Time evolution of the sQGP with new solutions of relativistic hydrodynamics

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Outline

1. Hydrodynamics
2. Constant EoS hydro compared to data
3. Solutions with a QCD EoS
4. Summary

Introduction

- Collective dynamics of the sQGP is observed at RHIC
  - Thermal spectra, momentum anizotropy v_2, scaling correlation radii
- Hydrodynamics: applicable in high energy A+A and p+p!
  PHENIX Collaboration, Nucl. Phys. A **757**, 184 (2005), nucl-ex/0410003
- Equations of hydro: highly non-linear, not straightforward to solve
- Exact, analytic solutions: important to determine initial and final state
- Non-relativistic hydro: many solutions, applicable, but inconsistent!
- Relativistic ideal hydro: formulated by Landau
  - Famous 1+1D solutions: Landau, Izv. Acad. Nauk SSSR **17**, 51 (1953)
    Hwa, Phys. Rev. D **10**, 2260 (1974)
    Bjorken, Phys. Rev. D **27**, 40 (1983)
  - Revival of interest, many new solutions, mostly 1+1D, few 1+3D
  - Using constant Equation of State!
- Time evolution of sQGP? Temperature dependent EoS?

Equations of hydrodynamics with a conserved charge

- There may be a conserved charge or number n (chem. pot. $\neq 0$)
- Basic eqs: continuity and energy-mom. conservation

$$\partial_\mu(nu^\mu) = 0 \text{ and } \partial_\nu T^{\mu\nu} = 0 \tag{1}$$

- Energy-momentum tensor in perfect fluid: $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$
- Equation on $T^{\mu\nu}$an be split into energy & momentum conservation

$$(\epsilon + p)\partial_\nu u^\nu + u^\nu \partial_\nu \epsilon = 0 \tag{2}$$

$$(\epsilon + p)u^\nu \partial_\nu u^\mu = (g^{\mu\nu} - u^\mu u^\nu)\partial_\nu p \tag{3}$$

- Equation of State (if $\kappa =$ const., $c_s^2 = 1/\kappa$): $\epsilon = \kappa p$
- This is a full set of equations for u^μ, n and p
- Introduce temperature with $p = nT$, temperature eq. from here:

$$T\partial_\mu u^\mu + \kappa u^\mu \partial_\mu T = 0 \tag{4}$$

- Write up solutions for u^μ, n and T instead!

Equations of hydrodynamics without conserved charges

- Energy-momentum conservation is the same
- Let us introduce entropy density σ
- Fundamental thermodynamical relations without a conserved charge

$$\varepsilon + p = T\sigma \;\Rightarrow\; d\varepsilon = Td\sigma \text{ and } dp = \sigma dT \tag{5}$$

- The same continuity equation for σ follows from here:

$$\partial_\nu(\sigma u^\nu) = 0, \tag{6}$$

- EoS can be used the same manner here, but different $p - T$ relation!

$$\epsilon = \kappa p \text{ and } p = T\sigma/(\kappa + 1) \tag{7}$$

- If $\kappa =$ const., we get the same equation on the temperature as with n:

$$T\partial_\mu u^\mu + \kappa u^\mu \partial_\mu T = 0. \tag{8}$$

- All $\kappa =$ const. solutions valid for both $\{u^\mu,\, n,\, T\}$ and $\{u^\mu,\, \sigma,\, T\}$.

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A known ellipsoidal solution with constant κ

- First exact, analytic and truly 3D relativistic solution
  Csörgő, Csernai, Hama *et al.*, Heavy Ion Phys. **A21**, 73 (2004), nucl-th/0306004

$$u^\mu = \frac{x^\mu}{\tau}, \quad \text{with } \tau = \sqrt{x_\mu x^\mu}, \tag{9}$$

$$n = n_0 \frac{V_0}{V} \nu(s), \quad T = T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \nu(s)^{-1}, \tag{10}$$

- $\nu(s)$ is an arbitrary function and

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}, \quad V = \tau^3, \tag{11}$$

- X, Y, Z: principal axes of expanding ellipsoid, $X(t) = \dot{X}_0 t$ etc.
- This solution is non-accelerating, ie. obeys $u^\nu \partial_\nu u^\mu = 0$.
- It can be written up for σ (i.e. no conserved n) as well.
- Constant κ!

