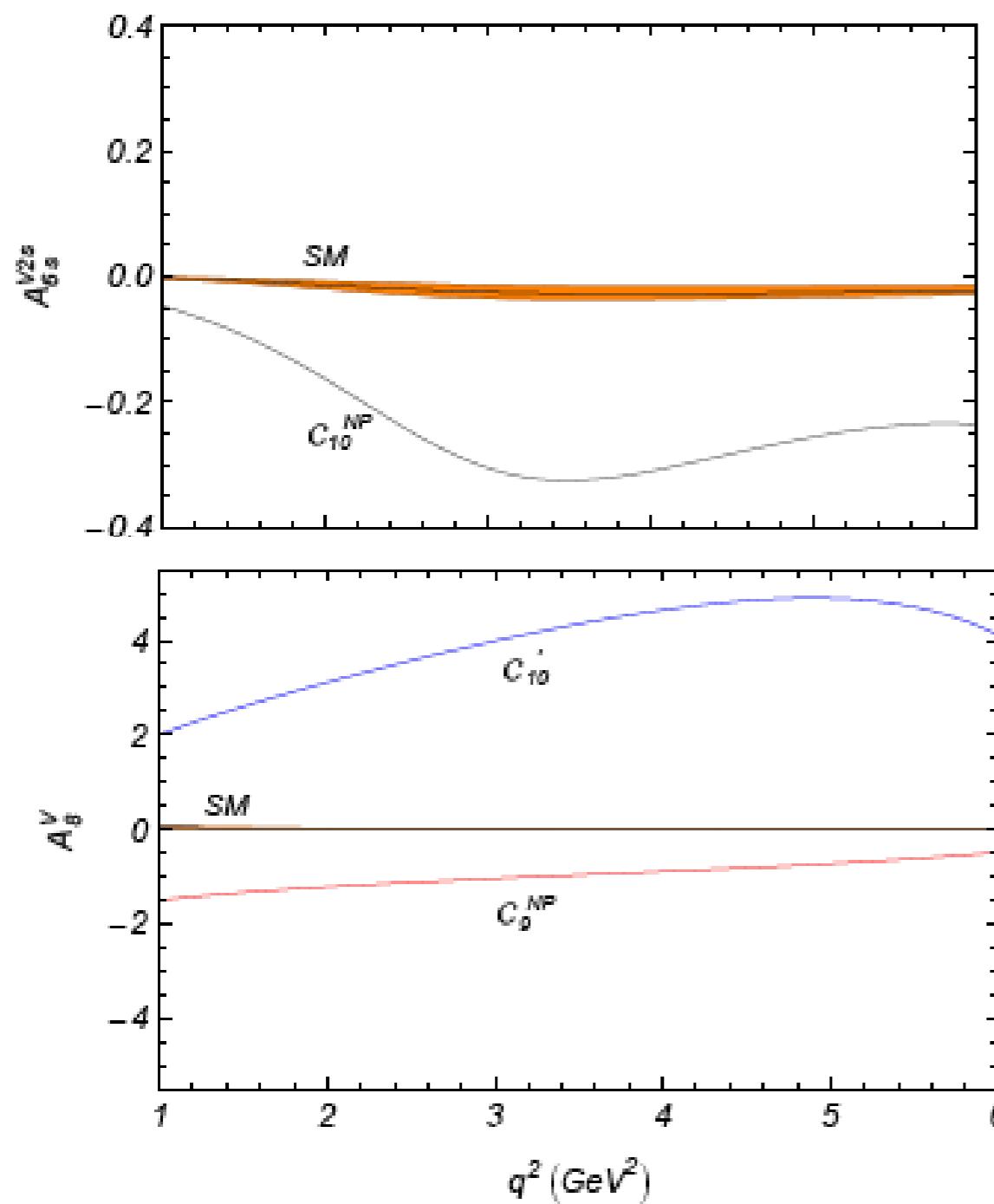
$$A_{V2s}^{6s} = \frac{I^{6s} - \bar{I}^{6s}}{I^{2s} + \bar{I}^{2s}}$$



Red bands: conservative estimate of uncertainty due to formfactors only

Relative error drops dramatically

Possible new physics effects versus experimental uncertainties



$$|C_{9,NP}| = 2, \Phi_9 = \pi/8; |C_{10,NP}| = 1.5, \Phi_{10} = \pi/8; |C'_{10}| = 2, \Phi_{10'} = \pi/8$$

New physics not outside the experimental 2σ range.

$$C_i = C_i^{\text{SM}} + |C_i^{\text{NP}}| \exp(i\phi_i), \quad C'_i = |C'_i| \exp(i\phi'_i)$$

* C^{SM} real * in reasonable models $|C^{\text{NP}}|$ much smaller than C^{SM}

Have we chosen the wrong observables ? Last LHCb-Theory workshop

A_6^V suppressed by small strong phase

A_8^V normalized by tiny quantity $J_8 + \bar{J}_8$

Altmannshofer,Paradisi,Straub, 2011

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A_6^V and A_8^V are chosen in an exemplary mode in Egede et al 2010.

Our main conclusions on the sensitivity to NP phases
in $C_9, C'_9, C_{10}, C'_{10}$ are based on all observables!

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Our main conclusions on the sensitivity to NP phases
in $C_9, C'_9, C_{10}, C'_{10}$ are based on all observables!

1. Poor experimental sensitivity not due normalization!

(conservation of information in full angular fit)

We use the standard normalization in the following to
demonstrate this.

$$A_i^{(a)} = \left(J_i^{(a)} - \bar{J}_i^{(a)} \right) \Bigg/ \frac{d(\Gamma + \bar{\Gamma})}{dq^2}$$

2. Favored observables (up to normalization)

- $A_{7,8,9}$ are T-odd asymmetries
(T reverses all particle momenta and spins)
 - * terms proportional $\cos(\theta_{\text{strong}}) \times \sin(\phi_{\text{weak}})$
 - * in contrast, T-even asymmetries: terms $\sin(\theta_{\text{strong}}) \times \cos(\phi_{\text{weak}})$
- A_6, A_8 are suppressed by α_s !
- $A_{5,6,8,9}$ can be extracted from $d\Gamma + d\bar{\Gamma}$
 - * not really relevant for the self-tagging mode $B_d \rightarrow K^{*0} (\rightarrow K^+ \pi^-) \ell^+ \ell^-$
 - * but useful for handling systematical errors

$$\frac{d^4\bar{\Gamma}}{dq^2 d\cos\theta_l d\cos\theta_K d\phi} = \frac{9}{32\pi} \bar{J}(q^2, \theta_l, \theta_K, \phi)$$

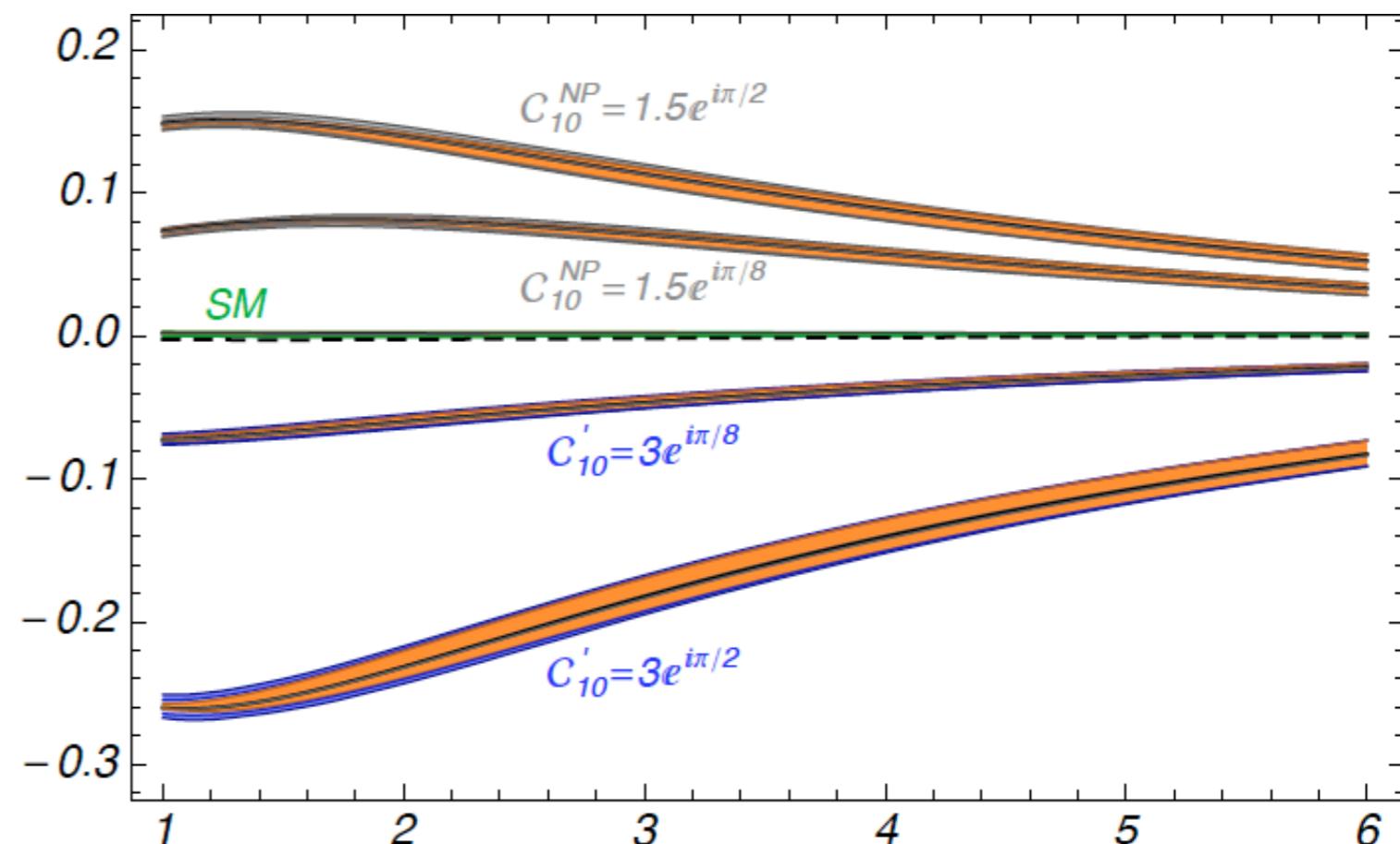
$$J_{1,2,3,4,7} \rightarrow \bar{J}_{1,2,3,4,7}, \quad J_{5,6,8,9} \rightarrow -\bar{J}_{5,6,8,9}$$

