Unusual Interactions of Pre- and Post-Selected Particles

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ABL

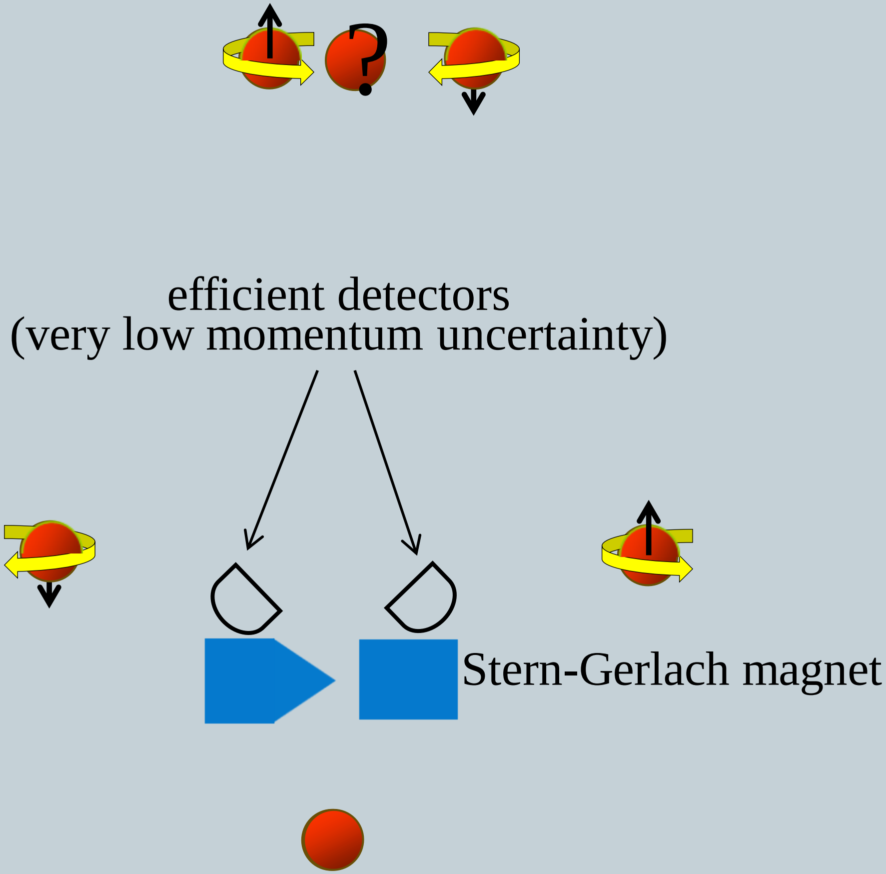
- In their 1964 paper Aharonov, Bergmann and Lebowitz introduced a time symmetric quantum theory.
- By performing both pre- and postselection ($|\psi(t')\rangle$ and $\langle\Phi(t'')|$ respectively) they were able to form a symmetric formula for the probability of measuring the eigenvalue c_j of the observable c:

$$P(c_j) = \frac{\langle\Phi(t'')|c_j\rangle\langle c_j|\psi(t')\rangle}{\sum_i \langle\Phi(t'')|c_i\rangle\langle c_i|\psi(t')\rangle}$$