Hadronic observables compared to data

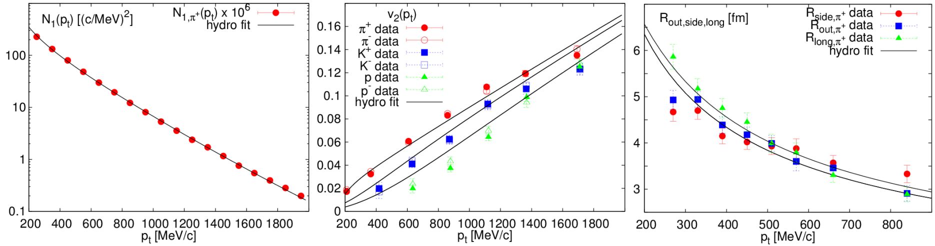
- How does the $\kappa =$const. solution compare to data?
- The hadronic source function is given from the solution:

$$S(x,p)d^4x = \mathcal{N}\frac{p_\mu\, d^3\Sigma^\mu(x) H(\tau) d\tau}{n(x)\exp\left(p_\mu u^\mu(x)/T(x)\right) - 1} \tag{12}$$

- Compared to PHENIX data successfully

Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010), arXiv:0909.4842

Data: PHENIX Coll., PRC**69**034909(2004), PRL**91**182301(2003), PRL**93**152302(2004)

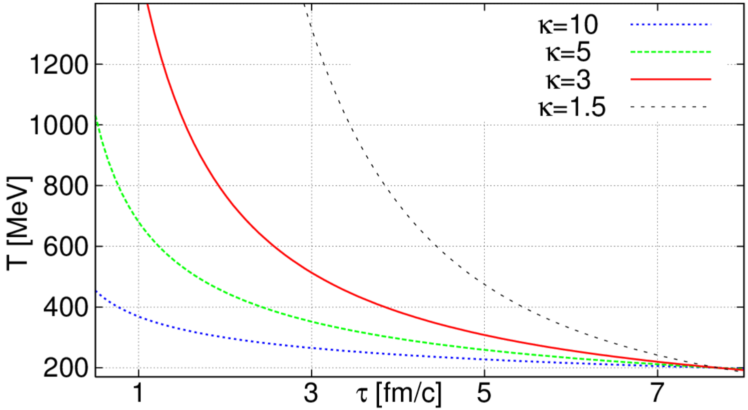


Time evolution of the sQGP matter?

- Hadronic data describe the final state
- Different EoS lead to different initial states!

Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010), arXiv:0909.4842

Csanád, Nagy, Csörgő, Eur. Phys. J. **ST**, 19 (2008), arXiv:0710.0327



Direct photon observables compared to data

- Photons are created throughout the evolution
- Their distribution reveals information about the EoS!
- The source function of photon creation is assumed as:

$$S(x,p)d^4x = \mathcal{N}\frac{p_\mu u^\mu}{\exp\left(p_\mu u^\mu(x)/T(x)\right) - 1}\, d^4x \qquad (13)$$

- Integrated over energy (i.e. four-momentum): emission $\propto T^4$
  - Analyzed systematic change with T power, based on

$$\text{rate}(A+B\to X) = n_A n_B \left\langle v\sigma_{A+B\to X}\right\rangle \propto T^6 \qquad (14)$$

- Transverse mom. distribution, elliptic flow and HBT radii calculable

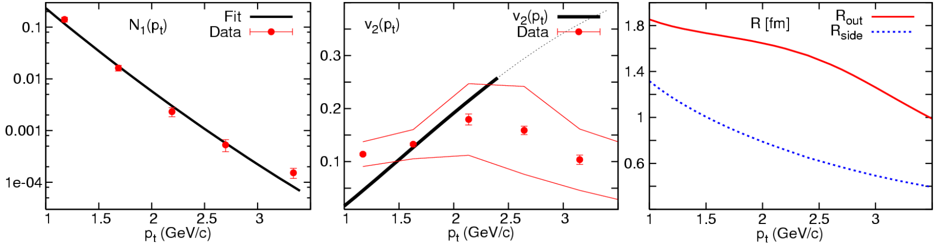
Csanád, Májer, Central Eur. J. Phys. **10** (2012), arXiv:1101.1279

