

# Neutrino masses in the universe

Yvonne Y. Y. Wong  
RWTH Aachen

Neutrino town meeting, CERN  
May 14 – 16, 2012

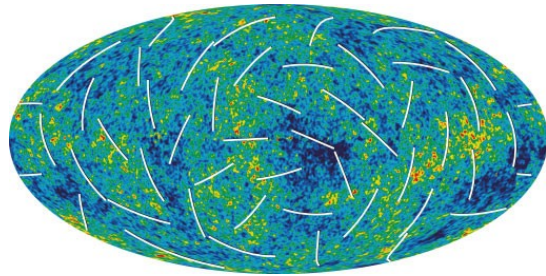
# Evidence for sterile neutrinos from precision cosmology?

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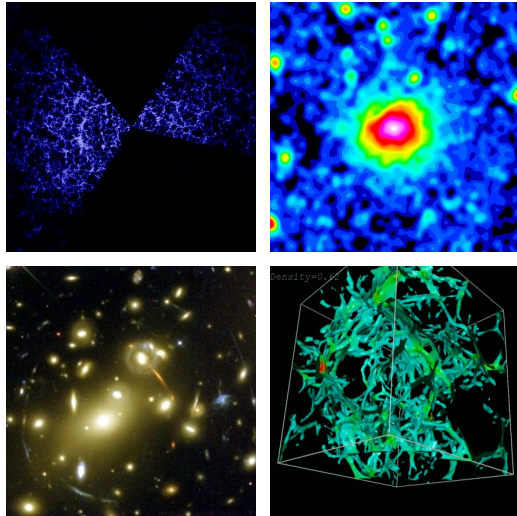
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# Precision cosmological probes...

## Probes of inhomogeneities

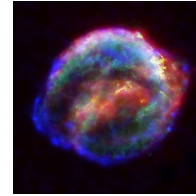


CMB temperature & polarisation anisotropies

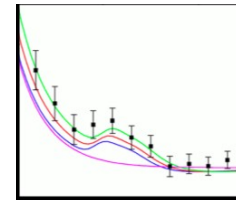


Large-scale matter distribution

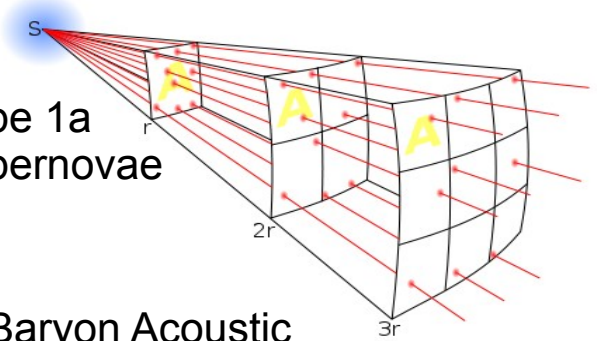
## Distance vs redshift



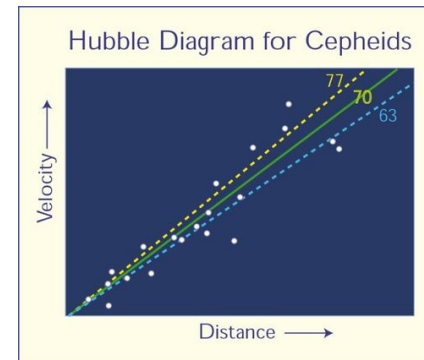
Type 1a supernovae



Baryon Acoustic Oscillation scale



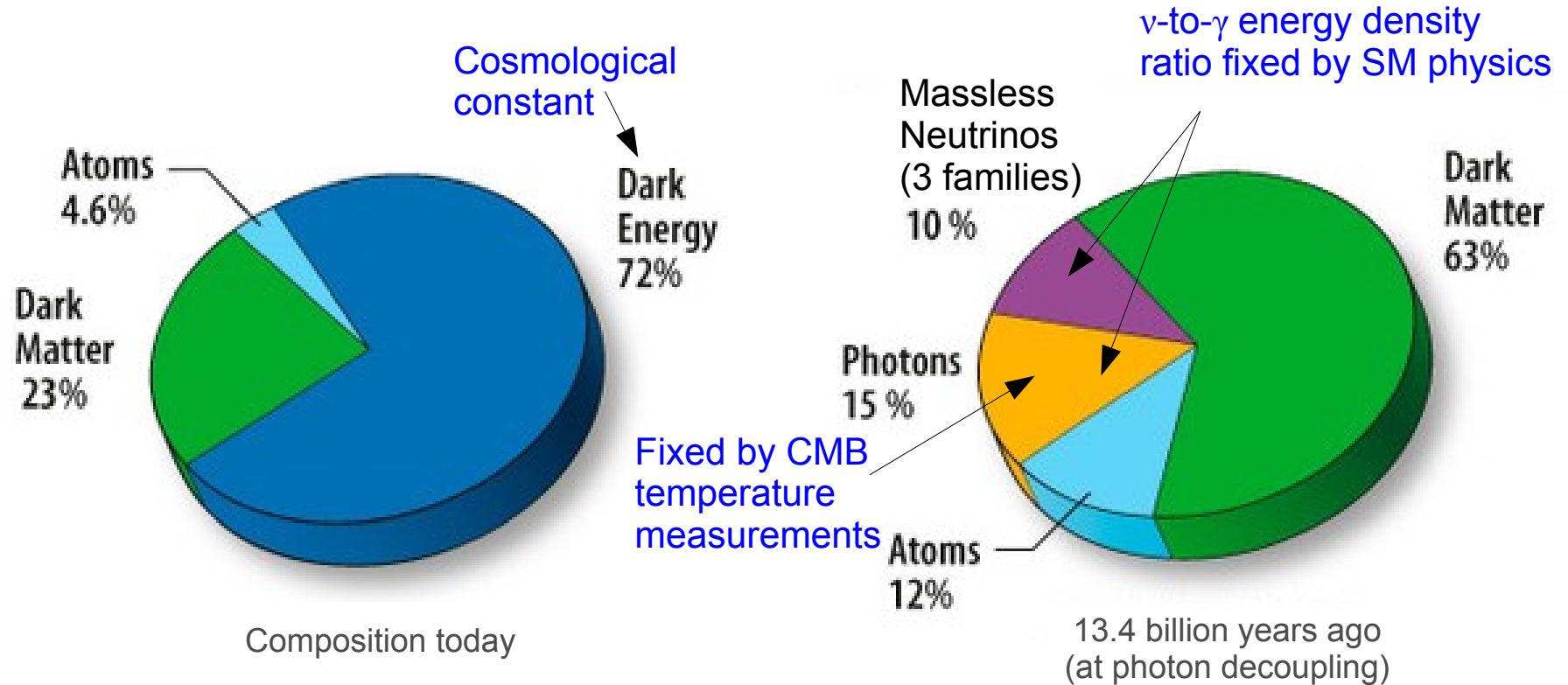
## Local Hubble expansion rate



$$H_0 = 100 h \text{ km/s/Mpc} \\ = 73.8 \pm 2.4 \text{ km/s/Mpc}$$

# The concordance flat $\Lambda$ CDM model...

- The **simplest** model consistent with **present observations**.



Plus flat spatial geometry+initial conditions from single-field inflation

# Neutrino energy density (standard picture)...

- Neutrino decoupling at  $T \sim O(1)$  MeV. ← Fixed by weak interactions

- After  $e^+e^-$  annihilation ( $T \sim 0.2$  MeV):

- **Temperature:**

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

Assuming instantaneous decoupling

- **Number density per flavour:**

$$n_\nu = \frac{6}{4} \frac{\zeta(3)}{\pi^2} T_\nu^3 = \frac{3}{11} n_\gamma$$

Photon temperature, number density, & energy density

- **Energy density per flavour:**

$$\rho_\nu = \frac{7}{8} \frac{\pi^2}{15} T_\nu^4 = \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$

$$\frac{3\rho_\nu}{\rho_\gamma} \sim 0.68$$

- If **massive**, then at  $T \ll m$ :  $\rho_\nu = m_\nu n_\nu \longrightarrow \Omega_{\nu,0} h^2 = \frac{m_\nu}{94 \text{ eV}}$

Hot dark matter (not within vanilla  $\Lambda$ CDM)

# Extending the “neutrino” sector...

- Any particle species whose production is associated with some **thermal process** and that **decoupled while relativistic at relatively late times** [ $T < O(100)$  MeV] will behave (more or less) like a neutrino as far as cosmological observations are concerned.

$$\begin{aligned} \sum_i \rho_{\nu,i} + \rho_X &= N_{\text{eff}} \left( \frac{7}{8} \frac{\pi^2}{15} T_\nu^4 \right) \\ &= (3.046 + \Delta N_{\text{eff}}) \rho_\nu^{(0)} \end{aligned}$$

Three SM neutrinos

Other light stuff:  
sterile neutrinos,  
hidden photons,  
etc.

Neutrino temperature per definition

Corrections due to  
non-instantaneous decoupling,  
finite temperature effects, and  
flavour oscillations

# Plan...

- Evidence of  $N_{\text{eff}} > 3$  from **CMB and large-scale structure** observations.
- Connection to the **short baseline sterile neutrino**.

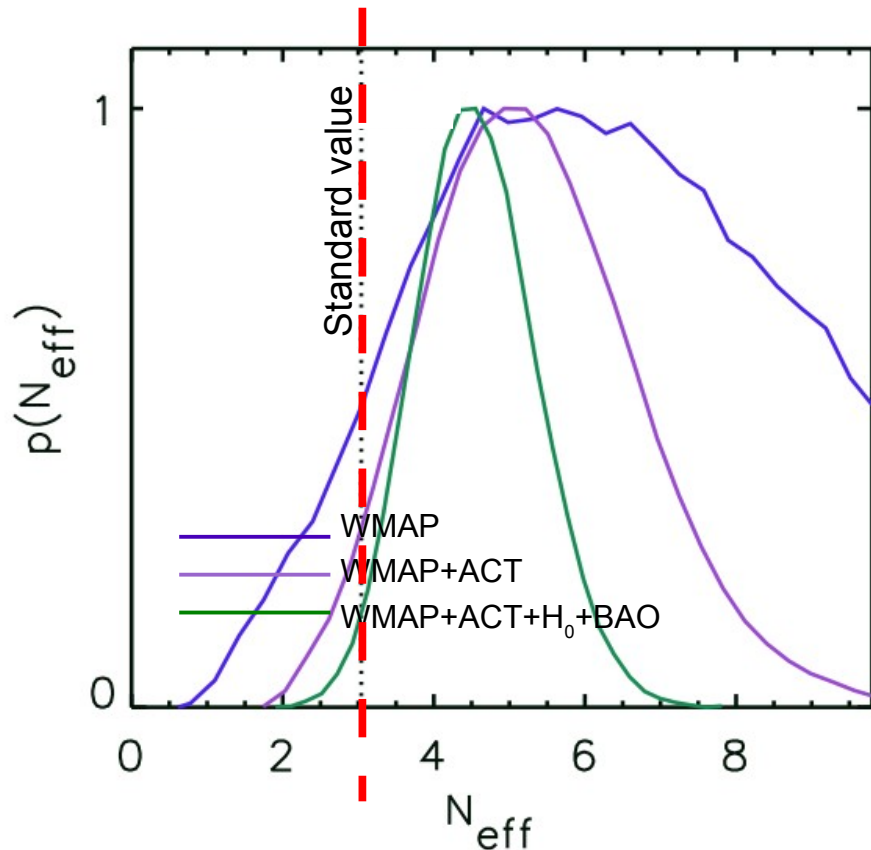
- Bonus slides: **Big bang nucleosynthesis**