A7

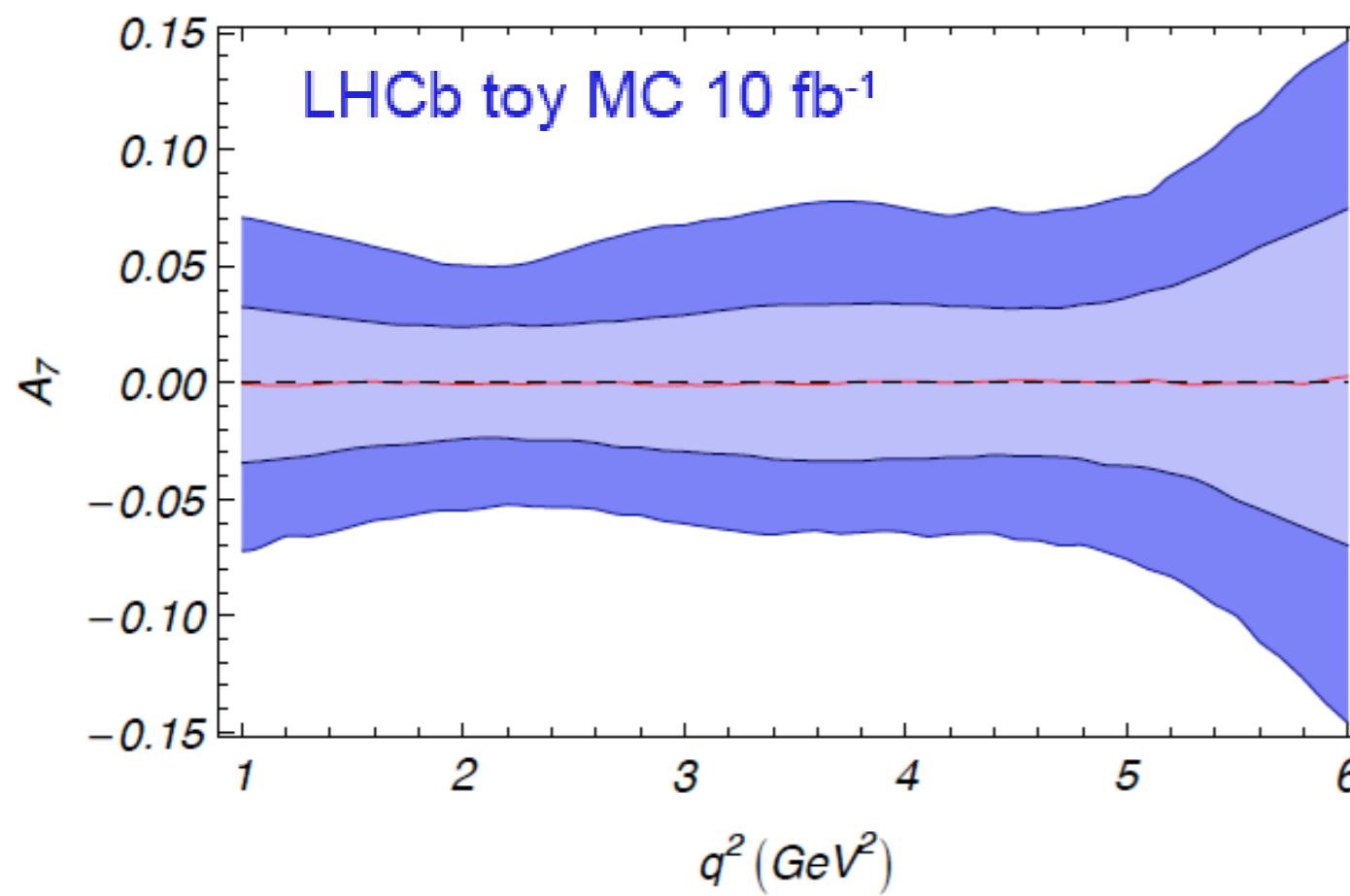
$$C_i = C_i^{\text{SM}} + |C_i^{\text{NP}}| \exp(i\phi_i),$$

$$C'_i = |C'_i| \exp(i\phi'_i)$$

* in reasonable models $|C^{\text{NP}}|$ much smaller than C^{SM}



experimental uncertainties

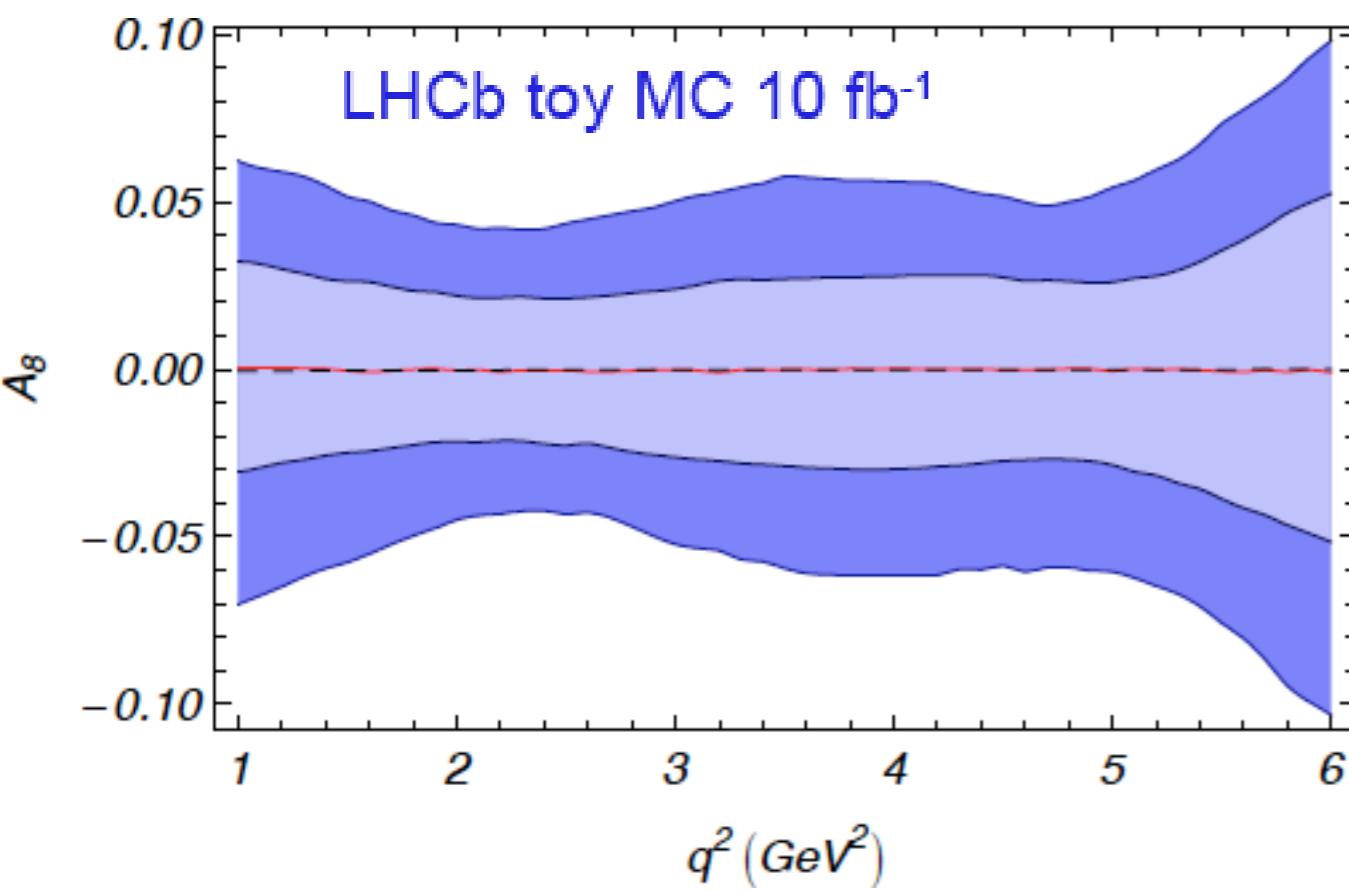
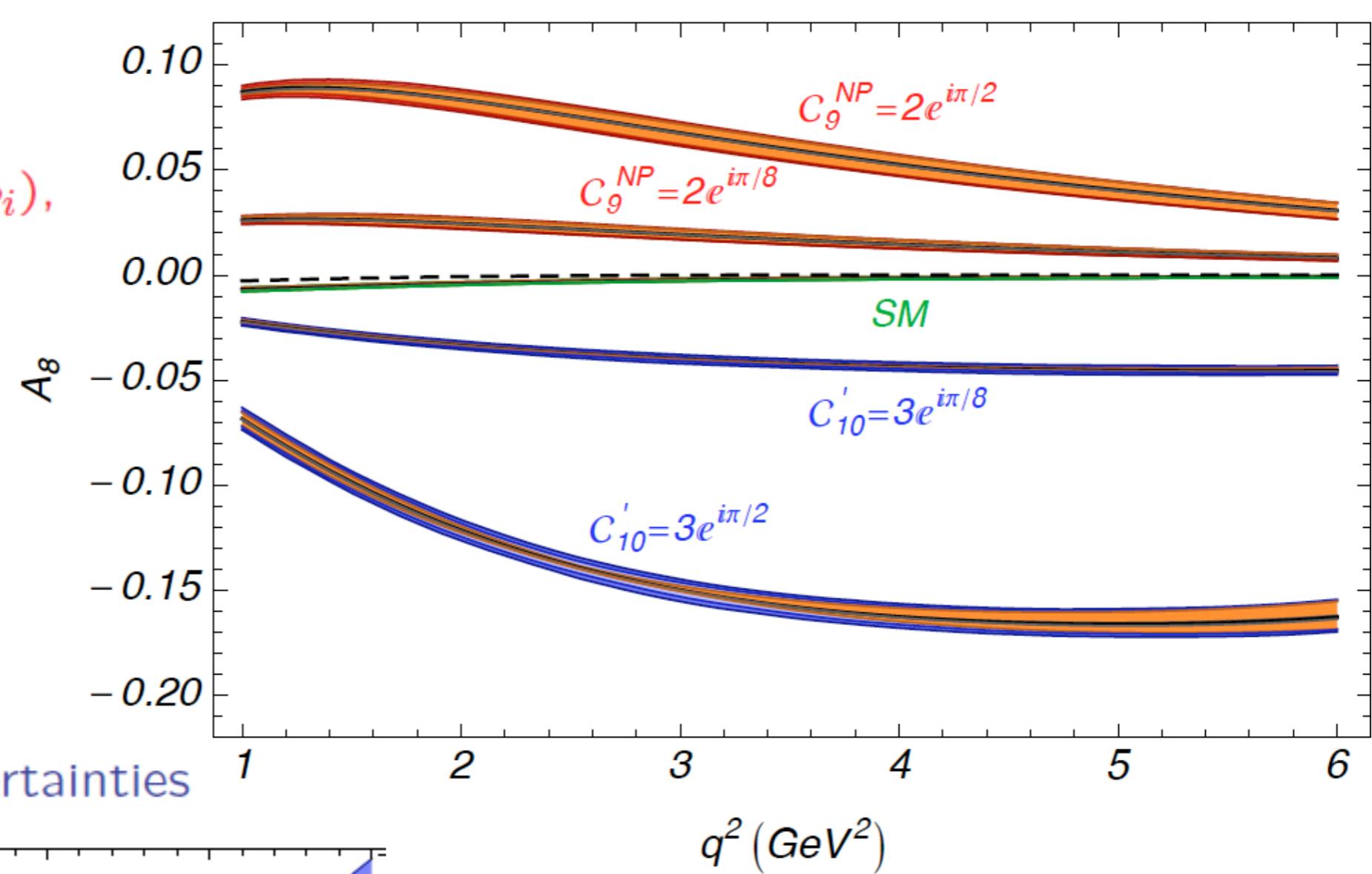


