Holographic Josephson Junction Networks

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based on

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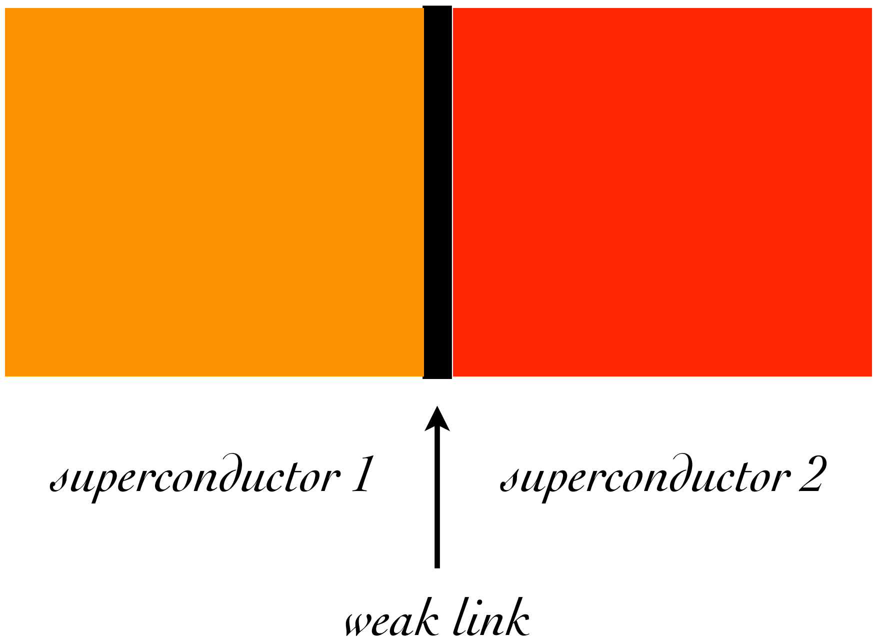
Many technological applications of superconductors and superconducting materials involve Josephson junctions (JJs)



Typical types of junctions: SIS, SNS (I: insulator, N: normal metal), or more exotic combinations (sIs, sId, dId,...)

Each combination comes with its own special phenomenology, e.g. the typical current-phase relation $I = I_{\max} \sin \vartheta$ may vary.

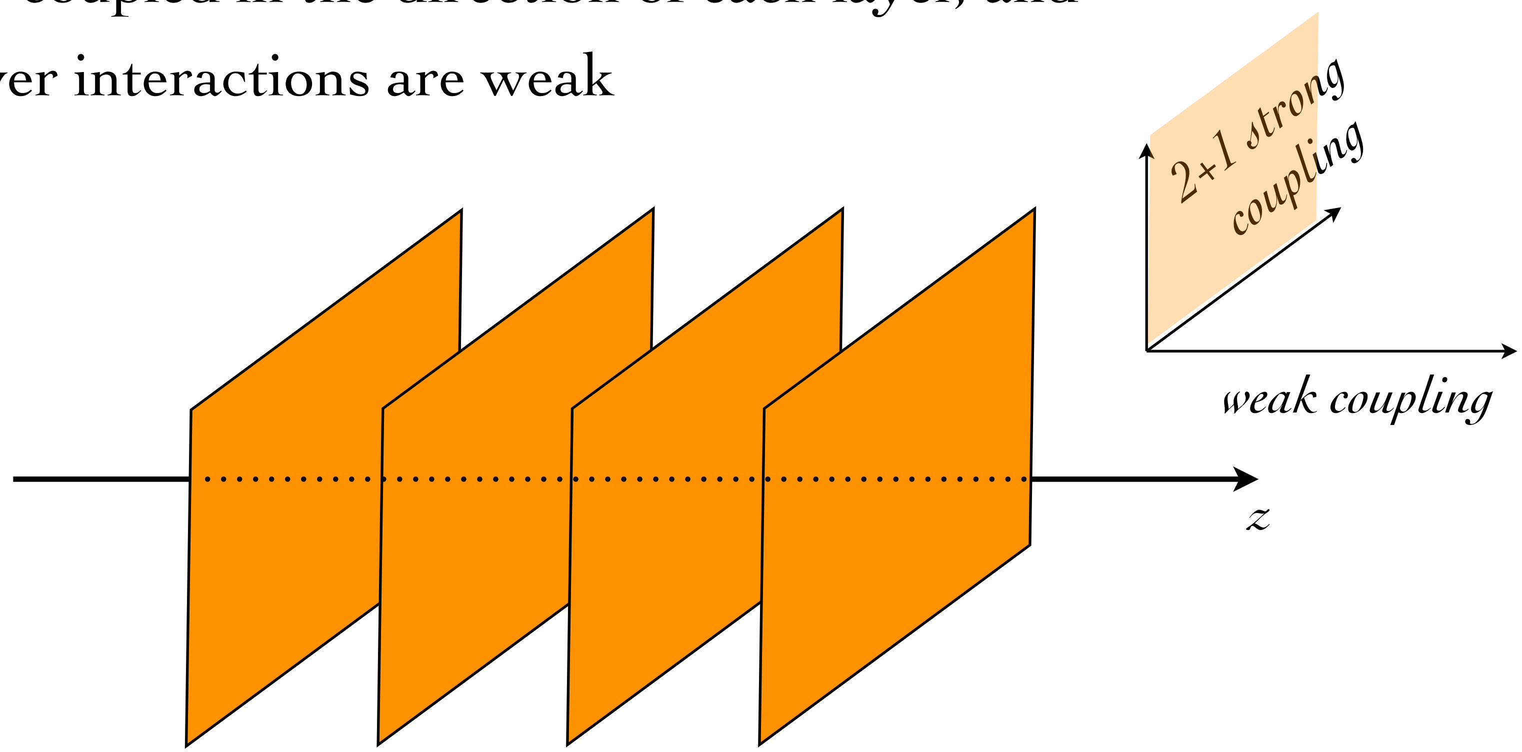
Our interest.