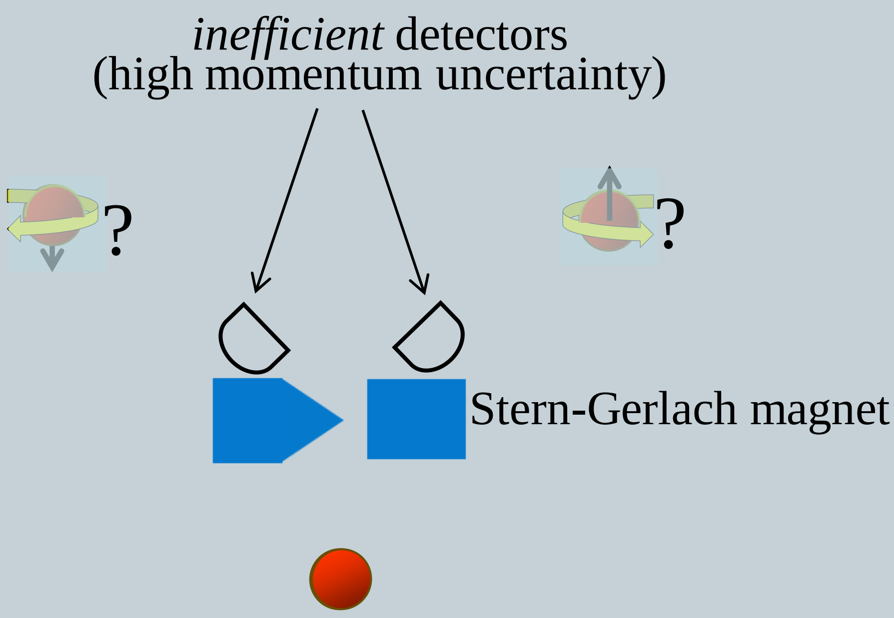
TSVF

- This idea was later widened to a new formalism of quantum mechanics: the Two-State-Vector Formalism (TSVF).
- The TSVF suggests that in every moment, probabilities are determined by two state vectors which evolved (one from the past and one from the future) towards the present.
- This is a hidden variables theory, in that it completes quantum mechanics, but a very subtle one as we shall see.

Strong Measurement



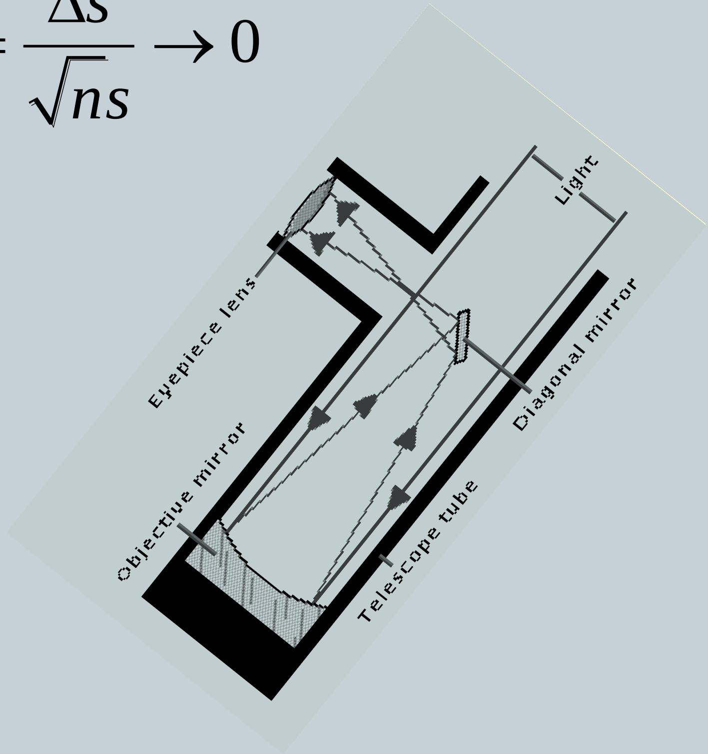
Weak Measurement - I

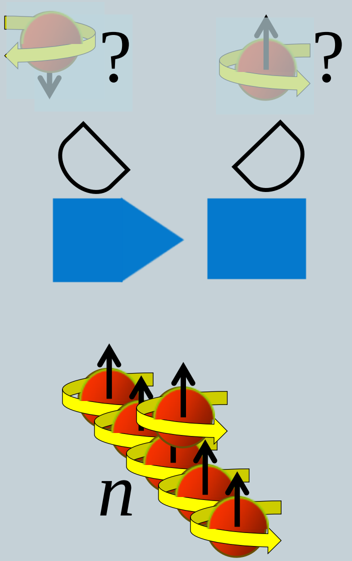


Why Weak Measurement?

