Higher order corrections in view of the LHC

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Fermilab

Menu

- Theoretical setup.
- Status of α_s and parton distribution functions
- Necessity of NLO
- MCFM
- Calculation of tree diagrams
- Progress with calculation of one loop diagrams

QCD improved parton model

Hard QCD cross section is represented as the convolution of a short distance cross-section and non-perturbative parton distribution functions. Physical cross section is formally independent of μ_F and μ_R

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Physical cross section

Parton distribution function

Renormalization scale μ_R

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Factorization scale μ_F

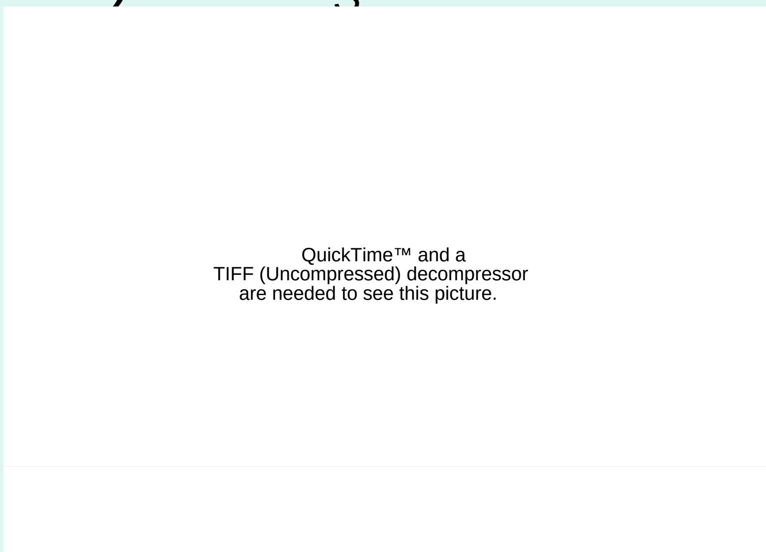
Short distance cross section, calculated as a perturbation series in α_S

Short-distance cross section

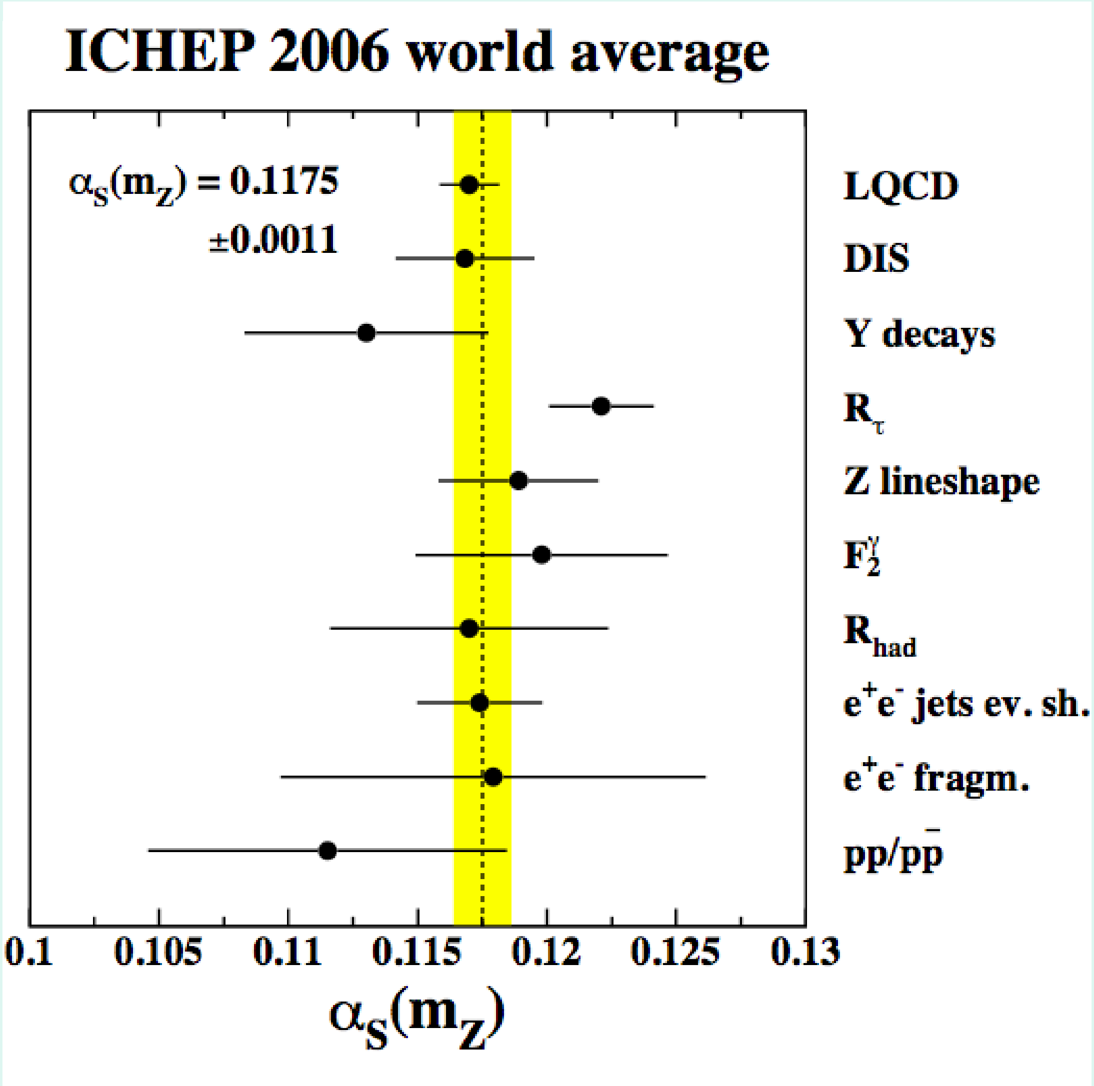
For a hard process the short distance cross section can be calculated
in various approximations.

Leading order (LO) tree graphs α_s^n
Next-to-leading order (NLO) α_s^{n+1}
Next-to-next_to_leading order (NNLO) α_s^{n+2}

Current state of the art can calculate large number of loops
and small number of legs or a smaller number of legs and a larger number of loops.



