Quantum Spacetime & NC Black Holes

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1. Why quantum differential geometry?

... as we approach the Planck scale probes massenergy destroys the geometry we wanted to observe as they form black holes

Hence sub-planck distances intrinsically unknowable and building science on continuum geometry is unfounded

(We also don't know if black holes evaporate, going down right slope)

Eg Continuum $\Rightarrow \infty$ zero point energy. Planck scale cut off still 10^{122} x obs.

2. Lessons from 3D quantum gravity

geometry

Write the pair $A=(e^i,\omega^i)$ of 3-bein and spin connection as an $e_3 = \mathbb{R}^3$ \rtimes su₂-valued connection. Ad-invariant inner product on $e_3 \Rightarrow$

C_{lassic}al Guantum G

$$
\mathcal{S}_{\mathrm{Chern-Simons}} = \int_{\Sigma\times\mathbb{R}} A\dot{\wedge}(\mathrm{d} A + \frac{1}{3}[A\wedge A]) = \mathcal{S}_{\mathrm{Cartan-Weyl}}
$$

i.e. view gravity as a TFT.

⇒ solutions of gravity with point sources at punctures *i* \Rightarrow Theory described by topology of Σ and `local model' quantum \mathbf{S}^1 experiments of $(\sum_{i=1}^n y_i - \sum_{i=1}^n z_i)$ at \mathbf{S}^1 $\frac{1}{8}$ on $C(3a_2)$ a group of motions $U(su_2) \ll C(SU_2)$ acting on $U(su_2)$ as quantum

 N ith cosmological constant its instead $U(\omega)$ $\frac{1}{\sqrt{q}}$ with cosmological constant its instead $\frac{1}{q}(\frac{\partial w_2}{\partial y_1}, \frac{\partial w_1}{\partial z_2})$ Curry On $C_q(vw_2)$. With cosmological constant its instead $U_q(su_2) \!\!\times\!\! C_q(SU_2)^{op}$ acting on $U_q(su_2)$ with $q \sim e^{-\frac{1}{m_p l_c}}$, where $-\frac{1}{mp^l c}$, where $l_c = \sqrt{-\Lambda}$

Different limits of 3D Quantum Gravity (w. B. Schroers)

Bicrossproduct model spacetime (SM+H. Ruegg '94)

$$
A = U(\mathbb{R} \times \mathbb{R}^3) \qquad H = U(so(1,3)) \times \mathbb{C}[\mathbb{R} \times \mathbb{R}^3]
$$

\n
$$
[x_i, t] = \lambda x_i, \quad [x_i, x_j] = 0 \qquad \text{if. Lukierski et al}
$$

\n
$$
[p^i, N_j] = -\frac{i}{2} \delta^i_j \left(\frac{1 - e^{-2\lambda p^0}}{\lambda} + \lambda \overline{p}^2 \right) + \lambda p^i p_j,
$$

space, time not simultaneously measurable

$$
\Delta N_i = N_i \otimes 1 + e^{-\lambda p^0} \otimes N_i + \lambda \epsilon_{ij}{}^k p^j \otimes M_k,
$$

\n
$$
\Delta p^i = p^i \otimes 1 + e^{-\lambda p^0} \otimes p^i
$$

 $||p||_\lambda^2 = \vec{p}^2 e^{\lambda p^0}$ $-\frac{2}{\lambda^2}$ $\frac{2}{\lambda^2}$ (cosh(λp^0) – 1) Wave operator on plane waves $e^{i\vec{x}\cdot\vec{p}}e^{itp_0}$

Variable Speed Light
$$
\left| \frac{\partial p^0}{\partial p^i} \right| = e^{\lambda p^0}
$$

 $\Delta_T \sim \lambda \Delta_{p^0} \frac{L}{c} \sim 10^{-44} \text{ s} \times 100 \text{ MeV} \times 10^{10} \text{ y}$ *c* $\sim 10^{-44}$ s × 100 MeV × 10¹⁰ y \sim 1 ms, Differential arrival time of gamma-ray bursts (SM+GAC'2000) 3. Quantum anomaly for differential calculus

Space of 1-forms, i.e. `differentials dx'

$$
\Omega^1 \qquad \qquad \text{a}((\text{db})\text{c}) = (\text{a}(\text{db}))\text{c} \qquad \qquad \text{bimodule'}
$$
\n
$$
\text{d}: A \to \Omega^1 \qquad \qquad \text{d}(\text{ab}) = (\text{da})\text{b} + \text{a}(\text{db}) \qquad \qquad \text{Leibniz rule'}
$$

 ${adb} = \Omega^1$ ker $d = \mathbb{C}.1$ connectedness (optional)

In quantum group case we ask it to be translation invariant: E.g. $A = \mathbb{C}[x] \implies \Omega^1 = \mathbb{C}[x]dx$ $df(x) = \frac{f(x + \lambda) - f(x)}{\lambda}$ $\frac{\partial}{\partial x}$ dx $(\mathrm{d}x)f(x) = f(x + \lambda)\mathrm{d}x$

Theorem (SM&E Beggs, 2004) For simple g there do not exist associative differential calculi of classical dimensions on $\mathbb{C}_q(G)$ that are bicovariant on $|U(\mathfrak{g})|$ that are ad-covariant

=> extra cotangent dimensions. General feature of NCG!