A8

$$C_i = C_i^{\text{SM}} + |C_i^{\text{NP}}| \exp(i\phi_i),$$

$$C'_i = |C'_i| \exp(i\phi'_i)$$

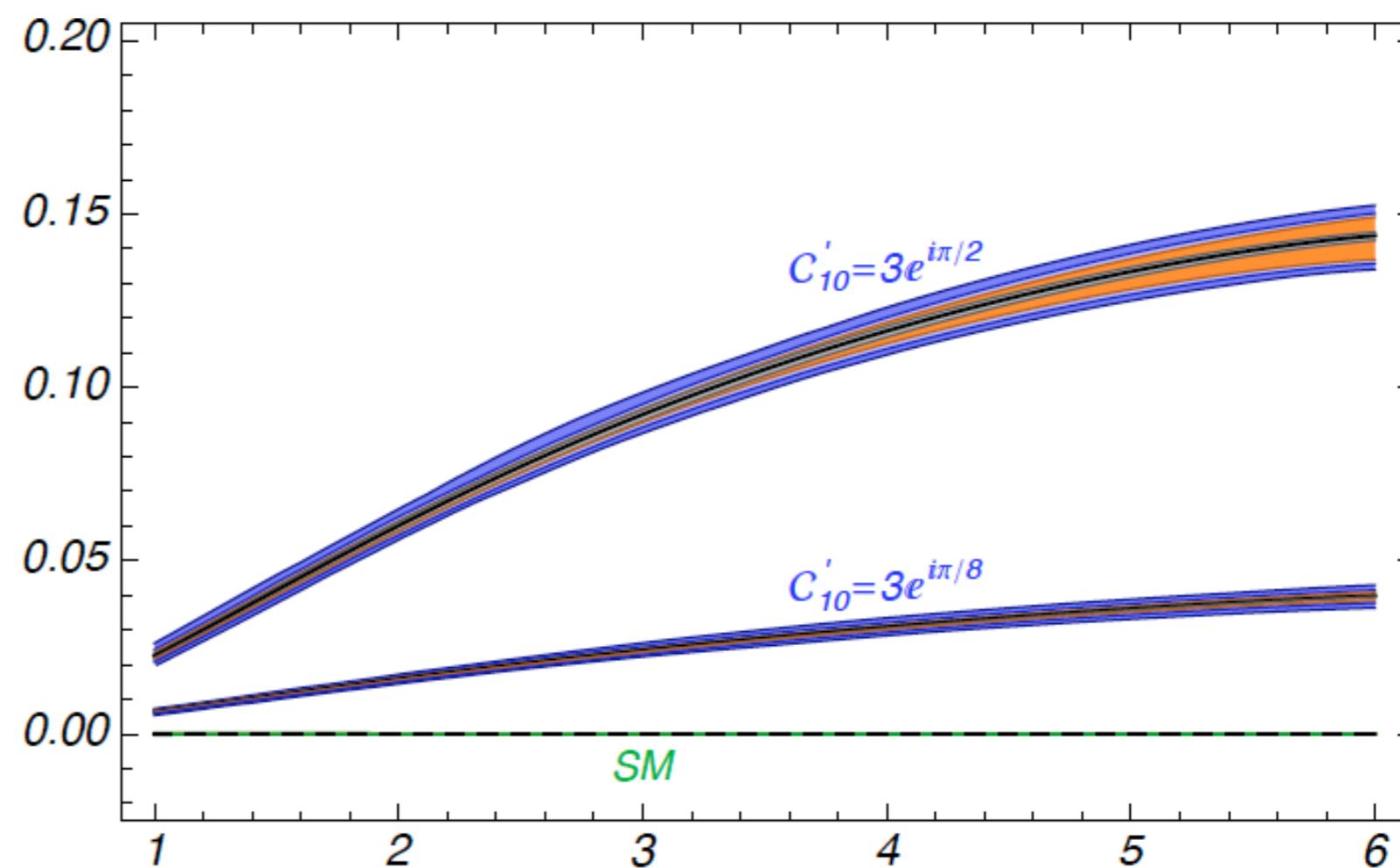
experimental uncertainties



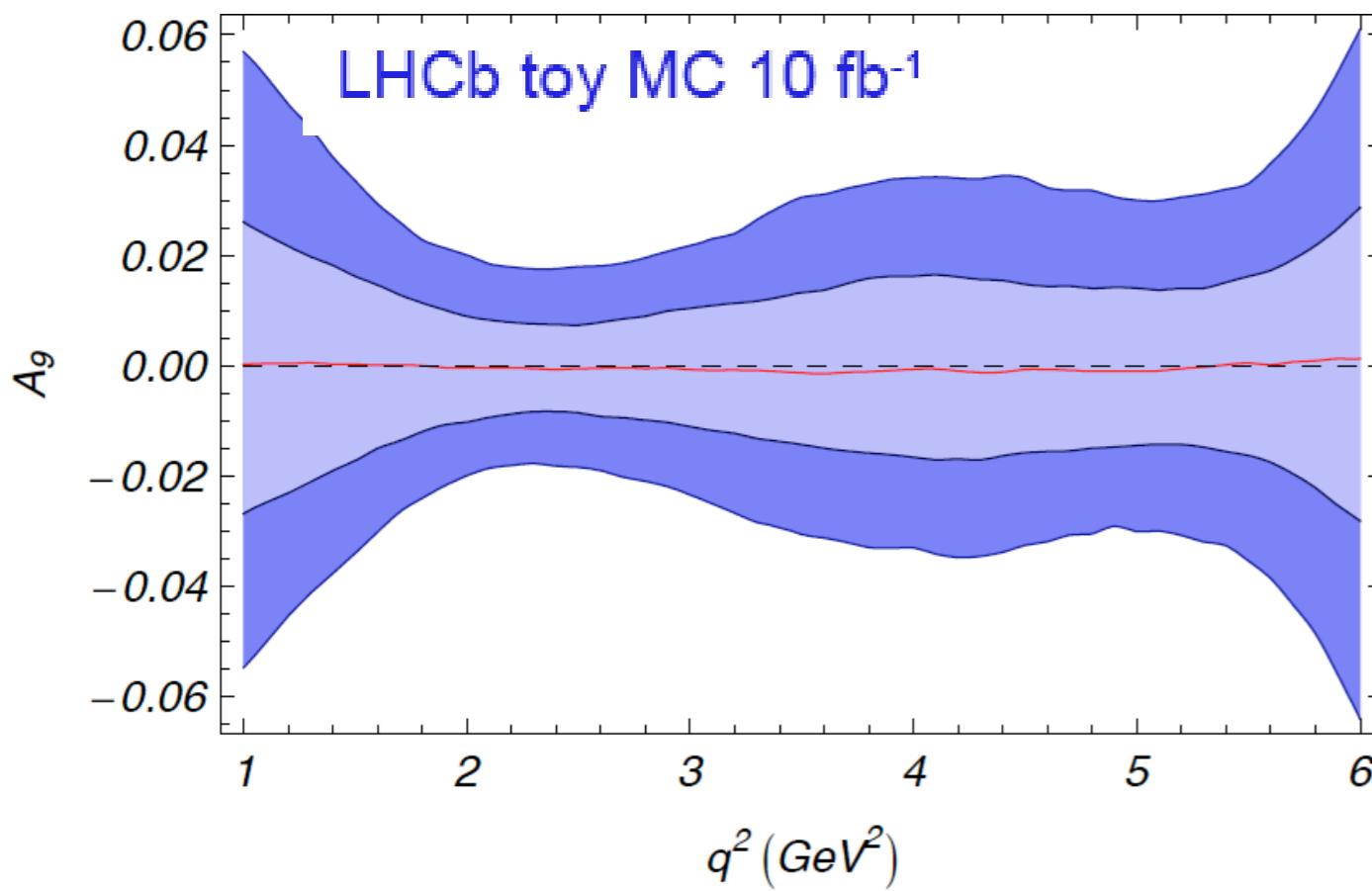
A9

$$C_i = C_i^{\text{SM}} + |C_i^{\text{NP}}| \exp(i\phi_i),$$

$$C'_i = |C'_i| \exp(i\phi'_i)$$



experimental uncertainties



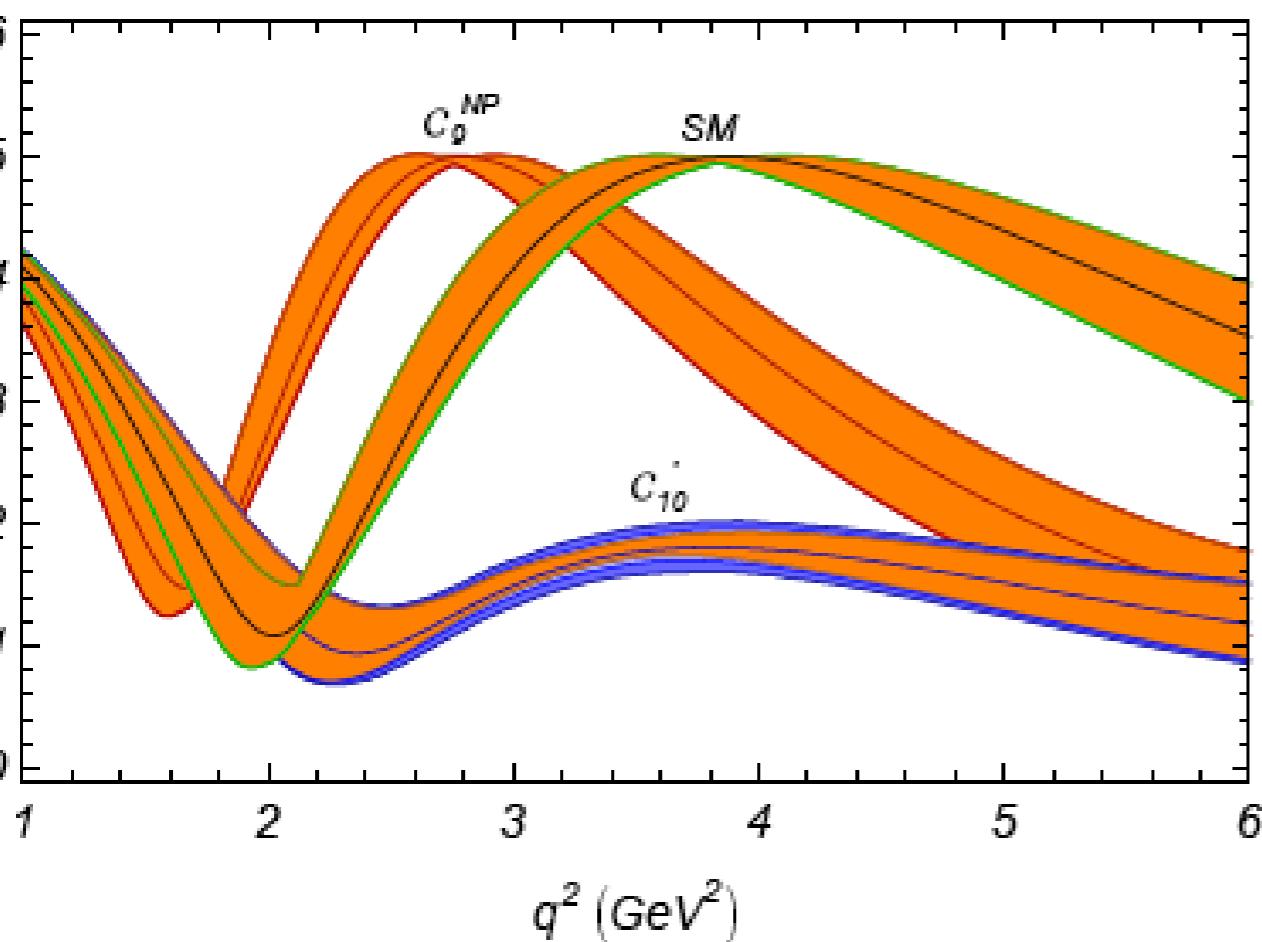
* LHCb measurement will be better than MC!

