

Electroweak physics at LHC

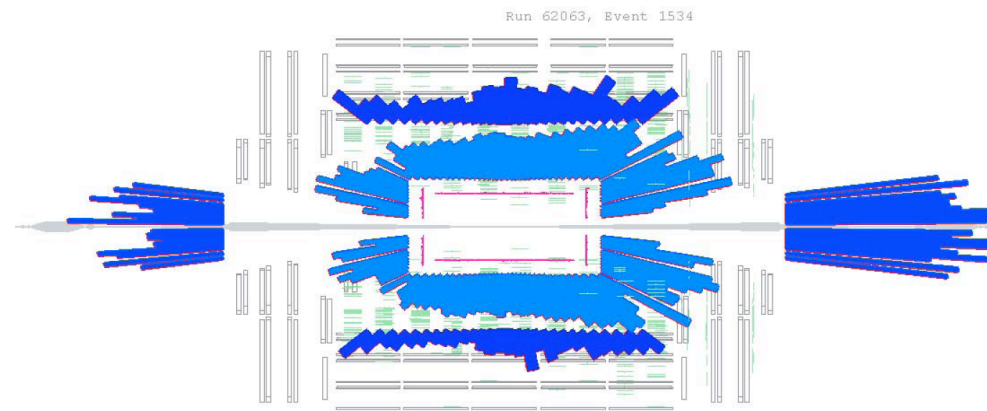
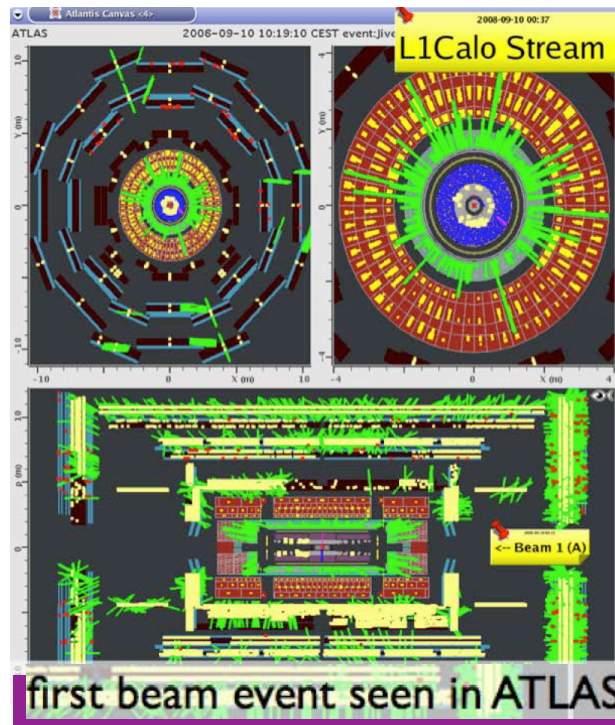
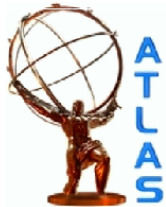
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On behalf of the ATLAS & CMS Collaborations

Physics @ LHC - 2008

Split, Croatia



first beam event seen in CMS



ElectroWeak physics at LHC



Large number of topics --> need to make a (personal) choice :

Outline

- W mass
- Top mass
- Forward Backward asymmetry in Z decay
- Associated production of Gauge Bosons

Current EW theory successfully tested at present energies

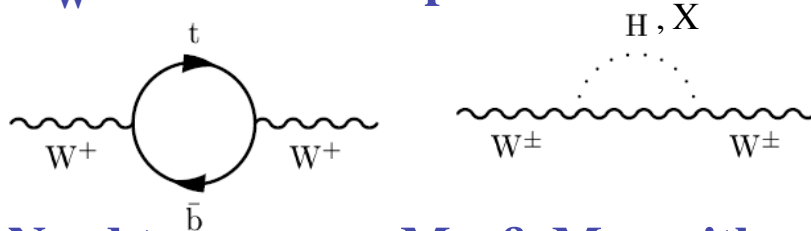
Aim @ LHC : deeper understanding in view of :

- * tightening indirect constraints (M_W , M_{top})
- * search for deviations (couplings)
- * understanding and calibrating detectors(i.e. see

K. Lohwasser talk)

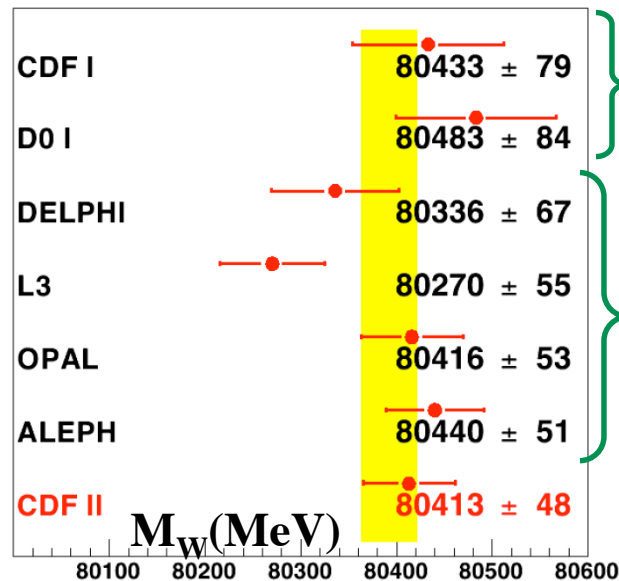
Each experiment @ LHC: 'Low' lumi $10 \text{ fb}^{-1}/\text{y}$
'High' lumi $100 \text{ fb}^{-1}/\text{y}$
TeVatron has collected so far $\approx 4.5 \text{ fb}^{-1}$

- ❖ M_W : fundamental parameter of the theory linked to M_{top} & to Higgs mass:



$$M_W = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_F \sin\theta_W \sqrt{1-\Delta r}}}$$

- ❖ Need to measure M_W & M_{top} with greatest precision



$$\Delta M_W^{\text{TeV I}} = 59 \text{ MeV}$$

$$\Delta M_W^{\text{LEP}} = 33 \text{ MeV}$$

World average(ICHEP08): $M_W = 80399 \pm 25 \text{ MeV}$

Great job !! Still LHC could improve it :

$$f(M_{\text{top}}^2, \ln M_H) \sim \text{few } \%$$

0.3‰

- ❖ $W(\&Z)$ cross-sections an order of magnitude more than TeVatron

$$\sigma_{Wlv}^{\text{NNLO 14 TeV}} \sim 20 \text{ nb} \rightarrow 1 \text{ fb}^{-1} \text{ gives } \sim 4\,000\,000 \text{ W events } (\epsilon_{\text{sel}} \sim 20\%)$$