• Anomaly => extra dimension => Laplacian as conjugate d_{α} $\left[\begin{array}{cc} m & n \end{array} \right] = 2a$ is α and α are a curricum anomaly is α $\lim_{\alpha \to 0} [x_i, x_j] = 2 \omega \pi j_k x_k \longrightarrow \text{Cauta function in } \alpha$ many classes of familiar non-commutative spaces likewise do not admit differential α ϵ_{ijk} uli $k + i \lambda \sigma_{ij}$, $u_i, v_j - i \lambda u_i$ $\begin{array}{c|c}\n\hline\n\end{array}$ $\Delta t_i + \frac{\partial \Lambda}{\partial \alpha} (\Delta \psi) \theta \Rightarrow \Delta = \frac{-}{\Lambda^2} [\Lambda_i / 1 + \lambda^2 \sum \partial^{i2} - 1] \sim_{\lambda \to 0} \sum \partial^{i2}$ $\begin{array}{cc} \Delta \end{array}$ and $\begin{array}{cc} \Delta \end{array}$ apply to the Poincar´e quantum group on the Poincar´e quant \mathbf{E} or enjourned $[x, x] = 2i$, $\epsilon_{\text{U}} x$ \rightarrow extra direction θ $[dx_i, x_j] = i\lambda \epsilon_{ijk} dx_k + i\lambda \delta_{ij} \theta$, $[x_i, \theta] = i\lambda dx_i$ flat the theorem does apply the theorem does appears that one similar that λ $d\psi = (\partial^i \psi) dx_i + \frac{\imath \lambda}{\mu} (\Delta \psi) \theta \Rightarrow \Delta = \frac{2}{\lambda^2} \left(\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial t^2} - 1 \right) \sim_{\lambda \to 0} \sum \partial^{i2}$ λ^2 (compared that the original the original λ^2) $\textbf{17.5:}$ spin moder $\left[\omega_l, \omega_j\right]$ and $\left[\omega_l, \omega_j\right]$ is the classical limit rather than the original κ = 1*/*λ. λ^2 (cotan-1 2 (λ^2) λ^2) by extra constant can be fixed by extra constant c flat flat the theorem does appears the theorem does appears that one similar that one similar that one similar
It appears that one similar that one simil E.g. spin model $[x_i, x_j] = 2i\lambda \epsilon_{ijk} x_k \Rightarrow$ extra direction Quantum anomalies for differential structure can typically be fixed by extra cotan- λ (or λ) in the λ λ version of λ version of λ version of λ $\Delta \psi = (\partial \psi) \Delta x_i + \frac{\partial}{\partial \psi} (\Delta \psi) \sigma \quad \rightarrow \quad \Delta - \frac{\partial}{\partial \psi}$ E.g. spin model $[x_i, x_j] = 2i\lambda \epsilon_{ijk}x_k$ \Rightarrow extra direction θ $d\psi = (\partial^i \psi) dx_i +$ $i\lambda$ $\frac{\partial \Lambda}{2} (\Delta \psi) \theta \Rightarrow \Delta =$ 2 $\overline{\lambda^2}$ $\sqrt{ }$ $\overline{1}$ $\overline{}$ $1 + \lambda^2$ i $\partial^{i2} - 1$ \setminus $\Big\vert \sim_{\lambda\rightarrow 0} \sum_{\lambda}$ i ∂^{i2}

O Similarly bicrossproduct model $[x_i, x_j] = 0$, $[x_i, t] = i \lambda x_i$ poincare covariance has an anomaly, forces extra direction *Date*: Revised December 2011. \bullet Similarly biomogeneoduct model $[x, x] = 0$ poincare covariance has an anomaly, forces extra di θ'

$$
[dx_i, x_j] = i\lambda \delta_{ij}\theta', \quad [\theta', x_i] = 0, \quad [\theta', t] = i\lambda \theta' \qquad \text{cf } \text{Sitarz}
$$

$$
[dx_i, t] = 0, \quad [x_i, dt] = i\lambda dx_i, \quad [dt, t] = i\lambda \theta' - i\lambda dt
$$

 ∂ $\begin{array}{ccccc}\n\text{Cov}_i & \text{Cov}_i & \text{Cov}_i & \text{Cov}_i \\
\text{Cov}_i & \text{Cov}_i & \text{Cov}_i & \text{Cov}_i \\
\text{Cov}_i & \text{Cov}_i & \text{Cov}_i & \text{Cov}_i \\
\text{Cov}_i & \text{Cov}_i & \text{Cov}_i & \text{Cov}_i\n\end{array}$ The form of dictions and on normal ordered from the distinctions and on normal ordered functions and on α $d\psi = \frac{\partial}{\partial x}\psi(x,t)dx_i + \partial_0\psi(t)dt + \frac{\partial}{\partial x}(\Box\psi(x,t))\theta'$ $\sum_{i=1}^{n}$ d can be deduced from the $\sum_{i=1}^{n}$ d can be deduced the set of $\sum_{i=1}^{n}$ d can be deduced functions and on $\sum_{i=1}^{n}$ d and $\mathrm{d}\psi =$ ∂ ∂x_i $\psi(x,t)\mathrm{d}x_i + \partial_0\psi(t)\mathrm{d}t$ + *ı*λ 2 $+\frac{i\lambda}{2}(\Box\psi(x,t))\theta'$ $\frac{i\lambda}{2}(\Box\psi(x,t))\theta' \implies$ same \Box as before in VSL prediction

freedom known as the the differential structure? • What is the physical meaning of this new degree of
freedom known as the the differential structure? What is the physical meaning of this new degree of

Fact: we can change to $[dt, t] = \beta i \lambda \theta' - i \lambda dt$ where β is any function on space, still gives calculus and Laplacian becomes: except that we have inserted a dimensionful constant \mathbf{e} in front of \mathbf{e} in front of \mathbf{e} \mathbf{c} **an** \mathbf{c} **hange to** $\begin{bmatrix} dt & t \end{bmatrix} = \beta_2 \lambda \theta$ I.e. we take a point of view of the original origin of the origin of the same of the wave equation of the wave of runcuon on space, sem gives caredius (2.1) dψ = 1, dμ = 1, d
| = 1, dμ = 1 ψ(*x, t*)d*xⁱ* + ∂0ψ(*t*)d*t* + !ψ(*t*)θ! function on space still gives calculus and Laplacian becom *γ µ x n xpace, still gives calculus a* ∂*x*² (2.3) [∆]0ψ(*t*) = νψ(*^t* ⁺ *^ı*λ) + *^µ*ψ(*^t* [−] *^ı*λ(^β $f \circ \text{dist}(t) = \beta \iota \lambda \theta' - \iota \lambda dt$ where β is any ∂*µ* + 2*µ* = β*, xⁱ* ∂ν ∂*xⁱ* **ES.** ind I anlacian becomes: es calculus and Laplacian becomes: $\left|t,t\right|$

$$
\Box \psi = \bar{\Delta} \psi(t + i\lambda) + 2\Delta_0 \psi, \quad \bar{\Delta} = \frac{\partial^2}{\partial x_i^2} - \frac{1}{2\beta} \frac{\partial \beta}{\partial x_i} \frac{\partial}{\partial x_i} + \frac{x_i \frac{\partial \mu}{\partial x_i} + 2\mu}{x_i \frac{\partial \nu}{\partial x_i} + \nu} = \mu
$$

$$
\Delta_0 \psi(t) = \frac{\nu \psi(t + i\lambda) + \mu \psi(t - i\lambda(\frac{\beta}{\mu} - 1)) - (\nu + \mu)\psi(t + i\lambda(1 - \frac{\beta}{\nu + \mu}))}{(i\lambda)^2}
$$

 $\lim_{\Delta t \to 0} 2\Delta_0 = \beta \frac{\sigma}{2\sqrt{2}}$ so \Box corresponds to $g = -\frac{1}{\beta} dt \otimes dt + dx_i \otimes dt_i$ \sqrt{a} order \sqrt{b} $\lim_{\varepsilon} 2\Delta_0 = \beta \frac{\sigma}{2\sigma^2}$ so $i\lambda \rightarrow 0$ $2\Delta_0 = \beta \frac{\partial^2}{\partial t^2}$ ∂*t*² so \Box corresponds to $g =$ $\frac{1}{\beta} dt \otimes dt + dx_i \otimes dx_i.$ $\overline{}$ are a specialization of more general results in $\overline{}$ or any Riemannian substitution of more general results in $\overline{}$ rrespol (2.4) *^g* ⁼ ¹