S Kluth, hep-ex/0609020



2006 World average $\alpha_s(M_z)$ =0.1175±0.0011

Parton distribution functions

Measurement of the non-perturbative parton distributions at lower energies allow extrapolations to higher values of μ and lower values of x using the DGLAP equation

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The evolution kernel is calculable as a perturbation series in α_s

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LO NLO NNLO

NNLO is known completely. (Moch et al, hep-ph/0403192)

Projected parton model uncertainties after HERAII

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HERA for LHC workshop,hep-ph/0601012

…and consequent improvement on uncertainty of jet cross section

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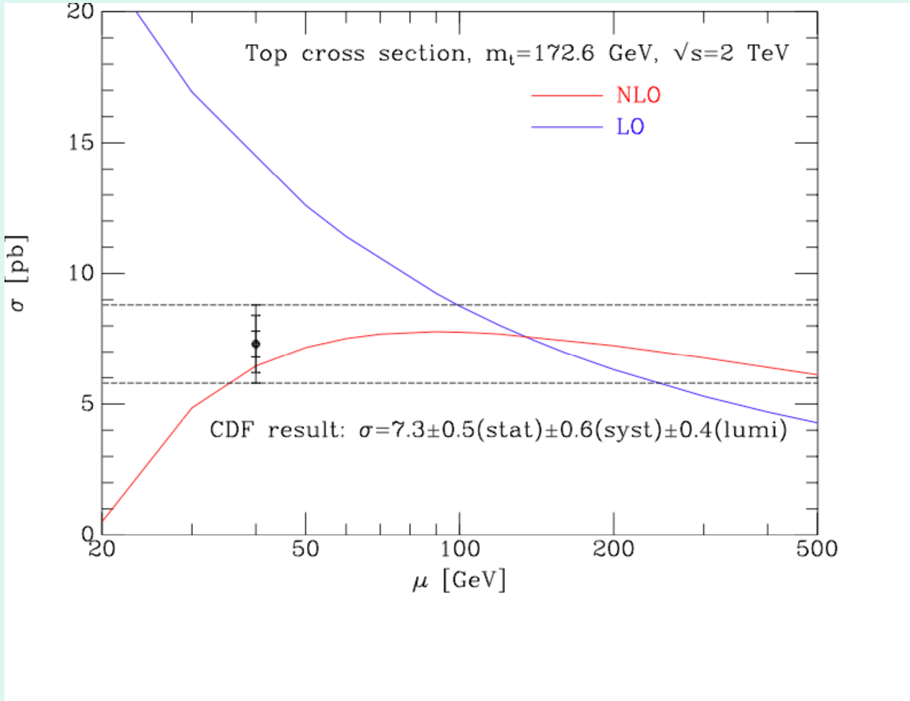
Dependence on unphysical scales

So the uncertainty in α_S is 1% and α_S~10%, and the uncertainty due to parton distributions is ~10-20% (at least for the measured values of x).

Why can we not predict physical cross sections to ~10-20%?

We need NLO (or better)



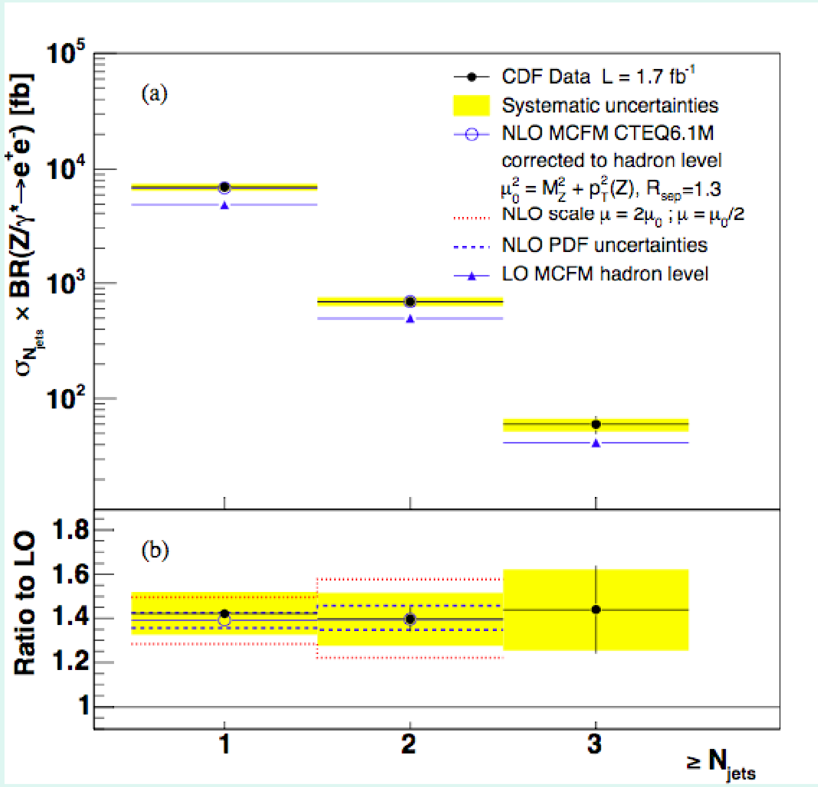
In order to get ~10% accuracy we need to include NLO.

- Less sensitivity to unphysical input scales, (eg. renormalization and factorization scales)
- NLO first approximation in QCD which gives an idea of suitable choice for μ.
- NLO has more physics, parton merging to give structure in jets, initial state radiation, more species of incoming partons enter at NLO.
- A necessary prerequisite for more sophisticated techniques which match NLO with parton showering.

Isn’t it just an overall K-factor?

Sometimes…..