Our conclusion:

First nontrivial sensitivity to CP phases most probably in CP conserving observables

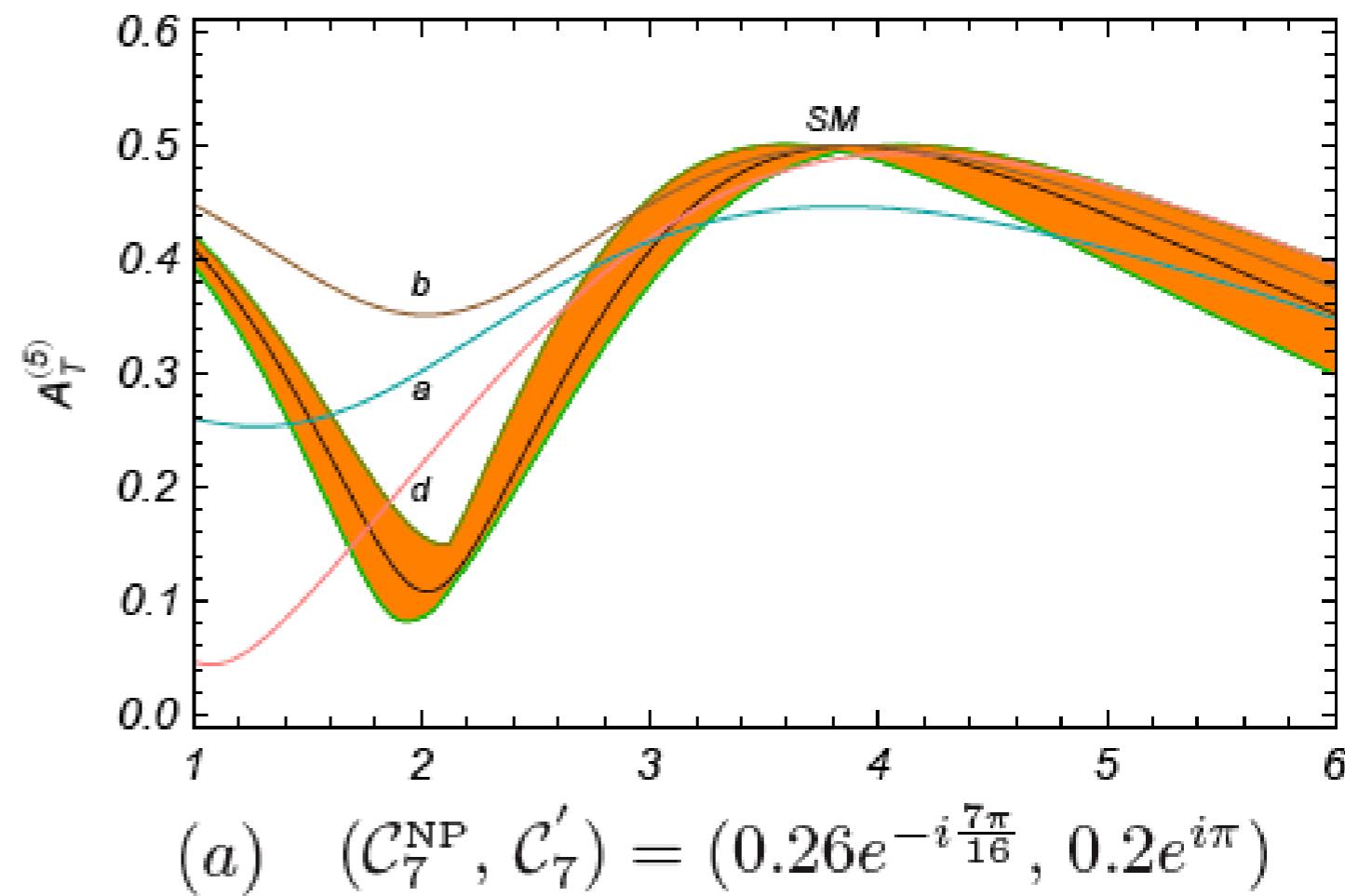
$$A_T^{(5)} = \frac{|A_{\perp}^L A_{\parallel}^{R*} + A_{\perp}^{R*} A_{\parallel}^L|}{|A_{\perp}^L|^2 + |A_{\perp}^R|^2 + |A_{\parallel}^L|^2 + |A_{\parallel}^R|^2}$$

$$A_T^{(5)} \Big|_{m_\ell=0} = \frac{\sqrt{16J_1^s{}^2 - 9J_6^s{}^2 - 36(J_3^2 + J_9^2)}}{8J_1^s}$$



NP in $\mathcal{C}'_{10} = 3e^{i\frac{\pi}{8}}$ and $\mathcal{C}_9^{\text{NP}} = 2e^{i\frac{\pi}{8}}$

$A_T^{(5)}$



(a) $(\mathcal{C}_7^{\text{NP}}, \mathcal{C}'_7) = (0.26e^{-i\frac{7\pi}{16}}, 0.2e^{i\pi})$

(b) $(0.07e^{i\frac{3\pi}{5}}, 0.3e^{i\frac{3\pi}{5}})$

(d) $(0.18e^{-i\frac{\pi}{2}}, 0)$

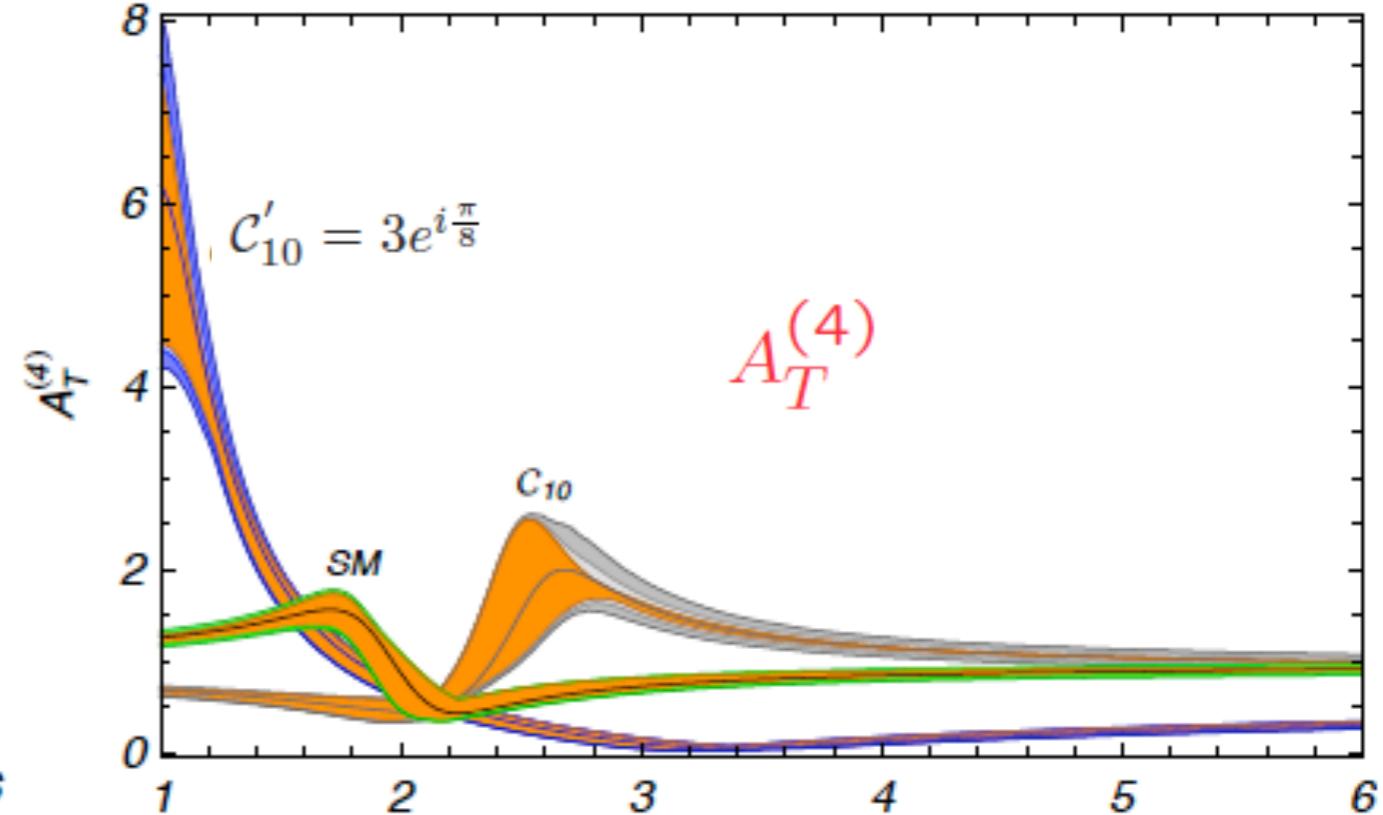
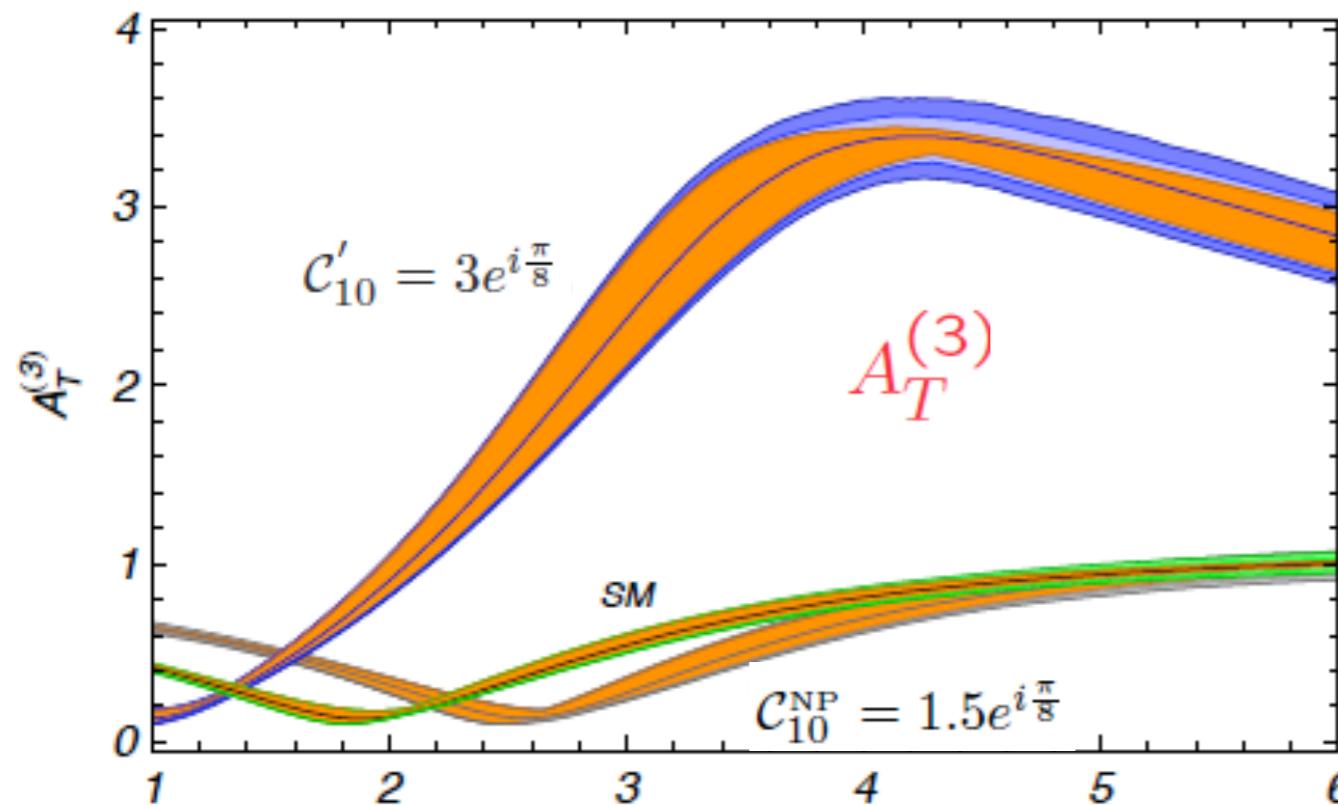
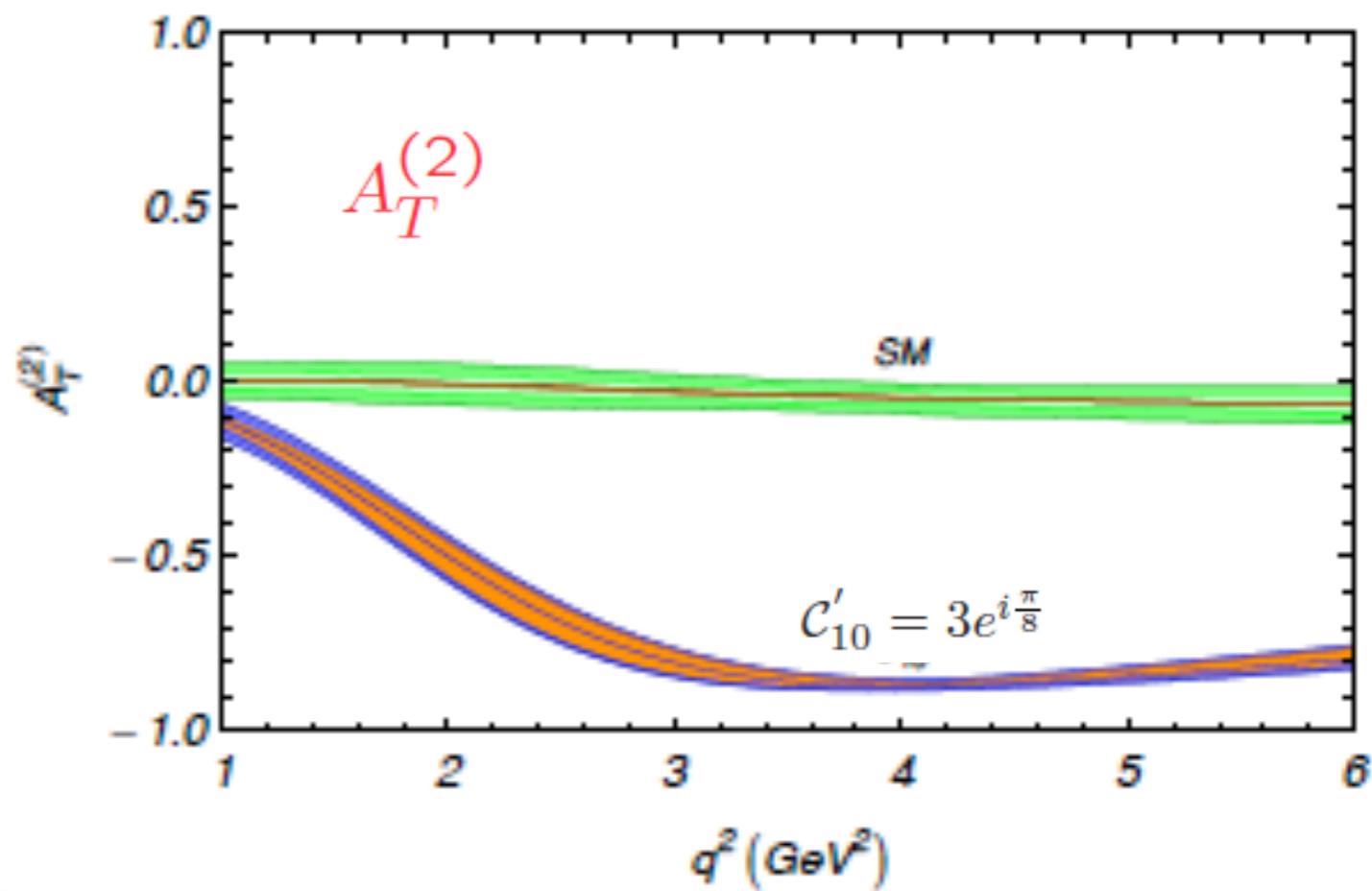
Very different behaviour for different NP contributions