Motivated by the physical significance of such configurations we would like to understand the properties of systems that exhibit:

i) a layered structure,

ii) are strongly coupled in the direction of each layer, and

iii) the interlayer interactions are weak



Dimensions can vary, more complicated networks can be imagined.

Physics of a single layer.

On each (2+1)-dimensional layer lives a strongly coupled (large-N) QFT.
We assume this theory has a dual weakly curved gravitational description
(a holographic superconductor, an example below...)

Physics of the interlayer coupling.

2-layer system: **w/o coupling** the total QFT=$\mathrm{QFT}_1 \oplus \mathrm{QFT}_2$ is described by the direct sum of actions $S = S_1 + S_2$.

The system has: 2 separate large-N gauge symmetries
2 separately conserved stress-energy tensors
2 separate sets of global R-symmetries
...

The dual gravity description is obvious: a trivial **bi-gravity (bi-string)** theory.

In **field theory** the ***unique*** type of inter-theory coupling that respects the separate gauge invariances of QFT_1 and QFT_2 is one effected by multi-trace deformations.

For example, if O_1 is a scalar single-trace operator of QFT_1 and O_2 a scalar single-trace operator of QFT_2 then one can consider interactions of the form

$$\int d^{(2+1)}x \, W(\mathcal{O}_1, \mathcal{O}_2)$$

where W is multi-trace, *e.g.* a double trace of the form

$$W = g_{12}\mathcal{O}_1\mathcal{O}_2$$

This coupling may be relevant or irrelevant, break relative symmetries etc...
g_{12} scales as $O(N^0)$ and preserves the 1/N expansion.

What happens in **gravity**? ➠ **designer multi-gravity**

The bi-gravity theory becomes non-trivial.
Such bi-gravity theories have a well-known large-N description in the AdS/CFT correspondence.

At **tree-level** in gravity the boundary multi-trace interactions map to **mixed boundary conditions** for the dual fields.

Beyond tree-level massive gravity...

An illustrative model

Assume a large-N (2+1)-dimensional QFT with a dual bulk gravitational description that can be reduced to the (3+1)-dimensional Einstein-abelian Higgs model:

$$S_{bulk} = \int d^{3+1}x \, \sqrt{-g} \left[R - \frac{1}{4} G(|\phi|) F^2 - (\nabla |\phi|)^2 - J(|\phi|)(\nabla\theta - qA)^2 - V_{bulk}(|\phi|) \right]$$

R: Ricci scalar,

A: abelian gauge field with F=dA its field strength,

ϕ: a charged complex scalar field with U(1) charge q, and $\phi = |\phi| e^{i\theta}$

G, J, V_{bulk} are model-dependent functions of $|\phi|$ (left arbitrary in our discussion).

