Rare leptonic radiative B_s decays

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1. Introduction

- LHCb is probing $B_s \to \mu^+\mu^-$ at the SM level!!!

  ★ Very clean prediction in the Standard Model (one non-perturbative parameter)

$$\mathcal{B}r^{(\mathrm{SM})}_{B_s\to\mu^+\mu^-} = \frac{1}{8\pi^5}\cdot\frac{M_{B_s}}{\Gamma_{B_s}}\cdot\left(G_F^2 M_W^2 m_\mu f_{B_s}|V_{ts}^* V_{tb}|\eta_Y Y(\bar{x}_t)\right)^2\left[1-4\frac{m_\mu^2}{M_{B_s}^2}\right]^{1/2}$$

Buras, Carlucci, Gori, Isidori

$$\mathcal{B}r^{(\mathrm{SM})}_{B_s\to\mu^+\mu^-} = \frac{3}{4\pi^3}\cdot\frac{\Delta M_{B_s}^{(\mathrm{Expt})}}{\Gamma_{B_s}}\cdot\frac{(G_F M_W m_\mu \eta_Y Y)^2}{\hat{\eta}\hat{B}_{B_s}S_0(\bar{x}_t)}\left[1-4\frac{m_\mu^2}{M_{B_s}^2}\right]^{1/2} = (3.3\pm0.2)\times10^{-9}$$

$$\mathcal{B}^{(\mathrm{LD})}_{B_s\to\mu^+\mu^-}\sim 6\times10^{-11}$$

Burdman, et al
Golowich, Hewett, Pakvasa, AAP, Yeghiyan

- Why study rare radiative decays like $B_s \to \gamma\mu^+\mu^-$ then?

  ★ Standard Model rate for $B_s \to \mu^+\mu^-$ is helicity suppressed: $\dfrac{\mathcal{B}(B_s\to\gamma\ell^+\ell^-)}{\mathcal{B}(B_s\to\ell^+\ell^-)}\propto\alpha\dfrac{m_B^2}{m_\ell^2}$

  ★ $B_s \to \mu^+\mu^-$ is not sensitive to vector-like New Physics (e.g. vector Z')

  ★ Three-particle final state: more variables for amplitude analysis

  ★ Soft photon: irreducible background to $B_s \to \mu^+\mu^-$

  ★ Hard photon: model-independent calculations in Soft-Collinear Effective Theory

Introduction: hadronic parameters

- Weak effective hamiltonian for $B_s \to \mu^+\mu^-$ is simple

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \; C_{10}^{\text{eff}} \; \bar{s}\gamma_\mu(1-\gamma_5)b \; \bar{\ell}\gamma^\mu\gamma_5\ell$$

- Other operators (e.g. Q_9) do not contribute due to vector current conservation

$$\langle 0|\bar{s}\gamma_\mu(1-\gamma_5)b|B_s(p)\rangle = -if_B p_\mu$$

$$\downarrow$$

$$ip_\mu\langle\mu^+\mu^-|\bar{\ell}\gamma^\mu\ell|B_s(p)\rangle = \langle\mu^+\mu^-|\bar{\ell}\not{\partial}\ell|B_s\rangle = 0$$

$$ip_\mu\langle\mu^+\mu^-|\bar{\ell}\gamma^\mu\gamma_5\ell|B_s(p)\rangle = \langle\mu^+\mu^-|\bar{\ell}\not{\partial}\gamma_5\ell|B_s\rangle \propto m_\mu$$

One non-perturbative parameter: lattice

Introduction: hadronic parameters

- Weak effective hamiltonian for $B_s \to \gamma\mu^+\mu^-$ is considerably more complicated

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi} V_{tb} V_{ts}^* \Big[\; C_9 \; \bar{s}\gamma_\mu (1-\gamma_5) b \; \bar{\ell}\gamma^\mu \ell$$
$$+ C_{10} \; \bar{s}\gamma_\mu (1-\gamma_5) b \; \bar{\ell}\gamma^\mu \gamma_5 \ell$$
$$- \frac{2C_7}{q^2} \; i m_b \; \bar{s}\sigma_{\mu\nu} q^\nu (1+\gamma_5) b \; \bar{\ell}\gamma^\mu \ell \Big]$$

Recall: only Q_{10} contributed to $B_s \to \gamma\mu^+\mu^-$ decay!