Further examples

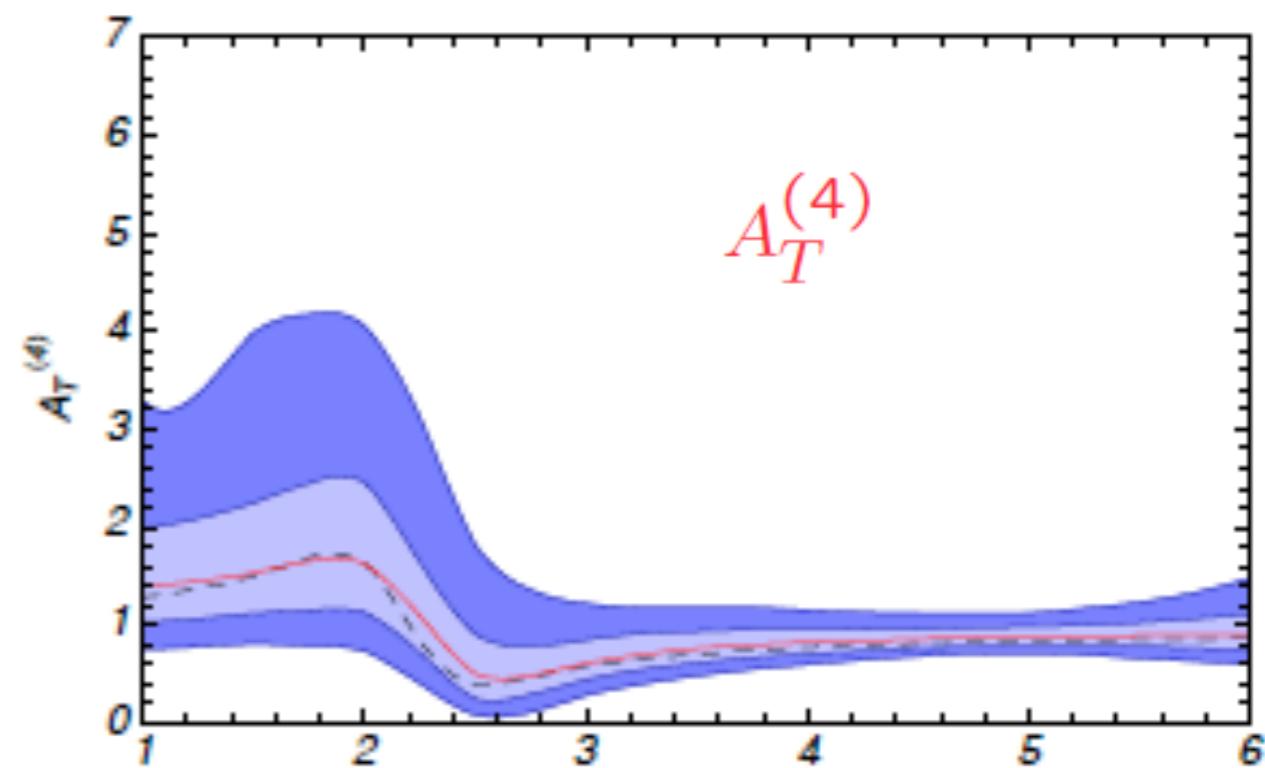
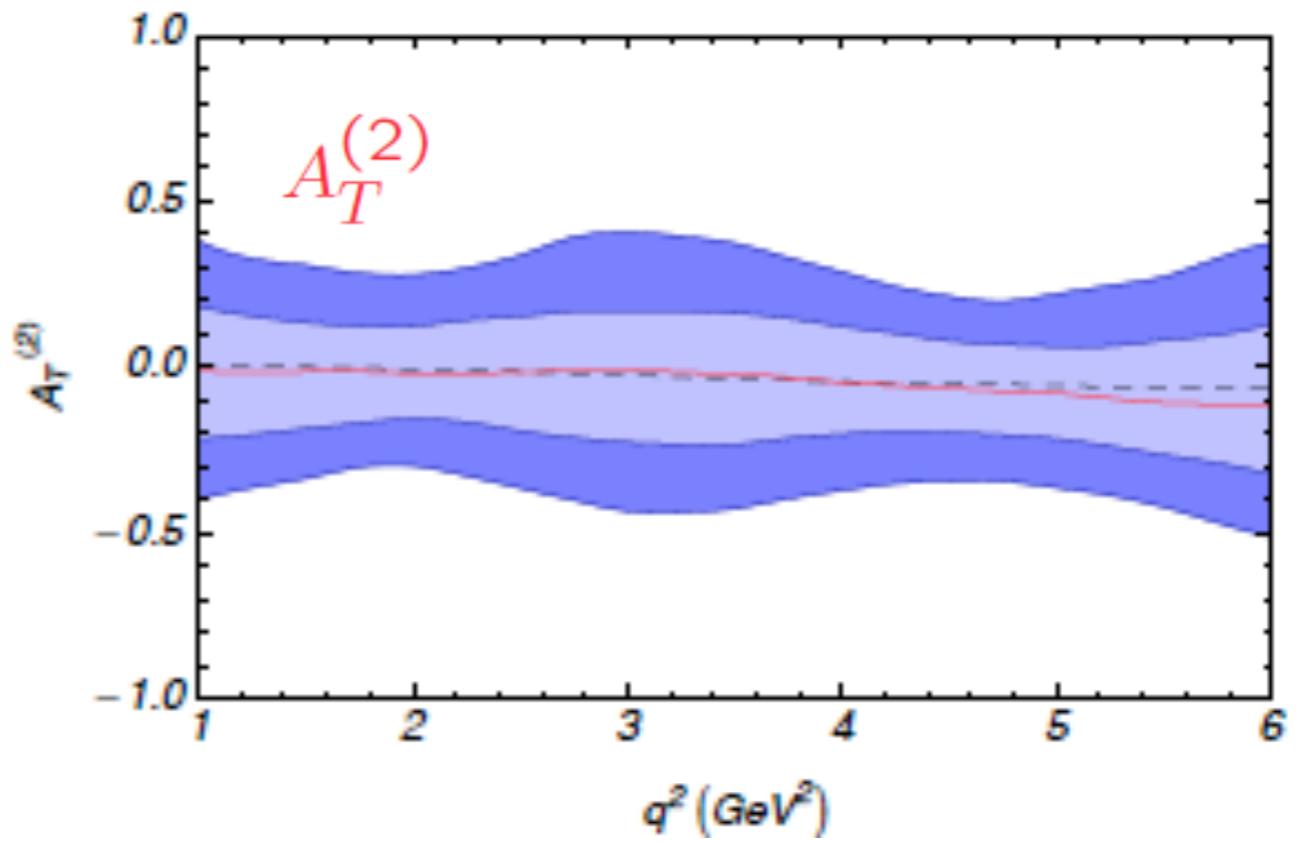
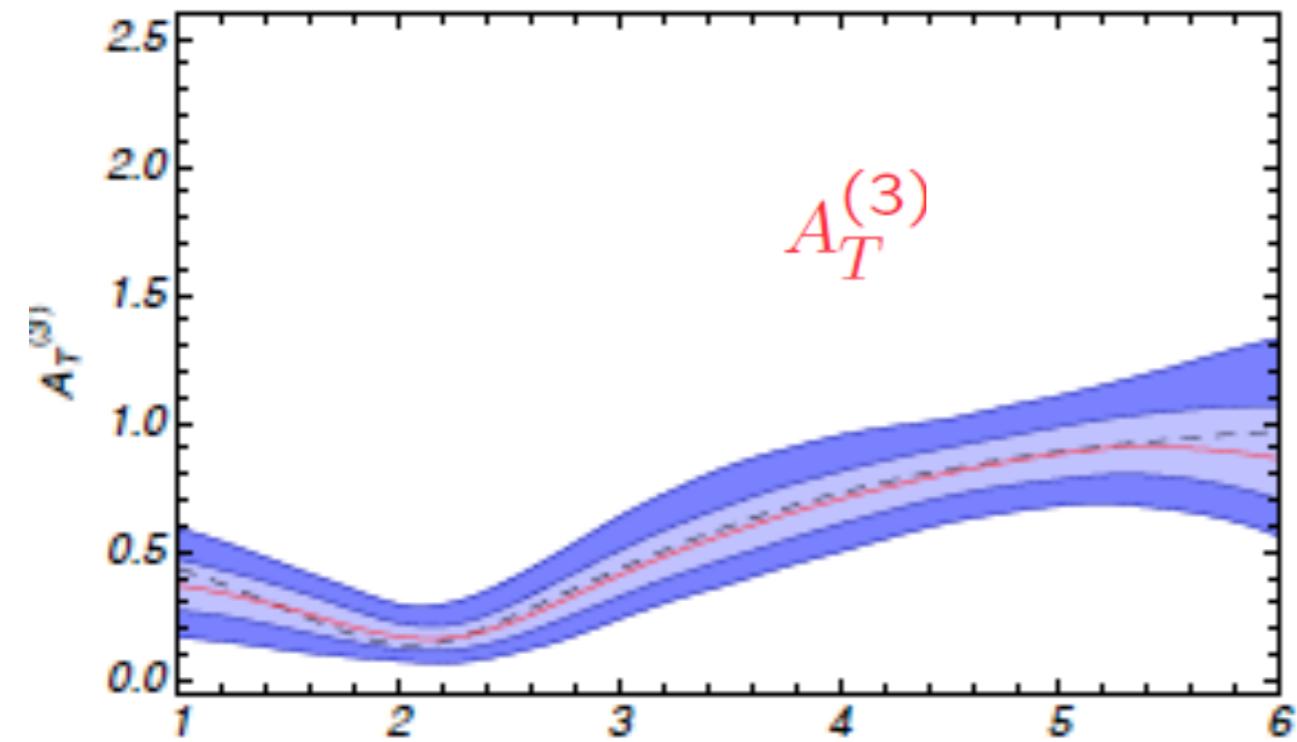
$$A_T^{(2)} = \frac{|A_{\perp}|^2 - |A_{\parallel}|^2}{|A_{\perp}|^2 + |A_{\parallel}|^2}$$