# 1. CMB+large-scale structure...

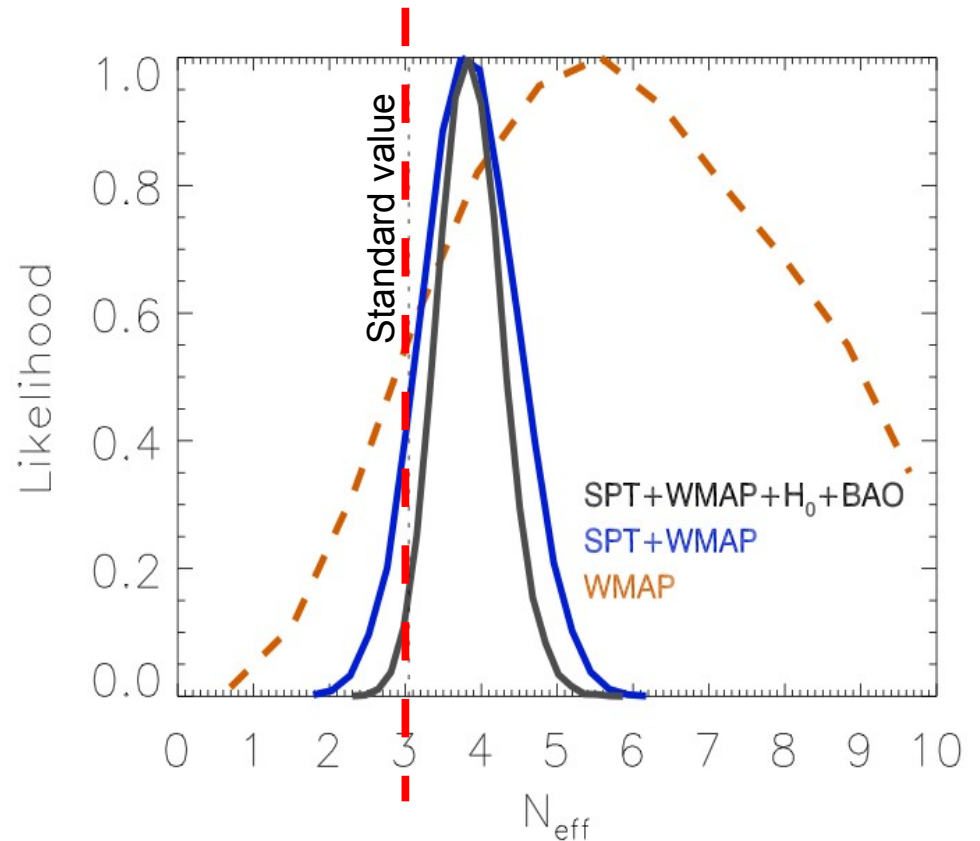


# Evidence for $N_{\text{eff}} > 3$ from CMB+LSS...

- Recent CMB+LSS data appear to **prefer  $N_{\text{eff}} > 3$ !**



Dunkley et al. [Atacama Cosmology Telescope] 2010



Keisler et al. [South Pole Telescope] 2011

- $N_{\text{eff}} > 3$  trend has been there since WMAP-5.
- Exact numbers depend on the **cosmological model**, and the **combination of data**.
- Many model+data combinations find  $N_{\text{eff}} > 3$  at 95% – 99% C.L.
- Central value  $N_{\text{eff}} \sim 4$ .

Model	Data	$N_{\text{eff}}$
$N_{\text{eff}}$	W-5+BAO+SN+ $H_0$	$4.13^{+0.87(+1.76)}_{-0.85(-1.63)}$
	W-5+LRG+ $H_0$	$4.16^{+0.76(+1.60)}_{-0.77(-1.43)}$
	W-5+CMB+BAO+XLF+ $f_{\text{gas}}+H_0$	$3.4^{+0.6}_{-0.5}$
	W-5+LRG+maxBCG+ $H_0$	$3.77^{+0.67(+1.37)}_{-0.67(-1.24)}$
	W-7+BAO+ $H_0$	$4.34^{+0.86}_{-0.88}$
	W-7+LRG+ $H_0$	$4.25^{+0.76}_{-0.80}$
	W-7+ACT	$5.3 \pm 1.3$
	→ W-7+ACT+BAO+ $H_0$	$4.56 \pm 0.75$
	W-7+SPT	$3.85 \pm 0.62$
	→ W-7+SPT+BAO+ $H_0$	$3.85 \pm 0.42$
	→ W-7+ACT+SPT+LRG+ $H_0$	$4.08^{(+0.71)}_{(-0.68)}$
	→ W-7+ACT+SPT+BAO+ $H_0$	$3.89 \pm 0.41$
	W-7+CL+SPT+BAO+ $H_0$	( $< 3.74$ )
$N_{\text{eff}} + f_\nu$	W-7+CMB+BAO+ $H_0$	$4.47^{(+1.82)}_{(-1.74)}$
	→ W-7+CMB+LRG+ $H_0$	$4.87^{(+1.86)}_{(-1.75)}$
$N_{\text{eff}} + \Omega_k$	W-7+BAO+ $H_0$	$4.61 \pm 0.96$
	→ W-7+ACT+SPT+BAO+ $H_0$	$4.03 \pm 0.45$
$N_{\text{eff}} + \Omega_k + f_\nu$	→ W-7+ACT+SPT+BAO+ $H_0$	$4.00 \pm 0.43$
$N_{\text{eff}} + f_\nu + w$	W-7+CMB+BAO+ $H_0$	$3.68^{(+1.90)}_{(-1.84)}$
	W-7+CMB+LRG+ $H_0$	$4.87^{(+2.02)}_{(-2.02)}$
$N_{\text{eff}} + \Omega_k + f_\nu + w$	→ W-7+CMB+BAO+SN+ $H_0$	$4.2^{+1.10(+2.00)}_{-0.61(-1.14)}$
	→ W-7+CMB+LRG+SN+ $H_0$	$4.3^{+1.40(+2.30)}_{-0.54(-1.09)}$

→ =  
>95% C.L.

- **One exception: cluster abundance** from ROSAT All-sky Survey/Chandra X-ray observatory prefers a more “standard” value of  $N_{\text{eff}}$

$$N_{\text{eff}} < 3.74 \text{ (95 \% C.L.)}$$

WMAP-7+Clusters+SPT+BA0+ $H_0$   
( $N_{\text{eff}}$  restricted to  $\geq 3$ )

Burenin & Vikhlinin 2012

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→ = uses cluster data

# How does it work...

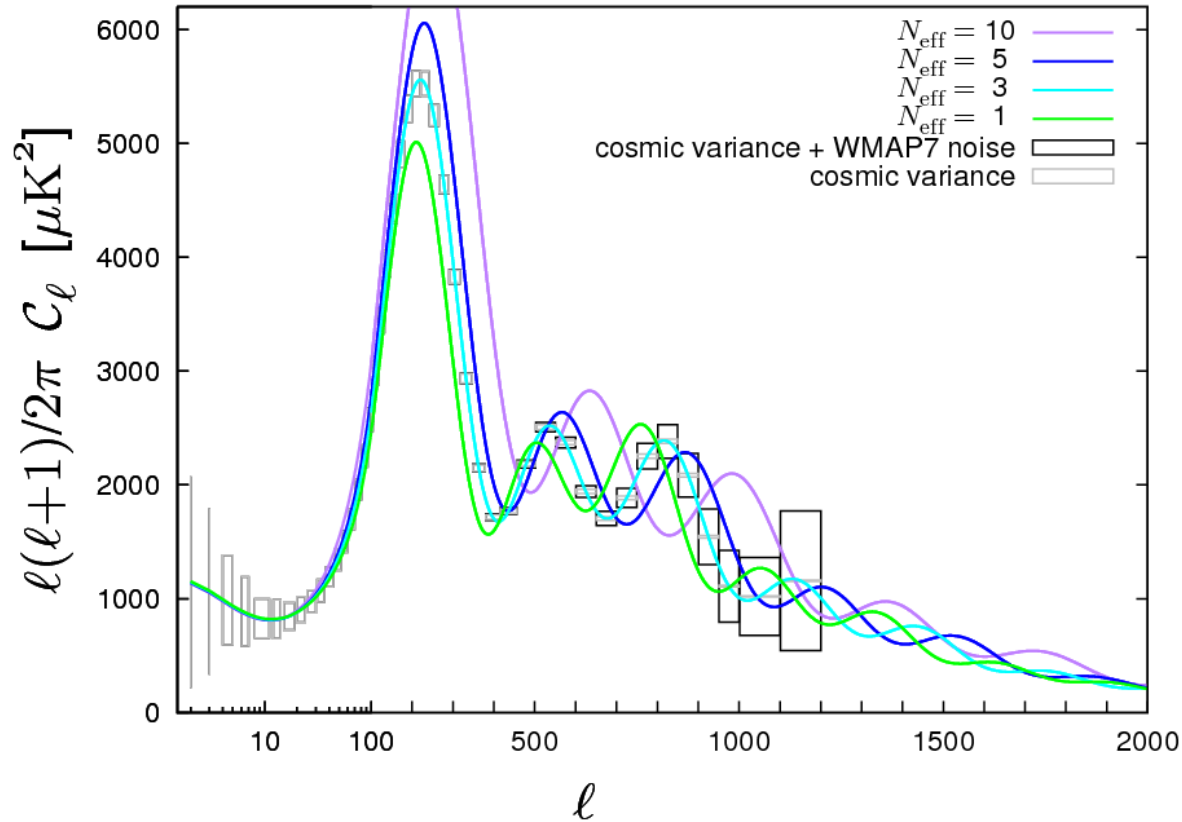
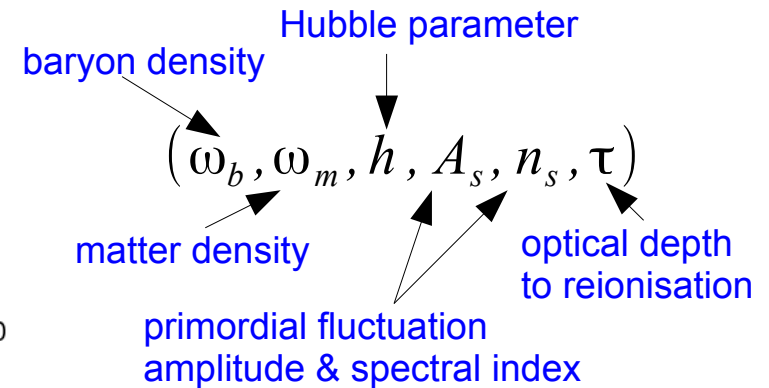


Figure courtesy of J. Hamann

- $N_{\text{eff}}$  looks easy to detect..
- But we also use the **same data** to measure at least **6 other cosmological parameters**:



- Plenty of **parameter degeneracies!**

# What the CMB really probes: equality redshift...

- Ratio of 3<sup>rd</sup> and 1<sup>st</sup> peaks sensitive to the redshift of **matter-radiation equality** via the early ISW effect.