- ❖ Design peak luminosity ~ an order of magnitude more than TeVatron

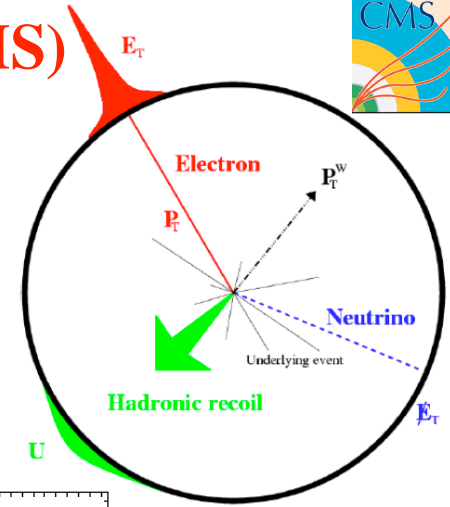


Mass of W boson: analysis steps (ATLAS & CMS)

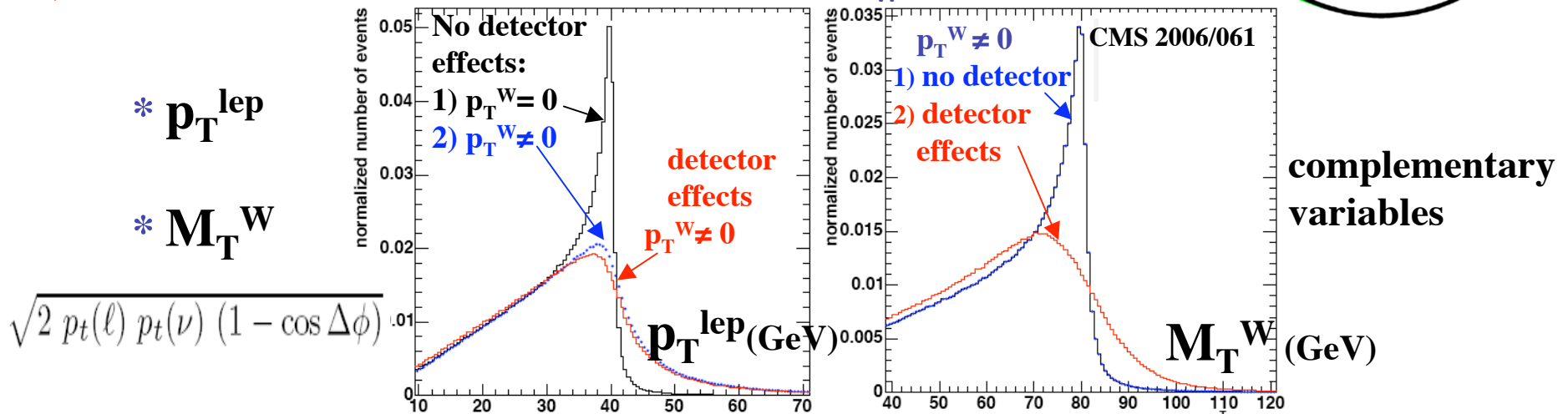


- 1) Select candidate W's
(in leptonic channel)

Isolated lepton $p_T^{\text{lep}} > 25 \text{ GeV}$,
 $|\eta| < 2.4$
 $E_T^{\text{Miss}} > 25 \text{ GeV}$
 No jets with $p_T > 30 \text{ GeV}$
 Recoil $< 20 \text{ GeV}$



- 2) Plot the two main variables sensitive to M_W :



- 3) Build templates (i.e. expected distributions of p_T^{lep} & $M_T^W = f(M_W)$)
- 4) χ^2 fit to find the template matching best the data $\rightarrow M_W$

- ❖ Z events play a crucial role: reduce experimental & theoretical uncertainties
 - * Z events 'modified' and used to build templates (CMS)
 - * Z events used to tune W MC (ATLAS)



Mass of W boson: build templates (CMS)



❖ Method 1: Scaled observable method

$$R'(X) = \text{corr} * R(X)$$

$$\frac{d\sigma^W}{dp_T^{lep}} = \frac{M_Z}{M_W} R'(X) \frac{d\sigma^Z}{dp_T^{lep}}$$

predicted
~ from theory
measured

$$R(X) = \frac{d\sigma^W}{d(p_T^{lep}/M_W)} \bigg/ \frac{d\sigma^Z}{d(p_T^{lep}/M_Z)}$$

$R(X)$: calculable using perturbative QCD

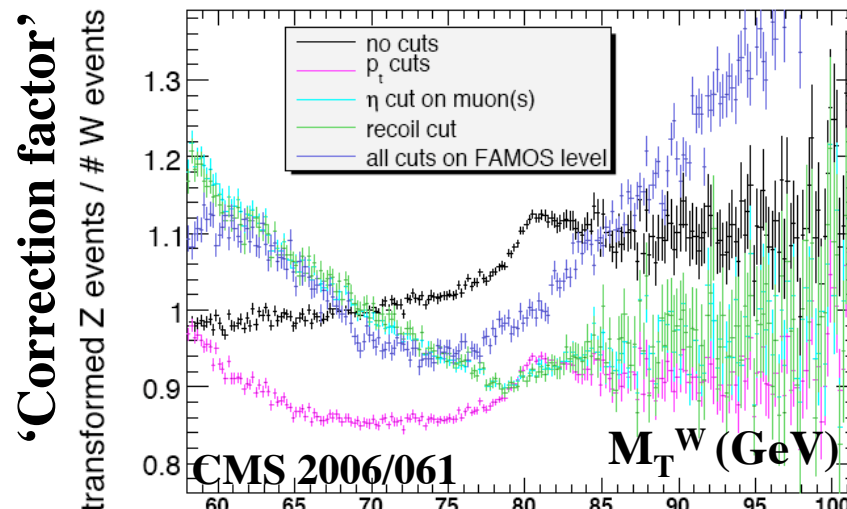
corr : W/Z acceptance and exp. resolution -> understanding detector response

1 fb⁻¹: $\Delta M_W = 40 \text{ (stat.)} \oplus 40 \text{ (exp.)} \oplus 40 \text{ (theo.) MeV}$

mainly lepton scale linearity

mainly p_T^W spectrum

❖ Method 2: Morphing (scaling the Z events rather than the Z observables)