Under gauge-gravity duality:

- A *maps to* a $U(1)$ current on the boundary QFT

- ϕ *maps to* a complex scalar operator O with scaling dimension Δ

  (*our `Cooper-pair' operator whose vev will break the U(1) leading to superfluidity/superconductivity*)

For asymptotically AdS$_4$ solution near the boundary

vev, source of $\mathcal{O}$

$$ds^2 \simeq r^2 dx^\mu dx_\mu + \frac{dr^2}{r^2}$$

$$\phi \simeq \frac{\alpha}{r^\Delta} + \ldots + \frac{\beta}{r^{d-\Delta}} + \ldots$$

$$A \simeq A^{(0)} + \ldots + \frac{A^{(1)}}{r}$$

Building a junction

On the QFT side the weak link is implemented via a multi-trace interaction

$$S_{tot} = S_1 + S_2 + \int d^{2+1}x \; W(\mathcal{O}_1, \mathcal{O}_2)$$

On the gravity side W translates to scalar field mixed boundary conditions.

With asymptotics

$$\phi_1 \simeq \frac{\alpha_1}{r_1^{\Delta_1}} + \ldots + \frac{\beta_1}{r_1^{d-\Delta_1}} + \ldots$$

$$\phi_2 \simeq \frac{\alpha_2}{r_2^{\Delta_2}} + \ldots + \frac{\beta_2}{r_2^{d-\Delta_2}} + \ldots$$

W translates to the mixed bc's

$$\beta_1 = \partial_{\alpha_1} W(\alpha_1, \alpha_2) \ , \quad \beta_2 = \partial_{\alpha_2} W(\alpha_1, \alpha_2)$$

Finding the ground state

Standard practice in field theory:

➠ compute and minimize the quantum effective potential.

Typically very hard.

More complicated at finite temperature and density but we can do it...

Gravity gives a tractable prescription.

For boost invariant planar solutions (*A*=*0*, *T*=*0*) in the above example

$$V(\alpha_1, \alpha_2) = W(\alpha_1, \alpha_2) + \sum_{i=1}^{2} \widetilde{W}_i(\alpha_i)$$

$$\widetilde{W}_i(\alpha_i) = \frac{s_i \Delta_i}{3} |\alpha_i|^{\frac{3}{\Delta_i}}$$

A holographic Josephson junction example

This quiver diagram refers to a QFT Lagrangian of the form

$$\mathcal{L}_{tot} = \mathcal{L}_1 + \mathcal{L}_2 + g|\mathcal{O}_1|^2 + g|\mathcal{O}_2|^2 + h\left(e^{i\vartheta}\mathcal{O}_1\mathcal{O}_2^\dagger + e^{-i\vartheta}\mathcal{O}_1^\dagger\mathcal{O}_2\right) \;, \quad h \in \mathbb{R} \;, \quad \vartheta \in [0, 2\pi)$$

which gives

$$V(\alpha_1, \alpha_2) = \sum_{i=1}^{2}\left(g|\alpha_i|^2 + \frac{s}{\delta}|\alpha_i|^\delta\right) + h\left(e^{i\vartheta}\alpha_1\alpha_2^* + e^{-i\vartheta}\alpha_1^*\alpha_2\right)$$

The ground state is determined by solving the algebraic extremization equations

$$g\alpha_1 + he^{-i\vartheta}\alpha_2 + \frac{s}{2}\alpha_1|\alpha_1|^{\delta-2} = 0$$

$$g\alpha_2 + he^{i\vartheta}\alpha_1 + \frac{s}{2}\alpha_2|\alpha_2|^{\delta-2} = 0$$

In the algebraically simple case $\delta = 4$

$$\alpha_2 = -h^{-1}e^{i\vartheta}\left(g + \frac{s}{2}|\alpha_1|^2\right)\alpha_1$$

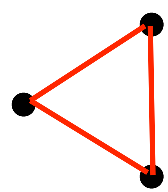
$$(1)\ \alpha_1 = 0\ , \quad (2)\ |\alpha_1|^2 = \frac{2}{s}\left(\pm h - g\right)\ , \quad (3)\ |\alpha_1|^2 = -\frac{1}{s}\left(g \pm \sqrt{g^2 - 4h^2}\right)$$

Networks from designer multi-gravity

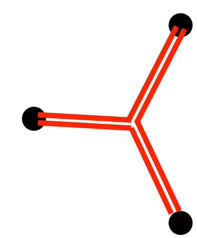
The framework can be generalized in a straightforward manner to describe networks of very diverse architecture and internal structure

vertices/sites	$\longrightarrow$	*large-N QFTs with gravity duals*
links	$\longrightarrow$	*multi-trace interactions mixed bcs*