$$\frac{\Delta s}{s} \Rightarrow \frac{\sqrt{n}\Delta s}{ns} = \frac{\Delta s}{\sqrt{ns}} \rightarrow 0$$

$$[\sigma_i, \sigma_j] = 2i\varepsilon_{ijk}\sigma_k$$





Weak Measurement - II

- The Weak Measurement can be described by the Hamiltonian:

$$H(t) = \frac{\lambda}{\sqrt{N}} g(t) A_s P_d$$

- In order to get blurred results we choose a pointer with zero expectation and $\delta >> \frac{\lambda}{\sqrt{N}}$ standard deviation.
- In that way, when measuring a single spin we get most results within the wide range $\frac{\lambda}{\sqrt{N}} \pm \delta$, but when summing up the N/2↑ results, most of them appear in the narrow range $\lambda\sqrt{N}/2 \pm \delta\sqrt{N}/\sqrt{2}$ agreeing with the strong results when choosing $\lambda >> \delta$.

Weak Value

- The "Weak Value" for a pre- and postselected (PPS) ensemble:

$$\langle A \rangle_w \equiv \frac{\langle \psi_{fin} | A | \psi_{in} \rangle}{\langle \psi_{fin} | \psi_{in} \rangle}$$

- It can be shown that when measuring weakly a PPS ensemble, the pointer is displaced by this value:

$$\Phi_{fin}(Q_d) \approx e^{-(i/\hbar)\langle A \rangle_w P_d} \Phi_{in}(Q_d) = \Phi_{in}(Q_d - \langle A \rangle_w)$$

No counterfactuals!

The Weak Interaction

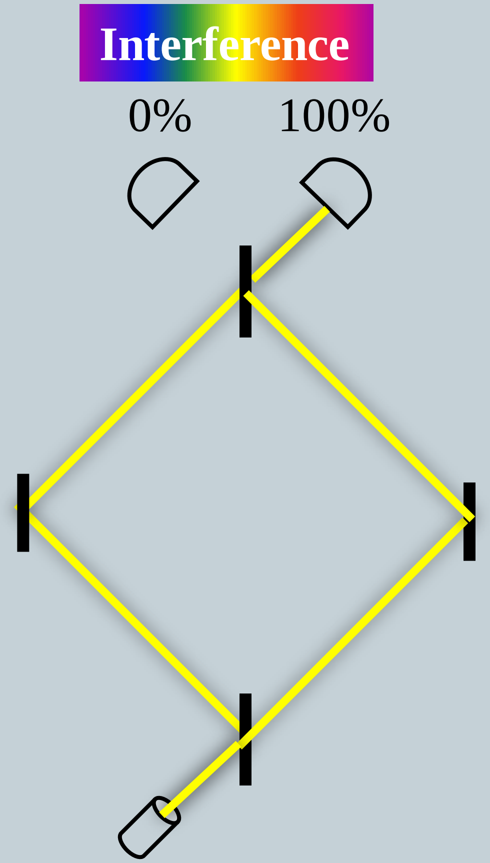
- We generalize the concept of weak measurement to the broader “weak interaction”.
- It can be shown that the Hamiltonian

  $H(1,2)=H_1(1)+H_2(2)+\lambda V(1,2)\equiv H_0(1,2)+\lambda V(1,2)$, when particle 1 is pre- and post- selected, results, to first order in λ , in the weak interaction :

$$V_W(2)=\lambda\frac{\left\langle \psi_2{}^0\middle|V\middle|\psi_1{}^0\right\rangle}{\left\langle \psi_2\middle|\psi_1\right\rangle}$$