but not always



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p_T 50 100 150 2

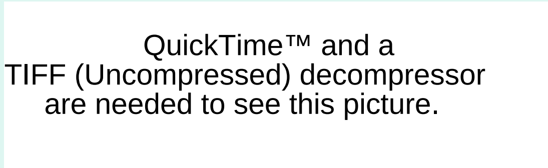
NLO-solid, LO-dashed

Z+jet production at the Tevatron

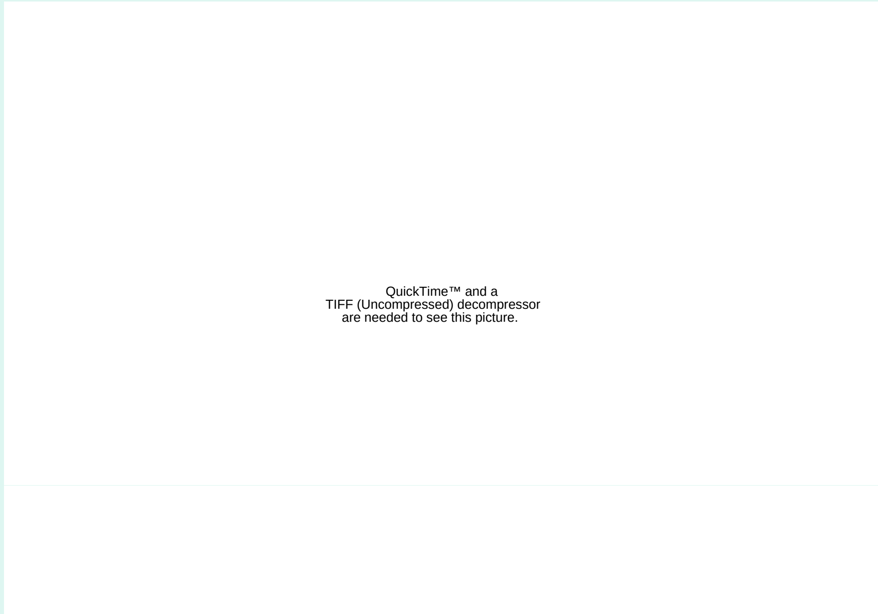
Wbb production at the L

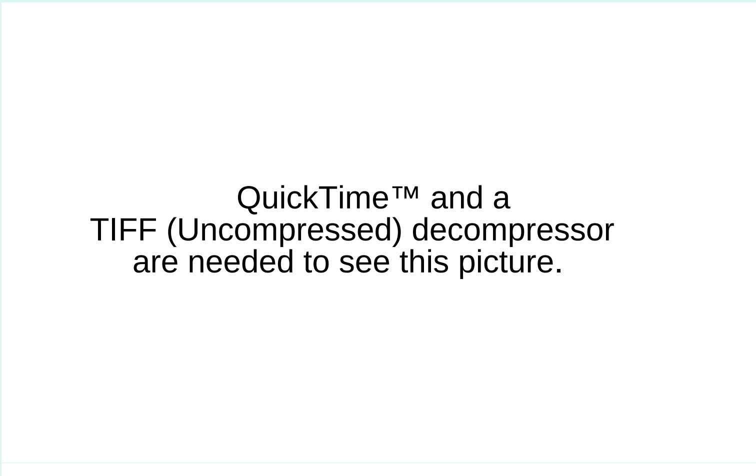
Influence of new processes at NLO

At LO



At NLO





For this case the influence of incoming gluons is negative http://mcfm.fnal.gov/

MCFM overview

Monte Carlo for FeMtobarn processes. At LHC few of the cross sections are expressed in fb, so MCFM. Parton level cross sections predicted to NLO in α_S. Currently released version 5.2, July 2007

Features- Less sensitivity to unphysical μ_R and μ_F, better normalization for rates, fully differential distributions.

Shortcomings- low parton multiplicity (no showering), no hadronization, hard to model detector effects.

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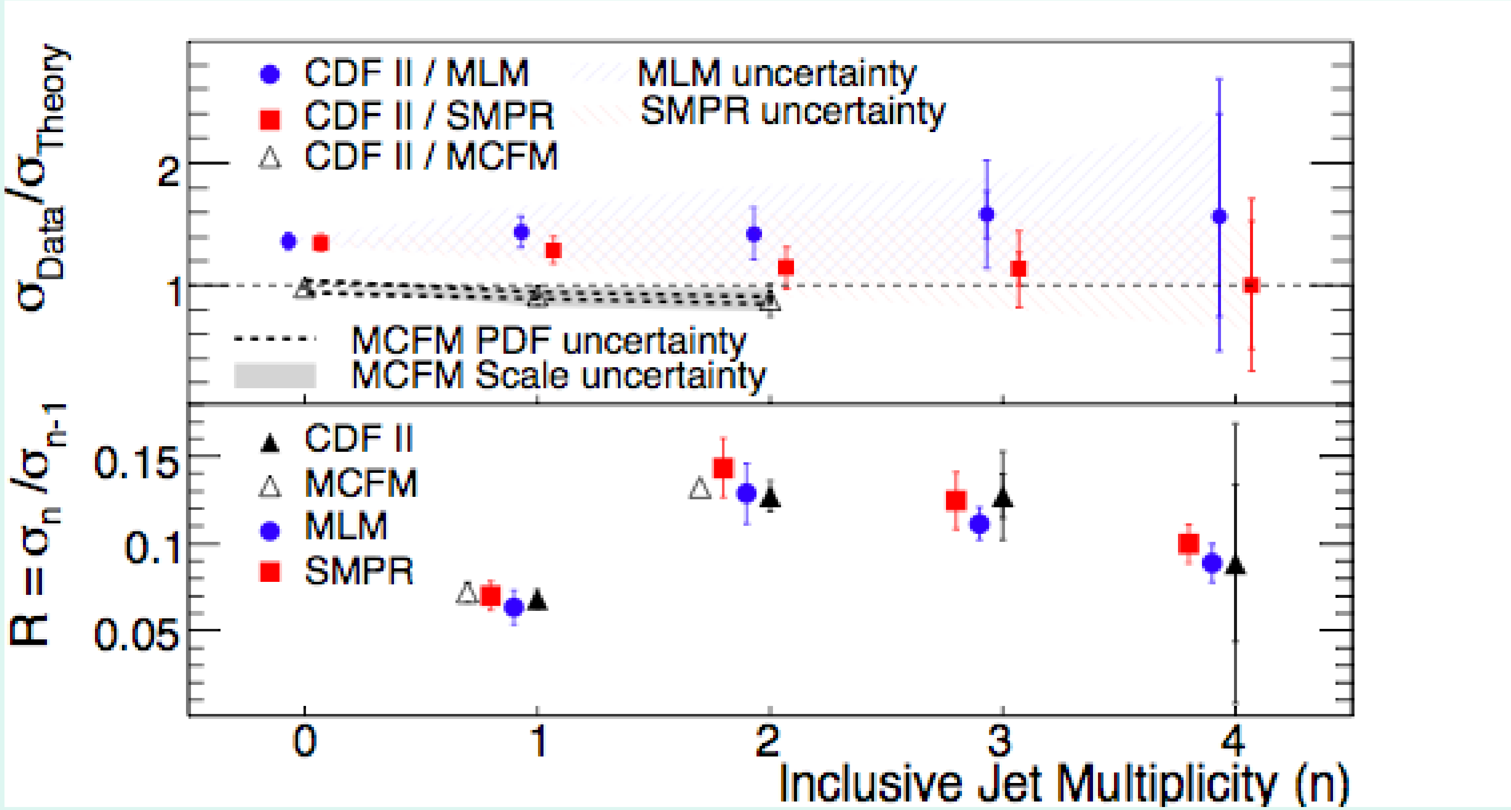
Work by John Campbell and Keith Ellis with appearances by guest celebrities, Fabio Maltoni, Francesco Tramontano, Scott Willenbrock & Giulia Zanderighi.