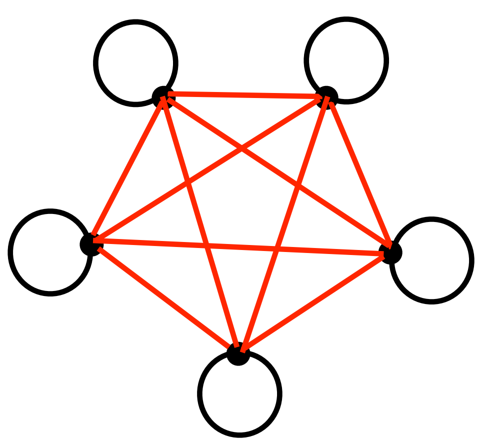
examples:



with 2-trace links



with 3-trace links



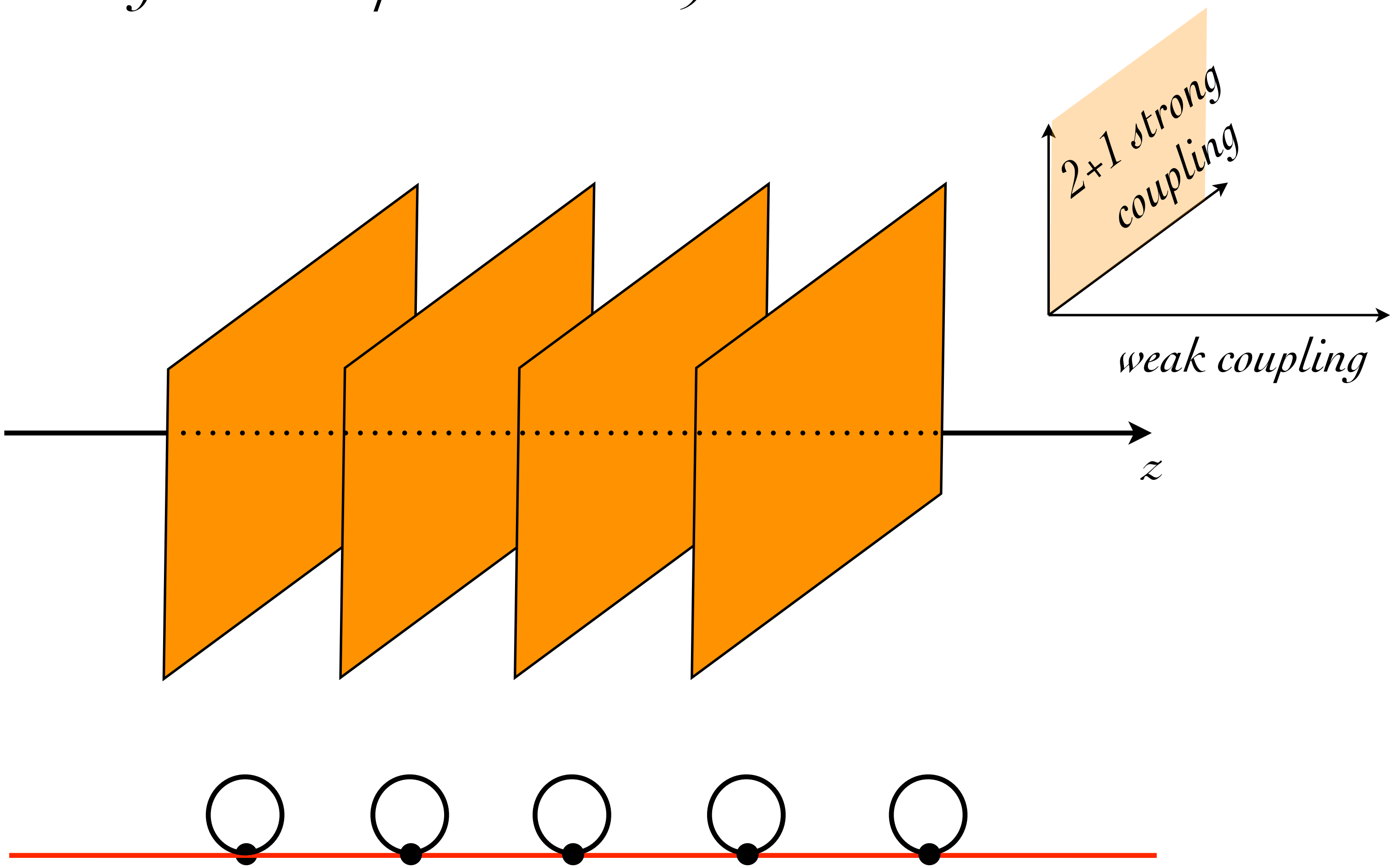
in quenched disorder computations via the replica trick