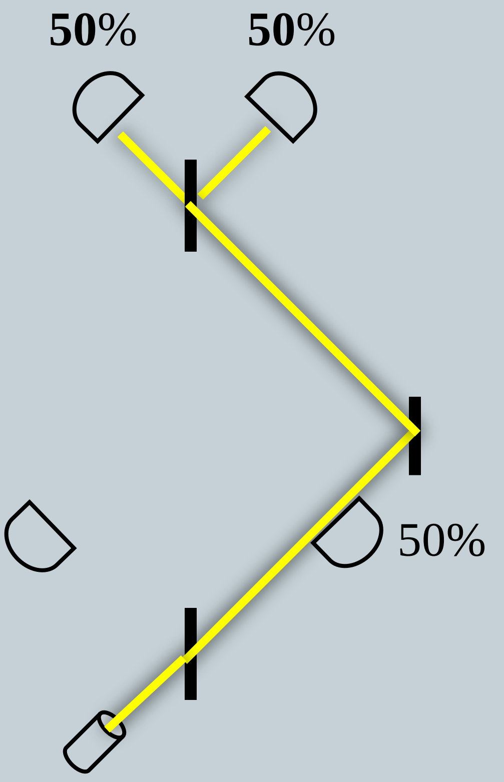
Which Path - I

Can we outsmart the "which path" uncertainty?



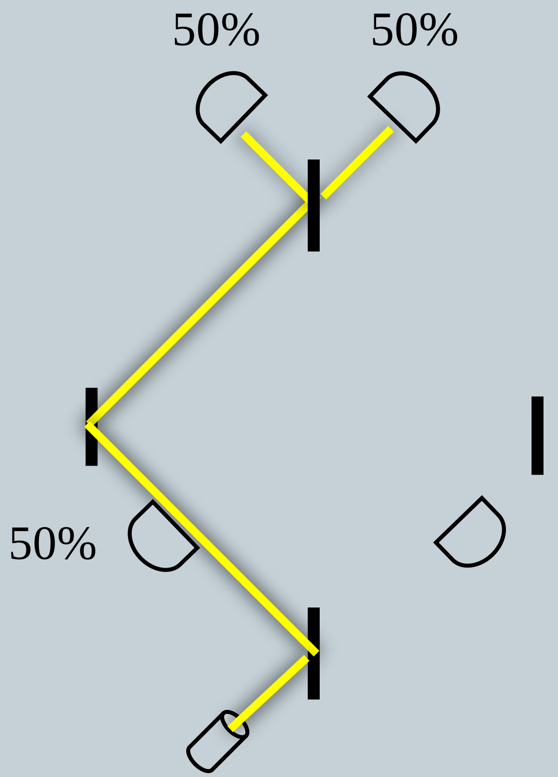
Which Path - II

Can we outsmart the “which path” uncertainty?



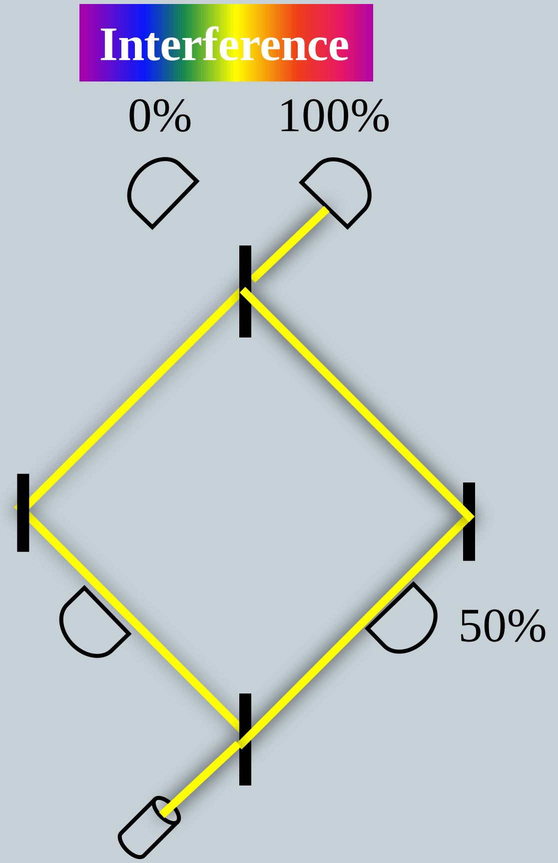
Which Path - III

Can we outsmart the “which path” uncertainty?



