MONOPOLES, HOLONOMY AND CHIRAL SYMMETRY BREAKING IN QCD

A. Di Giacomo

Pisa University and INFN, Italy.

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INTRODUCTION

QUARKS STRICTLY CONFINED IN NATURE

- $\frac{n_q}{n_p} \leq 10^{-27}$ EXPECT 10^{-12}
- $\frac{\sigma_q}{\sigma_{TOT}} \leq 10^{-15}$ EXPECT $O(1)$

NATURAL EXPLANATION : A SYMMETRY.
DECONFINEMENT A CHANGE OF SYMMETRY.

PURE GAUGE $SU(N)$ (NO QUARKS)

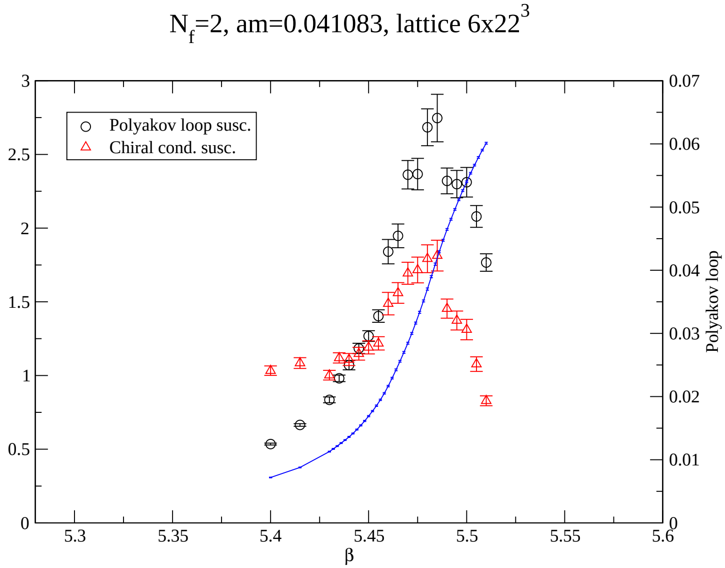
LAGRANGEAN BLIND TO THE CENTRE Z_N (SYMMETRY).
ORDER PARAMETER : POLYAKOV LINE $= \langle P \rangle$,
$P = Tr[(\tau \exp(ig \int_{x_0}^{x_0+\frac{1}{T}} A_0(\vec{x}, t) dt)]$ [HOLONOMY]

- IF $Z|0\rangle = |0\rangle$, $\langle P \rangle = \langle PZ \rangle = Z_i \langle P \rangle \rightarrow \langle P \rangle = 0$.
  $\langle P \rangle = \exp(-\frac{F_q}{T})$. $\langle P \rangle = 0 \rightarrow$ CONFINEMENT.
- IF Z_N SYMMETRY IS BROKEN (DECONFINEMENT)
  $\langle P \rangle \neq 0$

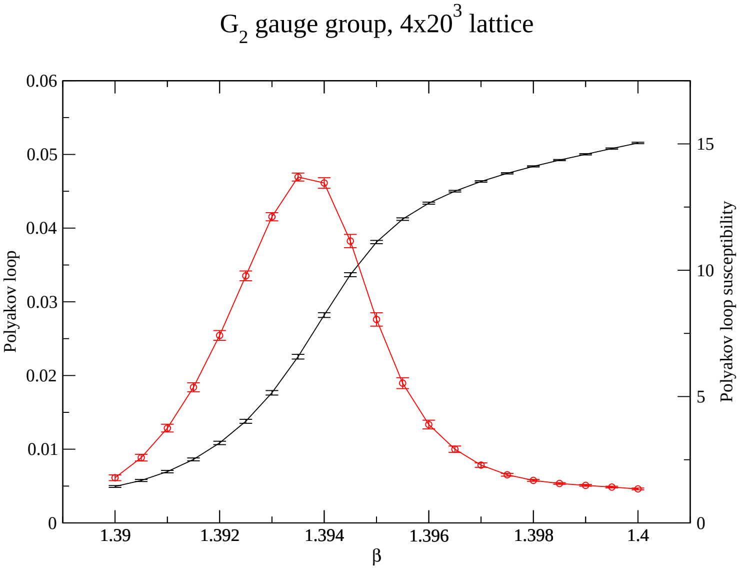
IS Z_N THE SYMMETRY FOR CONFINEMENT IN NATURE?
NO, BECAUSE

- IN PRESENCE OF QUARKS Z_N IS NOT A SYMMETRY.
- $\langle P \rangle$ IS A GOOD ORDER PARAMETER ALSO IN PRESENCE OF QUARKS AND FOR GROUPS LIKE G_2 WITH TRIVIAL CENTER. (see Fig.'s)

$\langle P \rangle$ in full QCD.