- ... this implies that the amplitude $A_{\ell\ell\gamma} = \langle \gamma(k), \ell^+(p_1), \ell^-(p_2) | H_{eff} | B(p) \rangle$ depends on

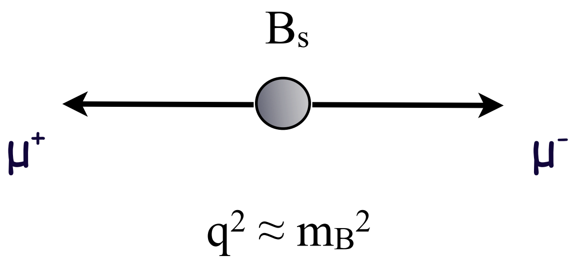
$$\begin{aligned}
M_{B_s} \langle \gamma(k,\epsilon) | \bar{s}\gamma_\mu b | B(p) \rangle &= e F_V(q^2) \; \epsilon_{\mu\nu\alpha\beta} \; \epsilon^{*\nu} p^\alpha k^\beta \\
M_{B_s} \langle \gamma(k,\epsilon) | \bar{s}\gamma_\mu \gamma_5 b | B(p) \rangle &= i e F_A(q^2) (g_{\mu\nu} \; p \cdot k - k_\mu p_\nu) \epsilon^{*\nu} \\
\langle \gamma(k,\epsilon) | \bar{s}\sigma_{\mu\theta} b | B(p) \rangle q^\theta &= i e F_{TV}(q^2) \; \epsilon_{\mu\nu\alpha\beta} \; \epsilon^{*\nu} p^\alpha k^\beta \\
\langle \gamma(k,\epsilon) | \bar{s}\sigma_{\mu\theta} \gamma_5 b | B(p) \rangle q^\theta &= e F_{TA}(q^2) (g_{\mu\nu} \; p \cdot k - k_\mu p_\nu) \epsilon^{*\nu}
\end{aligned}$$

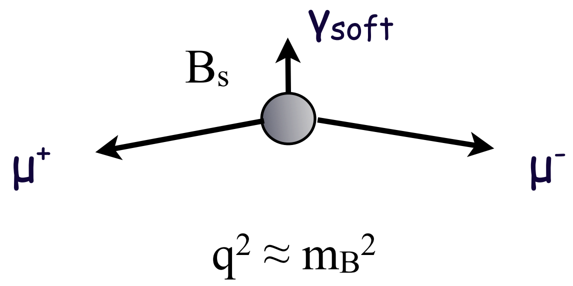
Four non-perturbative functions!

2. Soft photons in $B_s \to \gamma\mu^+\mu^-$ transitions

➢ Soft photons can make $B_s \to \gamma_{soft}\, \mu^+\mu^-$ look like $B_s \to \mu^+\mu^-$ ...

★ Experimentally, irreducible background





★ Theoretically, potentially large effect

➡ helicity suppression is absent in $B_s \to \gamma_{soft}\, \mu^+\mu^-$

➡ resonant enhancement of form-factors, e.g. vector part (Q_9):

$$M_{B_s}\langle\gamma(k,\epsilon)|\bar{s}\gamma_\mu b|B_s(p)\rangle = M_{B_s}\sum_{B_s^*\text{-pol's}}\frac{\langle 0|\bar{s}\gamma_\mu b|B_s^*\rangle\langle\gamma B_s^*|B_s\rangle}{q^2 - M_{B^*}^2}$$