PRD77 011108R

W+n jet rates from CDF



Both uncertainty on rates and deviation of Data/Theory from 1 are smaller tha other calculations. “Berends” ratio agrees well for all calculations, but unfortunately only available for n≤2 from MCFM.

Recent additions to MCFM

- WW+1jet (Campbell, RKE, Zanderighi, arXiv:0710.1832)
- H+2jet (Campbell, RKE, Zanderighi, hep-ph/0608194)

(Neither of these processes are yet included in the publically released code).

Campbell, Ellis,
Zanderighi

Higgs+2 jets at NLO

- Calculation performed using an effective Lagrangian, valid in the large m_t limit.

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Three basic processes at lowest order.

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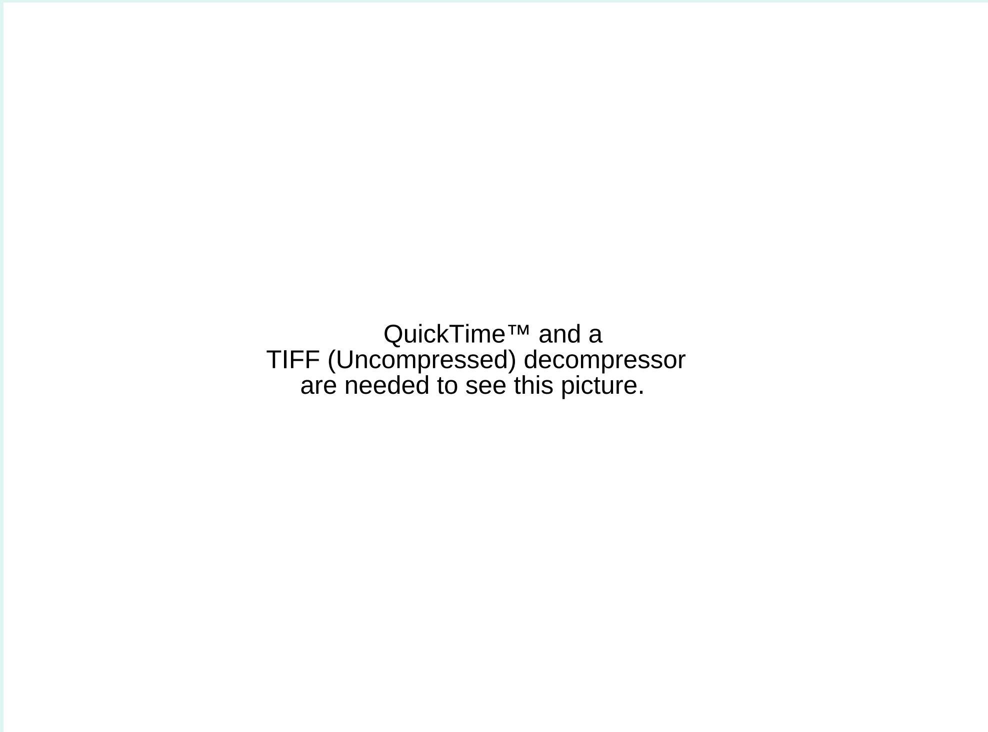
Higgs + 2 jets rapidity distribution versus WBF

- Shape of NLO result, similar to LO in rapidity.
- WBF shape is quite different at NLO.
- An irreducible background to WBF.



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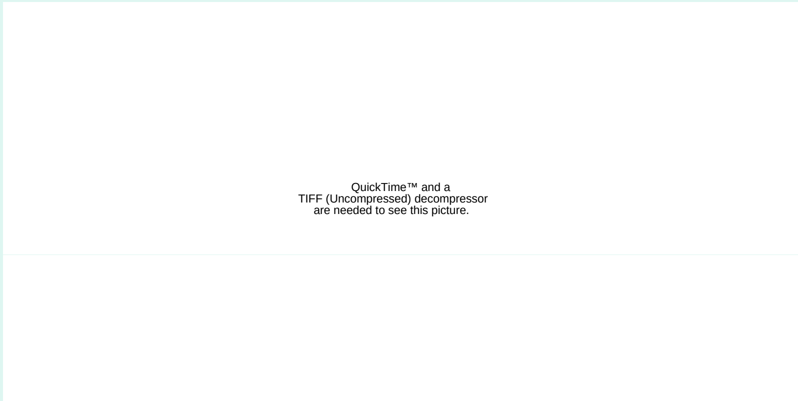
Ingredients for a NLO calculation

Example $e^+e^- \Rightarrow$ 2 jets

- Born process LO
- Interference of one-loop with LO
- Real radiation (also contributes to the two jet rate in the region of soft or collinear emission).



What is the bottle-neck?

- Consider for example W+n jets.
  (W+4 jets is a background to top production).
- W+n (LO) and W+(n+1)-parton amplitudes known since 1989
  Berends et al.
- Subtraction method understood 1980.
  Ellis, Ross & Terrano, Catani & Seymour
- NLO parton evolution known since 1980.

  Curci, Furmanski & Petronzio
- Bottleneck is the calculation of one loop amplitudes. In fact only the one-loop amplitudes for W+1 jet and W+2 jets are known.
  Bern et al (1997); Campbell, Glover & Miller (1997).

Tree graphs

The calculation of any tree graph is essentially solved.

- Berends-Giele recursion
- MHV based recursion
- BCF on-shell recursion
- Comparison of methods

Off-shell recursion

On-shell recursion method

Berends-Giele recursion

Building blocks are non-gauge invariant color-ordered off-shell currents. Off-shell currents with n legs are related to off-shell currents with fewer legs (shown here for the pure gluon case).