Exact degeneracy between the physical matter density  $\omega_m$  and  $N_{\text{eff}}$

$$1 + z_{\text{eq}} = \frac{\omega_m}{\omega_r} \frac{\omega_m}{\omega_y} \frac{1}{1 + 0.2271 N_{\text{eff}}}$$

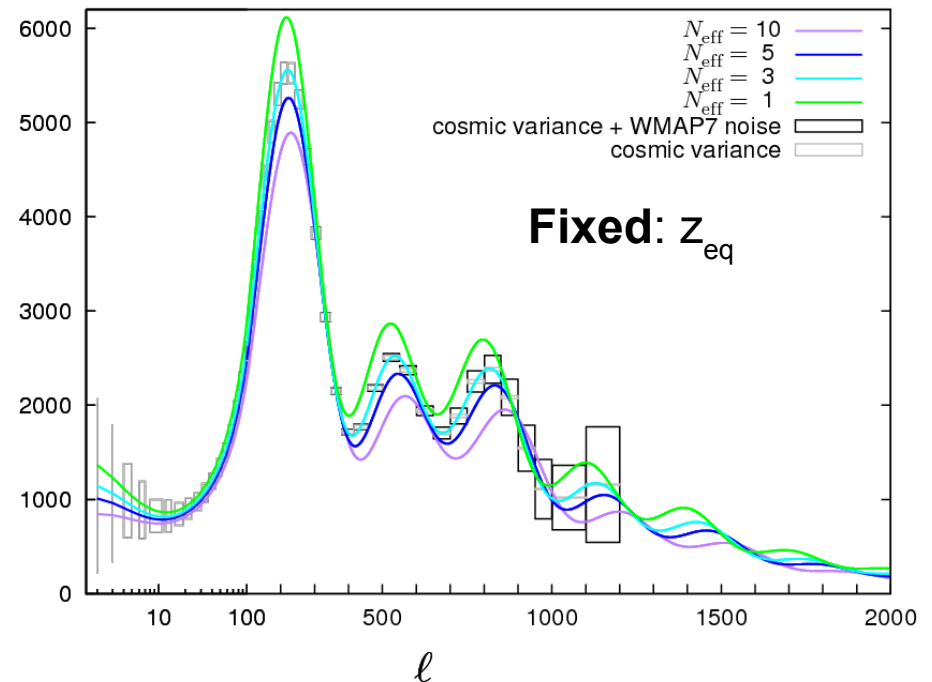
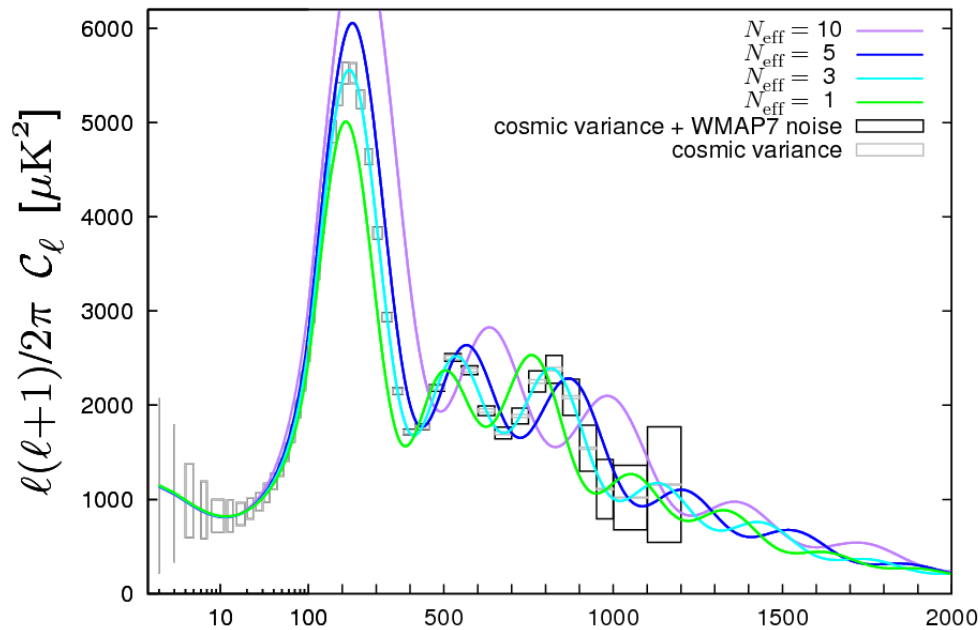


Figure courtesy of J. Hamann<sup>l</sup>

# What the CMB really probes: sound horizon...

- Peak positions depend on:

$$\theta_s = \frac{r_s}{D_A}$$

$r_s$  ← Sound horizon at decoupling  
 $D_A$  ← Angular distance to the last scattering surface

Flat  $\Lambda$ CDM

Fixed  $z_{eq}, \omega_b$

Exact degeneracy between  $\omega_m$  and the Hubble parameter  $h$ .

$$\theta_s \propto \frac{(\omega_m h^{-2})^{-1/2}}{\int_{a^*}^1 \frac{da}{\sqrt{\omega_m h^{-2} a^{-3} + (1 - \omega_m h^{-2})}}}$$

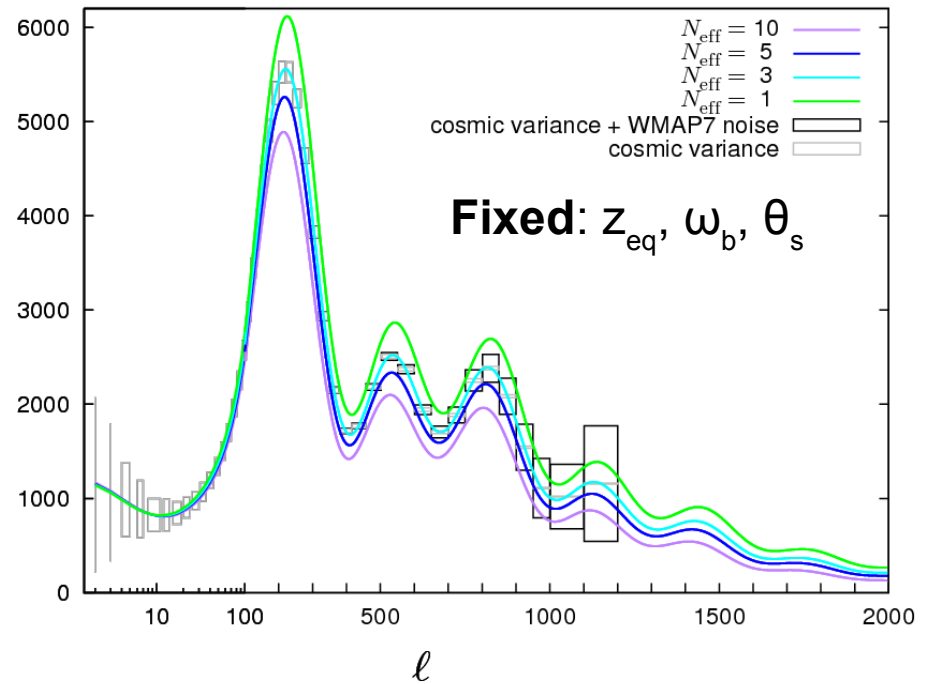
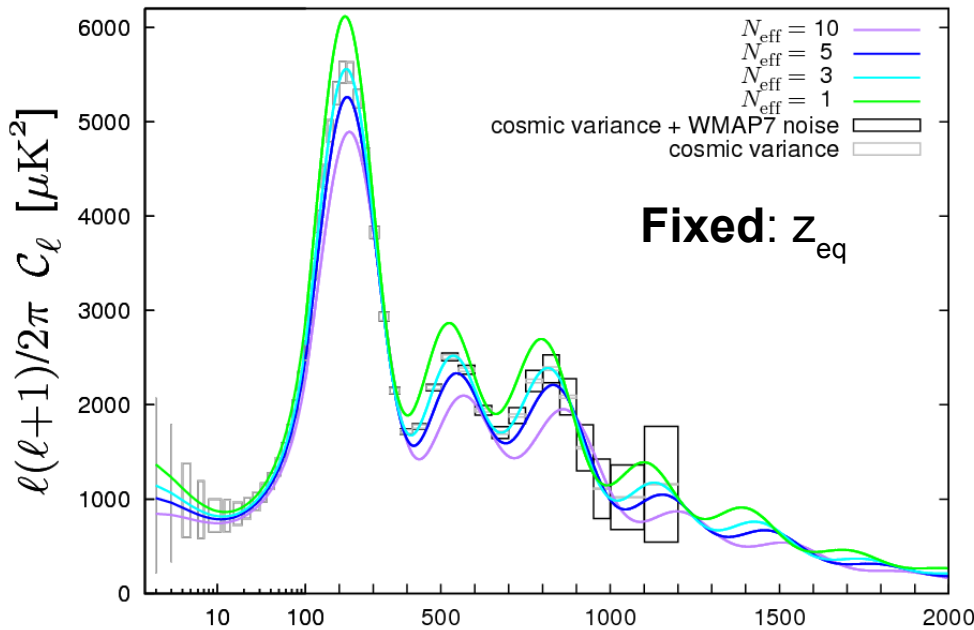
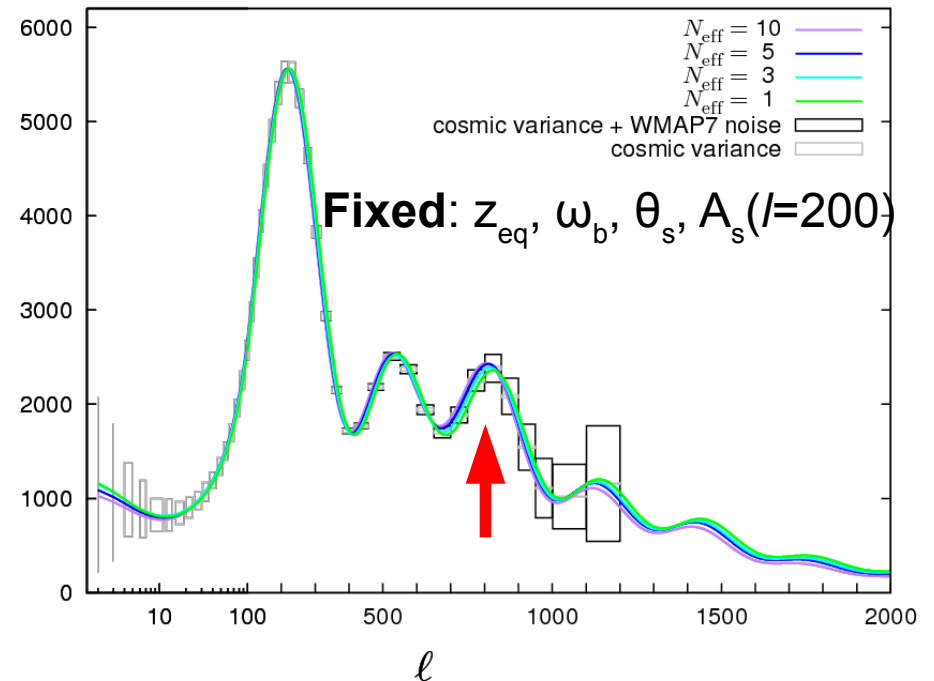
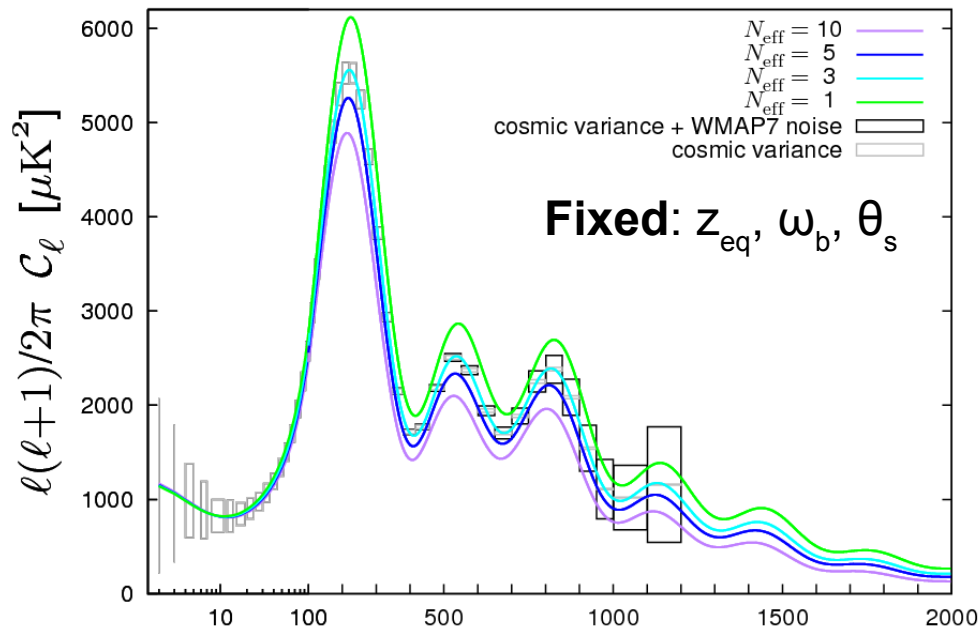