∂*x*² *i* ⁰ ψ(*t*) $\overline{10}$ ∂*xⁱ* \overline{p} pravity arises ∂*xⁱ n* of a free **aifferential calculus on flat quantum spacetime** The calculus remains a *a* $\frac{1}{2}$ and $\frac{1}{2}$ are still has th differential calculus on flat quantum spacetime α ity arises out of a freedom for the quantum avity anses out of a hidden for the => newtonian gravity arises out of a freedom for the quantum \mathbf{S} . Polar coordinates in the flat spacetime bic spacetime bics sp $T_{\rm eff}$ facts are a specialization of more general results in $[15]$ or any $R_{\rm eff}$ $\overline{\text{15}}$ out of a freedom for the quantum

modelled approximately as a metric of the form (2.4). It is elementary to compute the comp that for such metrics $\frac{1}{2}$ for suppose that the suppose that $\frac{1}{2}$ for $\frac{1}{2}$ H are a stat nas: The Wales of the Wales Here a static metric of this form has:

$$
\text{Ricci}_{00} = \phi \bar{\Delta}^{flat} \phi, \quad \bar{\Delta}^{flat} = \frac{\partial^2}{\partial x_i^2}, \quad \phi = \sqrt{-g_{00}} = \sqrt{-\beta^{-1}}.
$$

Interpret as n ^β ⁼ [−] ¹ **c**
*c*₂ <u>*c*₂ <u>d</u>₂ <u>d</u>₂ <u>d</u>₂ <u>d</u>₂ d₂ **d**₂ **d**₂ **d**₂ **d**₂</u> $\beta = -\frac{1}{c^2}(1 - \frac{2\Phi}{c^2})$ terpret as newtonian potential by $\beta = -\frac{1}{2}(1 - \frac{2\Phi}{2})$ where $|\Phi| < \infty$ \lt \lt **itely precas newtonian potential by** $p = -\frac{1}{c^2}(1-\frac{1}{c^2})$ where $p = 1 + 1$ Interpret as newtonian potential by $\beta = -\frac{1}{c^2}(1 - \frac{2\Phi}{c^2})$ where $|\Phi|$. with our level of a c^2 $\frac{c^2}{\sqrt{2}}$. $\frac{U_x}{i}$ $\frac{U_x}{i}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{2\Phi}{i}$ where $|\Phi| << c^2$ et as newtonian potential by $\beta = -\frac{1}{c^2}(1-\frac{2\Psi}{c^2})$ where $|\Phi| << c^2$ ergy tensor dominated by *T00 ≈ product* by *T00 ≈ product extensions (in trace reversed*) extensions (in trace reversed).
Einstein 'n trace reversed by the second tensor dominated by the second tensor dominated by the se pleteness only. Interpret as newtonian potential by $\beta = -\frac{1}{c^2}(1-\frac{1}{c^2})$ where $|\Psi| \leq C$ Interpret as newtonian potential by $\beta = -\frac{1}{c^2}(1 - \frac{2\Phi}{c^2})$ where $|\Phi| << c^2$

where c is the speed of light and for speed of light and for some spatially varying function \mathcal{L}

Assume static matter $T_{00} \approx \rho c^4$. Then $Ricci_{00} = \frac{8\pi G}{4} (T_{00} - \frac{1}{2} T a_{00})$ \longrightarrow $\bar{\Lambda}^{flat} \bar{\Phi} - 4\pi G \bar{\Phi}$ with $\Delta^{j \text{vac}} \Phi = 4\pi G \rho$ $\text{matter} \quad T_{00} \approx \rho c^4$ Than $\text{matter} \quad 100 \approx \rho c + 1 \text{ nen}$ $(T_{\rm eq} = 1T_{\rm deq})$ which means statistical means stress en- $T_{\rm eq}$ $(200 \t2^2 \t900)$ \rightarrow Δ^2 Ψ \rightarrow 47. Gp $\textsf{assume static matter} \quad T_{00} \; \approx \; \rho c^4 \;\; \textsf{Then}$ $Ricci_{00} = \frac{8\pi G}{T_{00}} = \frac{1}{T_{00}}$ $\rightarrow \frac{7}{\pi}$ *T III* π *A* \rightarrow *C* Ricci₀₀ = $\frac{8\pi G}{c^4}(T_{00} - \frac{1}{2}Tg_{00})$ \implies $\Delta^{flat}\Phi = 4\pi G\rho$ Λ esumes static matter $T \sim \omega_0^4$ Then **Example static matter** $T_{00} \approx pc + 1$ nen *^g*⁰⁰ ⁼ [−]φ² ≈ −*c*². Hence Einstein's equation in our approximation becomes \mathbf{r} matter $T_{00} \approx \rho c^2$. Ihen as in \mathcal{L} is a standard derivation which we include for commutation which we include for com-**ASSU** metter $T_{\text{eq}} \approx \rho c^4$ Then matter $I_{00} \approx \rho c + I$ $\Delta^{flat} \Phi = 4\pi G \rho$ itic matter T_{00} \approx ρc^4 . Then $\bar{\lambda}$ ^{*flat* $\bar{\lambda}$ </sub> *1* α} $2 - 300$ \equiv **1**∂β \equiv 100**p** me static ma $\text{ccin}_0 = \frac{8\pi G}{4} (T_0)$ $\frac{d}{2}T_0 - \frac{1}{2}Tg$ *i* $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ 2β ∂*xⁱ* ∂*xⁱ* $\bar{\Delta}^{flat} \Phi = 4\pi G$ $\Delta^{flat} \Phi = 4\pi G \rho$ Assume static matter T_{00} \approx ρc^4 Then

ergy tensor dominated by *^T*⁰⁰ [≈] ^ρ*c*⁴. Einstein's equations (in trace reversed Assume fields ψ slowly varying in space so g00 = −_φ2 ≈ −_α
Hence Einstein's equation in our approximation in our approximation in our approximation in our approximation in ∆¯ *f lat*Φ = 4π*G*ρ Next we consider how the associated spacetime Laplace-Beltrami wave operator Following inclus ψ sides ume field $\overline{\mathsf{ds}} \ \psi$ s − 1 2β ∂*xⁱ* ∂*xⁱ* $\frac{1}{2}$ in space so ω close the weak discriments ² ^β−¹∂β [≈] [∂]Φ*/c*² as long as the fields ^ψ are slowly varying ψ slowly varying in space so and in an ψ we now consider field of the form of the θ ² ^β−¹∂β [≈] [∂]Φ*/c*² as long as the fields ^ψ are slowly varying Assume fields ψ slowly varying in space so

$$
\bar{\Box}\psi = \left(\beta\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x_i^2} - \frac{1}{2\beta}\frac{\partial\beta}{\partial x_i}\frac{\partial}{\partial x_i}\right)\psi \approx \beta\frac{\partial^2}{\partial t^2}\psi + \bar{\Delta}^{flat}\psi
$$