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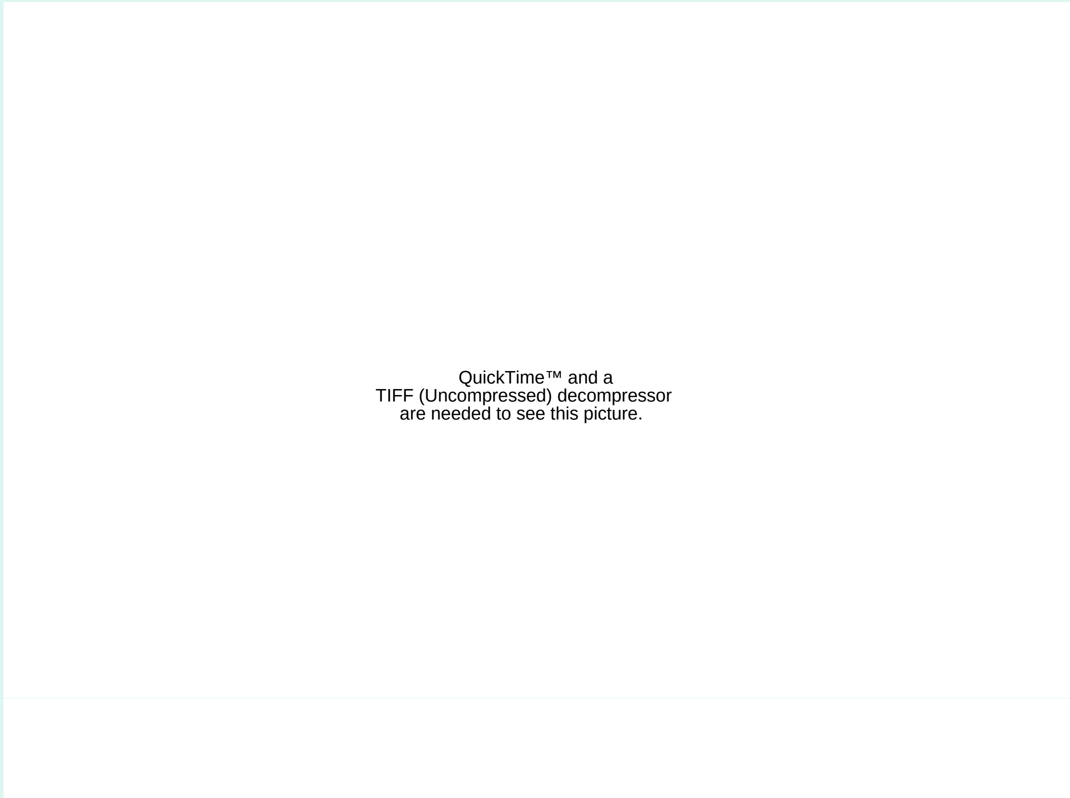
Despite the fact that it is constructing the complete set of Feynman diagrams, BG recursion is a very economical scher

Berends-Giele recursion

Kleiss and Kuijf showed in 1989 that the time for the calculation of gluonic amplitudes grows like N^4 using Berends-Giele off-shell recursion relations.

Economy comes from the fact that all previously calculated off-shell currents are re-used.



Dinsdale, Ternick, Weinzierl,   cf. Duhr, Hoche & Maltoni

Comparison of speed for numerical evaluation

Tree level amplitude with n external gluons may be written as

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Leading color matrix element squared is given by

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CPU time in seconds to calculate $\mathcal{M}_n$ using the various methods

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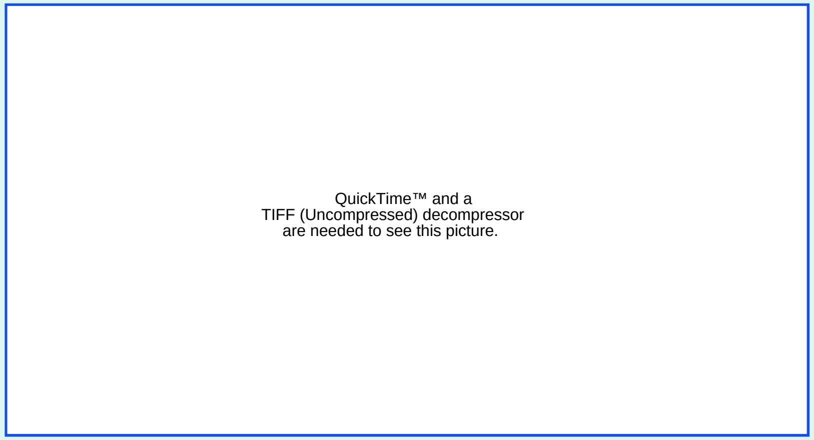
Conclusion on calculation of tree graphs

- Berends-Giele off-shell recursion is universal, fast enough and simple to program.

The calculation of one loop amplitudes

- The classical paradigm for the calculation of one-loop diagrams was established in 1979.
- Complete calculation of one-loop scalar integrals
- Reduction of tensors one-loop integrals to scalars.

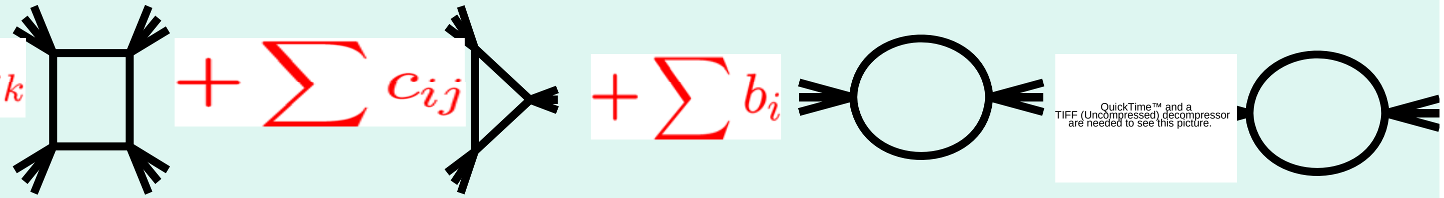


Neither will be adequate for present-day purposes.

Basis set of scalar integrals

Any one-loop amplitude can be written as a linear sum of boxes, triangles, bubbles and tadpoles

$$\mathcal{A}_N(\{p_i\}) = \sum d_{ijk} \; [\text{box}] + \sum c_{ij} \; [\text{triangle}] + \sum b_i \; [\text{bubble}] + [\text{tadpole}]$$



In addition, in the context of NLO calculations, scalar higher point functions, can always be expressed as sums of box integrals. Passarino, Veltman - Melrose (‘65)

- Scalar hexagon can be written as a sum of six pentagons.
- For the purposes of NLO calculations, the scalar pentagon can be written as a sum of five boxes.
- In addition to the ‘tH-V integrals, we need integrals containing infrared and collinear divergences.