1 fb⁻¹:

$$\Delta M_W = 40 \text{ (stat.)} \oplus$$

$$64 \text{ (exp.)} \oplus \geq 20 \text{ (theo.) MeV}$$

mainly PDF

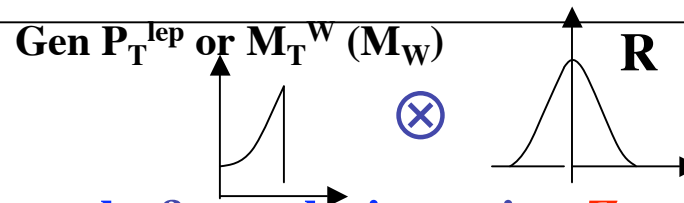
mainly E_T^{Miss} scale



Data driven methods developed by CMS in recent cross-section analyses will decrease the experimental systematic error



Mass of W boson: build templates (ATLAS)



❖ Templates for p_T^{lep} :

- Get the energy/momentum scale & resolution using $Z \rightarrow ll$ events:

Fit $Z \rightarrow ll$ data with Templates = [f(lept. scale, lept. resolution)]

- Tag and Probe to determine the lepton efficiency ($Z \rightarrow ll$ events)

❖ Templates for M_T^W :

- Model missing momentum response by looking at $Z \rightarrow ll$ events

control
of
experimental
systematics

❖ Results for 15 pb^{-1} :

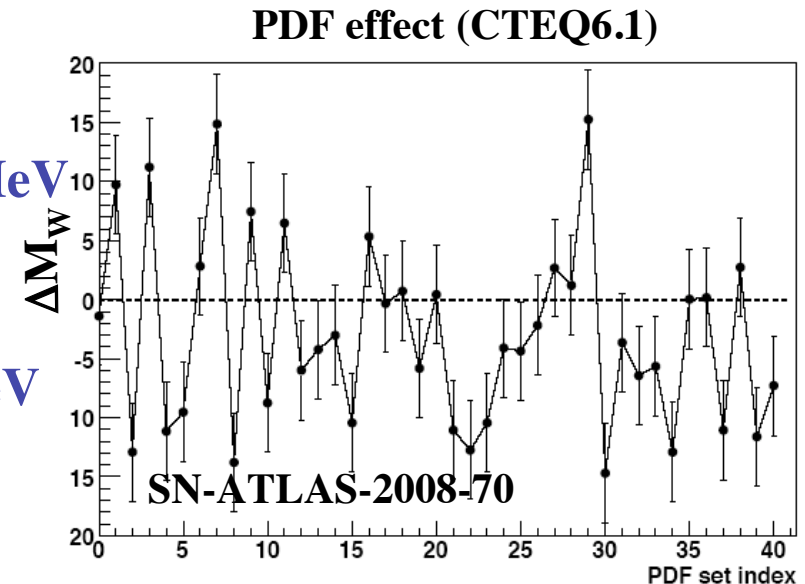
$W \rightarrow e \nu$ with p_T^{lep}

$$\Delta M_W = 120 \text{ (stat)} \oplus 114 \text{ (exp.)} \oplus 25 \text{ (PDF) MeV}$$

$W \rightarrow \mu \nu$ with M_T^W

$$\Delta M_W = 57 \text{ (stat)} \oplus 230 \text{ (exp.)} \oplus 25 \text{ (PDF) MeV}$$

How to control the theoretical uncertainties?



Mass of W boson: main theoretical uncertainties

- ❖ Transverse momentum distribution
 p_T^W affects p_T^{lep}

Study $p_T(l+l^-)$ vs M_{l+l^-} in γ^*/Z events

$10 \text{ fb}^{-1} \Delta M_W \approx 3 \text{ MeV}$

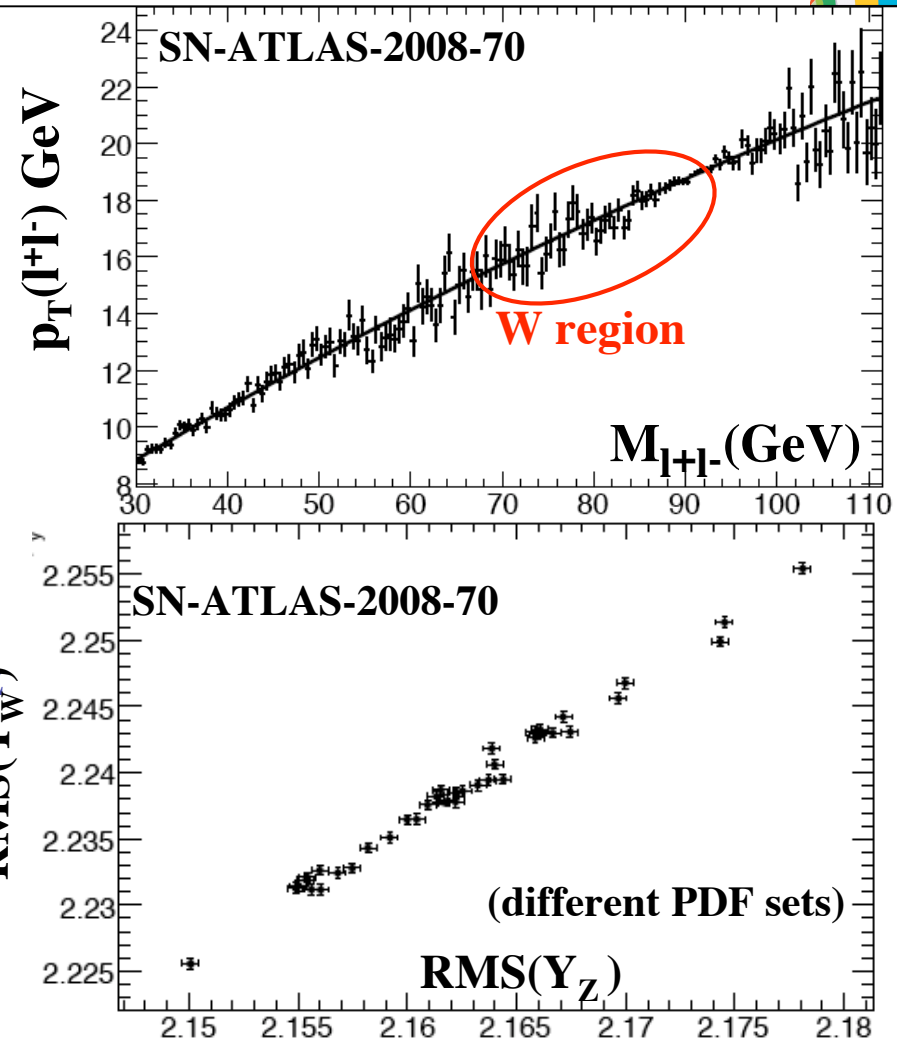
- ❖ PDF affect rapidity of $W(Y_W) \rightarrow M_W$