 $\frac{1}{2}$ **P** are slowly vary $\frac{1}{2}$ as $\frac{1}{2}$ as the fields $\frac{1}{2}$ as the fields $\frac{1}{2}$ are slowly variable $\frac{1}{2}$ and have the form $\psi = \Psi e^{-\imath t \frac{mc^2}{\hbar}}$ where Ψ slowly varying in x,t. Then $\frac{1}{2}$ where ^{*n*} wher \cdot e Ψ : **E** 210 WI *mc*² var ym g in x,t.Then !² ^Ψ and have the form $\psi = \Psi e^{-\imath t \frac{mc^2}{\hbar}}$ where Ψ slowly varying in x,t. Then

$$
\overline{\Box}\psi = \frac{m^2c^2}{\hbar^2}\psi \qquad \Longrightarrow \qquad \frac{1}{c^2}(1-\frac{2\Phi}{c^2})\left(\frac{m^2c^4}{\hbar^2}\Psi + 2i\frac{mc^2}{\hbar}\dot{\Psi} + \ddot{\Psi}\right) + \bar{\Delta}^{flat}\Psi = \frac{m^2c^2}{\hbar^2}\Psi
$$
\n
$$
\Longrightarrow \qquad \qquad i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\bar{\Delta}^{flat}\Psi + m\Phi\Psi
$$

postante interpretents.
Postante interpretents !*m*²*c*⁴ " *mc*² ϵ can define the ϵ the ϵ the other ϵ the other in comparison to the other set of ϵ a s Newtonian potential arpretation of β as Newtonian potential scription of a test particle of mass *m* moving in a gravitational potential Φ (created $\sum_{i=1}^n$ which we can define $\sum_{i=1}^n$ to the other in components. Confirms interpretation of β as Newtonian potential

where
$$
\psi(x,t) = \sum \psi_n(x)t^n
$$
 $\Delta_0^{hybrid} = \frac{1}{i\lambda} \left(\frac{\upsilon}{\partial t} - \partial_0\right)$
\nNow let $\psi = \Psi(x,t)e^{-i\frac{mc^2}{\hbar}t}$ $\tilde{m} = mc^2/\hbar$,

r)

*^c*² (1 + ^γ

xi

r

)*,* ^ν ⁼ [−] ¹

∂*xⁱ*

*c*2 (

should be viewed as, by definition, the exact noncommutative version, the exact noncommutative version of Newt
The exact non-commutative version of Newtonian of Newtonian of Newtonian of Newtonian of Newtonian of Newtonia

1

² [−] ^γ

 $\overline{}$ and we are minimum assumption that it still the minimum assumption that it still that it still that it still that it still the minimum assumption that it still the minimum assumption that it still that it stil

 Φ is the newtonian potential. Can solve Φ

 $\mathbf{\Psi}$ and constant energy in the model const

The first is a standard identity for the finite double double double double double double double difference and the second

^ψ(*^t* ⁺ *^ı*λ) [−] ²^γ

^r ln(^γ

*c*2 (

*c*²*r*

))

 Φ

∆*hybrid*

r

² ⁺ ^γ

⁰ ψ(*t* + *ı*λ)

gravity limit, the only assumption on \mathcal{A} was with regard to \mathcal{A} and \mathcal{A} also slowly varying varying varying \mathcal{A}

*c*2 (

2 − γ
2 − γ
2 − γ

 Φ

$$
i\hbar \frac{\sinh(\tilde{m}\lambda)}{\tilde{m}\lambda} \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2 e^{\tilde{m}\lambda}}{2m} \bar{\Delta}^{flat} \Psi + \left(mc^2 \left(1 - \frac{\sinh(\frac{\tilde{m}\lambda}{2})}{\frac{\tilde{m}\lambda}{2}} \right) - \frac{GMm}{r} \left(\frac{\tilde{m}\lambda + e^{-\tilde{m}\lambda} - 1}{\frac{\tilde{m}^2 \lambda^2}{2}} \right) \right) \Psi
$$

which we write as
$$
i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m_I} \bar{\Delta}^{flat} \Psi + (V_0 - \frac{GMm_G}{r}) \Psi
$$

 Φ

c² (1 + γ

 Φ

We see that the effect in additional *γ/r in β is an additional term which in β is an additional* term which in

)*, µ* ⁼ [−] ¹

^ψ(*t*) [−] ¹

*c*2 (

^µ ≈ −ρ*c*² is the trace and

² ⁺ ^γ

 Φ

⁰ *^g* + (∂0*f*) [∂]

)*, µ* ⁼ [−] ¹

We now suppose for the sake of discussion that τ is of order that τ is of order that τ

 Φ

*r*3(1 + γ¹ + γ¹ + γ¹ + γ¹

2 β−1∂β to the Laplacian as we did this in this in this in this in the classical analysis of the Newtonian analysis

2

gravity or of any other inverse square force in N or of any other interpreting α interpreting α

 γ suitably). This is important because otherwise the approximations made in the approximations made in the approximations made in the approximations made in the approximation of γ

1

form) read Ricci⁰⁰ = ⁸π*^G*

c $\frac{1}{\sqrt{2}}$ (*T*₀

−1
−1

²*T g*00) where *T* = *T ^µ*

<u>−</u>

derivatives.
Derivatives

!ψ(*t*) = !^β=−1*/c*²

⁰ *^f*)*^g* ⁺ *^f*(*^t* [−] *^ı*λ)∆*hybrid*

c

limit. Then our equation becomes

!ψ = *m*²*c*²ψ*.*

$$
m_I = m \frac{\sinh(\tilde{m}\lambda)}{\tilde{m}\lambda} e^{-\tilde{m}\lambda} \qquad m_G = m \left(\frac{\tilde{m}\lambda + e^{-\tilde{m}\lambda} - 1}{\frac{\tilde{m}\lambda}{2} \sinh(\tilde{m}\lambda)} \right)
$$

$$
V_0 = mc^2 \frac{\tilde{m}\lambda}{\sinh(\tilde{m}\lambda)} \left(1 - \frac{\sinh(\frac{\tilde{m}\lambda}{2})}{\frac{\tilde{m}\lambda}{2}} \right) = -\frac{mc^2}{24} (\tilde{m}\lambda)^2 + o((\tilde{m}\lambda)^4))
$$

geometry correction! suggests how vacuum energy might arise as a quantum where \mathbf{v}_i from what was observed. This is presumably contradicted by presumably contradicted by \mathbf{v}_i ϵ tion: we observe gravity macroscopically and rather it raises the question ϵ

 $\frac{1}{2}$. Effective masses and constant executive masses in the model in the behave differently approaching and above planck mass! and suggests that macroscopic massive quantum states may