Scalar one-loop integrals

- ‘t Hooft and Veltman’s integrals contain internal masses; however in QCD many lines are (approximately) massless. The consequent soft and collinear divergences are regulated by dimensional regularization.
- So we need general expressions for boxes, triangles, bubbles and tadpoles, including the cases with one or more vanishing internal masses.

Basis set of sixteen divergent box integrals

RKE, Zanderighi

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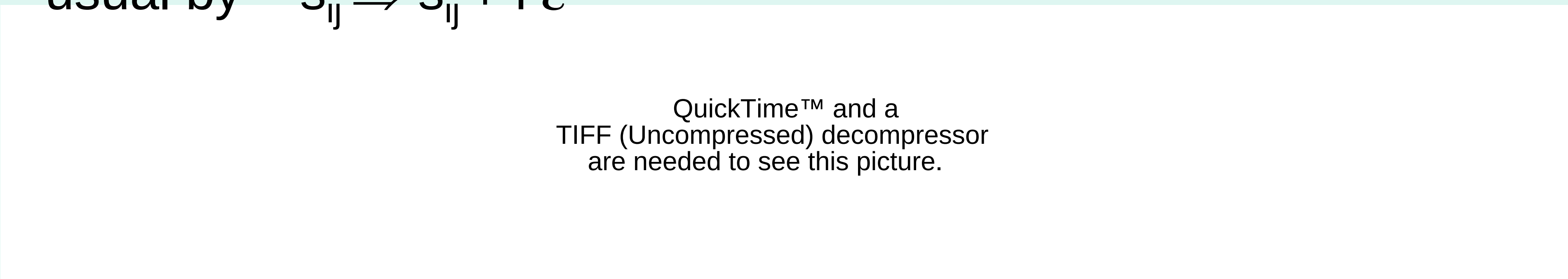
are needed to see this picture.

Example of box integral from qcdloop.fnal.gov

Basis set of 16 basis integrals allows
the calculation of any divergent box diagram.
Result given in the spacelike region. Analytic continuation as usual by $s_{ij} \Rightarrow s_{ij} + i\,\varepsilon$



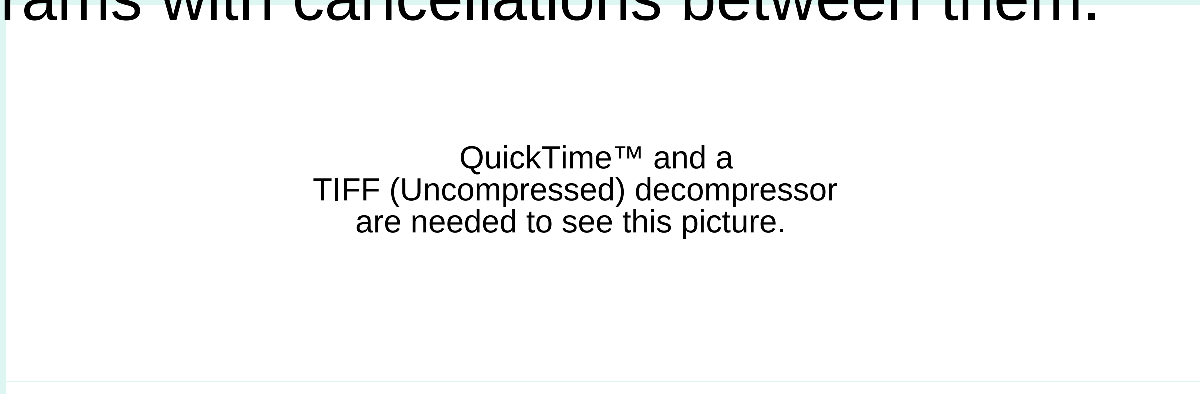
Limit $p_3^2 = 0$ can be obtained from this result, (limit $p_2^2 = 0$ cannot)

QCDLoop

- Analytic results are given for the complete set of divergent box integrals at http://qcdloop.fnal.gov
- Fortran 77 code is provided which calculates an arbitrary scalar box, triangle, bubble or tadpole integral.
- Finite integrals are calculated using the ff library of Van Oldenborgh. (Used also by Looptools)
- For divergent integrals the code returns the coefficients of the Laurent series $1/\varepsilon^2$, $1/\varepsilon$ and finite.
- Problem of one-loop scalar integrals is completely solved numerically and analytically!

Determination of coefficients of scalar integrals

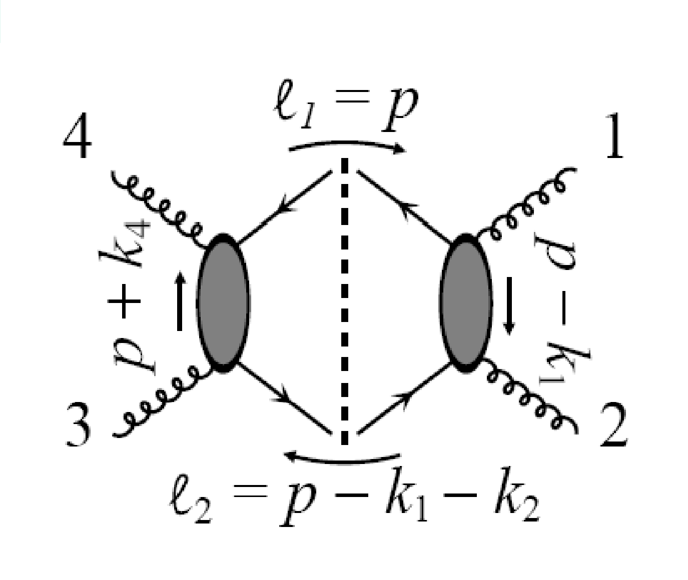
Feynman diagrams + Passarino-Veltman reduction cannot be the
answer as the number of legs increases. There are too many diagrams with cancellations between them.



Semi-numerical methods based on unitarity may be the answer. Note however that to date the majority of complete n-leg calculations for n>4 are based on Passarino-Veltman.