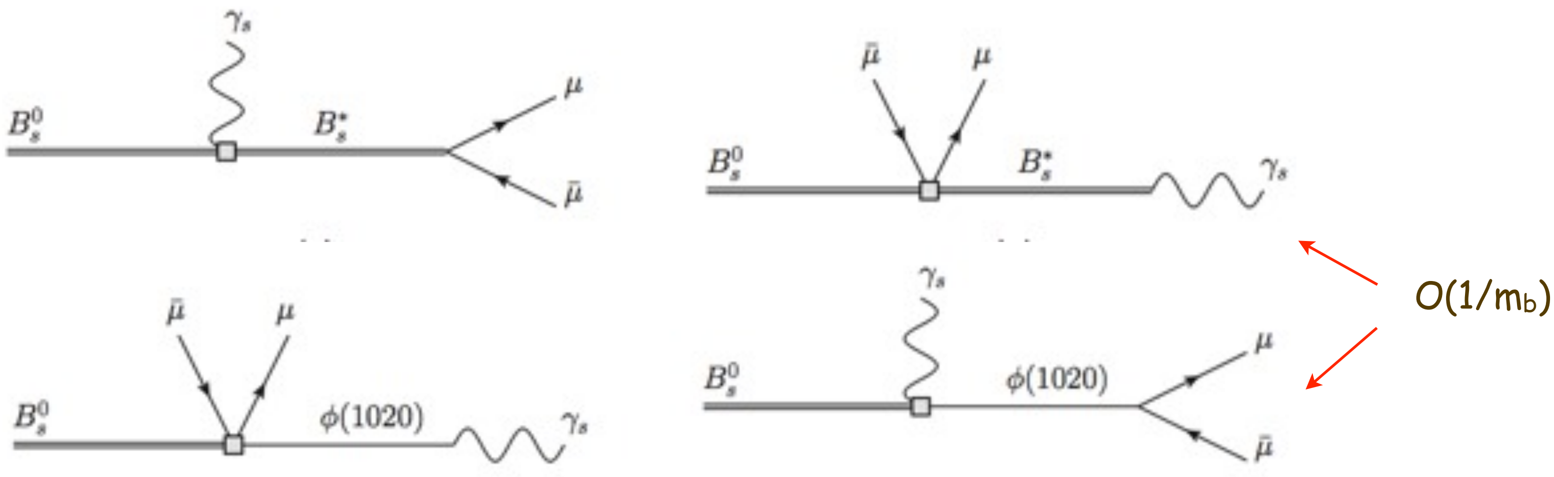
For $q^2 \approx M_B^2$: $$F_V(q^2) = \frac{f_{B_s^*} M_{B_s} g_{B_s^* B_s \gamma}}{M_{B_s^*}}\frac{1}{1 - q^2/M_{B_s^*}^2}$$

Becirevic, Haas, Kou

Soft photons in $B_s \to \gamma\mu^+\mu^-$ transitions

Aditya, Healey, AAP

- Calculate $B_s \to \gamma_{soft}\,\mu^+\mu^-$ in heavy meson chiral perturbation theory
  - ★ Diagrams with photon emission from the lepton legs are helicity-suppressed
  - ★ Structure-dependent contributions to $B_s \to \gamma_{soft}\,\mu^+\mu^-$ include



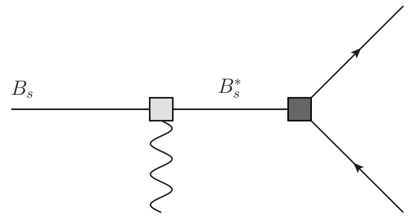
$+$ etc.

  - ★ Calculate leading-order contributions (in $1/m_b$)

Soft photons in $B_s \to \gamma\mu^+\mu^-$ transitions

Aditya, Healey, AAP

- Largest contribution to $B_s \to \gamma_{soft}\,\mu^+\mu^-$
  - ★ $B_s \to \gamma_{soft}\, B_s^*$ vertex depends on the magnetic moment

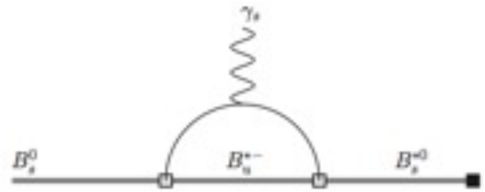


$$\mathcal{M}_{B_s \to B_s^* \gamma} = -ie\mu\eta^*_\alpha v_\beta k_\mu \epsilon^*_\nu \epsilon^{\mu\nu\alpha\beta}$$

  - ★ ... which can be obtained from the following Lagrangians...

$$\mathcal{L} = \mathcal{L}_h + \mathcal{L}_l = \frac{q_Q}{2m_Q}\bar{h}_v\sigma_{\mu\nu}h_v F^{\mu\nu} + \frac{\beta e}{4} Tr\left[\bar{H}_a H_b \sigma^{\mu\nu} F_{\mu\nu} Q^\xi_{ba}\right]$$