Direct photon observables compared to data

- Compared to PHENIX data successfully
- Predicted photon HBT radii

  Csanád, Májer, Central Eur. J. Phys. **10** (2012), arXiv:1101.1279

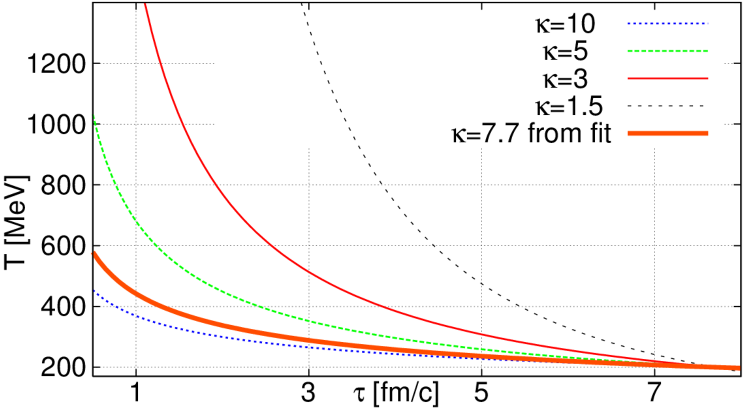
  Data: PHENIX Collaboration, arXiv:0804.4168 and arxiv:1105.4126



- Result for the average EoS: $c_s = 0.36 \pm 0.02_{stat} \pm 0.04_{syst}$
- Time interval: $\tau_{\rm ini} \approx 0.7$ fm$/c$, $\tau_{\rm final} = 7.7$ fm$/c$ (from hadron fits)

Time evolution result with κ =const.

- The $c_s = 0.36$ result means $\kappa = 7.7$
- Corresponds to to $T_{\rm ini}$ of $507 \pm 12_{stat} \pm 90_{syst}$ MeV at 0.7 fm/c



Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010), arXiv:0909.4842

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Solutions with arbitrary EoS?

- Constant EoS may not be realistic (temperature may change rapidly)
- If $\kappa(T)$, new solutions have to be found
- With a conserved charge (and $\epsilon = \kappa n T$), the temperature equation is:

$$T\partial_\mu u^\mu + \frac{d(\kappa T)}{dT} u^\mu \partial_\mu T = 0. \tag{15}$$

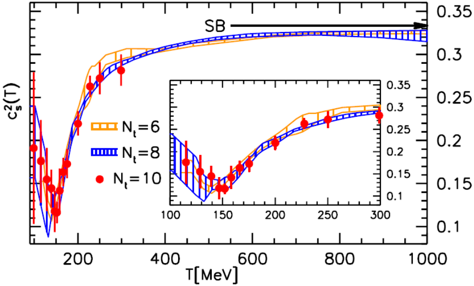
- Works only if $d(\kappa T)/dT > 0$!
- In case of no conserved charges (and $\epsilon = \kappa T\sigma/(\kappa+1)$):

$$T\partial_\mu u^\mu + \left[\kappa + \frac{T}{\kappa+1}\frac{d\kappa}{dT}\right] u^\mu \partial_\mu T = 0, \tag{16}$$

- Remarkable: these are not the same!
- They coincide if $\kappa =$ const., c.f. slide no. 5!
- If u^μ known, these can be solved for an arbitrary $\kappa(T)$!

Lattice QCD EoS

- lQCD EoS calculated for physical quark masses, continuum limit
  Borsányi, Fodor, Katz *et al.* JHEP **1011**, 077 (2010), arXiv:1007.2580

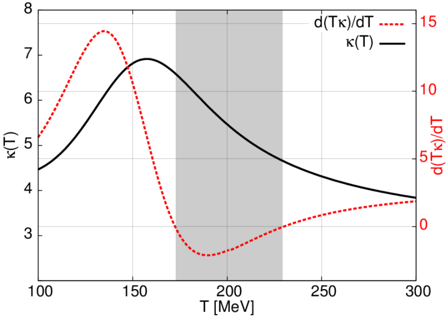


- Trace anomaly $I(T)/T^4$ is parametrized analytically in the cont. limit
- Pressure is given by $\frac{p(T)}{T^4} = \int \frac{dT}{T} \frac{I(T)}{T^4}$
- From $I = \epsilon - 3p \;\Rightarrow\; \kappa = I/p + 3$

A realistic EoS

- Take EoS from lQCD: trace anomaly $I(T)/T^4$ parametrized
- EoS in form of $\kappa(T)$ analytically given

  Csanád, Nagy, Lökös, arXiv:1205.5965



- Problem: $d(\kappa T)/dT \leq 0$ at $T = 173 - 225$ MeV
- Recall slide no. 14: conserved charges not compatible with this EoS!