Basic setup for one-loop diagrams, use of unitarity

$$T^\dagger - T = -2iT^\dagger T$$



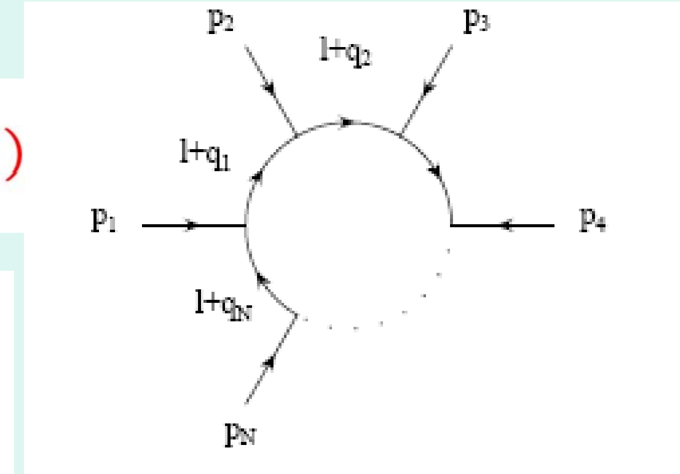
The use of unitarity allows us to recycle tree graph analytic results

$$-i\,\mathrm{Disc}\,A_4(1,2,3,4)\Big|_{s-\mathrm{cut}} = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\delta^{(+)}(\ell_1^2 - m^2)\, 2\pi\delta^{(+)}(\ell_2^2 - m^2) \times A_4^{\mathrm{tree}}(-\ell_1, 1, 2, \ell_2)\, A_4^{\mathrm{tree}}(-\ell_2, 3, 4, \ell_1)\,,$$

Bern, Dixon, Kosower
Britto, Cacahazo and Feng

Decomposing one-loop N-point amplitudes in terms of master integrals

$$\mathcal{A}_N(p_1, p_2, \ldots, p_N) = \int [d\,l]\ \mathcal{A}(p_1, p_2, \ldots, p_N; l)$$



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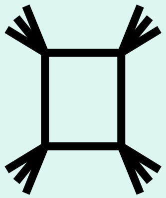
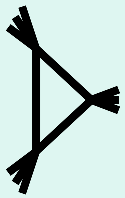
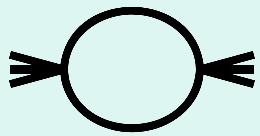
$$d_i = (l + q_i)^2 - m_i^2 = (l - q_0 + \sum_{j=1}^{i} p_i)^2 - m_i^2$$

$$\mathcal{A}_N(\{p_i\}) = \sum d_{ijk}$$  $$+ \sum c_{ij}$$  $$+ \sum b_i$$ 

Any Feynman amplitude can be expressed as a sum of scalar boxes, triangles, bubbles and tadpoles (not shown).

Decomposing in terms of ■▲●…

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are needed to see this picture.

- Without the integral sign, the identification of the coefficients is straightforward.
- Determine the coefficients of a multipole rational function

Algebraic reduction, subtraction terms

- Ossola, Papadopoulos and Pittau showed that there is a systematic way of calculating the subtraction terms at the integrand level.
- We can re-express the rational function in an expansion over 4,3,2, and 1 propagator terms.
- The residues of these pole terms contain the l-independent master integral coefficients plus a finite number of spurious terms.

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Residues of poles and unitarity cuts

Define residue function

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We can determine the d-coefficients, then the c-coefficients and so on

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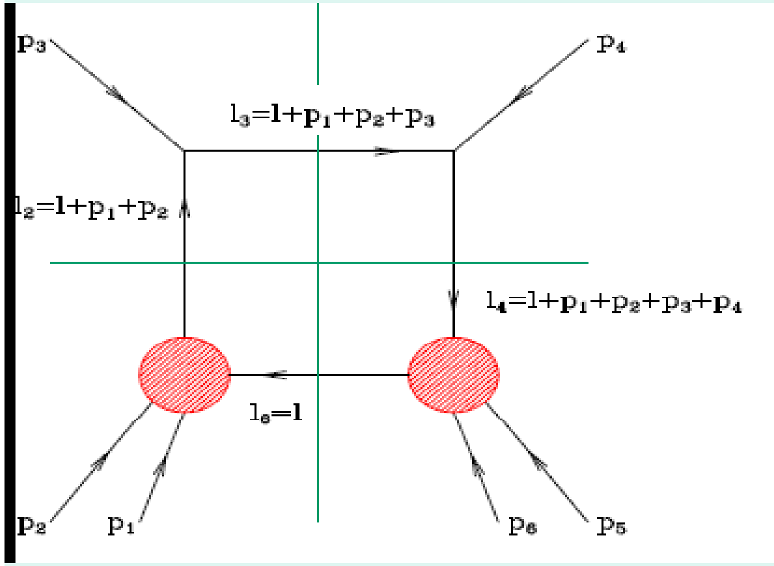
The box residue

$$\text{Res}_{2346}\left(\mathcal{A}_6(l^\pm)\right) = \mathcal{M}_4^{(0)}(l_6^\pm; p_1, p_2; -l_2^\pm) \times \mathcal{M}_3^{(0)}(l_2^\pm; p_3; -l_3^\pm)\mathcal{M}_3^{(0)}(l_3^\pm; p_4; -l_4^\pm)$$

$$\times \mathcal{M}_4^{(0)}(l_4^\pm; p_5, p_6; -l_6^\pm) \quad = \quad \bar{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l \cdot n_1$$



$$d_{ijkl} = \frac{\text{Res}_{ijkl}\left(\mathcal{A}_N(l^+)\right) + \text{Res}_{ijkl}\left(\mathcal{A}_N(l^-)\right)}{2}$$

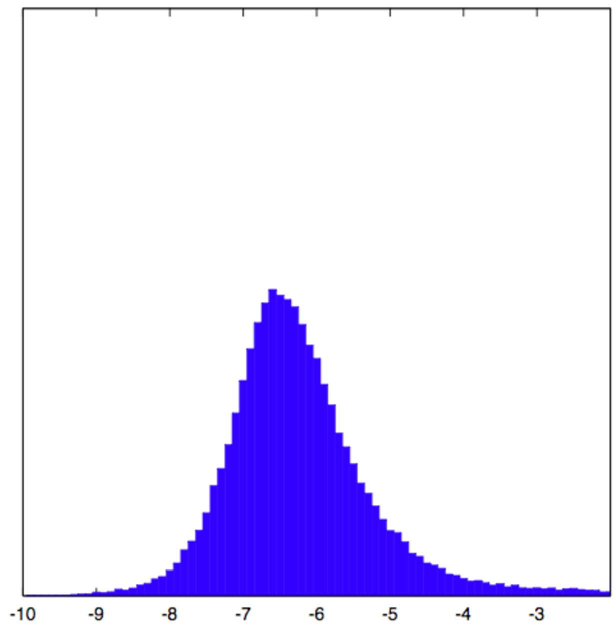
$$\tilde{d}_{ijkl} = \frac{\text{Res}_{ijkl}\left(\mathcal{A}_N(l^+)\right) - \text{Res}_{ijkl}\left(\mathcal{A}_N(l^-)\right)}{2i\sqrt{V_4^2 - m_l^2}}$$