  - ★ ... and is given by $\mu = -\dfrac{1}{3m_b} - \dfrac{1}{3}\beta + g^2\dfrac{m_K}{4\pi f_K^2}$



  - ★ Extract g and β from

$$\Gamma(D^{*+} \to D^+\gamma) = \frac{\alpha}{3}\left(\frac{2}{3m_c} - \frac{1}{3}\beta + g^2\frac{m_\pi}{4\pi f_\pi^2}\right)^2 |\vec{k}|^3,$$

$$\Gamma(D^{*+} \to D^0\pi^+) = \frac{g^2}{6\pi f_\pi^2}|\vec{p_\pi}|^3.$$

Soft photons in $B_s \to \gamma\mu^+\mu^-$ transitions

- Estimate $B_s \to \gamma_{soft}\, \mu^+\mu^-$ numerically:

  ★ Make "CDF q^2-cut": $\sqrt{q^2} > 4.669$ GeV (or $E_\gamma < 652$ MeV)

$$\mathcal{B}(B_s \to \mu^+\mu^-\gamma_{soft}) = 0.65 \times 10^{-9}$$

Aditya, Healey, AAP; preliminary

  ★ Recall that

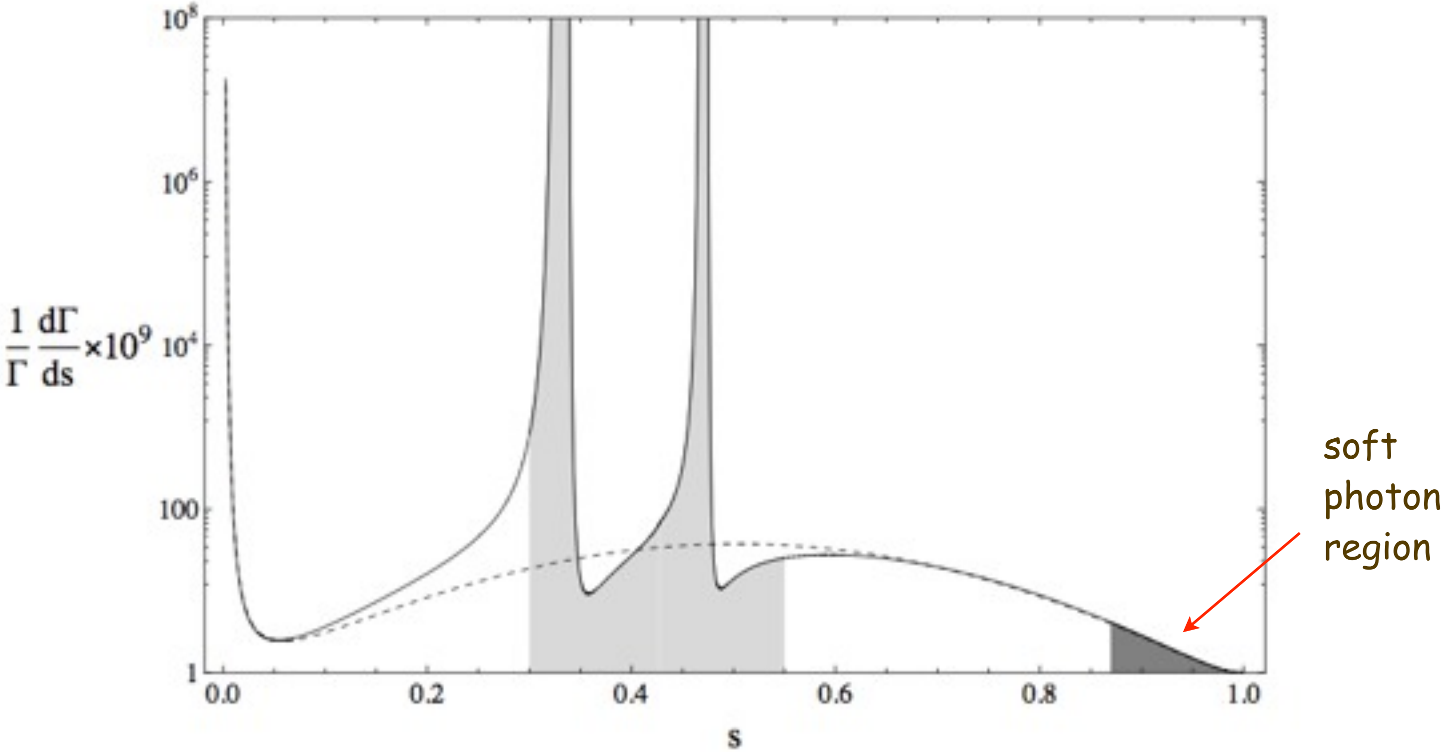
$$\begin{aligned} \mathcal{B}(B_s \to \mu^+\mu^-)_{SM} &= (3.3 \pm 0.2) \times 10^{-9} \\ \mathcal{B}(B_s \to \mu^+\mu^-)_{CDF} &< 4.6 \times 10^{-9}\ (95\%\ CL) \\ \mathcal{B}(B_s \to \mu^+\mu^-)_{LHCb} &< 4.5 \times 10^{-9}\ (95\%\ CL) \end{aligned}$$