Strong correlation between Y_W and Y_Z when varying PDF

$10 \text{ fb}^{-1} \Delta M_W \approx 1 \text{ MeV}$

10 fb^{-1} Total systematics $\Delta M_W < 10 \text{ MeV}$ (1 analysis)

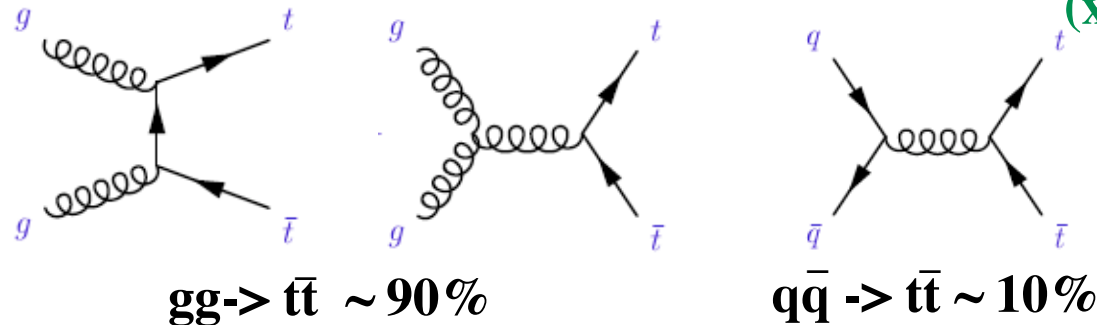
- For equal contribution to M_H (indirect) uncertainty :
if $\Delta M_W < 10 \text{ MeV} \rightarrow \Delta M_{\text{top}} < 1.5 \text{ GeV}$



Top Mass measurement

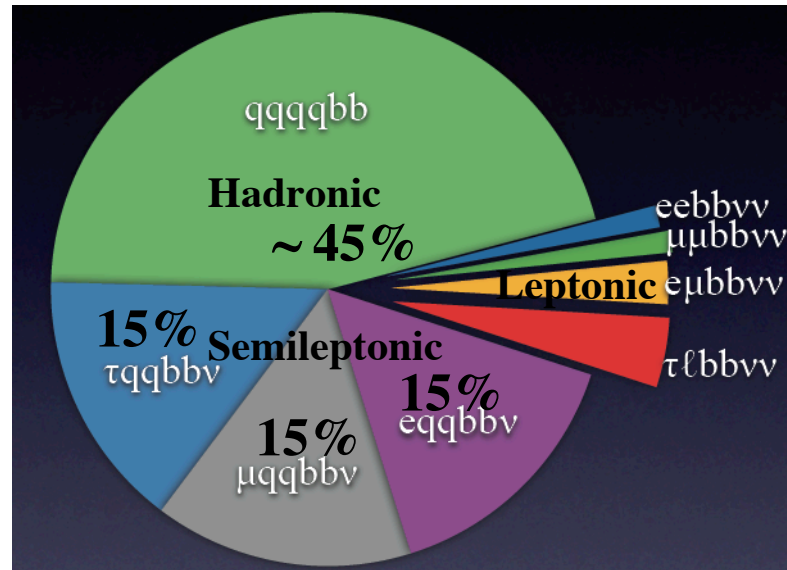
- Accurate measurement of M_{top} important :
 - * consistency check of SM
 - * constrain Higgs mass within SM
- Top abundantly produced @ LHC: $\sigma_{t\bar{t}}^{\text{NLO}} \sim 830 \text{ pb} \Rightarrow \sim 10^7 \text{ } t\bar{t} \text{ pairs/y @ } 10^{33} \text{ cm s}^{-1}$

(x 100 TeVatron !!)



- $\text{Br}(t \rightarrow Wb) \approx 100\%$.

Decays :



- Hadronic :
 - + high Br
 - + possible to reconstruct whole event
 - high bkg and combinatorics
- Leptonic :
 - + lower bkg & combinatorics
 - low Br,
 - final state difficult to reconstruct
- Semileptonic: good compromise ('golden')

Top Mass in semileptonic decays: analysis steps

1) Event selection:

1 isolated high p_T lept+ $E_t^{\text{miss}} + \geq 4$ jets of high p_T (2 b-jets)
 $\epsilon \approx 5\%$, $S/B \approx 10$, $\epsilon_b \approx 60\%$

2) Hadronic W mass reconstruction M_{jj} : several methods

Ex: first select jet pairs in a mass range then get M_{jj} , $\alpha_{1,2}$:

$$\chi^2 = \frac{(M_{jj} - M_W^{PDG})^2}{\Gamma_W^2} + \frac{(E_{j1}(1 - \alpha_1))^2}{\sigma_1^2} + \frac{(E_{j2}(1 - \alpha_2))^2}{\sigma_2^2}$$

$\alpha_{1,2}$ & $\sigma_{1,2}$ = light jet energy scale & resolution

3) Top quark reconstruction: several methods

Ex: choose b jet closest to $W \rightarrow jj$ + further cuts

4) Leptonic W mass reconstruction M_{lv}

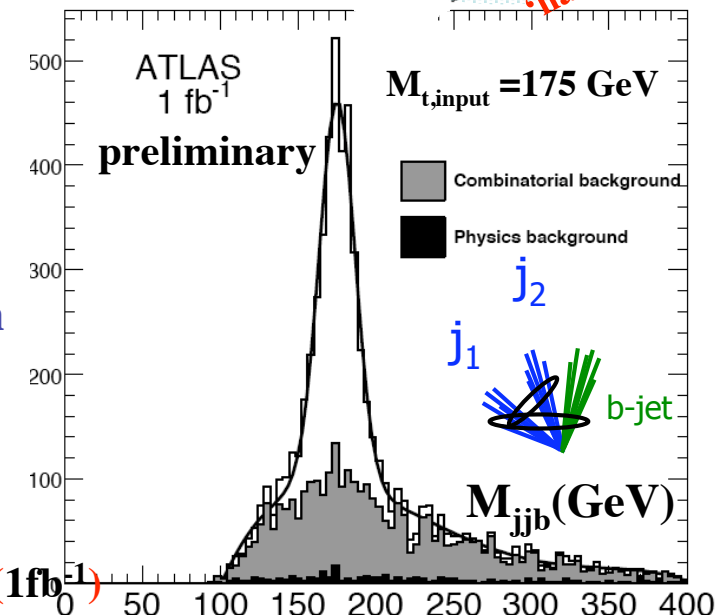
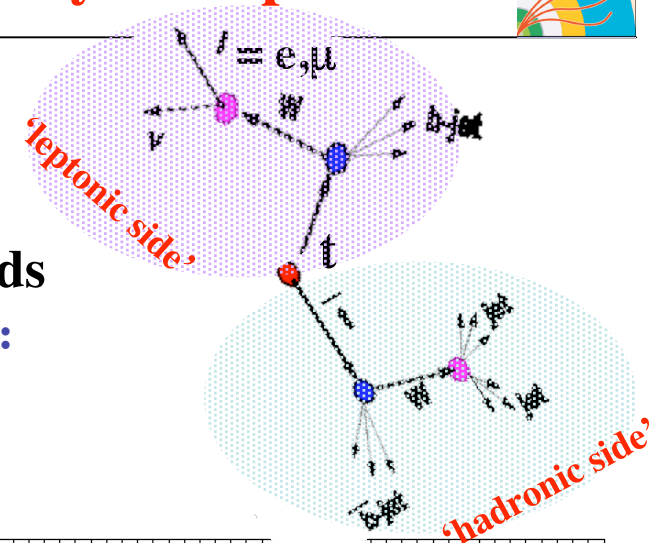
Using known W mass and 4-momentum conservation
 (then select solution such that $|M_{lvb} - M_{jjb}|$ is minimum after step 4)