$\langle P \rangle$ for G_2 GAUGE THORY.



THE SYMMETRY

$\langle P \rangle$ ACTS AS ORDER PARAMETER OF ANOTHER SYMMETRY, WHICH COINCIDES WITH Z_N BY CHANCE IN THE QUENCHED $SU(N)$ CASE.

- $\langle P \rangle$ (POLYAKOV LINE) ORDER PARAMETER FOR DUAL SUPERCONDUCTIVITY OF THE VACUUM.
- $\langle \Pi \rangle$ 'tHOOFT LINE (DUAL HOLONOMY) IS THE DISORDER PARAMETER
- CHIRAL SYMMETRY BREAKING LOCKED TO MONOPOLE CONDENSATION.
  $\langle \bar{\psi}\psi \rangle \propto \langle \Pi \rangle$.

'tHOOFT POLYAKOV MONOPOLES

$$L = -\frac{1}{4}\vec{G}_{\mu\nu}\vec{G}_{\mu\nu} + (D_\mu\vec{\Phi})^2 - \frac{\lambda}{4}(\vec{\Phi}^2 - \frac{\mu^2}{\lambda})^2$$

$\mu^2 > 0 \rightarrow$ HIGGS PHASE : $\vec{\Phi}^2 = \frac{\mu^2}{\lambda}$. $\quad \bar{\Phi} \equiv \frac{\mu}{\sqrt{\lambda}}$

- HEDGEHOG GAUGE $\equiv$ LANDAU GAUGE $[\partial_\mu A_\mu = 0]$
  $\vec{\Phi}\vec{\sigma} = \vec{\sigma}\hat{r}\bar{\Phi}H(\xi)$, $\quad \xi = rg\bar{\Phi} \quad H(\xi)_{\xi\rightarrow\infty} \rightarrow 1$

  $\vec{B}\vec{\sigma}_{\xi\rightarrow\infty} \approx \frac{\hat{r}}{2g}\frac{\hat{r}\vec{\sigma}}{r^2}$, $\Phi_{S_2}(B_3) = 0.$ $\rho \approx \frac{1}{g\bar{\Phi}}$, $M \approx \frac{2\pi}{g}\bar{\Phi}$.
  A NON TRIVIAL HOMOTOPY $S_2 \rightarrow SU(2)$.
- UNITARY GAUGE $\equiv$ MAXIMAL ABELIAN GAUGE
  $[D^0_\mu A^\pm_\mu \equiv \partial_\mu A^\pm_\mu \mp i[A^0_\mu, A^\pm_\mu] = 0$
  $(\vec{\Phi}\vec{\sigma})_{\xi\rightarrow\infty} \approx \bar{\Phi}\sigma_3 \quad (\vec{B}\vec{\sigma}) = \sigma_3\frac{1}{2g}\frac{\hat{r}}{r^2}$,$\Phi_{S_2}(B_3) = \frac{2\pi}{g}$.
- CHARGE CONJUGATION; $\Phi \rightarrow \Phi$ $A_\mu \rightarrow -A_\mu$ $G_{\mu\nu} \rightarrow -G_{\mu\nu}$, $D_\mu\Phi \rightarrow D_\mu\Phi$. ASYMPTOTIC HIGGS FIELD THE SAME FOR MONOPOLE AND ANTIMONOPOLE.

QCD MONOPOLES$_1$

MONOPOLES WITH $m = \pm 1$ STABILIZED BY HIGGS FIELD. IN LATTICE *QCD* STABLE MONOPOLES DETECTED IN MAX ABELIAN GAUGE, SMALL FLUCTUATIONS (MONOPOLE DOMINANCE) . WHAT ACTS AS HIGGS FIELD?

EQ.S OF MOTION

$$D_\nu G_{\mu\nu} = i[\Phi, D_\mu \Phi] \ ,$$
$$D_n G_{mn} = D_0 G_{m0} + i[\Phi, D_m \Phi]$$

IN THE STATIC CASE $(\partial_0 = 0)$, $D_0 G_{m0} = i[A_0, D_m A_0]$

$$D_n G_{mn} = i[\Phi, [D_m, \Phi] + i[A_0, [D_m, A_0]]$$

A_0 ACTS AS EFFECTIVE Φ .

A_0 TIME INDEPENDENT BY A G.T.DEPENDING ON x_0

$$P = \exp(i \frac{A_0 g}{T})$$

NON TRIVIAL HOLONOMY $\leftrightarrow$ STABLE MONOPOLES.