The residues of the poles = sum over factorized tree amplitudes

Ellis, Giele, Kunszt

Result for six gluon amplitude

- Results shown here for the cut-constructible part
- The relative error for the finite part of the 6-gluon amplitude compared to the analytic result, for the (+ + - - - -) helicity choice. The horizontal axis is the log of the relative error, the vertical axis is the number of events in arbitrary linear units.
- For most events the error is less than 10^{-6}, although there is a tail extending to higher error, which can be treated by using higher



Giele, Kunszt,Melnikov, arXiv 0801.223

Numerical Unitarity Method in D-dimensions

If we can control the complete D dependence of the internal lines, we can obtain the complete amplitude, not just the cut-constructible part.

i) spin-polarizations live in D_s

ii) loop momentum live in D. (D_s>D)

$$\mathcal{A}_{(D,D_s)}(\{p_i\},\{J_i\}) = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{\mathcal{N}^{(D_s)}(\{p_i\},\{J_i\};l)}{d_1 d_2 \cdots d_N}.$$

$$\sum_{i=1}^{D_s-2} e_\mu^{(i)}(l) e_\nu^{(i)}(l) = -g_{\mu\nu}^{(D_s)} + \frac{l_\mu b_\nu + b_\mu l_\nu}{l \cdot b},$$

$$l^2 = \bar{l}^2 - \tilde{l}^2 = l_1^2 - l_2^2 - l_3^2 - l_4^2 - \sum_{i=5}^{D} l_i^2$$

Two key features

Dependence on Ds is linear

$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

- Choose two integer values $D_s = D_1$ and $D_s = D_2$ to reconstruct the full D_s dependence.
- Suitable for numerical implementation.
- $D_s=4-2\varepsilon$ 't Hooft Veltman scheme, $D_s=4$ FDHS

The loop momentum effectively has only 4+1 component

$N(l) = N(l_4, \mu^2)$ $\quad \mu^2 = -l_5^2 - \ldots - l_D^2$

maximum 5 constraints: we need to consider also pentagon cuts.

Reduction in D-dimensions

The parametrization of the N-particle amplitude

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}}$$

$$+ \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

Parametrization of the residues

Pentuple residue:

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(0))
n

Box residue: $\overline{d}^{\mathrm{FDH}}_{ijkn}(l) = d^{(0)}_{ijkn} + d^{(1)}_{ijkn} s_1 + (d^{(2)}_{ijkn} + d^{(3)}_{ijkn} s_1) s_e^2 + d^{(4)}_{ijkn} s_e^4$

QuickTime™ and a
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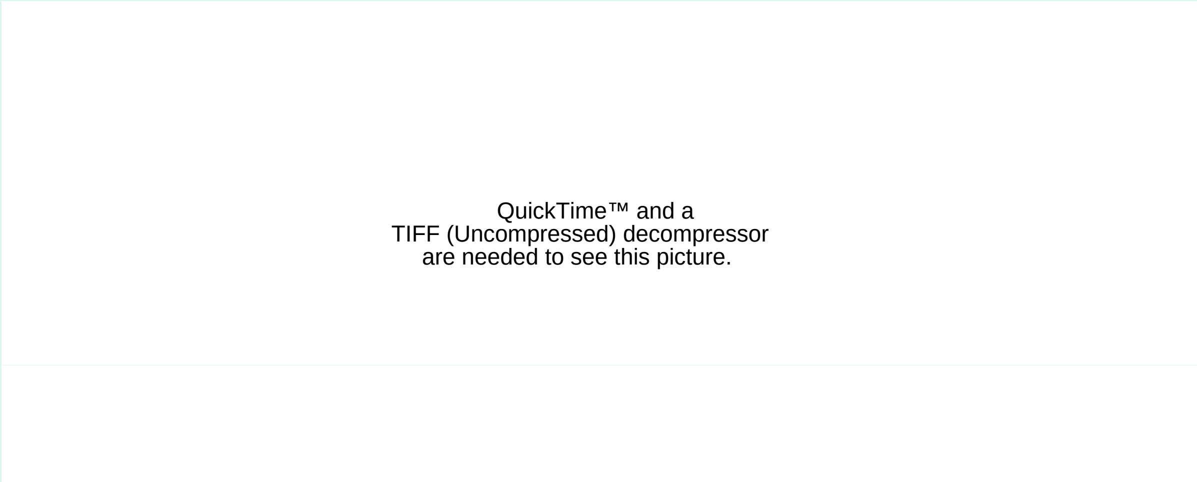
are needed to see this picture.

Three extra structures for triple, three for double and zero for single cuts, only even powers of

Giele, Kunszt,Melnikov, arXiv 0801.2237

Extension to full amplitude

- Keep dimensions of virtual unobserved particles integer and perform calculations in more than one dimension.
- Arrive at non-integer values D=4-2ε by polynomial interpolation.
- Results for six-gluon amplitudes agree with original Feynman diagram calculation of RKE, Giele, Zanderighi.



Scaling property of tree and loop amplitudes

Results for the complete one-loop 20 gluon amplitude!

Satisfy a number of non-trivial checks.

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are needed to see this picture.

Giele and Zanderighi arXiv:0805.2152

Grows like N^9

Summary

- $\alpha_S(M_Z)$ is known to $< 1\%$
- Parton distributions are known well enough to predict most cross sections to 20%, ($0.005<x<0.3$)
- Calculation of tree graphs is a solved problem, for all practical purposes. Berends-Giele recursion is numerically the best method.
- Open theoretical problem is thus the calculation of one-loop amplitudes. There is currently great intellectual fervor regarding the calculation of one-loop corrections.
- Unitarity based methods have achieved important results for one-loop diagrams, but not all semi-numerical methods have been tested in real physical calculations.
- Remaining challenge is to assemble into a program with efficient phase space sampling.
- The hope is to have several semi-automatic methods of calculating one-loop amplitudes (time scale about 1 year).