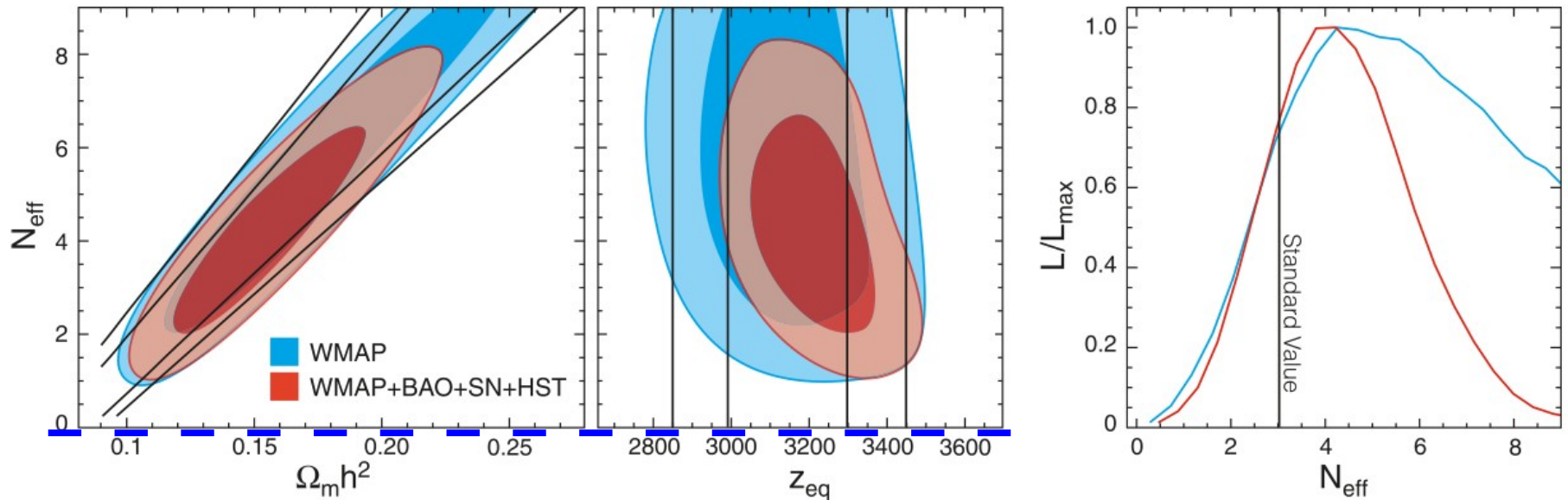
Figure courtesy of J. Hamann<sup>ℓ</sup>

# What the CMB really probes: anisotropic stress...

- Apparent (i.e., not physical) partial degeneracies with **primordial fluctuation amplitude  $A_s$**  and **spectral index  $n_s$** .
- However, **free-streaming** particles have **anisotropic stress**.
- **First real signature of  $N_{\text{eff}}$  in the 3rd peak!**



- Measurement of the third peak (since WMAP-5) gives **lower limit on  $N_{\text{eff}}$  from WMAP alone** (without supplementary large-scale structure data).

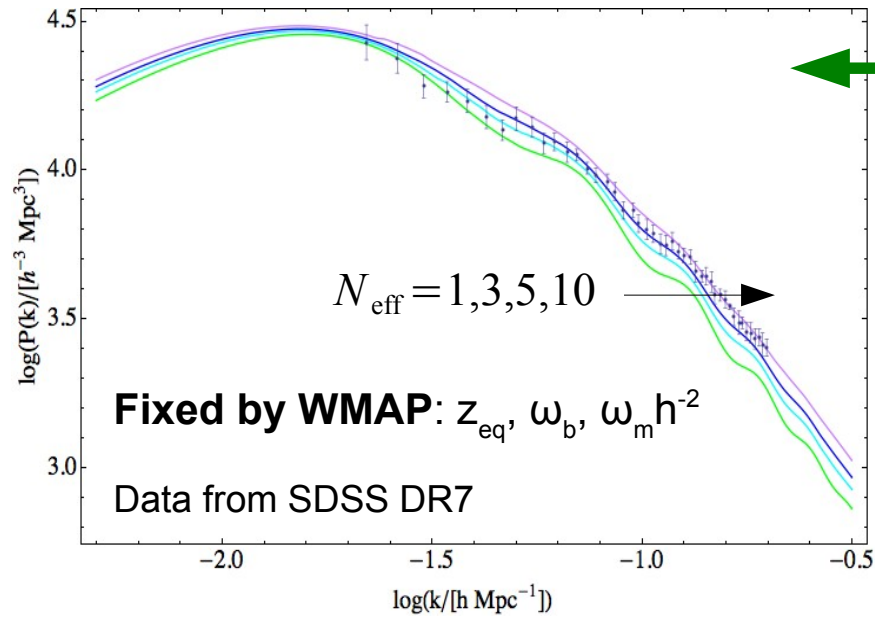


Komatsu et al. [WMAP5] 2008

- **Upper limit** requires combination of WMAP with other observations to break the remaining  **$N_{\text{eff}} - \omega_m - h$  parameter degeneracies**.
  - Pinning down either  $\omega_m$  or  $h$  will do!
    - from local ( $z < 0.1$ ) expansion rate measurements



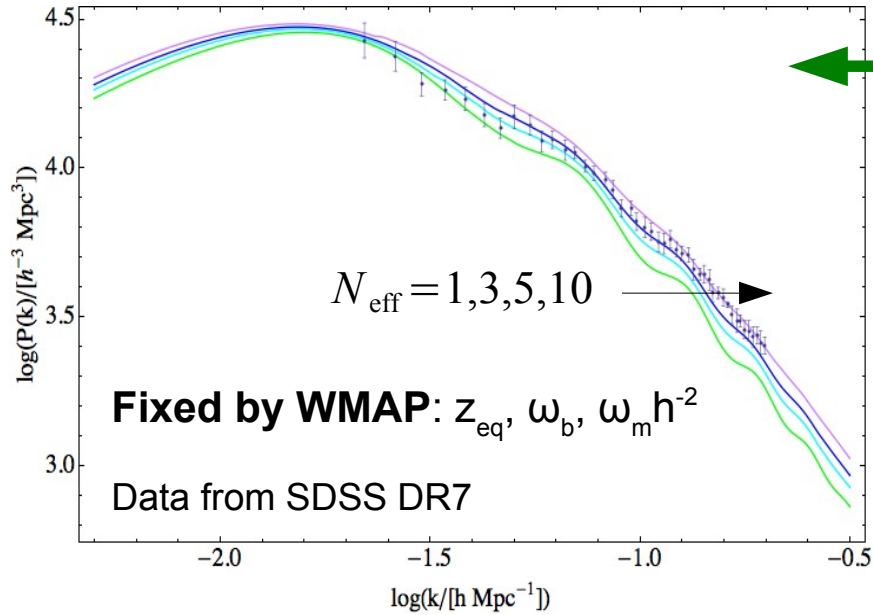
# Breaking the remaining parameter degeneracies...