  ★ The $s = (\sqrt{q^2})/M_B$ spectrum can also be calculated

$$\frac{d\Gamma}{ds} = \frac{G_F^2 |V_{tb}V_{ts}^*|^2 \alpha^3 \mu^2 f_B^2 M_B^5}{768\pi^4} \frac{(1-s)^3}{s^2(2\hat{\Delta}+1-s)^2} \sqrt{1-\frac{4\rho^2}{s}} \Big[(s+2\rho^2)(2C_7^2 \hat{m}_b{}^2 (s^2+1) + 2C_9 C_7 \hat{m}_b s(s+1) + C_9^2 s^2) + C_{10}^2 s^2 (s-4\rho^2) \Big]$$

Aditya, Healey, AAP; preliminary

Pretend that we know low-q^2 asymptotic



Aditya, Healey, AAP;
very preliminary

$$\mathcal{B}(B_s \to \mu^+\mu^-\gamma) = 2.0 \times 10^{-9}$$

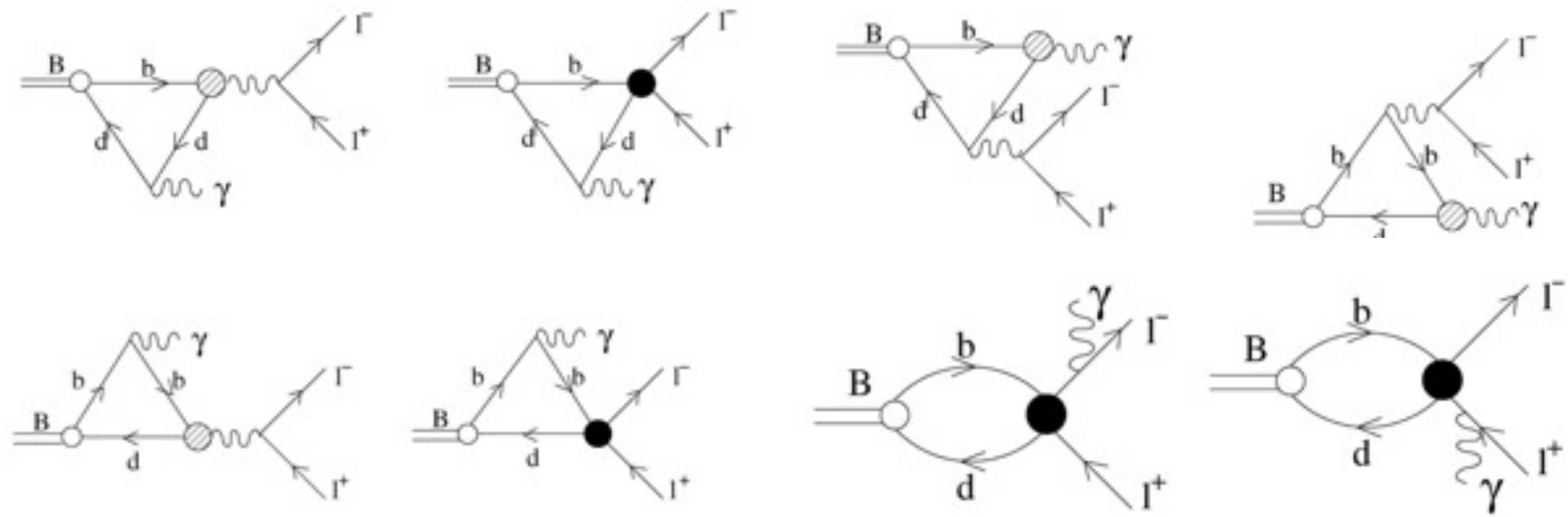
What if we stop pretending?