5) Top quark mass

Gaussian+polynomial fit $M_{\text{top}} = 175.0 \pm 0.2_{\text{stat}} \text{ GeV} (1\text{fb}^{-1})$

Kinematic fit to the full event (less sensitivity to FinalStateRadiation)

$$M_{\text{top}}^{\text{fit}}(\chi^2=0) = 174.8 \pm 0.4_{\text{stat}} \text{ GeV}$$



Top Mass in semileptonic decays: systematics

Systematic uncertainty	χ^2 minimization method
Light jet energy scale	0.2 GeV/%
b jet energy scale	0.7 GeV/%
ISR/FSR	$\simeq 0.3$ GeV
b quark fragmentation	≤ 0.1 GeV
Background	negligible
Method	0.1 to 0.2 GeV

Most important systematics :
jet energy scale (JES) of

* b-jet (1 fb^{-1} $\Delta M_{\text{top}}(\text{b-JES}) \approx 1\text{-}3.5 \text{ GeV}$
depending on b-JES $\approx 1\text{-}5\%$)

@LHC (1 fb^{-1} , 1 analysis) :

Current value (TeV ICHEP2008): $M_{\text{top}} = 172.4 \pm 0.7_{\text{stat}} \pm 1.0_{\text{syst}} \text{ GeV}$

- If no use of b-tag ('early data')

* higher combinatorics

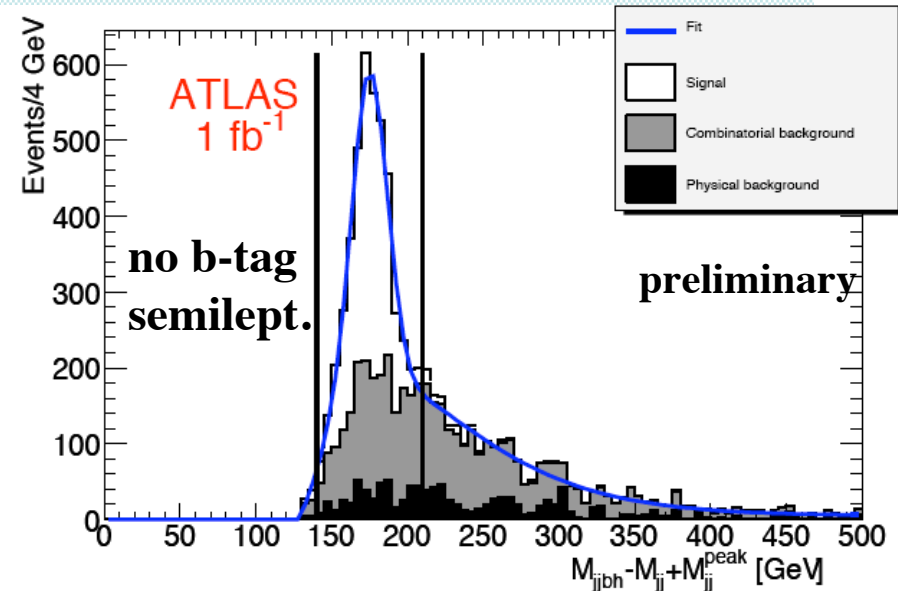
---> additional cuts

$\epsilon \approx 1.6\%$, purity $\approx 50\%$

* issue: constraint bkg shape with data

$M_{\text{top}} = 172.4 \pm 1.5_{\text{stat}} \text{ GeV}$

Systematics \approx as above except from $\Delta M_{\text{top}}(\text{bkg})$ which goes to $\sim 1 \text{ GeV}$





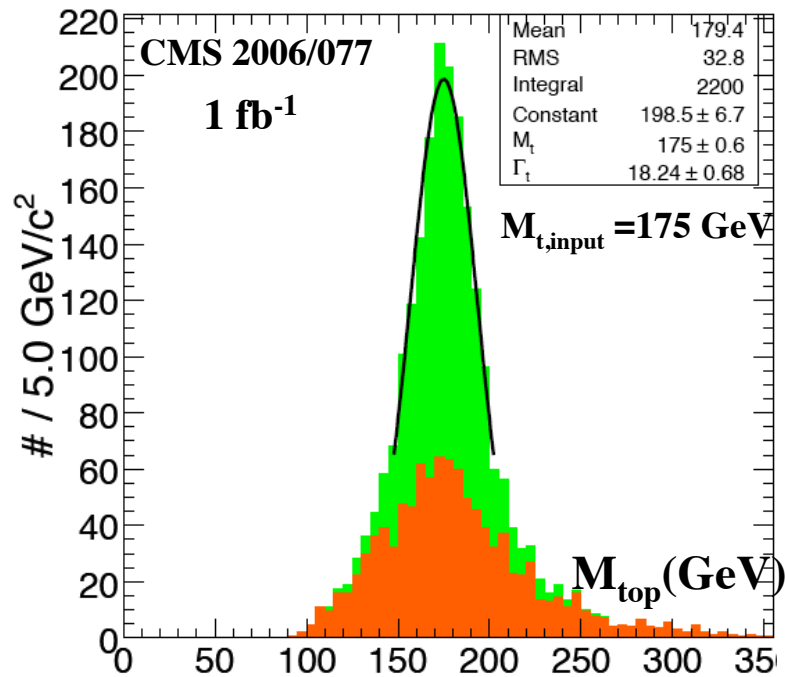
Top Mass fully hadronic & Top Mass di-leptonic decays