CONFINEMENT BOSE CONDENSATION.

QCD MONOPOLES$_2$

THE MAX ABELIAN GAUGE IS FIXED BY MAXIMIZING F
$\delta F \equiv \delta \Sigma_{n\mu} Tr[U_\mu(n)\sigma_3 U^\dagger_\mu(n)\sigma_3] = 0 \quad (1)$
IF MONOPOLES ARE LOW DENSITY AND DOMINATE
$U_i(n) \approx 1$ AND EQ (1) BECOMES
$\delta \Sigma_n Tr[U_0(n)\sigma_3 U^\dagger_0(n)\sigma_3] = 0$ OR $U_0(n) = \exp(igA_0 a\sigma_3)$
THE POLYAKOV LINE IS DIAGONAL : MAX ABELIAN AND POLYAKOV GAUGE APPROXIMATELY COINCIDE IN THE CONFINED PHASE.
$\langle P \rangle$ IS GAUGE INVARIANT AND $= 0$ AT LARGE VOLUMES IN CONFINED PHASE. $-\frac{i}{2} Tr[\sigma_3 P] = \sin(\frac{gA_0^{(3)}}{T})$ IN MAX ABELIAN GAUGE.
$gA_0^{(3)}$ RELATED TO SIZE AND MASS OF MONOPOLES.
EXTENDED CHECKS ON LATTICE UNDER WAY.

INSTANTONS AND MONOPOLES

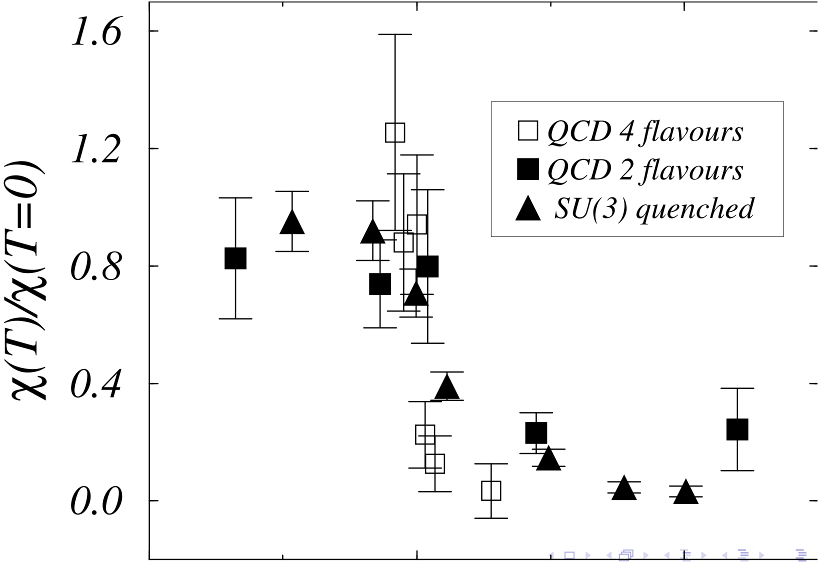
FINITE T INSTANTONS (CALORONS) KNOWN AS EXACT SOLUTIONS ON LATTICE [Lee, vanBaal]
IF HOLONOMY IS NON TRIVIAL EACH OF THEM CONTAINS A $M - \bar{M}$ PAIR OF BPS TYPE.
VICEVERSA, IF A MONOPOLE IS PRESENT (A NON TRIVIAL HOLONOMY) THERE IS A FINITE PROBABILITY TO HAVE AN INSTANTON [Rubakov effect]

WORKING HYPOTHESIS :

NUMBER OF INSTANTONS $\propto$ NUMBER OF MONOPOLES
HOLONOMY $\langle P \rangle$ ALSO CONTROLS CHIRAL SYMMETRY.

A GAS OF INSTANTONS, EACH WITH A BPS PAIR $M - \bar{M}$.
MONOPOLES WEAKLY INTERACTING.
(UNDER CHECK IN COLLABORATION WITH F.PUCCI AND M. HASEGAWA.)
DENSITY OF INSTANTONS ACROSS T_c (FIG)

B.Alles,M.DElia,A. Di Giacomo, Phys.Lett.B483, 139 (2000)