← **Large-scale matter power spectrum**  
(probes baryon fraction)

$$f_b \equiv \frac{\omega_b}{\omega_m} \leftarrow \omega_b \text{ fixed by CMB}$$

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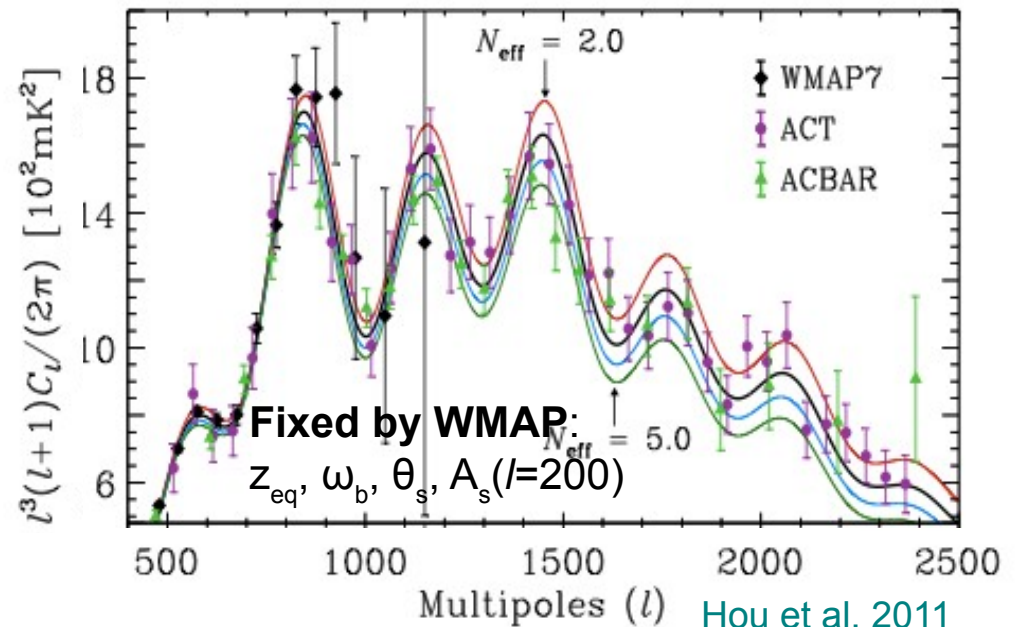
$$f_b \equiv \frac{\omega_b}{\omega_m} \leftarrow \omega_b \text{ fixed by CMB}$$

**CMB damping tail**  
(probes photon diffusion scale)

ACT since 2010  
SPT since 2011

$$\frac{\theta_d}{\theta_s} = \frac{r_d}{r_s} \propto \omega_m^{1/4}$$

Fixed by WMAP



## 2. Connection to the short baseline sterile neutrino...

# Experimental anomalies & the sterile $\nu$ interpretation...

- **Best-fits** parameters: e.g., Kopp, Maltoni & Schwetz 2011; Giunti & Laveder 2011

## Reactor experiments only

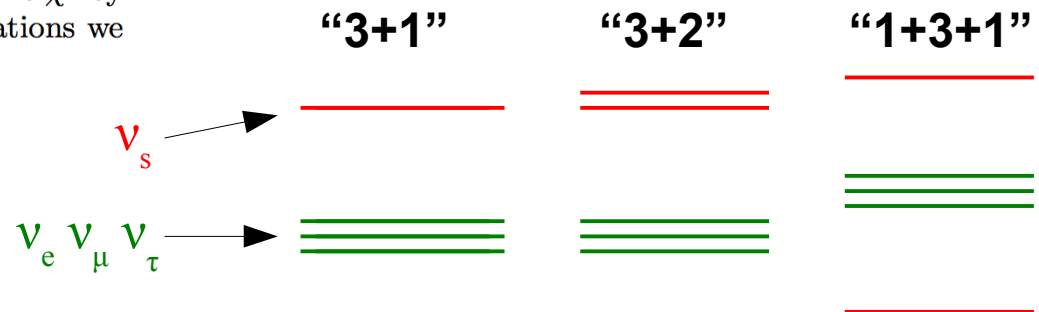
	$\Delta m_{41}^2$ [eV <sup>2</sup> ]	$ U_{e4} $	$\Delta m_{51}^2$ [eV <sup>2</sup> ]	$ U_{e5} $	$\chi^2/\text{dof}$
3+1	1.78	0.151			50.1/67
3+2	0.46	0.108	0.89	0.124	46.5/65

## Global short baseline (including LSND+MiniBooNE)

	$\Delta m_{41}^2$	$ U_{e4} $	$ U_{\mu 4} $	$\Delta m_{51}^2$	$ U_{e5} $	$ U_{\mu 5} $	$\delta/\pi$	$\chi^2/\text{dof}$
3+2	0.47	0.128	0.165	0.87	0.138	0.148	1.64	110.1/130
1+3+1	0.47	0.129	0.154	0.87	0.142	0.163	0.35	106.1/130

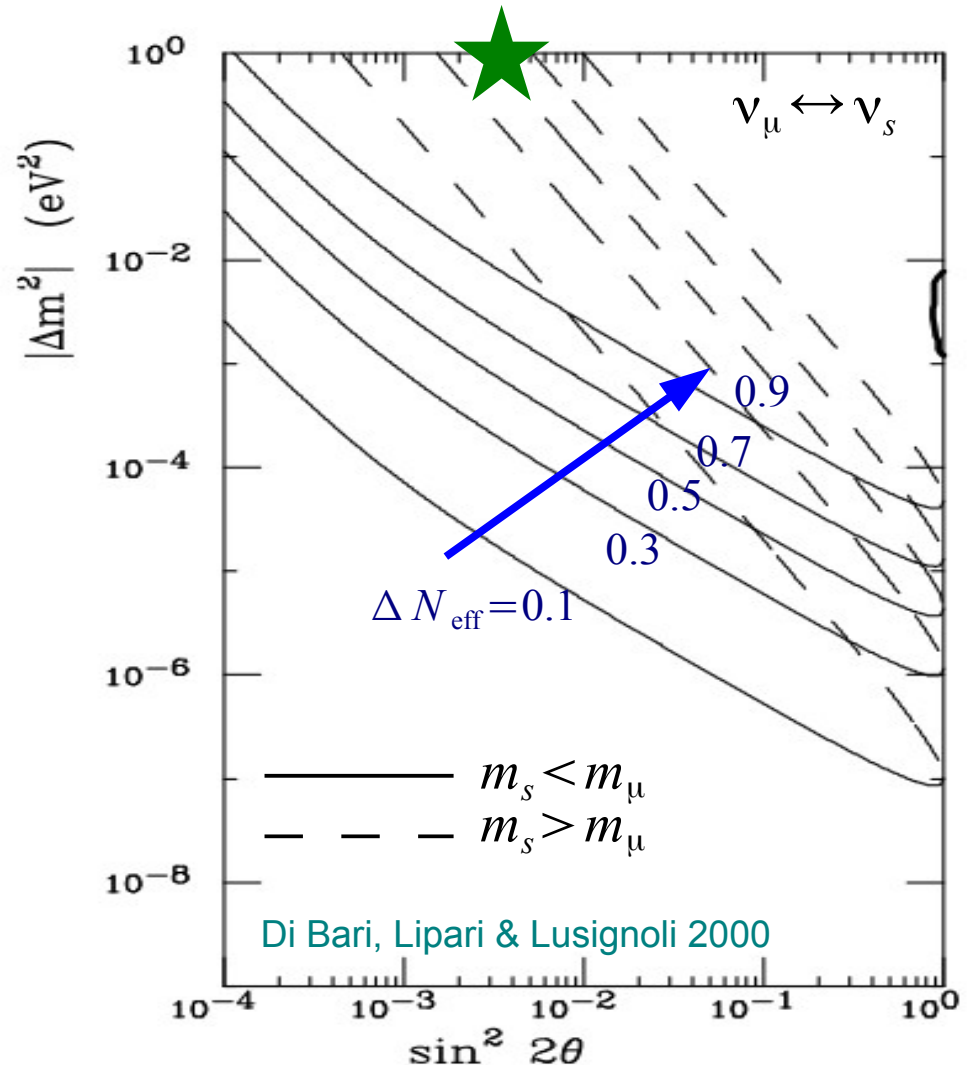
**Table I:** Best fit points for the 3+1 and 3+2 scenarios from reactor anti-neutrino data. The total number of data points is 69 (Bugey3 spectra plus 9 SBL rate measurements; we have omitted data from Chooz and Palo Verde, which are not very sensitive to the model parameters, but would dilute the  $\chi^2$  by introducing 15 additional data points). For no oscillations we have  $\chi^2/\text{dof} = 59.0/69$ .

**Table II:** Parameter values and  $\chi^2$  at the global best fit points for 3+2 and 1+3+1 oscillations ( $\Delta m^2$ 's in eV<sup>2</sup>).



# Light sterile neutrinos and $N_{\text{eff}} \dots$

- SBL-preferred  $\Delta m^2$  and mixing favour the **production and thermalisation** of sterile neutrinos in the early universe via  $\nu_\alpha \leftrightarrow \nu_s$  **oscillations** +  $\nu_\alpha$  **scattering**.
  - Can easily produce an **excess** relativistic energy density of  $\Delta N_{\text{eff}} \sim 1$ .
  - Sterile states have the **same temperature** as the SM neutrinos.

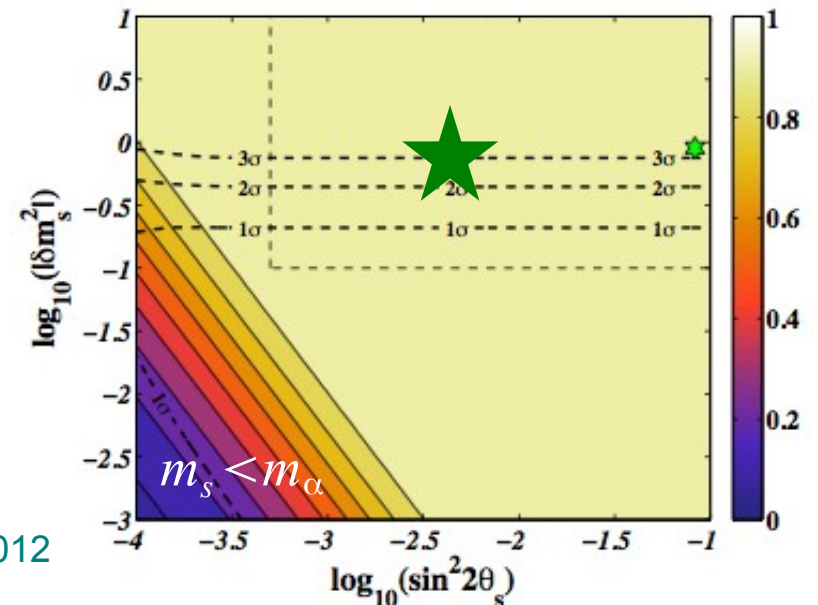
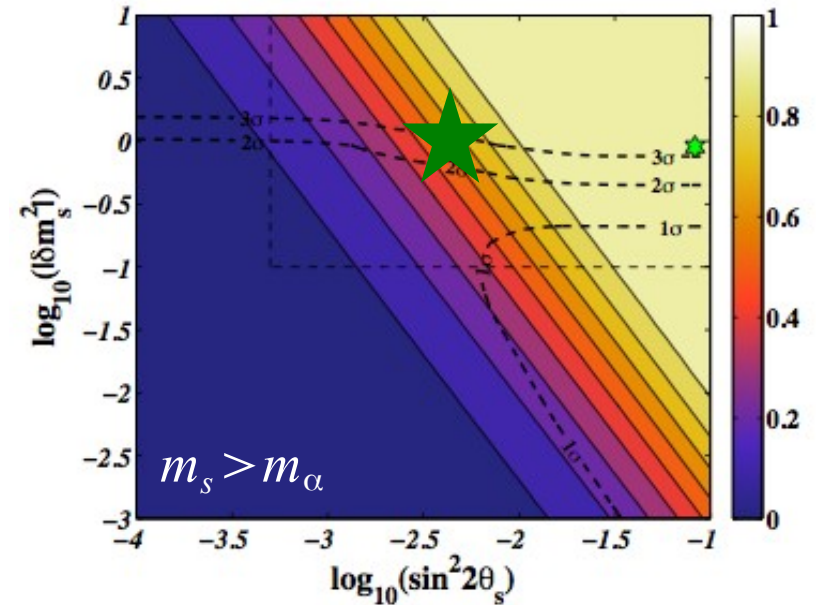