Starting $S/B < 10^{-6}$

Selection $S/B \approx 1/9$ $\epsilon = 2.7\%$

Likelihood on masses and angles
to perform the pairing + top choice



$M_{top} = 175.0 \pm 0.6(\text{stat}) \pm 4.2(\text{syst}) \text{ GeV}$

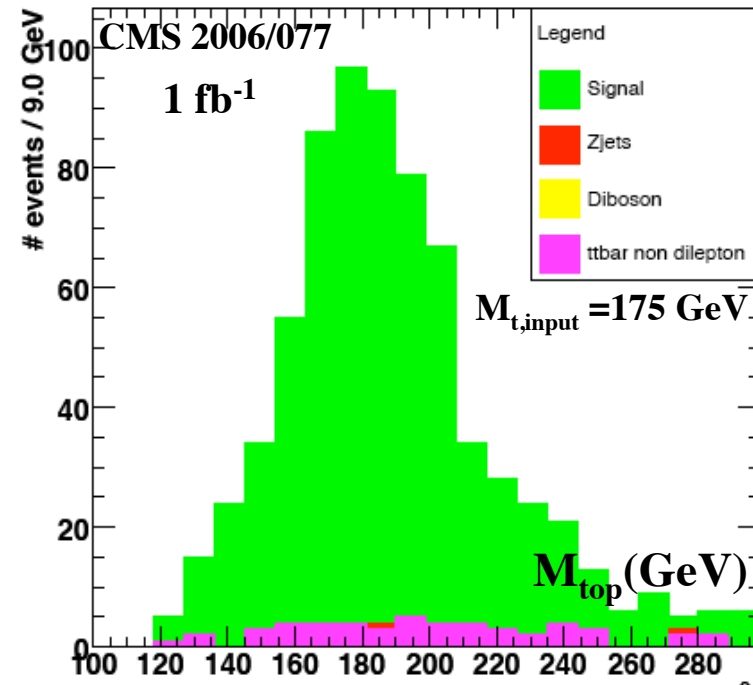
Systematics: JES & ISR/FSR

QCD background

Starting $S/B \approx 5 \cdot 10^{-3}$

Kinematical reconstruction of event
pairing with likelihood

$S/B \approx 12$ $\epsilon = 1.2\%$



$M_{top} = 178.5 \pm 1.5(\text{stat}) \pm 2.9(\text{syst}) \text{ GeV}$

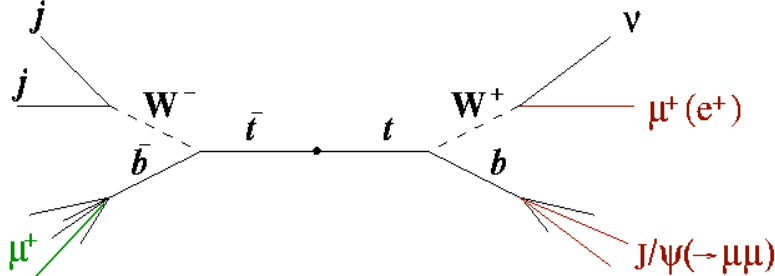
Main systematics: JES, ISR/FSR



Top Mass : additional methods



❖ Final states with J/ψ (100 fb^{-1}) :

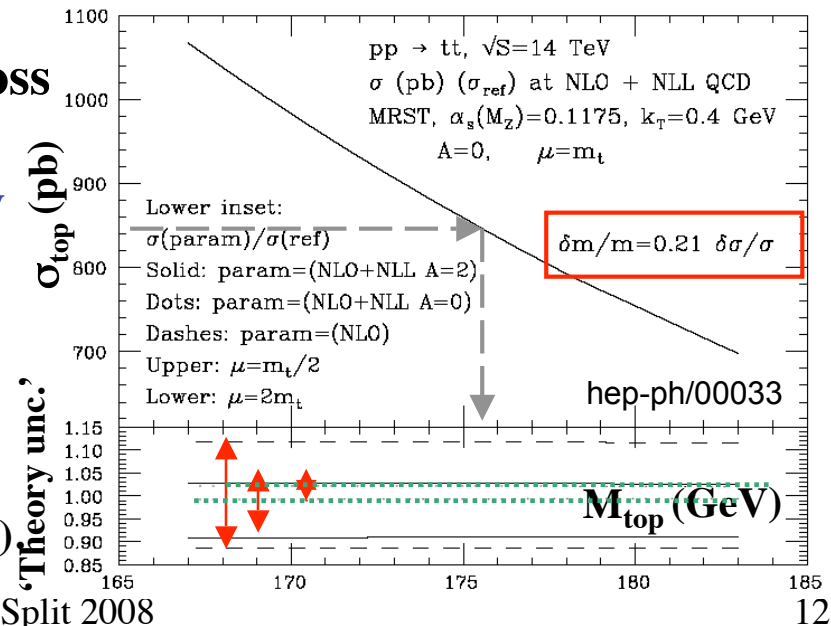
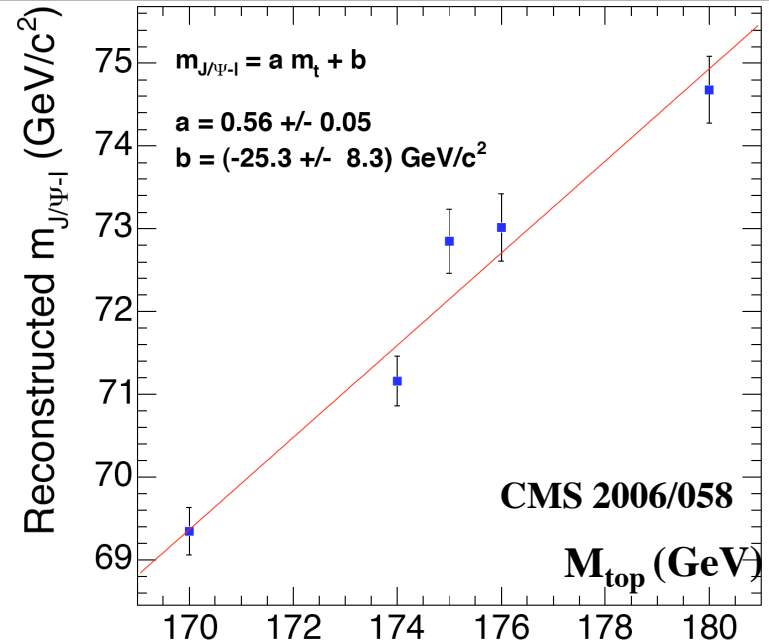


- Correlation between $M_{\text{lep } J/\psi}$ and M_{top}
--> No systematics on b-jet scale !
- $\sim 1000 \text{ evts}/100 \text{ fb}^{-1}$ --> $\Delta M_{\text{top}} \sim 1 \text{ GeV}$
(main syst. b-fragmentation)

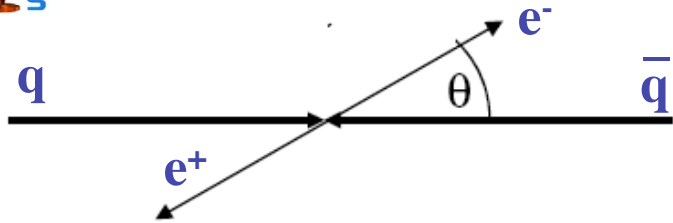
❖ M_{top} from the measured $t\bar{t}$ production cross section

- Uncertainty $\Delta M_{\text{top}} \geq 2 \text{ GeV}$ from theory calculation
- Can be reduced with computation of higher order, resummation of NLL corrections, PDF measurement.