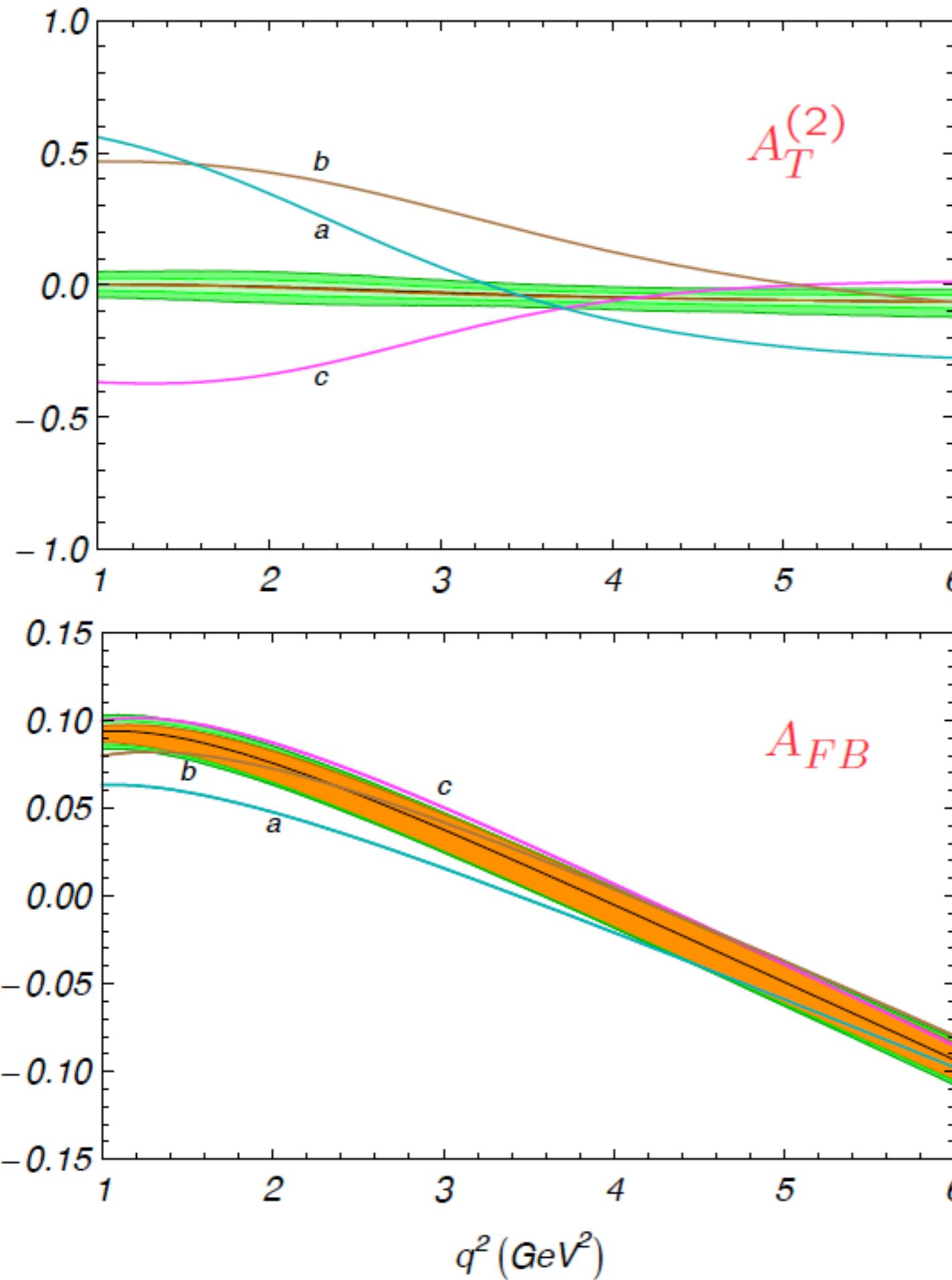
$$A_T^{(3)} = \frac{|A_{0L}A_{\parallel L}^* + A_{0R}^*A_{\parallel R}|}{\sqrt{|A_0|^2|A_{\perp}|^2}}$$

$$A_T^{(4)} = \frac{|A_{0L}A_{\perp L}^* - A_{0R}^*A_{\perp R}|}{|A_{0L}A_{\parallel L}^* + A_{0R}^*A_{\parallel R}|}$$



experimental uncertainties
LHCb toy MC 10 fb^{-1}





$A_T^{(2)}$ versus A_{FB}

- a* $(\mathcal{C}_7^{\text{NP}}, \mathcal{C}'_7) = (0.26e^{-i\frac{7\pi}{16}}, 0.2e^{i\pi})$
- b* $(0.07e^{i\frac{3\pi}{5}}, 0.3e^{i\frac{3\pi}{5}})$
- c* $(0.03e^{i\pi}, 0.07)$

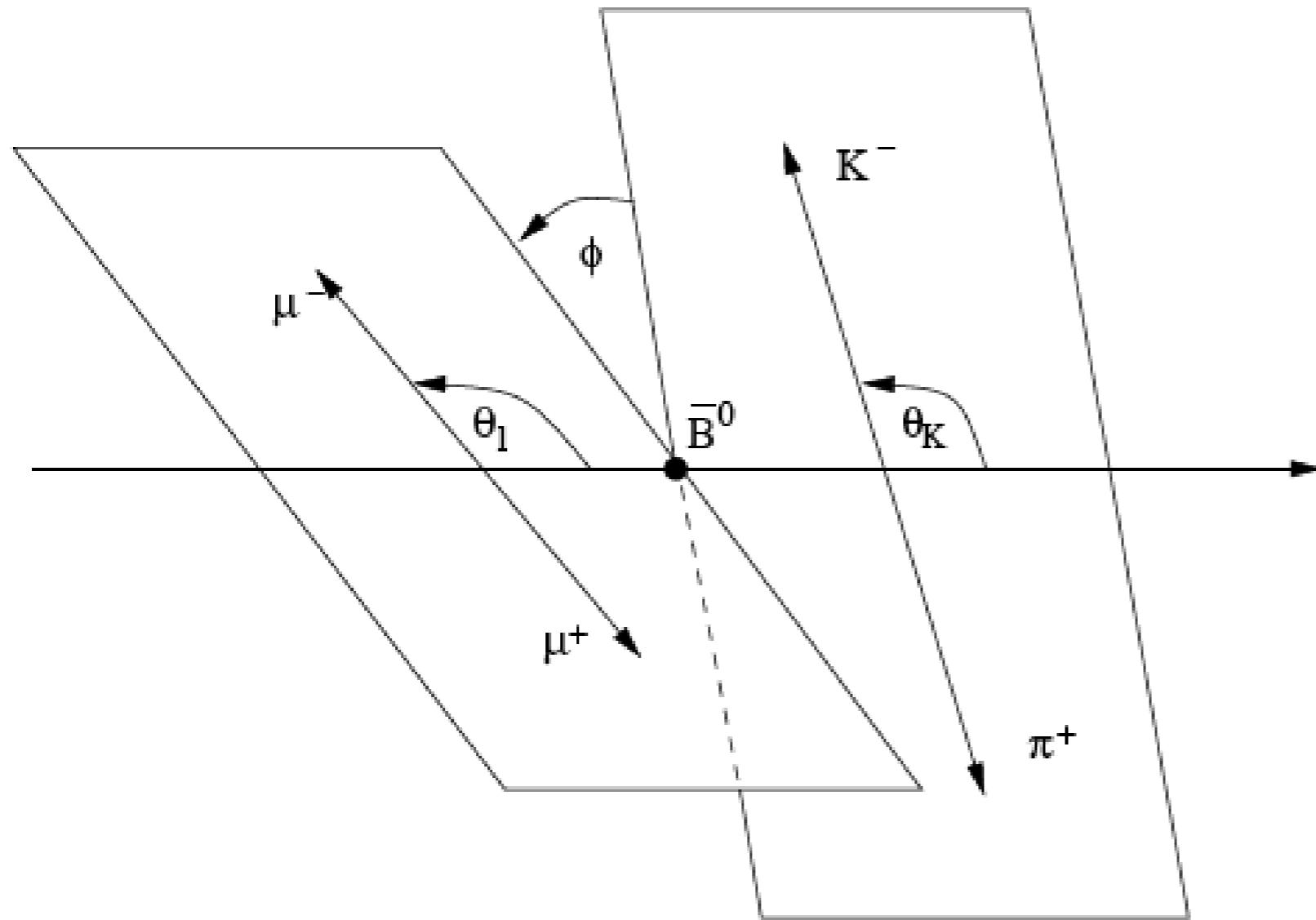
Extra

- NLO corrections included
- Λ/m_b corrections estimated for each amplitude as $\pm 10\%$ and $\pm 5\%$
this uncertainty fully dominant
- Input parameters:

m_B	$5.27950 \pm 0.00033 \text{ GeV}$	λ	0.2262 ± 0.0014
m_K	$0.896 \pm 0.040 \text{ GeV}$	A	0.815 ± 0.013
M_W	$80.403 \pm 0.029 \text{ GeV}$	$\bar{\rho}$	0.235 ± 0.031
M_Z	$91.1876 \pm 0.0021 \text{ GeV}$	$\bar{\eta}$	0.349 ± 0.020
$\hat{m}_t(\hat{m}_t)$	$172.5 \pm 2.7 \text{ GeV}$	$\Lambda_{\text{QCD}}^{(n_f=5)}$	$220 \pm 40 \text{ MeV}$
$m_{b,\text{PS}}(2 \text{ GeV})$	$4.6 \pm 0.1 \text{ GeV}$	$\alpha_s(M_Z)$	0.1176 ± 0.0002
m_c	$1.4 \pm 0.2 \text{ GeV}$	α_{em}	$1/137.035999679$
f_B	$200 \pm 30 \text{ MeV}$	$a_1(K^*)_{\perp, \parallel}$	0.20 ± 0.05
$f_{K^*, \perp}(1 \text{ GeV})$	$185 \pm 10 \text{ MeV}$	$a_2(K^*)_{\perp}$	0.06 ± 0.06
$f_{K^*, \parallel}$	$218 \pm 4 \text{ MeV}$	$a_2(K^*)_{\parallel}$	0.04 ± 0.04
$\xi_{K^*, \parallel}(0)$	0.16 ± 0.03	$\lambda_{B,+}(1.5 \text{ GeV})$	$0.485 \pm 0.115 \text{ GeV}$
$\xi_{K^*, \perp}(0)^{\dagger}$	0.26 ± 0.02		

$\xi_{K^*, \perp}(0)$ has been determined from experimental data.