5.3. Minimally coupled Schwarzschild black hole. In contrast to Section 6, <u>here we give a support and the set of the black of the black of the black to the black the black of the black to the black of the </u> 4. Minimally coupled quantum black hole importantly ∆0, but adjust the wave operator to !*BH*ψ(*t*)=2∆0ψ(*t*) + [∆]¯ *LB*ψ(*^t* ⁺ ^λ) [−] ¹

hole, namely built on the particular conduct spacetime $\{r, t\} = \gamma \lambda$ $\beta = -\frac{1}{c^2(1$ $c^2(1-\frac{\gamma}{r})$ $\gamma = \frac{2GM}{c^2}$ Schwarzschild radius $\beta = -$ 1 $\frac{1}{2GM}$ Schwarzschild radius $c^2(1-\frac{1}{r})$ c^2 + $-$ [∂]*r*² ⁺ *^eieⁱ* $\gamma = \frac{2GM}{2}$ $c²$ We take as before flat quantum spacetime $[x_i, t] = \imath \lambda_p x_i$ and

 \mathbf{M} and \mathbf{M} is \mathbf{M} will now be the Schwarzschild radius for a set of mass \mathbf{M} *Ne* also minimally couple $\Delta_{\mathbb{R}^3} \mapsto \Delta_{LB}$ for $\bar{\Delta}_{\mathbb{R}^3} \mapsto \bar{\Delta}_{LB}$ is the discretion of \mathbf{C} coupled \mathbf{C} \mathbf{C} is \mathbf{C} is non-to-spacial wave operator. We also `minimally couple' $\ \bar\Delta_{\mathbb R^3} \mapsto \bar\Delta_{LB}$ for BH spatial metric \blacksquare $\overline{}$

■ Black hole quantum wave operator \overline{P} is not the spatial part of the black-hole wave operator. However, \overline{P} $\Box \psi(t) = 2\Delta_0 \psi(t) + \Delta_{LB} \psi(t+i\lambda_p) - \frac{1}{2\beta} (\Phi(t+i\lambda_p) - \frac{1}{2\beta})$ $\frac{1}{\sqrt{2}}$ $\Delta_0 e^{i\omega t} = \frac{1}{c^2} D(\omega, r) e^{i\omega t}$ \mathcal{C}^2 \sum Plack hole quantum work operator. \blacktriangleright Black hole quantum wave operator \Box *i*_k (1) Ω Λ $\partial_{\alpha}(1)$ $\overline{\Lambda}$ \mathbb{P}^{\prime} **2** in (5.7. **For the Schwarzschildle** \mathbb{P}^{\prime} **we have** 2β **in (5.7. in (5.2)** 1 $\frac{1}{c^2}D(\omega,r)e^{i\omega t}$ $\Box\psi(t)=2\Delta_{0}\psi(t) + \bar{\Delta}_{LB}\psi(t + \imath\lambda_{p}) - \frac{1}{2\pi}$ $\frac{1}{2\beta}(\bar{\mathrm{d}}\beta,\bar{\mathrm{d}}\psi)(t + \imath\lambda_p)$

where

where
\n
$$
D(\omega, r) = \frac{1}{\lambda_p^2} \left(\sinh(\omega \lambda_p) + e^{-\omega \lambda_p} (1 - \frac{\gamma}{r}) \left(1 - e^{\omega \lambda_p} - \frac{\gamma}{r} \ln \left(\frac{e^{\omega \lambda_p} r - \gamma}{r - \gamma} \right) \right) \right)
$$

Effect 2: Gravl time dilation/redshift is frequency dependent **Effect 2:** Grayl time dilation/redshift is frequency deper

$$
2D(\omega, r) = \frac{\omega^2}{(1 - \frac{\gamma}{r})} \left(1 - \frac{2}{3} \frac{\omega \lambda_p \gamma}{r(1 - \frac{\gamma}{r})} + O((\omega \lambda_p)^2) \right)
$$

ots that a higher frequency will be less redshift

suggests that a higher frequency will be less redshifted. $\overline{}$ **ss** ο
tha \overline{p} ^r) *D*(*n*_i_D), *n* cyaer y will be 3 ESS TEC

An emission + n'th harmonic at radius r won't be a harmonic. The deficit in distance per base cycle over which the harmonic completes its harmonic when received and this might be very \mathbf{r} $\overline{\mathbf{a}}$ $\overline{\mathbf{b}}$ $\overline{\mathbf{c}}$ ω"" $\sqrt{ }$ $rac{1}{\sqrt{2}}$ $\overline{2}$ $\frac{1}{\sqrt{1-\frac{1}{2}}}$ $\overline{}$ sensitively detected. One cycle error accumulates after distance *i* are different the matrix included and the similar theories of the interestion.
The property harmonic when received and this might be very figure for the entire length *L* of the journey (for our back-of-envelope estimate) we need *L* ∼ c^2 ω^2 3*r n*γ λ*^p* or approximately *ⁿ*^γ *L* ∼ **Une c** $\frac{C}{v^2} \frac{3r}{\rho_0}$

c

0.1 nm (X ray). $\gamma = 0.1 \implies L \sim 0.1$ light vears for the entire $\left(\frac{1}{2}a\right)$, $\frac{1}{2}a = 0.1$ and $\frac{1}{2}a = 0.1$ and $\frac{1}{2}a = 0.1$ and $\frac{1}{2}a = 0.1$ e.g. 0.1 nm (X ray), $\quad \frac{\gamma}{\gamma} = 0.1 \implies L \sim 0.1$ light years in order to accumulate order to a control cycle of phase error. For a α 1 nanometer (X-ray) wave**e.g. 0.1 mm (** \land **ray),** $\frac{1}{r} = 0.1$ → $L \sim 0.1$ light years modest by astronomical standards even if $\mathcal F$ well beyond current reaches the figure would be figure would be γ r e.g. 0.1 nm (X ray), $\frac{\gamma}{r} = 0.1$ ⇒ $L \sim 0.1$ light years r

 \overline{a} **l** Detect non-harmonicity by a resonant cavity? in order to accumulate one full control. For a 0.1 nanometer (X-ray) wave-term (X-ray) wave-term (X-ray) wavebe worse using infra red lasers but on the other hand it may not at all be necessary to Astrophysical harmonic emission? expect the frequency dependence of the frequency dependence of the redshift to apply to other gravitational po
The redshift to apply to other gravitational potentials, we have a positive to other gravitational potentials, be worse using infra red lasers but on the other hand it may not at all be necessary to

Effect 3: \sim (1 For posctrage percies $\omega > 0$ a `skin' of width $\gamma(1'-e^{-\omega\lambda_P})$ just below the event horizon where $\omega > 0$ $\gamma(1\sqrt{e^{-\omega\lambda_P}})$ $\Im D(\omega, r) \neq 0$ Effect 3: $\gamma(1 - e^{-\omega\lambda_P})$ iust below the event horizon where $\Im D(\omega,r) \neq \Im \mathcal{D}(\omega,r) \neq 0$