Which Path - IV

Can we outsmart the "which path" uncertainty?



Which Path Measurement Followed by Interference

Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer

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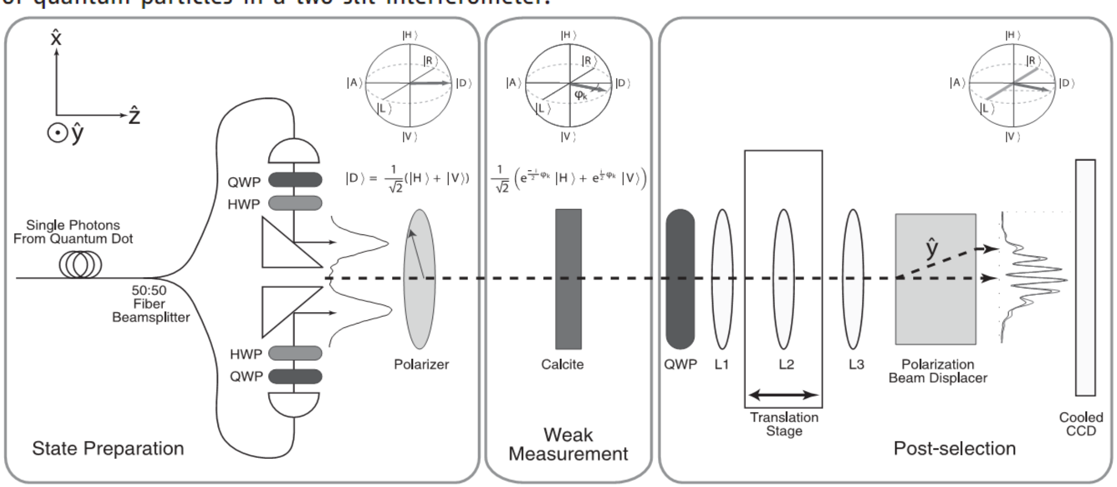
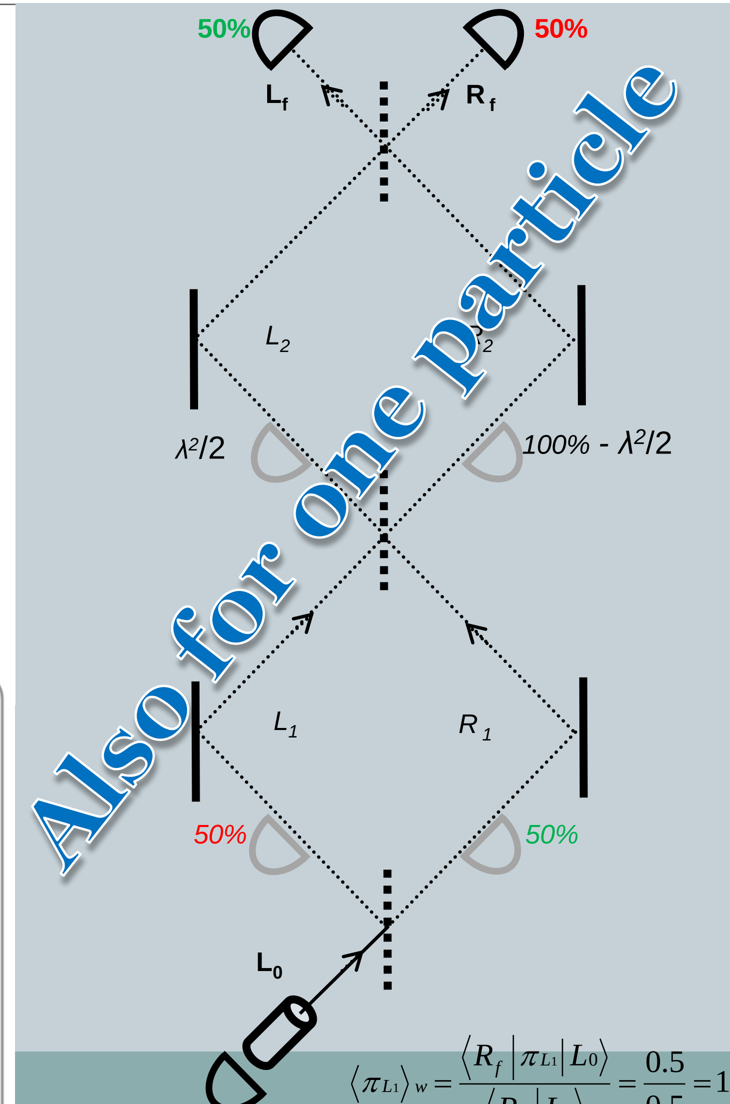
A consequence of the quantum mechanical uncertainty principle is that one may not discuss the path or "trajectory" that a quantum particle takes, because any measurement of position irrevocably disturbs the momentum, and vice versa. Using weak measurements, however, it is possible to operationally define a set of trajectories for an ensemble of quantum particles. We sent single photons emitted by a quantum dot through a double-slit interferometer and reconstructed these trajectories by performing a weak measurement of the photon momentum, postselected according to the result of a strong measurement of photon position in a series of planes. The results provide an observationally grounded description of the propagation of subensembles of quantum particles in a two-slit interferometer.