More on kinematics:



z axis: Direction of anti- K^{*0} in rest frame of anti- B_d

θ_l : Angle between μ^- and z axis in $\mu\mu$ rest frame

θ_K : Angle between K^- and z axis in anti- K^* rest frame

ϕ : Angle between the anti- K^* and $\mu\mu$ decay planes

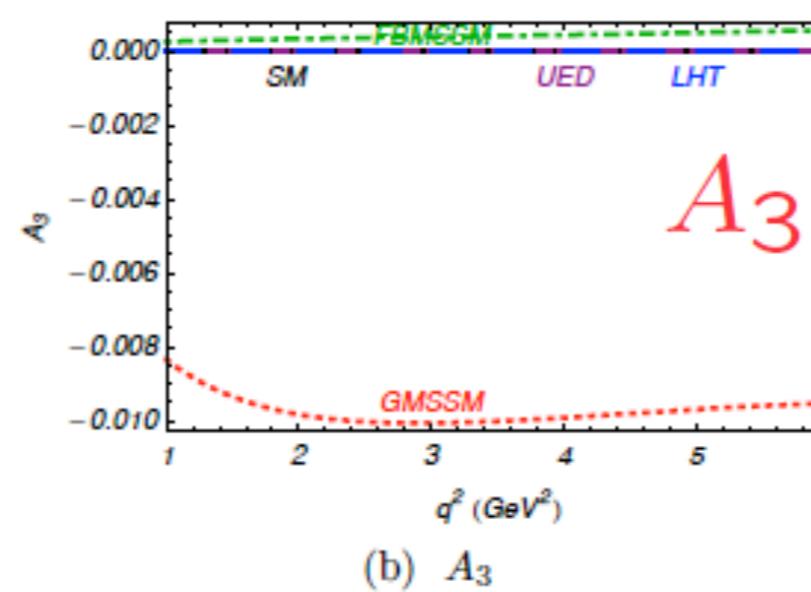
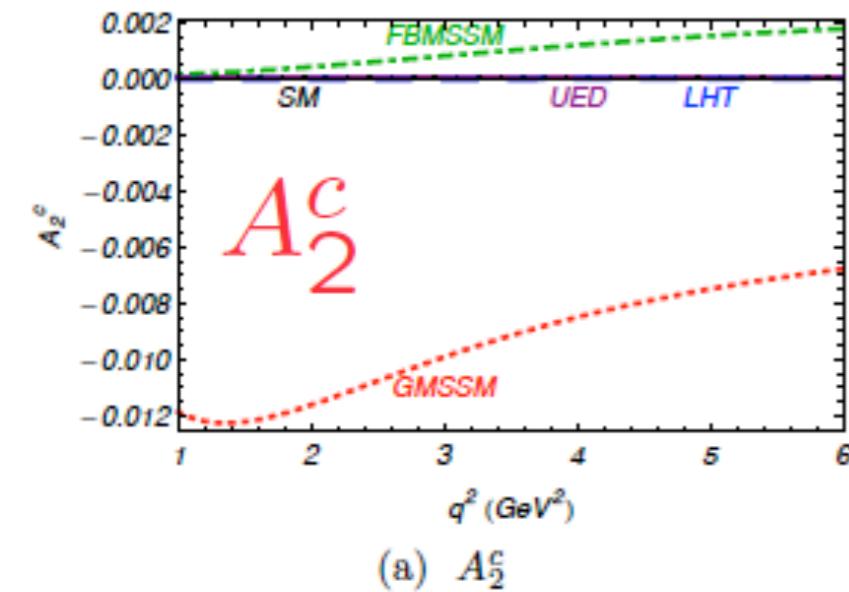
$$\mathbf{e}_z = \frac{\mathbf{p}_{K^-} + \mathbf{p}_{\pi^+}}{|\mathbf{p}_{K^-} + \mathbf{p}_{\pi^+}|}, \quad \mathbf{e}_l = \frac{\mathbf{p}_{\mu^-} \times \mathbf{p}_{\mu^+}}{|\mathbf{p}_{\mu^-} \times \mathbf{p}_{\mu^+}|}, \quad \mathbf{e}_K = \frac{\mathbf{p}_{K^-} \times \mathbf{p}_{\pi^+}}{|\mathbf{p}_{K^-} \times \mathbf{p}_{\pi^+}|}$$

$$\cos \theta_l = \frac{\mathbf{q}_{\mu^-} \cdot \mathbf{e}_z}{|\mathbf{q}_{\mu^-}|}, \quad \cos \theta_K = \frac{\mathbf{r}_{K^-} \cdot \mathbf{e}_z}{|\mathbf{r}_{K^-}|}, \quad \sin \phi = (\mathbf{e}_l \times \mathbf{e}_K) \cdot \mathbf{e}_z, \quad \cos \phi = \mathbf{e}_K \cdot \mathbf{e}_l$$

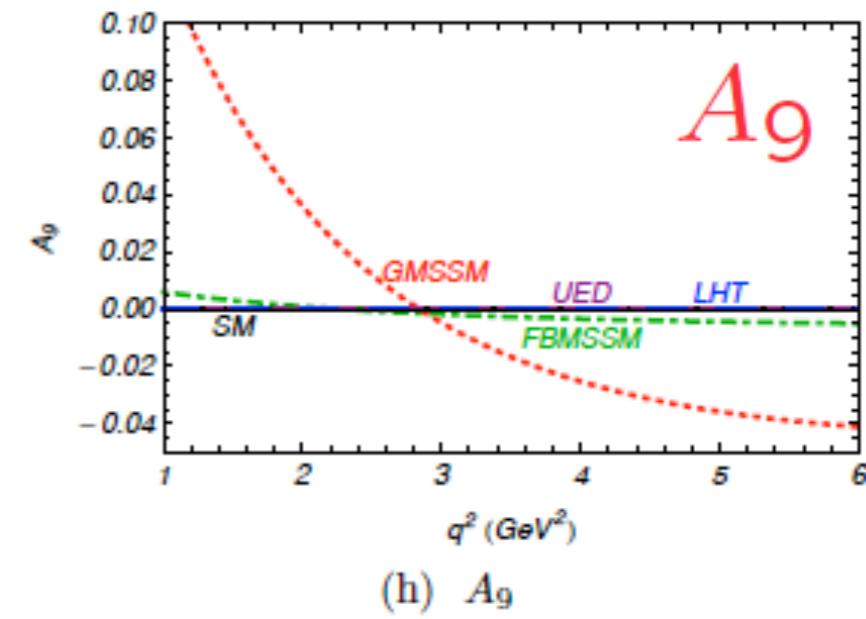
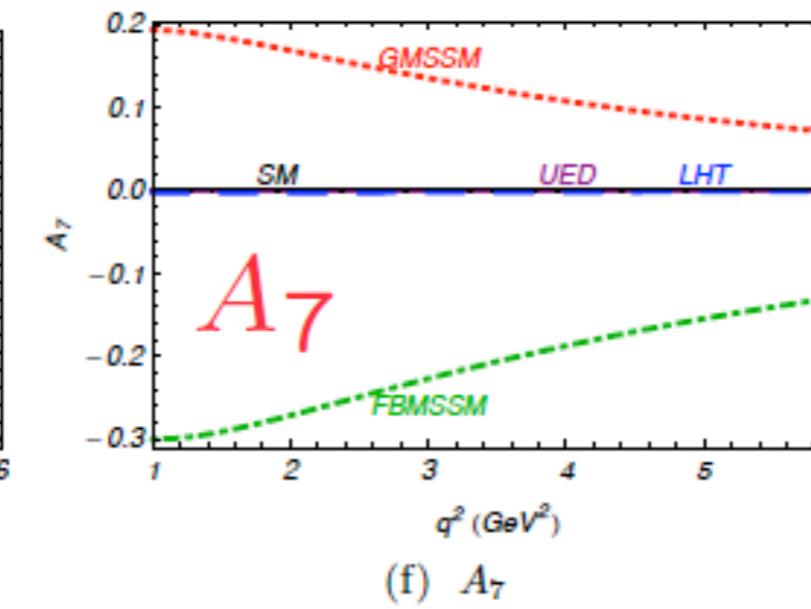
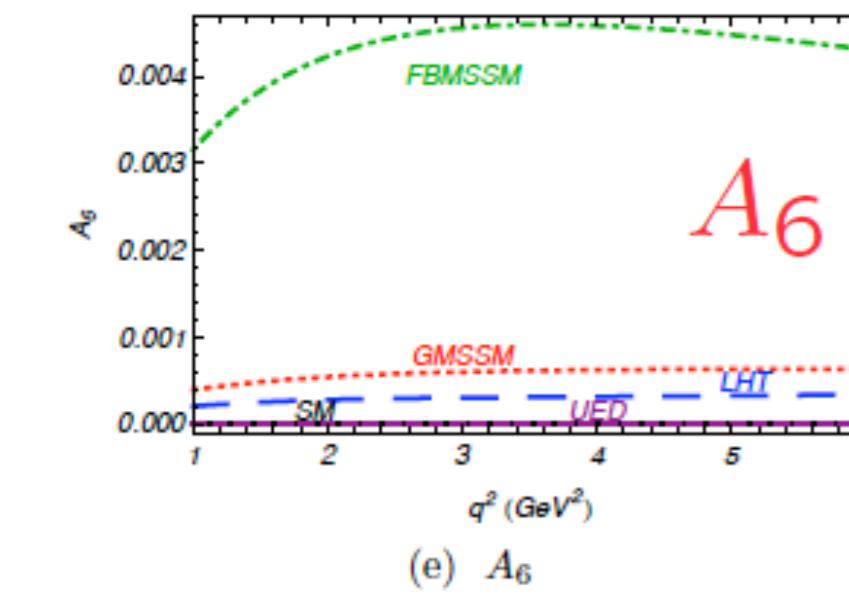
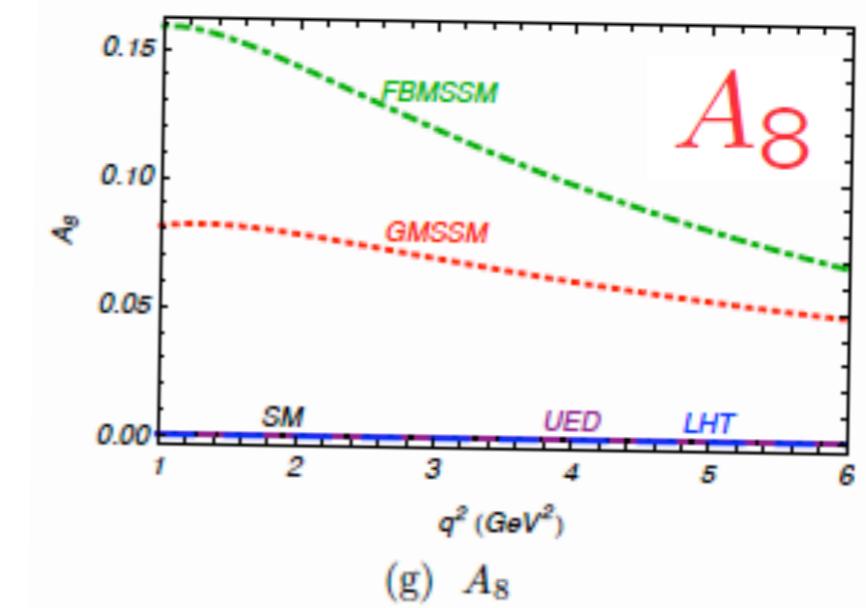
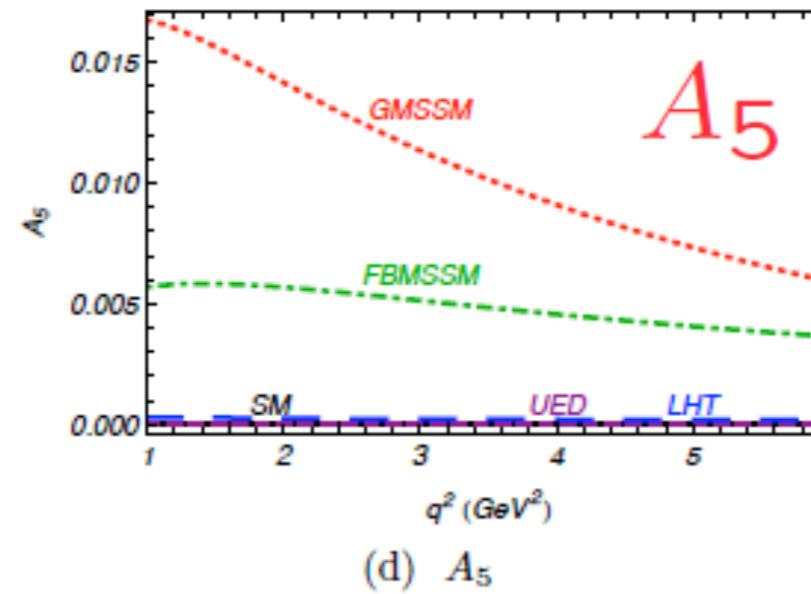
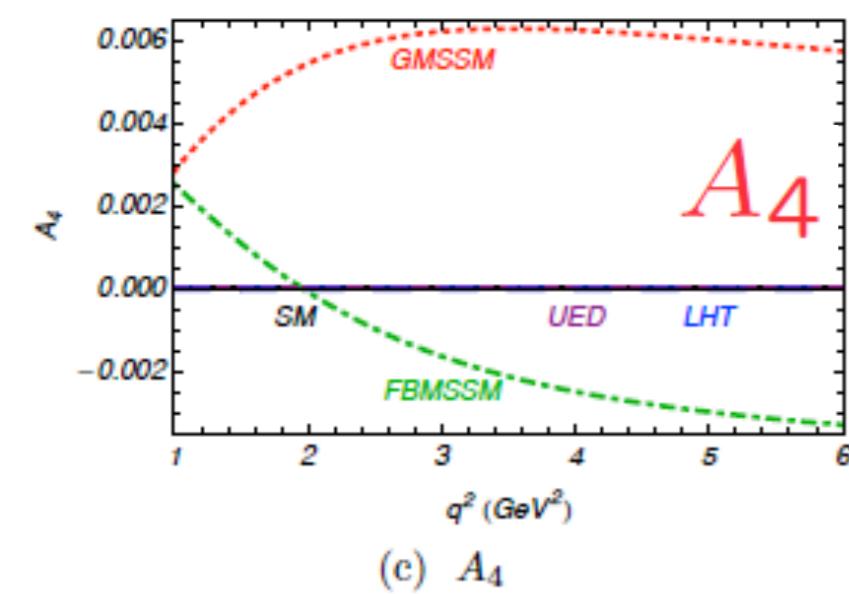
Benchmarks as introduced by Buras et al. 2008