New `potential':

$$V = W(\alpha_1, \ldots, \alpha_k) + \sum_{i=1}^{k} \widetilde{W}_i(\alpha_i)$$

Linear Josephson junction array as a simple model of a layered SC

(deconstructing an extra space dimension)



$$V=\sum_n\left(g|\alpha_n|^2+\frac{s}{\delta}|\alpha_n|^\delta+h\left(e^{i\vartheta}\alpha_n\alpha^*_{n+1}+e^{-i\vartheta}\alpha^*_n\alpha_{n+1}\right)\right)$$

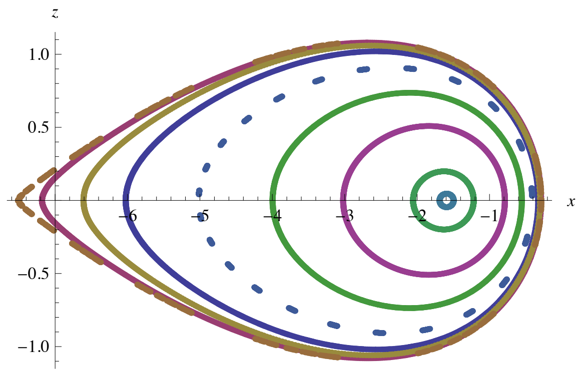
The vacuum is determined by solving the algebraic eqs of a discrete dynamical system

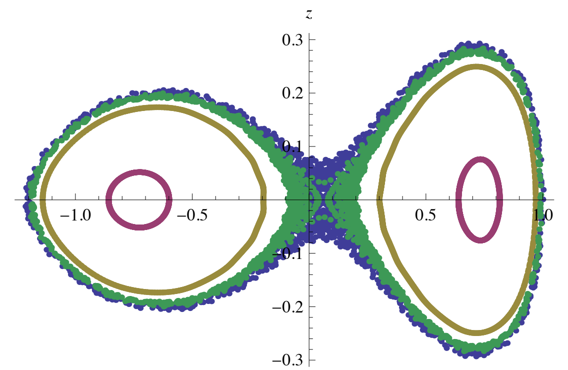
$$g\alpha_n + h\left(e^{i\vartheta}\alpha_{n-1} + e^{-i\vartheta}\alpha_{n+1}\right) + \frac{s}{2}\alpha_n|\alpha_n|^{\delta-2} = 0$$

The algebraically simple case δ=*4* has been studied extensively in the literature of dynamical systems (*see e.g. review by Tsironis, Hennig '99*) with applications in diverse condensed matter systems.

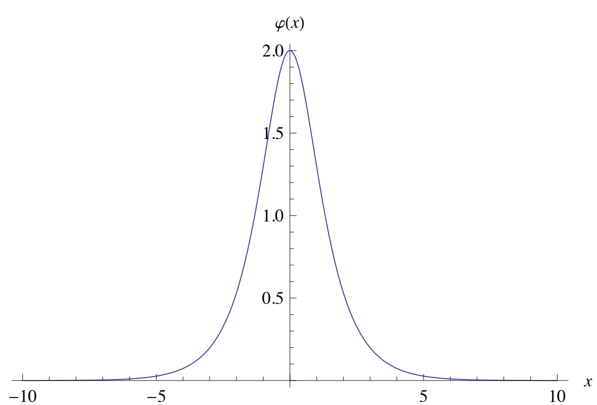
Rich solution space: complexity

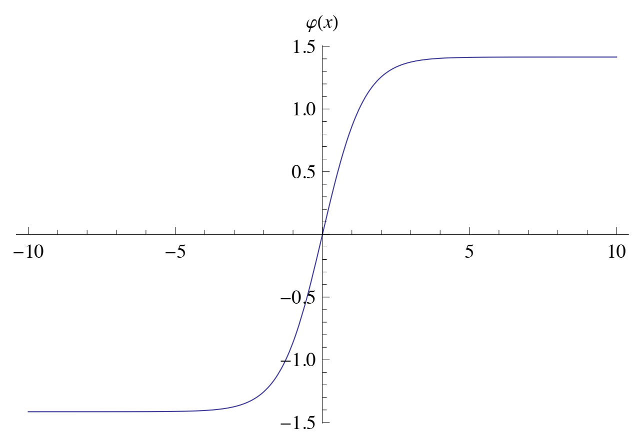
1) Chaos and bifurcation.