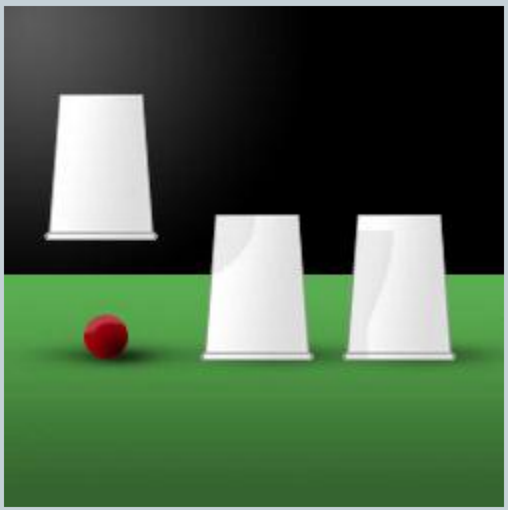


$$\langle \pi_{L1} \rangle_w = \frac{\langle R_f | \pi_{L1} | L_0 \rangle}{\langle R_f | L_0 \rangle} = \frac{0.5}{0.5} = 1,$$

A Quantum Shell Game

$$|\psi_{in}\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle + |C\rangle)$$

$$\Pi_i \equiv |i\rangle\langle i|$$



$$\langle \Pi_A \rangle_W = 1 \quad \langle \Pi_B \rangle_W = 1 \quad \langle \Pi_C \rangle_W = -1$$

$$|\psi_{fin}\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle - |C\rangle)$$

Aharonov Y., Rohrlich D., "Quantum Paradoxes", Wiley-VCH (2004)

Tunneling

$$H = \frac{p^2}{2m} - \delta(x)$$

$$\psi_{in}(x) = \sqrt{\alpha} e^{-\alpha|x|}$$

$$\psi_{fin}(x) = (\delta^2 \pi)^{-1/4} e^{-(x-x_0)/2\delta^2}$$

$$Where \quad \delta > \alpha\hbar^2 / m\varepsilon, \quad x_0 >> \delta^3 m\varepsilon / \hbar^2$$