	Model					
	SM	FBMSSM	GMSSM	LHT	UED	
$C_7^{\text{eff}}(\mu)$	-0.306	$0.031 + 0.475i$	$-0.186 + 0.002i$	-0.308	$-0.001i$	-0.297
$C_8^{\text{eff}}(\mu)$	-0.159	$-0.085 + 0.149i$	$-0.062 + 0.004i$	-0.159		-0.137
$\Delta C_9^{\text{eff}}(\mu)$	4.220	4.257	4.231	4.295	$+0.006i$	4.230
$C_{10}^{\text{eff}}(\mu)$	-4.093	-4.063	-4.241	-4.566	$-0.040i$	-4.212
$C_7'^{\text{eff}}(\mu)$	-0.007	$0.008 + 0.003i$	$0.155 + 0.160i$	-0.007		-0.007
$C_8'^{\text{eff}}(\mu)$	-0.004	$-0.000 + 0.001i$	$0.330 + 0.336i$	-0.004		-0.003
$C_9'^{\text{eff}}(\mu)$		0.002	$0.018 + 0.018i$			
$C_{10}'^{\text{eff}}(\mu)$		0.004	$0.003 + 0.003i$			
$(\mathcal{C}_S - \mathcal{C}'_S)(\mu)$		$-0.044 - 0.056i$	$0.000 + 0.001i$			
$(\mathcal{C}_P - \mathcal{C}'_P)(\mu)$		$0.043 + 0.054i$	$0.001 + 0.001i$			

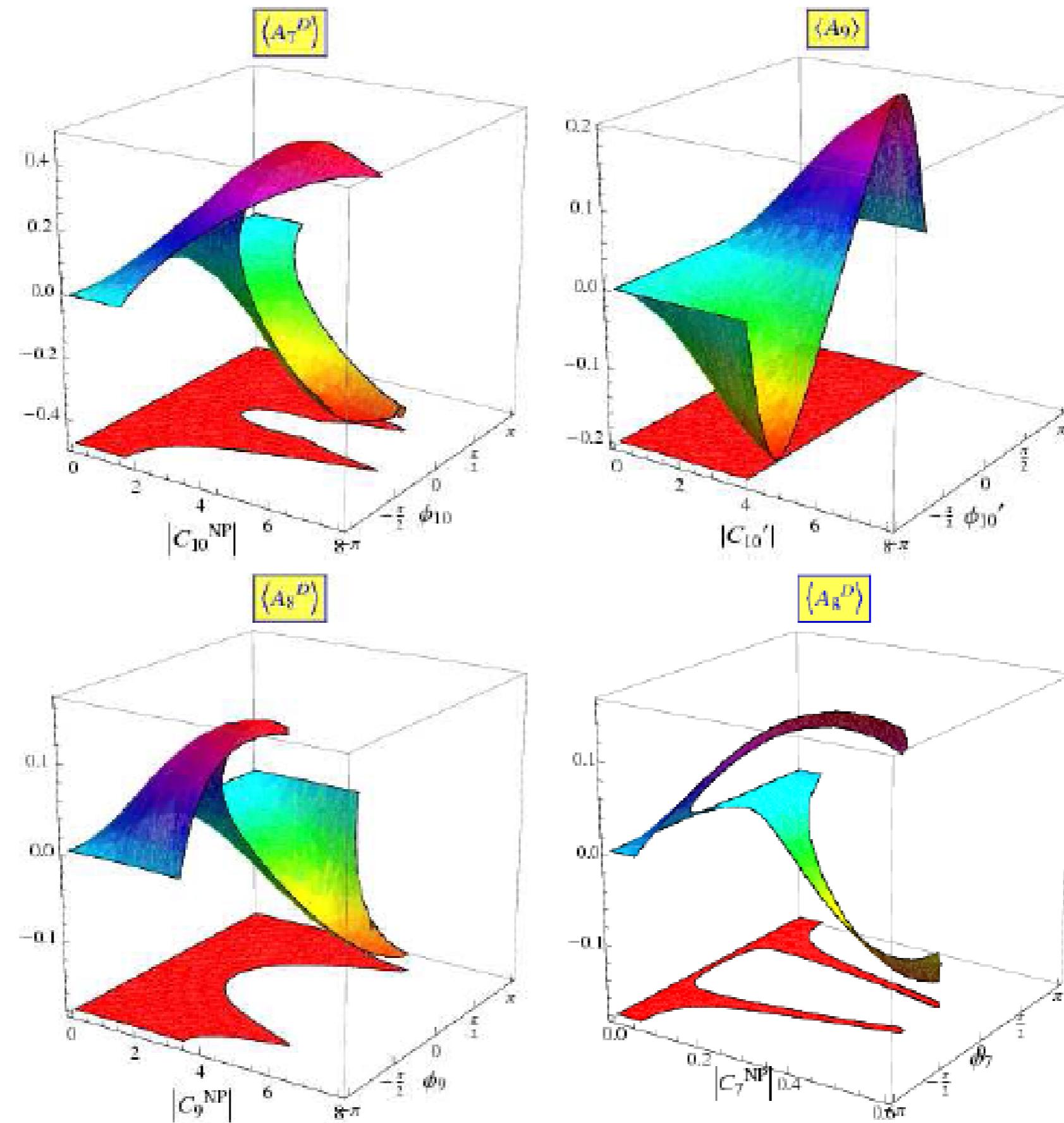
NP Wilson coefficients at $\mu = m_{b,\text{PS}}(2 \text{ GeV}/c^2) = 4.52 \text{ GeV}/c^2$.



$A_7 \ A_8 \ A_9$ favored



New physics phases not very much constrained (Bobeth,Hiller,Piranishvili 2008)



	generic NP	C_{10}^{NP} only	$C_{10}'^{\text{NP}}$ only	C_9^{NP} only
$\langle A_{\text{CP}} \rangle$	$[-0.12, 0.10]$	$[3, 8] \cdot 10^{-3}$	SM-like	$[-0.02, 0.02]$
$\langle A_3 \rangle$	$[-0.08, 0.08]$	SM-like	SM-like	SM-like
$\langle A_4^D \rangle$	$[-0.04, 0.04]$	$[-4, -1] \cdot 10^{-3}$	$[-3, -1] \cdot 10^{-3}$	$[-0.01, 0.01]$
$\langle A_5^D \rangle$	$[-0.07, 0.07]$	$[-0.04, 0.04]$	$[-0.02, 0.04]$	$[5, 9] \cdot 10^{-3}$
$\langle A_6 \rangle$	$[-0.13, 0.11]$	$[-0.05, 0.05]$	$[-9, -3] \cdot 10^{-3}$	SM-like
$\langle A_7^D \rangle$	$[-0.76, 0.76]$	$[-0.48, 0.48]$	$[-0.38, 0.38]$	SM-like
$\langle A_8^D \rangle$	$[-0.48, 0.48]$	$[2, 7] \cdot 10^{-3}$	$[-0.28, 0.28]$	$[-0.17, 0.17]$
$\langle A_9 \rangle$	$[-0.62, 0.60]$	SM-like	$[-0.20, 0.20]$	SM-like
$\mathcal{B}(\bar{B}_s \rightarrow \bar{\mu}\mu)$	$< 1.4 \cdot 10^{-8}$	$< 6.3 \cdot 10^{-9}$	$< 1.3 \cdot 10^{-8}$	SM

The ranges of the integrated CP asymmetries $\langle A_i^{(D)} \rangle$ for $(q_{\min}^2, q_{\max}^2) = (1, 6) \text{ GeV}^2$ applying the experimental constraints at 90% C.L.

- Transversity amplitude $A_T^{(1)}$

Defining the helicity distributions Γ_{\pm} as $\Gamma_{\pm} = |H_{\pm 1}^L|^2 + |H_{\pm 1}^R|^2$

one can define (Melikhov,Nikitin,Simula 1998)

$$A_T^{(1)} = \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+} \quad A_T^{(1)} = \frac{-2\text{Re}(A_{\parallel} A_{\perp}^*)}{|A_{\perp}|^2 + |A_{\parallel}|^2}$$

Very sensitive to right-handed currents (Lunghi,Matias 2006)

Big surprise:

$A_T^{(1)}$ is not invariant under the symmetries of the angular distribution

- $A_T^{(1)}$ cannot be extracted from the full angular distribution
- LHCb: practically not possible to measure the helicity of the final states on a event-by-event basis (neither as statistical distribution)
- Not a principal problem, but $A_T^{(1)}$ not an observable at LHCb or at Super B (measure three-momentum and charge)