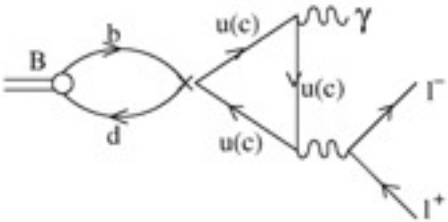
3. Hard photons in $B_s \to \gamma\mu^+\mu^-$ transitions

- Model-dependent: calculate form-factors for $B_s \to \gamma\mu^+\mu^-$ in a quark model

  - ★ We guess (calculate?) q^2-dependence of the four hadronic form-factors

Kruger, Melikhov;
Geng, Lih, Zhang;
Dincer, Sehgal;
Aliev, Ozpineci, Savci





Most complete and thorough study: D. Melikhov, N. Nikitin PRD70, 114028 (2004)

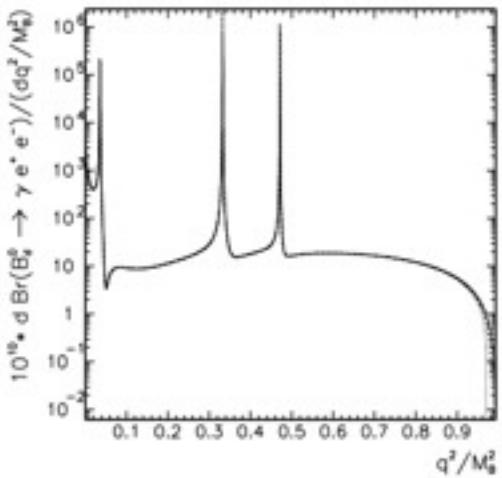
Hard photons in $B_s \to \gamma\mu^+\mu^-$ transitions

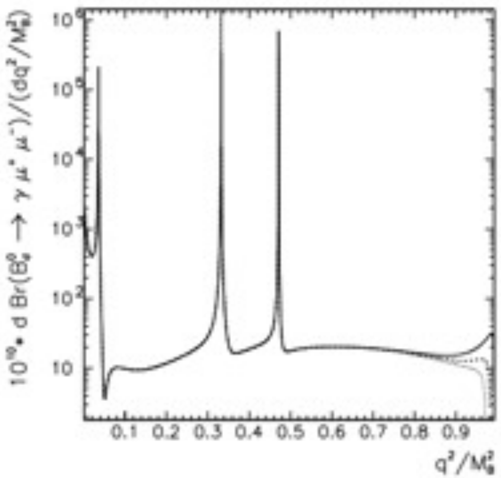
★ Cutting off low-E_γ region, can predict total branching fractions