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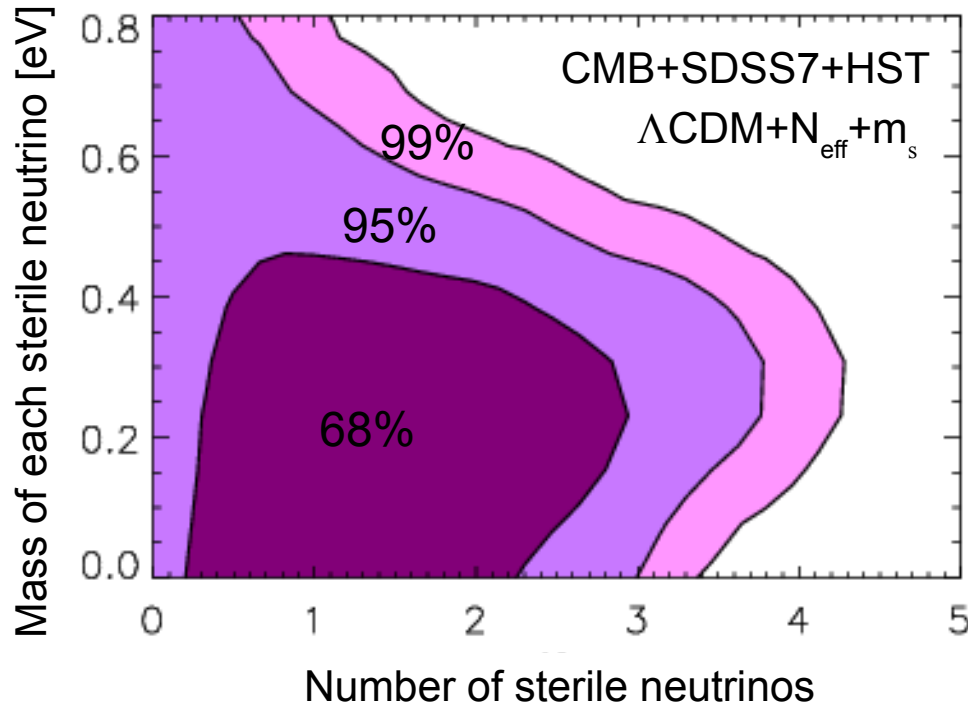
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→ Sterile states have the **same temperature** as the SM neutrinos.



# Can the short baseline sterile neutrino explain $N_{\text{eff}} > 3$ ?

- **Short answer:** Not so easy.
- **Reason:** eV mass neutrinos **violate CMB+LSS hot dark matter bounds.**



- 3+1 thermalised sterile:  
 $m_s < 0.48 \text{ eV}$  (95% C.I.)

Lab best-fit:  $m_s \sim 1 \text{ eV}$

- 3+2 thermalised sterile:  
 $m_{s1} + m_{s2} < 0.9 \text{ eV}$  (95% C.I.)

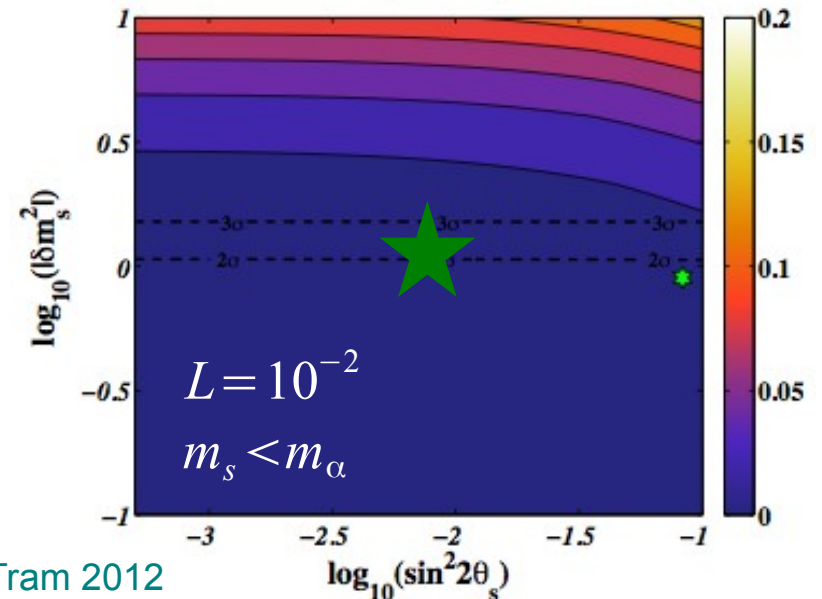
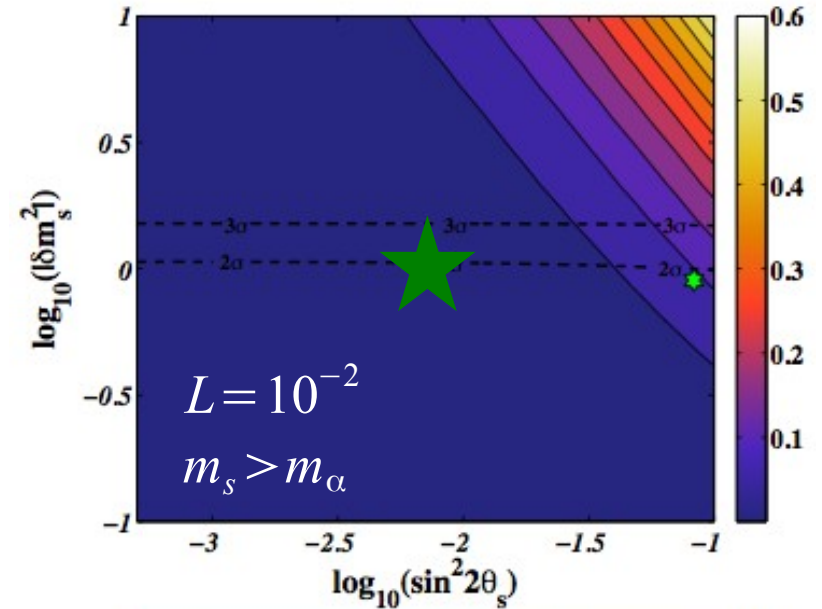
Lab best-fit:  $m_{s1} \sim 0.7 \text{ eV}$ ,  $m_{s2} \sim 0.9 \text{ eV}$

# Is there a way out? Plan A...

- **Suppress** sterile neutrino thermalisation using, e.g., **a large lepton asymmetry** ( $L \gg B \sim 10^{-10}$ ).

Foot & Volkas 1995

- Generating a large lepton asymmetry requires **new physics**.
- If complete suppression, then  $N_{\text{eff}} > 3$  must be explained by some **other physics** (sub-eV thermal axions, hidden photons, etc.?)





**Grin, Smith, and Kamionkowski**, Axion constraints in non-standard thermal histories, arXiv:0711.1352 [astro-ph]; **Kawasaki, Nakayama, and Senami**, Cosmological implications of supersymmetric axion models, arXiv:0711.3083 [hep-ph]; **Feng, Tu and Yu**, Thermal Relics in Hidden Sectors, arXiv:0808.2318 [hep-ph]; **Nelson and Walsh**, Chameleon vector bosons, arXiv:0802.0762 [hep-ph]; **Ackermann, Buckley, Carroll, and Kamionkowski**, Dark Matter and Dark Radiation, arXiv:0810.5126 [hep-ph]; **Mahato**, Torsion, Dirac Field, Dark Matter and Dark Radiation, gr-qc/0603134; **Jäckel, Redondo, and Ringwald**, Signatures of a hidden cosmic microwave background, arXiv:0804.4157 [astro-ph]; **Hasenkamp**, Dark radiation from the axino solution of the gravitino problem, arXiv:1107.4319 [hep-ph]; **Kobayashi, Takahashi, Takahashi, and Yamaguchi**, Dark Radiation from Modulated Reheating, arXiv:1111.1336 [astro-ph.CO]; **Feng, Rentala and Surujon**, WIMPless Dark Matter from an AMSB Hidden Sector with No New Mass Parameters, arXiv:1111.4479 [hep-ph]; **Hooper, Queiroz, and Gnedin**, Non-Thermal Dark Matter Mimicking An Additional Neutrino Species In The Early Universe, arXiv:1111.6599 [astro-ph.CO]; **Menestrina and Scherrer**, Dark Radiation from Particle Decays during Big Bang Nucleosynthesis, arXiv:1111.0605 [astro-ph.CO]; Aslanbeigi, **Robbers, Foster, Kohri, and Afshordi**, Phenomenology of Gravitational Aether as a solution to the Old Cosmological Constant Problem, arXiv:1106.3955 [astro-ph.CO]; **Chen and Lin**, Cosmon as the Modulon: Non-Gaussianity from Dark Energy, arXiv:1104.0982 [hep-ph]; **Das and Weiner**, Late Forming Dark Matter in Theories of Neutrino Dark Energy, astro-ph/0611353; **Nakayama, Takahashi, and Yanagida**, A theory of extra radiation in the Universe, arXiv:1010.5693 [hep-ph]; **Fischler and Meyers**, Dark Radiation Emerging After Big Bang Nucleosynthesis?, arXiv:1011.3501 [astro-ph.CO]; **Dreiner, Hanussek, Kim, and Sarkar**, Gravitino cosmology with a very light neutralino, arXiv:1111.5715 [hep-ph]; **Foot**, Mirror dark matter cosmology – predictions for Neff[CMB] and Neff[BBN], arXiv:1111.6366 [astro-ph.CO]; **Jeong and Takahashi**, Light Higgsino from Axion Dark Radiation, arXiv:1201.4816 [hep-ph]; **Kaplan, Krnjaic, Rehermann, and Wells**, Dark Atoms: Asymmetry and Direct Detection, arXiv:1105.2073 [hep-ph]; **Cicoli**, Large extra dimensions and light hidden photons from anisotropic string vacua, arXiv:1111.0790 [hep-th];

# Is there a way out? Plan B...

- Failing to suppress  $\nu_s$  thermalisation, exploit **parameter degeneracies** in the CMB+LSS to **engineer a good fit**.
- No room for play within the  $\Lambda$ CDM model, but **extensions of  $\Lambda$ CDM** can help to **relax** the hot dark matter constraint on  $m_s$ :
  - Non-standard dark energy equation of state.
  - Modified gravity.
  - Non-flat spatial geometry.
  - Even more massless degrees of freedom.
  - ...

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1 x 1 eV sterile neutrino  
can be reasonably  
accommodated.

1 x 2eV or 2 x 1 eV is  
still problematic...