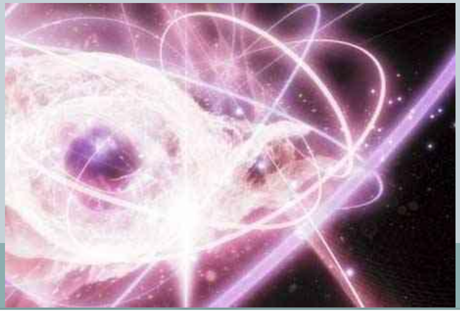
$$\Rightarrow E_k \approx -\frac{\alpha^2 \hbar^2}{2m}$$

Aharonov Y., Rohrlich D., "Quantum Paradoxes", Wiley-VCH (2004)

The Correspondence Principle

- Every quantum system is described by quantum numbers.
- When they become large, the system approaches its classical limit.
- This correspondence was first described by Bohr in the 1920' regarding the atom, but has a broader meaning.
- For example, the appropriate quantum number for the classical energy of an oscillator $E = \frac{1}{2} m A^2 \omega^2$

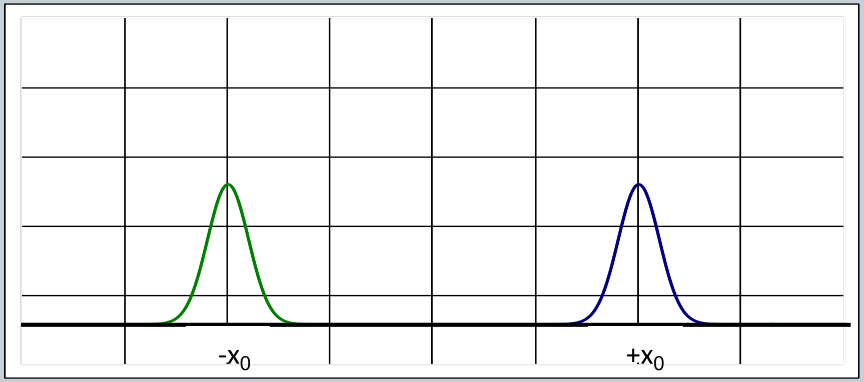
$$n = \frac{E}{\hbar\omega} - \frac{1}{2} = \frac{m\omega A^2}{2\hbar} - \frac{1}{2} \approx \frac{1}{2\hbar} = 4.74 \cdot 10^{33} \Rightarrow$$ High excitations



Challenging The Correspondence Principle

- Let $H = \frac{1}{2}(x^2 + p^2)$
- We Pre- and Post-select : $\psi_i = \pi^{-1/4}\exp[-(x-x_0)^2/2)]$

  and $\psi_f = \pi^{-1/4}\exp[-(x+x_0)^2/2)]$ where $x_0 >> 1$, $[\frac{x}{x_0}, p] = \frac{i}{x_0} \approx 0$.



- The weak values can be calculated to be:

  $x_w = 0$ and $p_w = -ix_0$.

Challenging The Correspondence Principle

- We argue that using the idea of weak interaction, this weird result gets a very clear physical meaning.
- When interacting with another oscillator $\psi_t = \exp(-p^2)$ through $H_{\text{int}} = \lambda p_1 p_2 g(t)$, it changes its *momentum* rather then its *position*:

$$\exp(i\int \lambda p_1 p_2 g(t)dt)\exp(-p_2{}^2) = \exp(\lambda x_0 p_2)\exp(-p_2{}^2) =$$

$$= \exp(\lambda x_0 p_2)\exp(-p_2{}^2) = \exp[-(p_2 - \lambda x_0/2)^2]\exp(\lambda^2 x_0{}^2/4)$$

Summary

- Weak measurements enable us to see and feel the TSVF.
- They also present the uniqueness of quantum mechanics.
- By using them we overcome the uncertainty principle in a subtle way and enjoy both which-path measurement and interference.
- Weak values, as strange as they are, have physical meaning:
  - In case of many measurements followed by proper postselections: Weak interaction or deviation of the measuring device.
  - Otherwise: An error due to the noise of the measurement device.
- In that way we avoid counterfactuals and obtain determinism in retrospect.


Thank You!

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