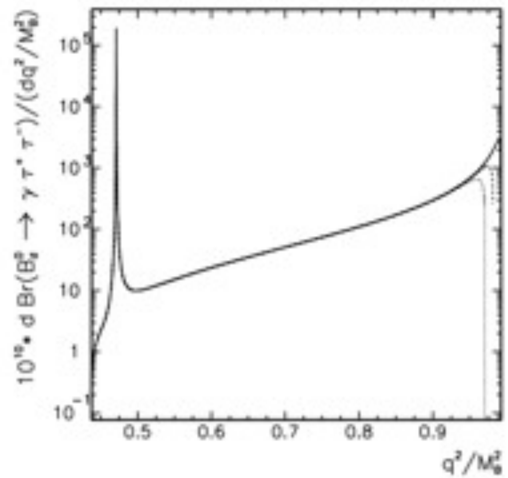
m_ℓ	m_e			m_μ			m_τ		
$E^\gamma_{\min}$ (MeV)	20	50	80	20	50	80	20	50	80
$Br(B_d \to \ell^+\ell^-\gamma) \times 10^{10}$ [This work]	3.95	3.95	3.95	1.34	1.32	1.31	3,39	2.37	1.87
$Br(B_s \to \ell^+\ell^-\gamma) \times 10^{9}$ [This work]	24.6	24.6	24.6	18,9	18.8	18.8	11.6	8.10	6.42
$Br(B_d \to \ell^+\ell^-\gamma) \times 10^{10}$ [5]	1.01	1.01	1.01	0.66	0.62	0.61	3.39	2.38	1.88
$Br(B_s \to \ell^+\ell^-\gamma) \times 10^{9}$ [5]	3.30	3.29	3.29	2.16	2.06	2.00	11.6	8.15	6.47
$Br(B_s \to \ell^+\ell^-\gamma) \times 10^{9}$ [6]	20	20	20	12	12	12	...	...	...

D. Melikhov, N. Nikitin
PRD70, 114028 (2004)

★ ... and differential distributions







Hard photons in $B_s \to \gamma\mu^+\mu^-$ transitions

- Next step: model-independent calculation of $B_s \to \gamma\mu^+\mu^-$

  ★ Can write a factorization theorem which allows calculation of form-factors perturbatively (in terms of light cone B_s meson wave function)

$$A(B_s \to \gamma\mu^+\mu^-) = \underbrace{C(E_\gamma/m_b)}_{\text{Wilson coefficient}} \int \frac{dn\cdot k}{4\pi} \underbrace{J(n\cdot k)}_{\text{Jet function}} \underbrace{\Psi_B(n\cdot k)}_{\text{Light cone wave function (from } B_s \to \gamma\mu\nu)}$$

  ★ The results will be available soon

Aditya, Blechman, Healey, Paz, AAP

What to do with all that?

- New Physics studies: more variables to look at

★ NP could induce new terms in the effective Hamiltonian

Alok, Datta, Duraisamy, Ghosh, London

$$\mathcal{H}_{\rm eff}^{VA} = -\frac{4G_F}{\sqrt{2}}\frac{\alpha_{em}}{4\pi}V_{ts}^*V_{tb}\Big\{R_V\,(\bar{s}\gamma^\mu P_L b)\,\bar{\mu}\gamma_\mu\mu + R_A\,(\bar{s}\gamma^\mu P_L b)\,\bar{\mu}\gamma_\mu\gamma_5\mu$$
$$+ R_V'\,(\bar{s}\gamma^\mu P_R b)\,\bar{\mu}\gamma_\mu\mu + R_A'\,(\bar{s}\gamma^\mu P_R b)\,\bar{\mu}\gamma_\mu\gamma_5\mu\Big\},$$
$$\mathcal{H}_{\rm eff}^{SP} = -\frac{4G_F}{\sqrt{2}}\frac{\alpha_{em}}{4\pi}V_{ts}^*V_{tb}\Big\{R_S\,(\bar{s}P_R b)\,\bar{\mu}\mu + R_P\,(\bar{s}P_R b)\,\bar{\mu}\gamma_5\mu$$
$$+ R_S'\,(\bar{s}P_L b)\,\bar{\mu}\mu + R_P'\,(\bar{s}P_L b)\,\bar{\mu}\gamma_5\mu\Big\},$$
$$\mathcal{H}_{\rm eff}^{T} = -\frac{4G_F}{\sqrt{2}}\frac{\alpha_{em}}{4\pi}V_{ts}^*V_{tb}\Big\{C_T(\bar{s}\sigma_{\mu\nu}b)\bar{\mu}\sigma^{\mu\nu}\mu + iC_{TE}(\bar{s}\sigma_{\mu\nu}b)\bar{\mu}\sigma_{\alpha\beta}\mu\ \epsilon^{\mu\nu\alpha\beta}\Big\}$$