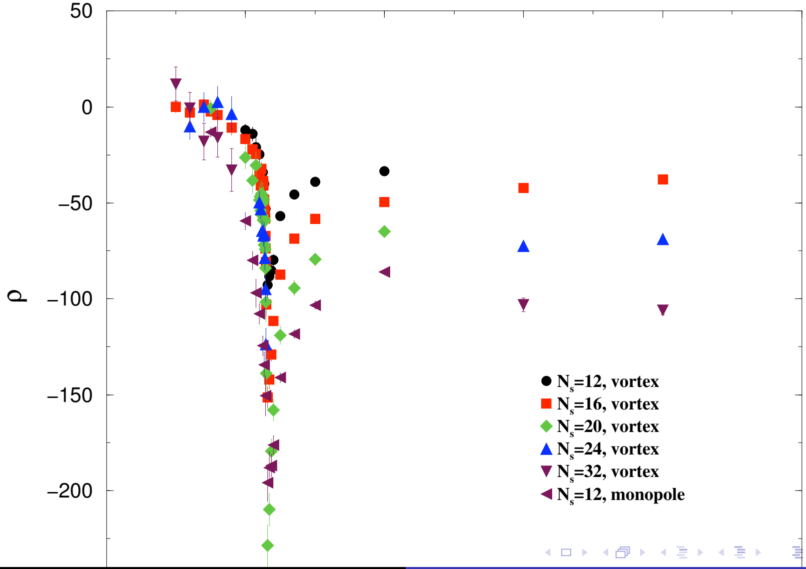


'tHOOFT LINE

A DUAL HOLONOMY $\langle \Pi \rangle$ DEFINED BY 'tHOOFT LOOPS $B(C)$
$A(C)B(C') = B(C')A(C)exp(i\frac{2\pi n_{CC'}}{N})$.
$\langle \Pi \rangle$ DUAL PAR. TRANSP. ACROSS LATTICE, SAY ALONG z.
$\langle \Pi \rangle \langle P \rangle = 0$

ADDING A 'tHOOFT LINE IS LIKE CREATING A $M - \bar{M}$ PAIR AT THE ENDS.
CONDENSATION $\rightarrow$ DEBYE SCREENING =CLUSTER PROPERTY
$\langle \Pi \rangle = \langle a^\dagger b^\dagger \rangle$ $a^\dagger, b^\dagger$ CREATION OPERATORS OF M , $\bar{M}$
COMPARE TO DIRECT DEFINITION.

L.Del Debbio, A. Di Giacomo, B.LuciniNucl.Phys. B594 287(2001)



GENERIC GAUGE GROUP

A MONOPOLE SPECIES FOR EACH SIMPLE ROOT i , $i = 1, 2, ...r$, (r THE RANK). MAXIMIZE
$F \equiv \Sigma_i \Sigma_{\mu n} Tr[U_\mu(n)\lambda_3^i U_\mu^\dagger(n)\lambda_3^i]$
λ_3^i THE 3 (DIAGONAL) COMPONENT OF $SU(2)$ OF THE ROOT i. OBTAIN MAX-ABELIAN FOR THE SINGLE i $SU(2)$'S
$\partial_\mu A_\mu^{i\pm} \mp i[A_\mu^{i3}, A_\mu^{i\pm}] = 0$
MONOPOLE DOMINANCE : $U_m(n) \approx 1$, $U_0(n)$ DIAGONAL.
GAUGE $A_0 = const$. HOLONOMY P THE PRODUCT OF THE r $SU(2)$ HOLONOMIES P_i . $\sin(\frac{gA_0^i}{T}) = -\frac{i}{2} Tr[\lambda_3^i P_i]$.
$SU(N)$: ALL i'S EQUIVALENT (WEYL SYMMETRY). CAN DIFFER FOR G_2
'tHOOFT LINE FOR EACH DIFFERENT i.

CONCLUSIONS

- IF MONOPOLES CONDENSE $\langle \Pi \rangle$ DISORDER PARAMETER.
  $\langle P \rangle$ BY DUALITY THE ORDER PARAMETER
- STABLE MONOPOLES IN GAUGE THEORIES CAN EXIST IF HOLONOMY IS NON TRIVIAL. THEY DO NOT EXIST IN DECONFINED PHASE.
- PHYSICAL MEANING OF $\langle P \rangle$: MONOPOLE DOMINANCE $\rightarrow P = \exp(i\frac{gA_0}{T}\sigma_3)$ $A_0\sigma_3$ THE ASYMPTOTIC HIGGS FIELD OF MONOPOLES AND ANTIMONOPOLES. $|A_0 g| = \frac{1}{\rho}$, $2\pi|\frac{A_0}{g}| \approx M_{mon}$.
- $N_{inst} \propto N_{mon}$ MONOPOLES BPS WEAKLY INTERACTING
- CHIRAL SYMMETRY BREAKING IS LOCKED TO CONFINEMENT.

WORK IN PROGRESS.