(Has same exponential growth with frequency that leads to Planckian bound in spatial the flat bicrossproduct spacetime model, the exponential puts of the exponential \mathbf{p}_i is a put spacetime model, \mathbf{p}_i

Effect 6: treats pos and neg frequencies differently

5. Quantization of $M \times \mathbb{R}$

Let (M,\bar{g}) be a Riemannian manifold of dimension n and τ a vector field $A = C(M) \rtimes \mathbb{R}$ $[f, t] = \lambda \tau(f)$ (2.5) a Nichianinan manifol<mark>u</mark> O $A = C(M) \rtimes \mathbb{R}$ $[J, U] = \lambda \tau(J)$ (M,\bar{g})

is a noncommutative version of $M \times \mathbb{R}$. Let $(\bar{\Omega}^1, \bar{d})$ be classical, Section 2 we have an extended differential calculus (Ω¹ *,* d) and other structures $(\bar{\Omega}^1, \bar{\mathrm{d}})$

the Lie derivative, $\bar{\Delta}$ a 2nd order operator and $\alpha = \frac{2}{n} \text{div}(\tau) - 1$. $\bar{\mathcal{L}}$ the Lie derivative, $\bar{\Delta}$ a 2nd order operator and $\alpha = \frac{2}{\pi} \text{div}(\tau) - 1$. $\bar{\Delta}$

Main Theorem For any function β **and conformal Killing vector** field τ , extending $\overline{\Omega^1(M)}$ by $\mathrm{d}t, \overline{\theta'}$ with relations field τ , extending $\bar{\Omega}^1(M)$ by $\mathrm{d}t, \theta'$ with relations

$$
[f, \omega] = \lambda(\omega, \bar{d}f)\theta', \quad df = \bar{d}f + \frac{\lambda}{2}(\bar{\Delta}f)\theta' \qquad \forall f \in C(M), \ \omega \in \bar{\Omega}^1(M)
$$

$$
[\omega, t] = \lambda(\bar{\mathcal{L}}_{\tau} - id)\omega - \lambda^2(\frac{n-2}{4})(\bar{d}\alpha, \omega)\theta' - \frac{\lambda^2}{2}(\bar{\mathcal{L}}_{\tau}\zeta^*, \omega)\theta'
$$

$$
[\theta', t] = \alpha\lambda\theta', \quad [f, dt] = \lambda df, \quad [dt, t] = \beta\lambda\theta' - \lambda dt
$$

gives a differential calculus $\Omega^1(C(M) \rtimes \mathbb{R})$ \Rightarrow \Box CMP 2012)

6. Summary

1) Position-momentum duality visible in 2+1

Einstein eqn ~ posn-mom symmetry (SM Class Quan Grav 1988)

- 2) Noncommutative space generates in own evolution out of an anomaly for differential calculus, wave operator is associated to an induced extra dimension
- 3) Differential calculus is a new degree of freedom, origin of gravity
- 4) BH model shows resolution of singularities, freq dept redshift

 \mathcal{L}_max , where \mathcal{L}_max

kappa-Minkowski papers:

Further Reading (not a bibliography)

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G. Amelino-Camelia and S. Majid, Waves on Noncommutative Spacetime and Gamma-Ray Bursts, *Int. J. Mod. Phys. A***15** (2000) 4301-4323

S. Majid, Quantum anomalies and Newtonian gravity on quantum spacetime, 9pp. *arXiv:1109.6190 (hep-th)*

Nogo theorem for quantum differential calculus:

E.J. Beggs and S. Majid, Semiclassical Differential Structures, *Pac. J. Math.* **224** (2006) 1-44

Black holes papers:

S. Majid, Almost commutative Riemannian geometry, I: wave operators, 50pp. *Commun. Math. Phys.* **310** (2012) 569-609

S. Majid, Scaling limit of the noncommutative black hole, *J. Phys. Conf. Ser.* **284** (2011) 012003 (12pp)

Born reciprocity and semidualisation:

S. Majid and B. Schroers, q-Deformation and semidualisation in 3D quantum gravity, *J. Phys A* **42** (2009) 425402 (40pp)

Continuum Assumption \Rightarrow infinite vacuum energy because in QFT we include modes of arbitrary large momentum. If we truncate at the Planck scale then we still get nonsense:

- *•* Finite size of the universe ⇒ minimum momentum or mass-energy $m_{min} = 10^{-66}$ g.
- *•* Planck cut-off [⇒] maximum momentum *^mplanck* = 10−5*^g*
- *•* How many oscillators are there? Around $(m_{\text{planck}}/m_{\text{min}})^3 = (r_{\text{univ}}/l_{\text{planck}})^3$.
- *•* ⇒ estimated vacuum density

$$
\frac{m_{\text{planck}}}{r_{\text{univ}}^3} \times \left(\frac{r_{\text{univ}}}{I_{\text{planck}}}\right)^3 = 5 \times 10^{93} g/cm^3.
$$

Compare with the experimental value: 70% of the mass-energy of the known Universe seems to be in the form of a uniform density around 10−29*g/cm*3. Naive cut-off disagrees with experiment by a factor of 10^{122} – the 'Dark energy problem' or puzzle of the cosmological constant. We need an actual theory!

<u>Let *H*</u> = *H*₂ be a discussed into the set of the set Semidualization theorem (SM 1988)

Let $H = H_1 \Join H_2$ be a quantum group factorist rotations' H_1 and 'quantum momentum' H_2 . Let $H = H_1 \Join H_2$ be a quantum group factorising into 'quantum

- **D** $H_1 \Join H_2$ acts canonically on H_2^* 'quantum spacetime'. $\mathcal{S} = \mathcal{S} \mathcal{S} = \mathcal{S} \mathcal{S} \mathcal{S}$
	- **2** There is a new quantum group H_2^* ►∕H1 (the 'semidual'). 1t acts canonically on *H*₂.
	- **3** The Heisenberg-Weyl algebra $H_2^*){\rtimes} (H_1{\bowtie} H_2)$ of the first \blacksquare model is the same as as the Heisenberg-Weyl algebra $(H_2^*M_1)\ltimes H_2$ of the second.

 $i.e.$ the combined rotations-momentum-position algebra is invariant under position \leftrightarrow momentum.