A lot of theoretical activity. Many recent results ('08)



Forward-backward Asymmetry in γ^*/Z decay

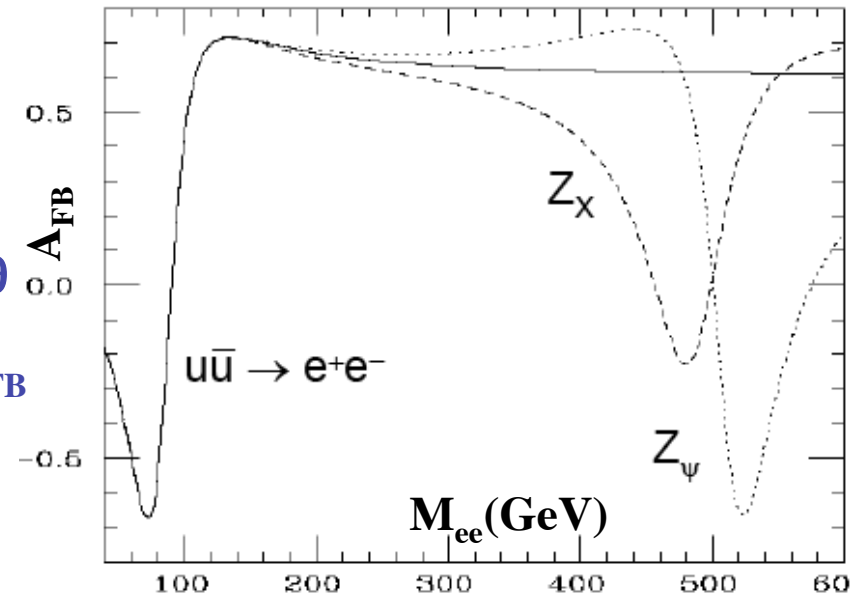


$$\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta} = \left[\frac{3}{8} (1 + \cos^2\theta) + A_{FB}(s) \cos\theta \right]$$

- Parity violation in neutral current --> asymmetry in the angular distribution of leptons

$$A_{FB}(s) = \frac{1}{\sigma} [\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)]$$

- $A_{FB}(s)$ depends on $g_V^{q,l}, g_A^{q,l}, Q_{q,l}$
--> probes V-A
extraction weak mixing angle $\sin^2\theta$
- Exchange of new particles would alter A_{FB}



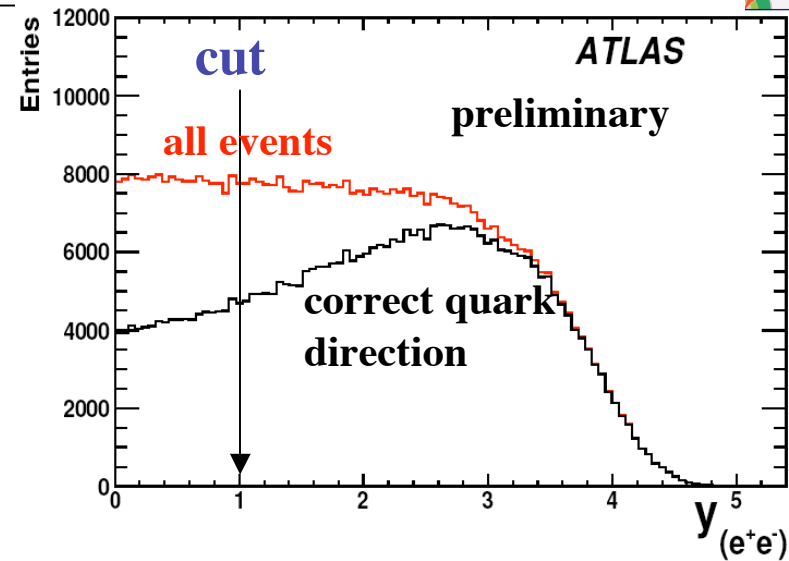
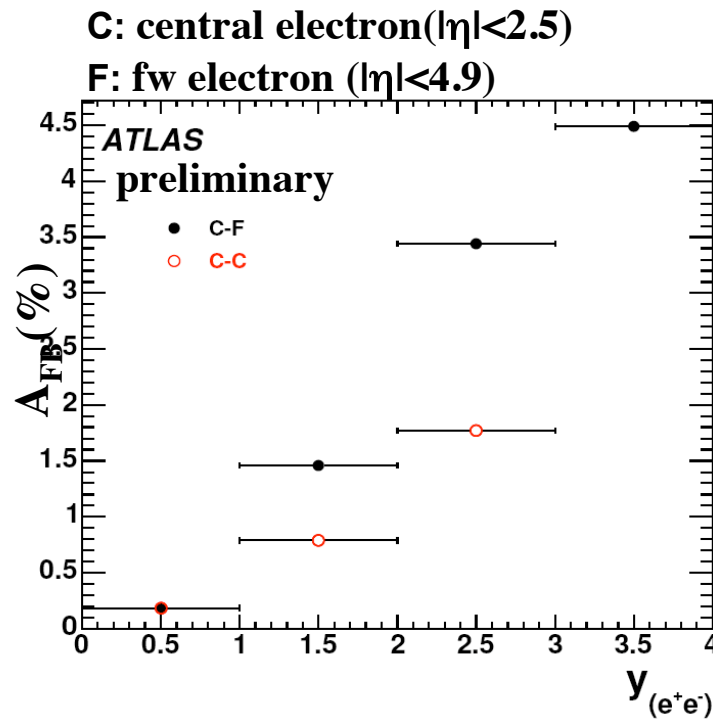
- @ TeVatron : statistics dominated.
Major uncertainties : PDF, detector resolution