# Is there a way out? Plan B...

Modified gravity scenario to explain accelerated expansion in lieu of dark energy



- **An example:** accommodating 1eV sterile neutrinos with **f(R) gravity**:

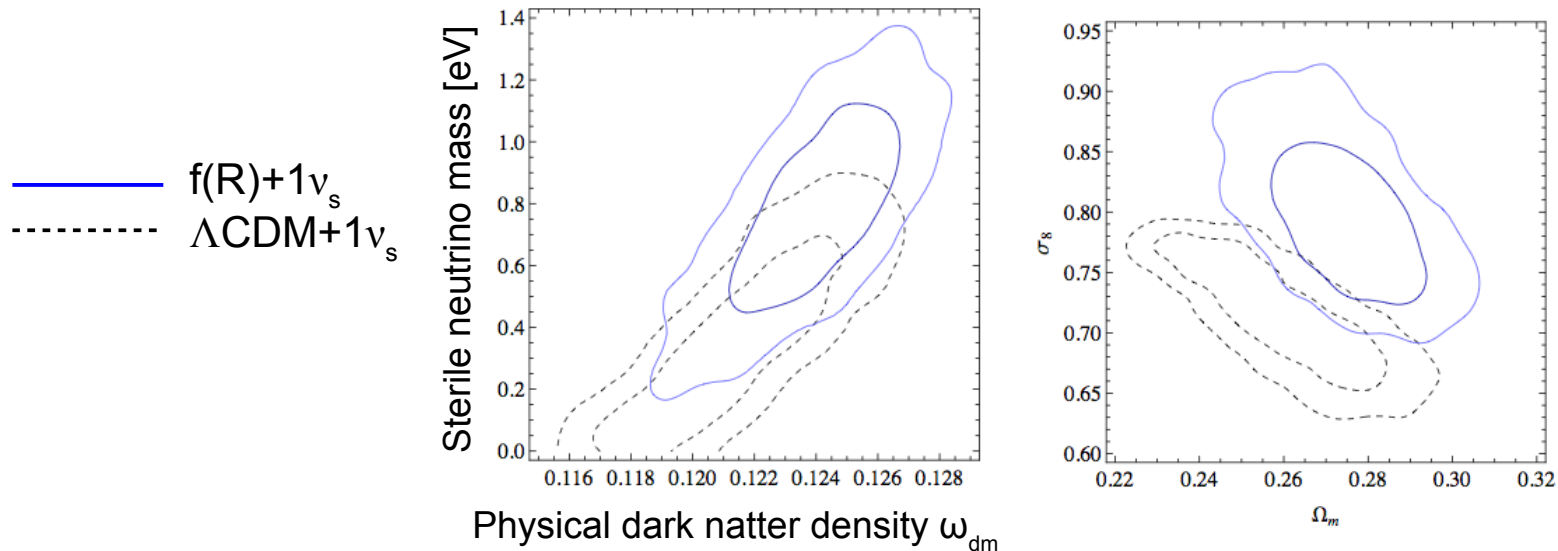
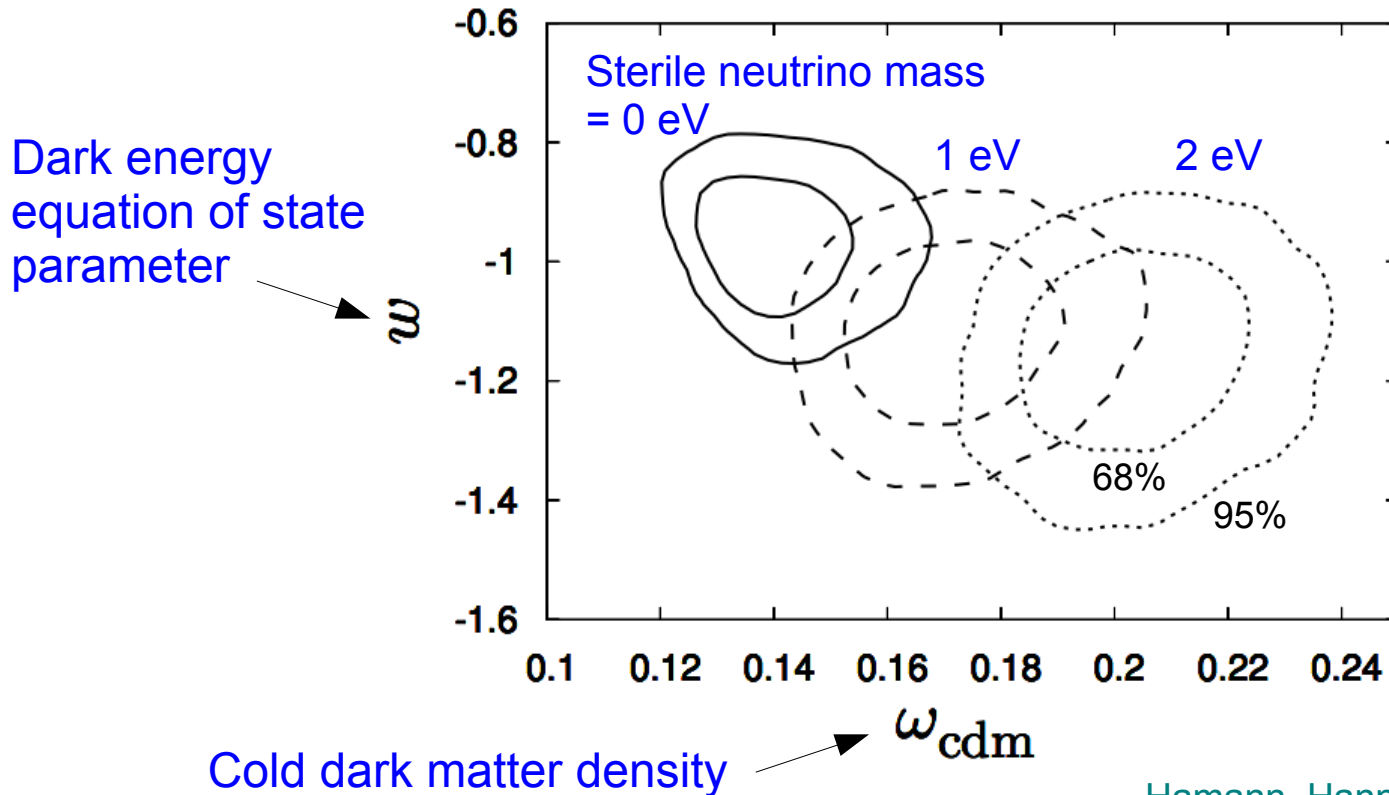


FIG. 1: 1 and  $2\sigma$  contours of the sterile neutrino mass (left) and  $\sigma_8$  (right) for the cases with three massless and one massive neutrinos in the  $\Lambda$ CDM model (dashed black) and  $f(R)$  gravity (solid blue).  $\chi_{\text{eff}}^2 = 3774.1$  and  $3767.0$ , respectively.

# Necessary side effects...

- Exploiting parameter degeneracies also implies that other (unrelated) **cosmological parameter values** will **change**.



# Planck and $N_{\text{eff}}$ ...


- If  $N_{\text{eff}}$  is as large as 4, it will be settled **almost immediately** by Planck (launched May 14, 2009; public data release early 2013).

68% sensitivities

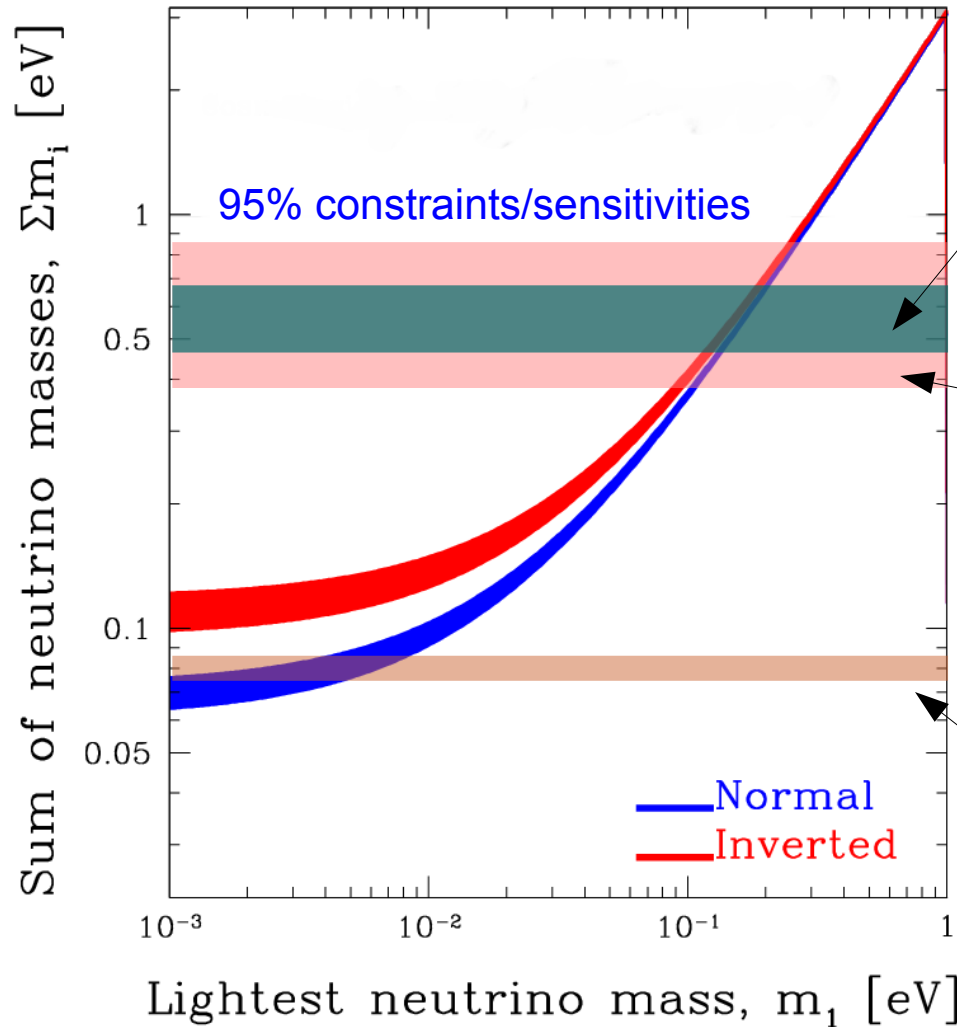
Experiment	$f_{\text{sky}}$	$\theta_b$	$w_T^{-1/2}$ [ $\mu\text{ K}'$ ]	$w_P^{-1/2}$ [ $\mu\text{ K}'$ ]	$\Delta N_\nu$ TT	$\Delta N_\nu$ TT+TE+EE	$\Delta N_\nu$ (free $Y$ ) TT+TE+EE
Planck	0.8	7'	40	56	0.6	<u>0.20</u>	<u>0.24</u>
ACT	0.01	1.7'	3	4	1	0.47	0.9
ACT + Planck					0.4	0.18	0.24
CMBPOL	0.8	4'	1	1.4	0.12	0.05	0.09

Bashinsky & Seljak 2004

Helium fraction  
as a free parameter



# Planck and neutrino mass...



Current constraints

Band = model complexity

Planck alone (1 year) **2013**  
(+ current LSS probes → not much improvement)  
Perotto et al. 2006

Not quite good enough to rule in or rule out 1 eV sterile neutrinos definitively...