4 Applied to 3D quantum gravity we also swap $m_p \leftrightarrow l_c$. Applied to 3D quantum gravity we alse

5. NCG associated to a Riemannian manifold Γ high to be the classical Laplace-Beltrami operator but will need a little Γ <u>s. Inclu associated to a Nichialihian inamioid</u>

Let (M,\bar{g}) be a Riemannian manifold dim n, inverse metric (,), levi-civita connection $\bar{\nabla},$ and $\bar{\Delta}$ a second order diffl op such that Let (M, \bar{q}) be a Riemannian manifold dim n inverse metric (i) a new one of the internation of the individual metric (,),
Let (x^{i_1}, y) be a neutrical matricular metric intervals in the classical metric of the classical metric of a $\frac{1}{2}$ of the polarization formula that the polarization formula $\frac{1}{2}$ $s = \frac{1}{\sqrt{2\pi}}$ to have connected by and \triangle a second order dim op sach that $\bar{\nabla}.$ and $\bar{\Delta}$

$$
\bar{\Delta}(fg) = (\bar{\Delta}f)g + f(\bar{\Delta}g) + (\bar{\mathrm{d}}f, \bar{\mathrm{d}}g), \quad \forall f, g \in C(M)
$$

 $t_{\rm max}$. This is the Leibniz rule. This is $\sqrt{2}$ (a.g.), the classical differential di **Letting** the classical calculus Ω (*NI*) has a i Lemma 2.1. *Let M be a Riemannian manifold with notations as above. Then* Lemma The classical calculus $\overline{\Omega}^1(M)$ has a noncommutative extension (`ito calculus')

 $\Omega^1 = \overline{\Omega}^1 \oplus C(M)\theta'$ *with* θ' *central and*

$$
f \bullet \omega = f\omega, \quad \omega \bullet f = \omega f + \lambda(\omega, \bar{d}f)\theta', \quad df = \bar{d}f + \frac{\lambda}{2}(\bar{\Delta}f)\theta'
$$

$$
f \in C(M), \omega \in \bar{\Omega}^1
$$

Lemma There is a well-defined linear map

 $\phi: \bar{\Omega}^1 \bar{\otimes} \bar{\Omega}^1 \rightarrow \Omega^1 \hat{\otimes} \Omega^1, \quad \phi(\omega \bar{\otimes} \eta) = \omega \hat{\otimes} \eta - \lambda \theta' \hat{\otimes} \bar{\nabla}_{\omega} \eta, \quad \forall \omega, \eta \in \bar{\Omega}^1$ *where the new product is understood.* from the classical $\bar{\otimes}$ over $C(M)$ to the new $\hat{\otimes}$ wrt •

Now suppose
$$
\bar{\Delta}
$$
 extends to 1-forms (eg Laplace-Beltrami):
\n
$$
\bar{\Delta}(f\omega) = (\bar{\Delta}f)\omega + f\bar{\Delta}\omega + 2\bar{\nabla}_{\bar{d}f}\omega
$$
\n
$$
\bar{\Delta}((\omega, \eta)) = (\bar{\Delta}\omega, \eta) + (\omega, \bar{\Delta}\eta) + 2(\bar{\nabla}\omega, \bar{\nabla}\eta)
$$
\n
$$
[\bar{\Delta}, \bar{d}]f = \text{Ricci}_{\bar{\Delta}}(\bar{d}f) \qquad \forall f \in C(M), \omega, \eta \in \bar{\Omega}^1
$$

Lemma
$$
\zeta
$$
 a classical vector field on M .
\n
$$
\bar{\Delta}f = \bar{\Delta}_{LB}f + \zeta(f), \quad \bar{\Delta}\omega = \bar{\Delta}_{LB}\omega + \bar{\nabla}_{\zeta}\omega
$$
\n
$$
\text{Ricci}_{\bar{\Delta}} = \text{Ricci} + \bar{\nabla}_{\zeta} - \bar{\mathcal{L}}_{\zeta}
$$

fulfils our conditions (we will need this greater generality in the next **paramers** fulfils our conditions (we will heed this greater generality if fulfils our conditions (we will need this greater generality in the next section)

 $\sqrt{\text{Theorem}}$ for any $K : \bar{\Omega}^1 \to \bar{\Omega}^1$ and $\nabla \theta'$ central $\lambda_{0}/\hat{\lambda}_{0}$ is usually discussed in the context of a metric connection but we connect $\lambda_{0}/\hat{\lambda}_{0}$ $t=\phi(\nabla\omega)+\frac{1}{2}\theta\otimes(\Delta-K)\omega, \quad \forall \omega\in \Omega^*\subset \Omega^*.$ $\bar{\nabla}$ $\hat{\Omega}'$ $\Omega' \hat{\Omega}$ $P = \theta' \hat{\otimes}_\omega, \quad \sigma(\theta' \hat{\otimes}_\omega) = \omega \hat{\otimes} \theta' \quad \sigma(\theta' \hat{\otimes} \theta') = \theta' \hat{\otimes} \theta'$ (γ) − αναφαίρεται η αν Lemma 2.3. *For any classical tensor ^K* : ^Ω¯ ¹ [→] ^Ω¯ ¹ *the classical Levi-Civita con-*Theorem for any $K : \bar{\Omega}^1 \to \bar{\Omega}^1$ and $\nabla \theta'$ central $\nabla\omega = \phi(\bar\nabla\omega) + \frac{\lambda}{2}$ 2 $\theta' \hat{\otimes} (\bar{\Delta} - K) \omega, \quad \forall \omega \in \bar{\Omega}^1 \subset \Omega^1.$ P° (**Proof**) 2.2 we have P° we have P° $\sigma(\omega\otimes\theta')=\theta'\otimes\omega,\quad \sigma(\theta'\otimes\omega)=\omega\otimes\theta',\quad \sigma(\theta'\otimes\theta')=\theta'\otimes\theta'$ [⊗]^ˆ (∆¯ [−] *^K*)(*f*ω) $\frac{1}{\alpha}$ **λ** with**}** $\hat{\mathbf{C}}$ $\partial \theta$ λ^2
 λ^2 (Discrete λ^2) $\frac{1}{\sqrt{2}}$ expresses by working via the properties of $\frac{1}{\sqrt{2}}$ is $\frac{1}{\sqrt{2}}$ is $\frac{1}{\sqrt{2}}$ Theorem for any $K : \bar{\Omega}^1 \to \bar{\Omega}^1$ and $\nabla \theta'$ central $\nabla \omega = \varphi(\nabla \omega) + \frac{1}{2} \theta \otimes (\Delta - \Lambda) \omega, \quad \forall \omega \in \Omega$ $\sigma(\omega\hat{\otimes}\eta) = \eta\hat{\otimes}\omega + \lambda\bar{\nabla}_{\omega}\eta\hat{\otimes}\theta' - \lambda\theta'\hat{\otimes}\bar{\nabla}_{\eta}\omega + \lambda(\omega,\eta)\nabla\theta' +$ λ^2 $\frac{\gamma}{2}(\text{Ricci}_{\bar{\Delta}}+K^T)(\omega,\eta)\theta' \hat{\otimes}\theta'$ $\sigma(\omega\hat{\otimes}\theta')=\theta'\hat{\otimes}\omega,\quad \sigma(\theta'\hat{\otimes}\omega)=\omega\hat{\otimes}\theta',\quad \sigma(\theta'\hat{\otimes}\theta')=\theta'\hat{\otimes}\theta'$