★ ... which could be probed in CP-violating and forward-backward asymmetries

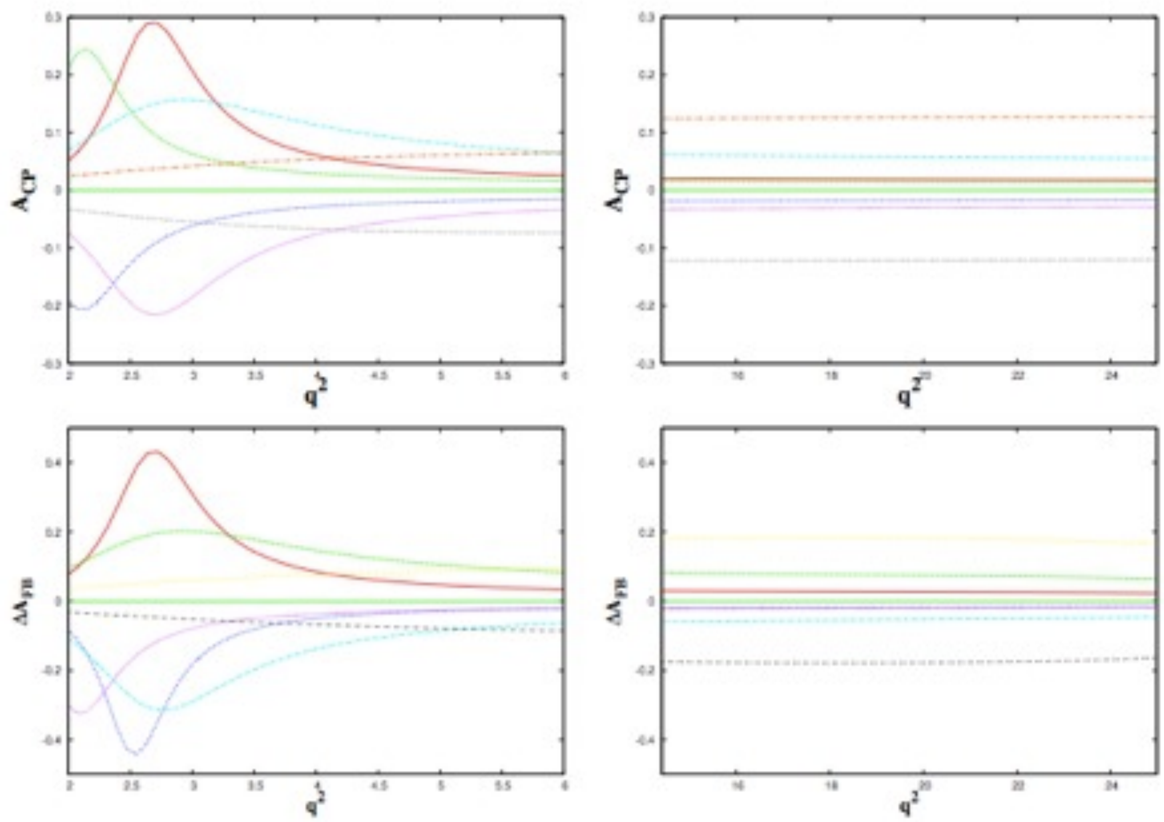
$$A_{CP}(q^2) = \frac{(dB/dz) - (d\overline{B}/dz)}{(dB/dz) + (d\overline{B}/dz)} \quad \text{and} \quad A_{FB}(q^2) = \frac{\int_0^1 d\cos\theta_\mu \frac{d^2B}{dq^2 d\cos\theta_\mu} - \int_{-1}^0 d\cos\theta_\mu \frac{d^2B}{dq^2 d\cos\theta_\mu}}{\int_0^1 d\cos\theta_\mu \frac{d^2B}{dq^2 d\cos\theta_\mu} + \int_{-1}^0 d\cos\theta_\mu \frac{d^2B}{dq^2 d\cos\theta_\mu}}$$

What to do with all that?

➢ New Physics studies: more variables to look at

Alok, Datta, Duraisamy, Ghosh, London

★ E.g., if NP only induces only R_V and R_A couplings, then for different values



Things to take home

- There are good reasons to study rare radiative leptonic B-decays
  - important irreducible background to $B_s \to \mu^+\mu^-$ (for soft photons)
  - probes different models of New Physics than $B_s \to \mu^+\mu^-$ (incl. vectors)
  - provides more variables to study NP and hadronic parameters (hard photons)