2) Solitons (*pinned superconductivity*), kinks (*junction*)

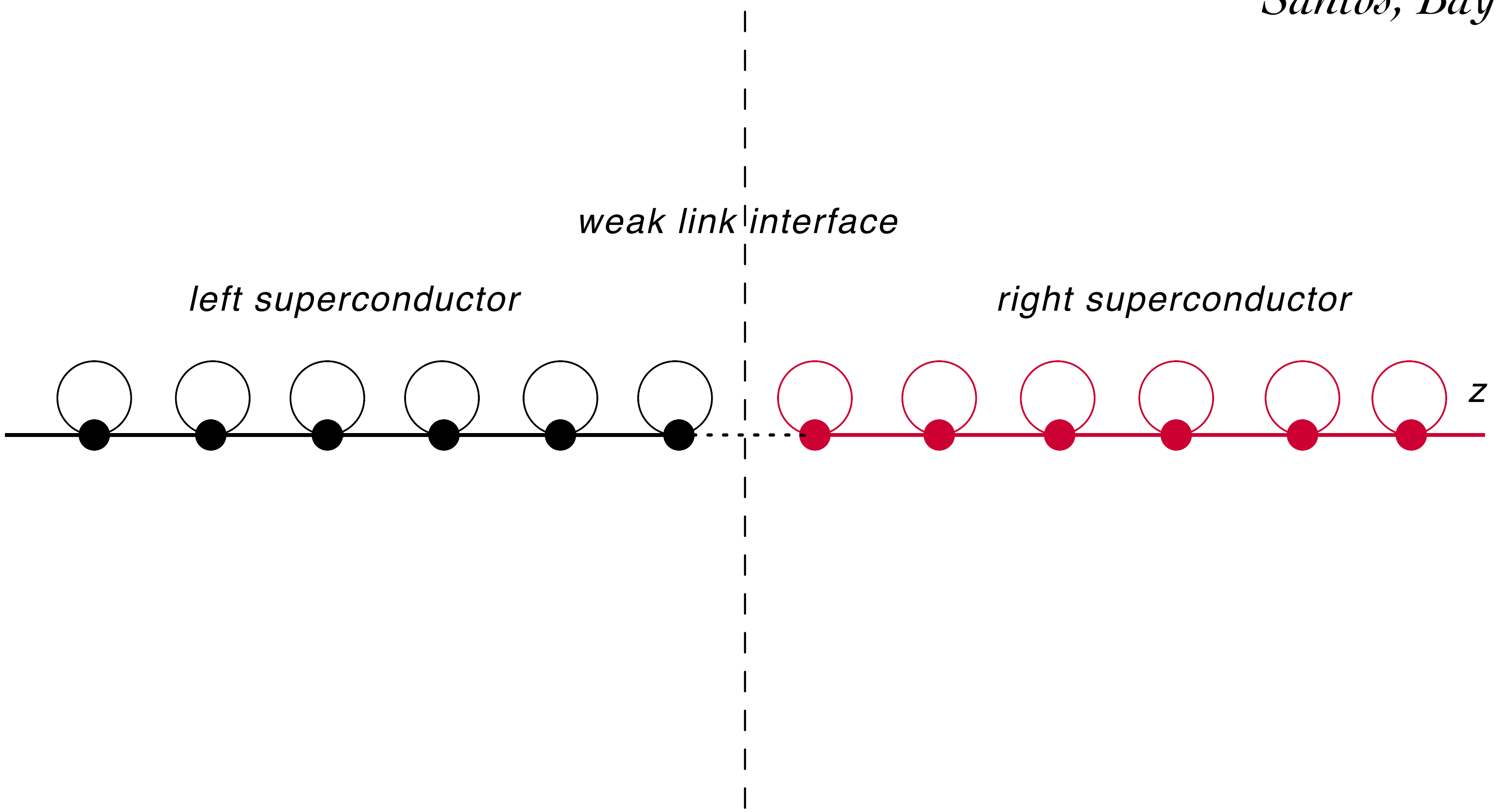




Diverse possibilities and applications

alternative to other approaches based on inhomogeneous holo-SC solutions (e.g. Horowitz, Santos, Bay '11)

(Un)conventional Josephson junctions



Outlook

Many possibilities for further work, *e.g.*

1) Physics of unconventional JJs

2) Magnetic fields/charge density/temperature

3) JJNs with different architectures

4) Time-dependence

5) ...