*{*ω*,* η*}* = λθ! *h* connection on the πe calculus, $\frac{1}{2}$ → $\frac{1}{2}$ $\frac{1}{2}$ → $\frac{1}{2}$ → $\frac{1}{2}$ -1 **S** a bimodule connection on the B
Notation of \hat{P} and \hat{P} and \hat{P} *is a bimodule connection on the ito calculus,*

$$
\nabla : \Omega^1 \to \Omega^1 \hat{\otimes} \Omega^1, \quad \sigma : \Omega^1 \hat{\otimes} \Omega^1 \to \Omega^1 \hat{\otimes} \Omega^1
$$

Propn take $\bar{\Delta} = \bar{\Delta}_{LB}$, $K = \text{Ricci}$, $\nabla \theta' = 0$ then *so that* [∇] *in Proposition 2.3 has zero torsion, provided* [∆]¯ *is the Laplace-Beltrami* ${\bf Propn}$ take $\Delta = I$.
1 i ΔB , $K = \text{Ricci}, \ \nabla \theta' = 0$ (2.5) ^σ(ω⊗^ˆ ^η) = ^φ(η⊗¯ω) + ^λ∇¯ ^ωη⊗^ˆ ^θ! ⁺ ^λ(ω*,* ^η)∇θ! ⁺ $\frac{2}{\pi}$ → $\frac{1}{\pi}$

\n- $$
\sigma^2 = \text{id}
$$
 iff Ricci = 0
\n- $\sigma_{12}\sigma_{23}\sigma_{12} = \sigma_{23}\sigma_{12}\sigma_{23}$ iff (M, \bar{g}) is flat
\n

(some kind of `braided 2-category' associated to any Riemannian manifold?) *{*θ! *,* d*f}* = [dθ! *, f*]*, {*d*b,* ^d*f}* ⁺ ^λ(d(d¯*f,* d¯*b*))θ! ⁺ ^λ(d¯*b,* d¯*f*)dθ! = 0*.* C and C and D and $\text{D$ (some kind of `braided 2-category' associated to any Riemannian manifold?)

Example: static spherically symmetric spacetimes briefly consider an approach to the black hole and other spacetimes where we first <u>example. Static spherically symmetric spac</u> calculation.

$$
M=\mathbb{R}^3\setminus\{0\},\quad \bar{g}=h(r)^2\bar{\rm d}r\bar{\otimes}\bar{\rm d}r+\bar{\omega}^T\bar{\otimes}\bar{\omega}
$$

$$
\tau = \frac{r}{h(r)} \frac{\partial}{\partial r}, \quad \alpha = \frac{2}{h(r)} - 1 \qquad \qquad g_{spacetime} = \beta^{-1} \overline{\mathrm{d}} t \overline{\otimes} \overline{\mathrm{d}} t + \overline{g}
$$

$$
[x_i, x_j] = 0, \quad [x_i, t] = \frac{\lambda}{h} x_i, \quad [\omega_i, x_j] = \lambda e_{ij} \theta', \quad [\mathrm{d}r, x_i] = \frac{\lambda}{h(r)^2} \frac{x_i}{r} \theta', \quad [\theta', x_i] = 0
$$

$$
[\omega_i, t] = \lambda \left(\frac{1}{h} - 1\right) \omega_i, \quad [\theta', t] = \lambda \left(\frac{2}{h} - 1\right) \theta', \quad [x_i, \mathrm{d}t] = \lambda \mathrm{d}x_i, \quad [\mathrm{d}t, t] = \beta \lambda \theta' - \lambda \mathrm{d}t.
$$

$$
[\mathrm{d}r, t] = \lambda (\mathrm{d}\left(\frac{r}{h}\right) - \mathrm{d}h)
$$

e.g.
$$
h = \frac{1}{\sqrt{1 - \frac{\gamma}{r}}}, \quad \tau = r\sqrt{1 - \frac{\gamma}{r}}\frac{\partial}{\partial r}, \quad \alpha = 2\sqrt{1 - \frac{\gamma}{r}} - 1, \quad \beta = -\frac{1}{c^2(1 - \frac{\gamma}{r})}
$$

where we adjoin h, h **finds right** *n n* ble differential al e we adioin h,h^{-1} black hole differential algebra' mutation relations and β. where we adjoin h,h^{-1} black hole differential algebra'

^r! ^d*r*! \sum wave operator in the constructed but hand to \sqrt{f} wave operator \Box constructed but hard to compute wave operator

$$
\implies \qquad df = \bar{d}f + (\partial^0 f)dt + \frac{\lambda}{2}(\Box f)\theta'
$$

 \overline{C} constructs the wave operator \overline{C} on $C(M)$, \overline{D} constructs the wave operator \square on $C(M) \rtimes_{\tau} \mathbb{R}$

On normal-ordered
$$
f = \sum_n f_n t^n
$$
, $f_n \in C(M)$

$$
\partial^0 f(t) = \frac{f(t) - f(t - \lambda)}{\lambda} \qquad \Box f(t) = (\bar{\Delta}f)(t + \lambda \alpha) + 2\Delta_0 f(t)
$$

$$
\Delta_0 f(t) = \frac{\nu f(t + \lambda \alpha) + \mu f(t - \lambda(\frac{\beta}{\mu} - \alpha)) - (\nu + \mu) f(t + \lambda(\alpha - \frac{\beta}{\nu + \mu}))}{\lambda^2}
$$

if functions μ, ν solve $\tau(\mu) = \beta - (1 + \alpha)\mu$, $\tau(\nu) = \mu - \alpha\nu$. *Then the calculus* $\frac{1}{2}$ is the normal-ordered element field in the normal-order element for $\frac{1}{2}$ $\frac{$ (can always do this locally)

$$
\begin{array}{ccc}\n\mathbf{A}=\bar{\Delta}_{LB}-\frac{1}{2}\bar{g}^{-1}(\beta^{-1}\bar{\mathrm{d}}\beta)&\Rightarrow&\Box&\text{deforms wave operator for} \\
\mathbf{static}\hspace{2mm}\mathbf{static}\hspace{2mm}\mathsf{metric}&\beta^{-1}\mathrm{d}t\otimes\mathrm{d}t+\bar{g}\n\end{array}
$$

[∂]⁰ *^f* (*t*) ⁼ So we quantise any static metric with spatial part admitting a conformal killing vector field! (SM, CMP 2012) ^ν *^f* (*^t* ⁺ λα) ⁺ ^µ*^f* (*^t* [−] ^λ(^β ^µ [−] ^α)) [−] (^ν ⁺ µ) *^f* (*^t* ⁺ ^λ(^α [−] ^β