A new $\kappa(T)$ type of solution without conserved charge

- A new solution for arbitrary $\kappa(T)$:
  Csanád, Nagy, Lökös, arXiv:1205.5965

$$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3}, \tag{17}$$

$$u^\mu = \frac{x^\mu}{\tau}, \tag{18}$$

$$\frac{\tau_0^3}{\tau^3} = \exp \int_{T_0}^{T} \left(\frac{\kappa(\beta)}{\beta} + \frac{1}{\kappa(\beta)+1} \frac{d\kappa(\beta)}{d\beta} \right) d\beta \tag{19}$$

β is the integration variable here, i.e. T

- Arbitrary $\kappa(T)$ functions may be used, the QCD one as well

A new $\kappa(T)$ type of solution with conserved charge

- A new solution with conserved charge n, for arbitrary $\kappa(T)$:
  Csanád, Nagy, Lökös, arXiv:1205.5965

$$n = n_0 \frac{\tau_0^3}{\tau^3}, \tag{20}$$

$$u^\mu = \frac{x^\mu}{\tau}, \tag{21}$$

$$\frac{\tau_0^3}{\tau^3} = \exp \int_{T_0}^{T} \left(\frac{1}{\beta} \frac{d\kappa(\beta)\beta}{d\beta} \right) d\beta \tag{22}$$

- Arbitrary $\kappa(T)$ functions may be used
- For some choices of the $\kappa(T)$ function this solution becomes ill-defined
- If $d(\kappa T)/dT \leq 0$, the last equation cannot be inverted
- Same problem as mentioned earlier

A new $\kappa(p)$ type of solution without conserved charge

- A partly implicit new solution:
  Csanád, Nagy, Lökös, arXiv:1205.5965

$$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3}, \tag{23}$$

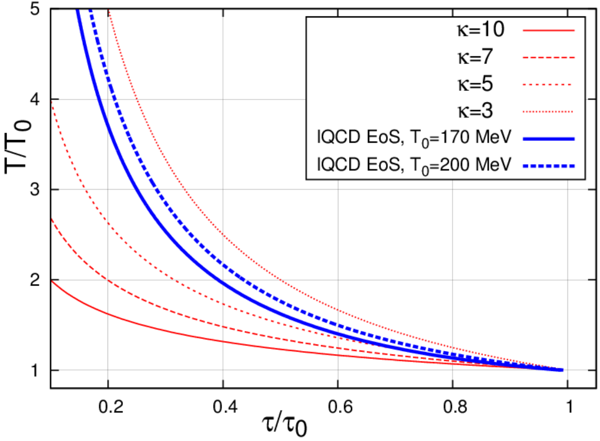
$$u^\mu = \frac{x^\mu}{\tau}, \tag{24}$$

$$\frac{\tau_0^3}{\tau^3} = \int_{p_0}^{p} \left(\frac{\kappa(\beta)}{\beta} + \frac{d\kappa(\beta)}{d\beta} \right) \frac{d\beta}{\kappa(\beta)+1} \tag{25}$$

- This solution may be used if κ is given as a function of pressure p.

Results for the lattice QCD EoS

- Let's assume the lQCD EoS
- $T(t)$ can be calculated for the above solutions (see arXiv:1205.5965)



- Result applicable very widely: plug in $\tau_{\text{final}}/\tau_{\text{ini}}$, get $T_{\text{final}}/T_{\text{ini}}$

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- Constant EoS solutions work when describing data
- Large temperature change: variable κ more realistic
- Found new exact solutions with arbitrary $\kappa(T)$ or $\kappa(p)$
- Lattice QCD EoS applicable when no conserved charges
- Calculated $T(\tau)$
- Only assuming a $\tau_{\rm final}/\tau_{\rm ini}$, one gets $T_{\rm final}/T_{\rm ini}$