Planck+Euclid weak lensing **2020+**

Hannestad, Tu & Y<sup>3</sup>W 2006

# Summary...

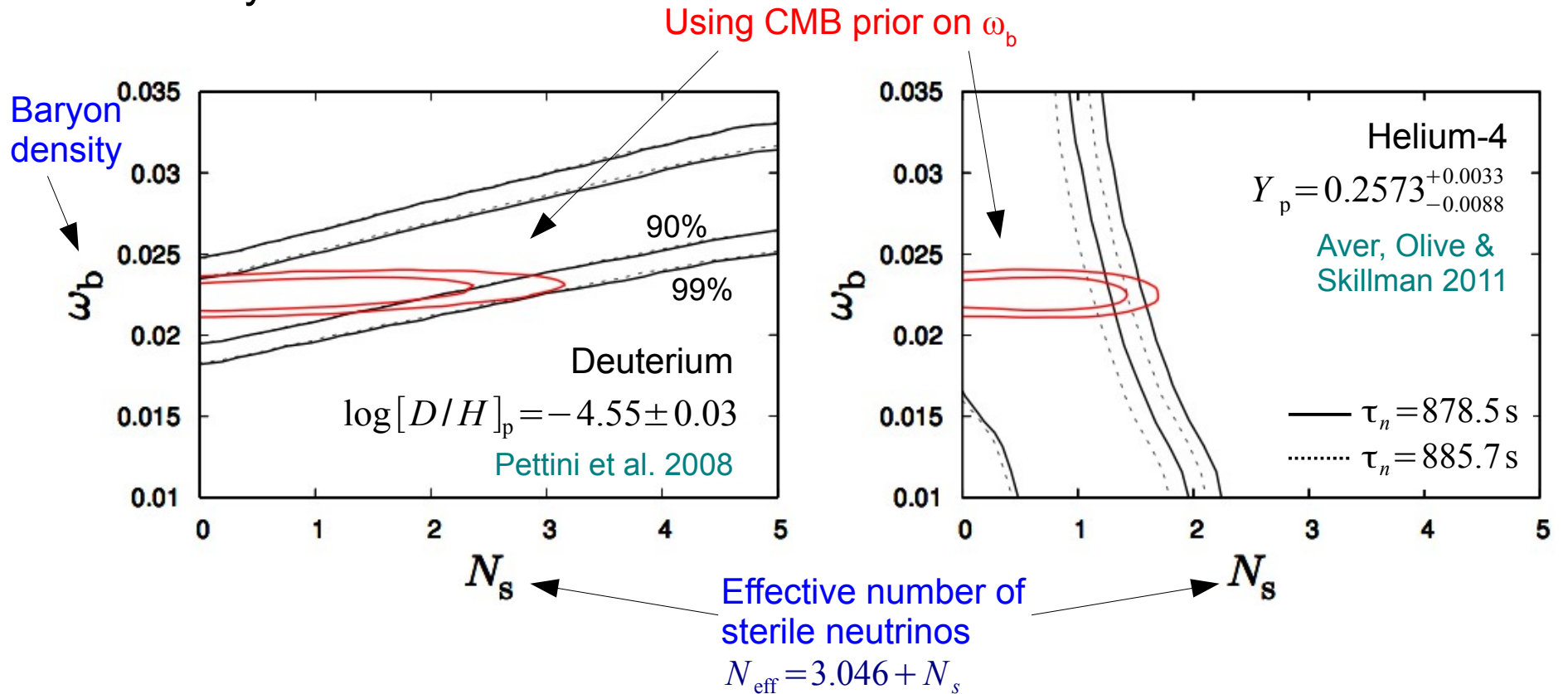
- Current precision cosmological data show a preference for extra **relativistic degrees of freedom** (beyond 3 neutrinos).
- **Sterile neutrino** interpretation of short baseline neutrino anomalies does not quite fit into the simplest picture though...
  - 3+2: **Too many** for BBN
  - 3+1, 3+2: **Too heavy** for CMB/LSS
- Non-trivial **extensions to  $\Lambda$ CDM** can reasonably accommodate 1 x 1 eV fully thermalised sterile neutrino species.
- **Planck with tell** (at least part of the story).



## 3. Extra slides: BBN

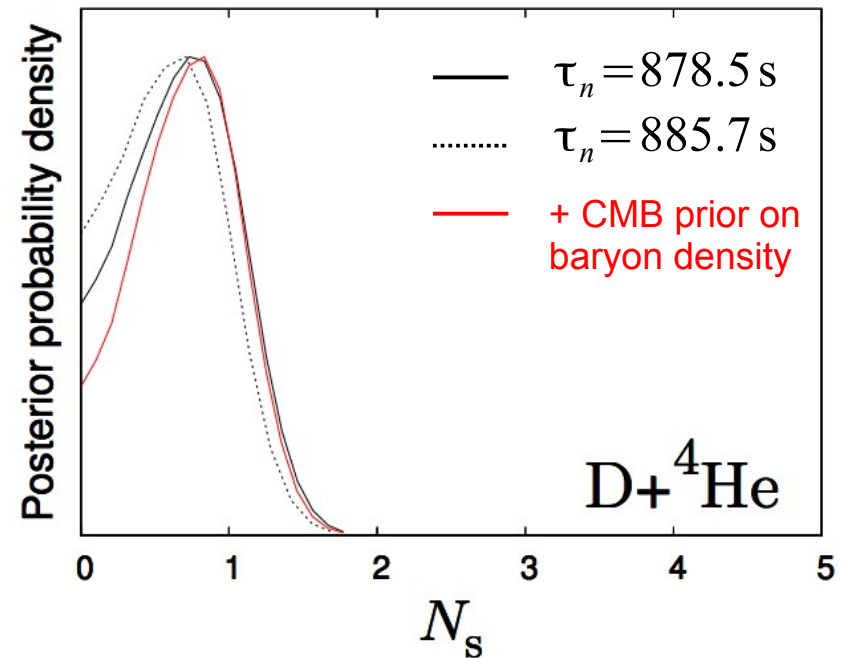
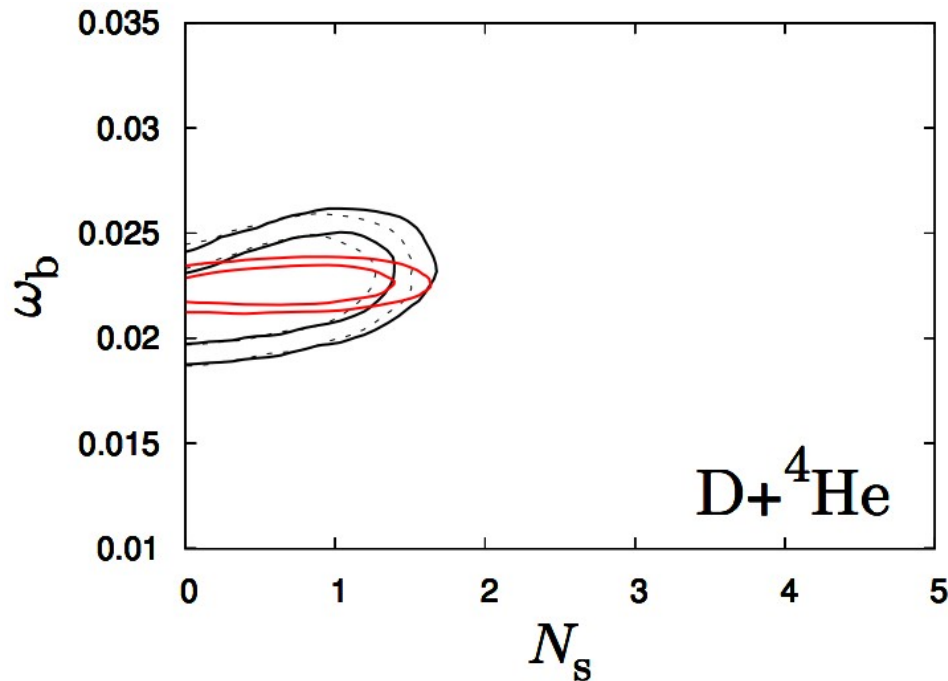
# Evidence for $N_{\text{eff}} > 3$ from BBN...

- Light element abundances are sensitive to excess relativistic energy density.



# Evidence for $N_{\text{eff}} > 3$ from BBN...

- Mild preference for  $N_{\text{eff}} > 3$  (or  $N_s > 0$ ) from Deuterium+Helium-4.
- But  $N_s = 2$  is **strongly disfavoured**.



# Quick fix: degenerate BBN...

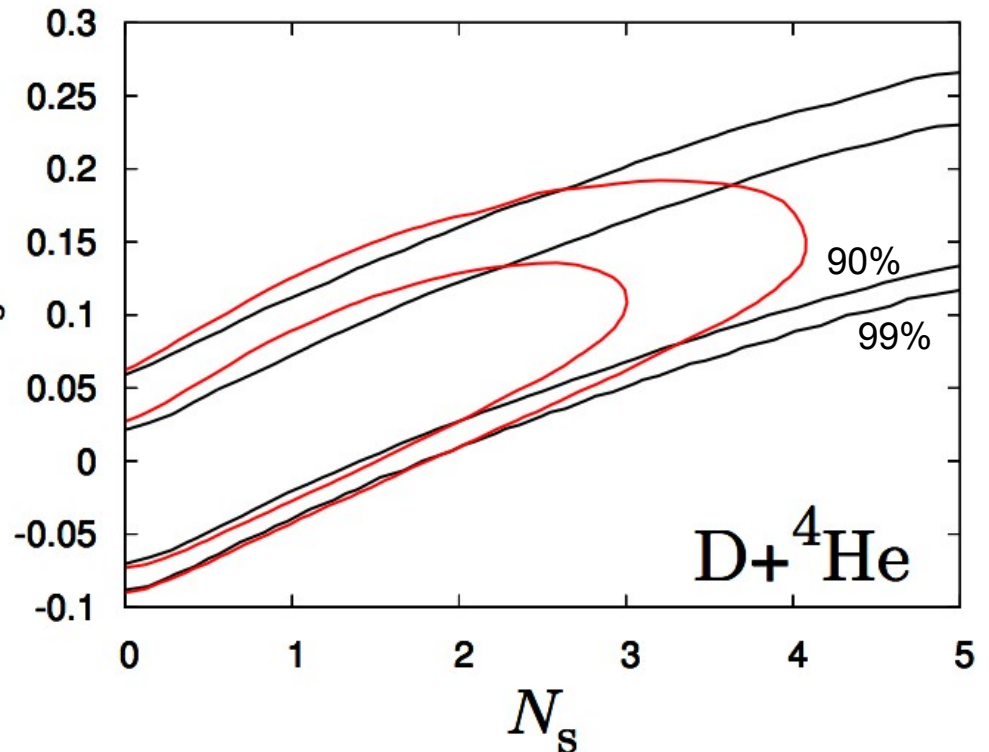
- Introduce a **neutrino chemical potential** (= O(0.1) **lepton asymmetry**).
- Then even  $N_s = 3$  is **allowed** by BBN.

Lepton asymmetry

$$L \equiv \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma}$$

$$= \frac{1}{12\zeta(3)} \left(\frac{T_\nu}{T_\gamma}\right)^3 (\pi^2 \xi + \xi^3)$$

Neutrino chemical potential  $\xi$



**Question:** How to simultaneously get  $L = O(0.1)$  and  $B = O(10^{-10})$ ?