Forward-backward Asymmetry in Z decay@LHC



- @ LHC : the quark direction is not known
--> take the Z direction
cut on Y_Z improves this assumption



☀ Forward electron identification important !!
($\epsilon_{fw\ ele} \sim 50\%$ $R_{ject} = 100$ --> $\Delta A_{FB}(stat) = 1.5 \cdot 10^{-4}$)

- Electron channel favoured wrt to μ channel due to higher angular acceptance ($|\eta| < 4.9$)
- Extraction of weak mixing angle $\sin^2\theta$ around the Z pole ($85 < M_{ee} < 97$ GeV) :

$$A_{FB} = b \{ a - \sin^2\theta_{eff}^{lept} \} \quad \text{with } a \text{ and } b \text{ from MC} \quad (a = 0.23 \pm 0.03 \quad b = 1.8 \pm 0.3)$$



$\sin^2\vartheta_{\text{eff}}^{\text{lept}}$ @LHC



ATLAS preliminary : 100 fb⁻¹

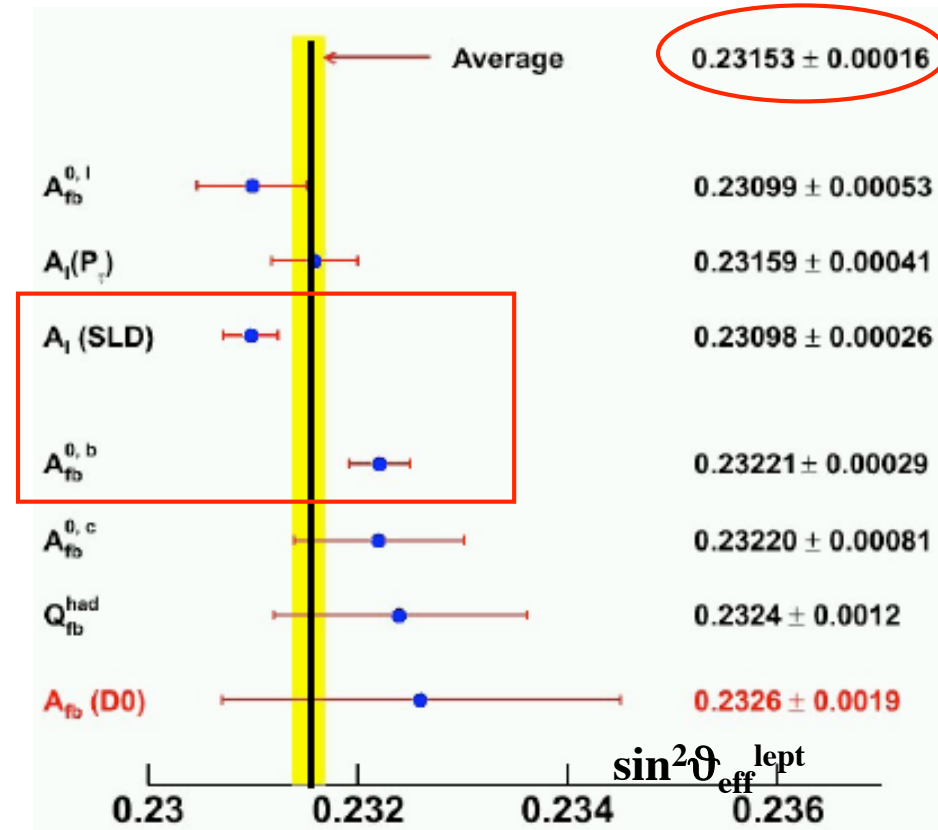
Uncertainties	$\delta \sin^2_{\text{eff}}$
Energy scale	1.5×10^{-5}
Reco. Efficiency	1.9×10^{-5}
Energy resolution	1.1×10^{-6}
Charge ID	1.4×10^{-5}
Bkg. subtraction	$< 10^{-5}$
Theory(PDF) <i>a and b parameters</i>	2.4×10^{-4} 3×10^{-5}
Statistical error	1.5×10^{-4}

Main uncertainty PDF.

Conversely can be used to constrain PDF

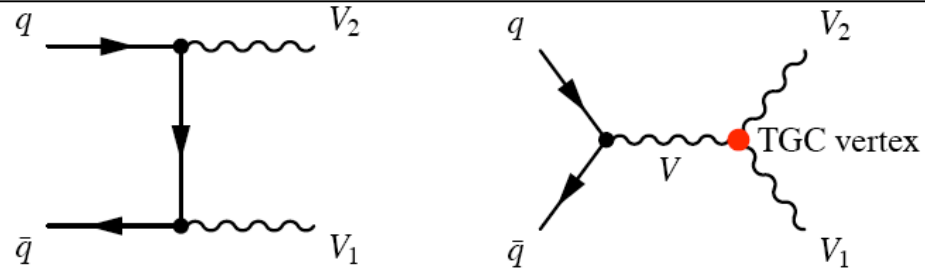
The weak mixing angle very well measured at LEP & SLD

3.2 σ



Associated production of Gauge Bosons : dibosons

Diboson LO SM diagrams:



TGC = Triple Gauge Coupling = Self interaction among 3 gauge bosons (V)

- Direct test of non-Abelian structure of SM (demonstrated @ LEP) at the highest energy
- If no Higgs found -> dibosons important in understanding EWSB
- Background for Higgs & New Physics

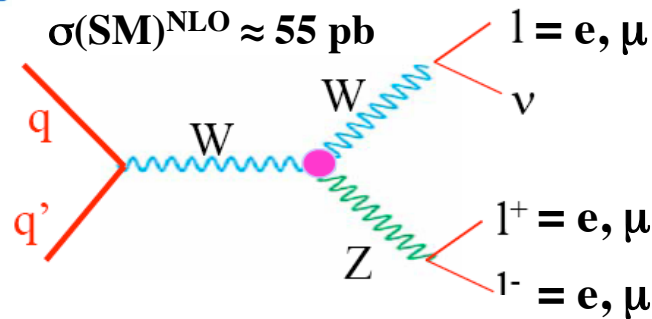
$$\begin{aligned}
 q \bar{q}' &\rightarrow W^{(*)} \rightarrow W \gamma : WW \gamma \\
 q \bar{q}' &\rightarrow W^{(*)} \rightarrow WZ : WWZ \\
 q \bar{q} &\rightarrow Z/\gamma^{(*)} \rightarrow WW : WW \gamma, WWZ \\
 q \bar{q} &\rightarrow Z/\gamma^{(*)} \rightarrow Z \gamma : \boxed{ZZ \gamma, Z \gamma \gamma} \\
 q \bar{q} &\rightarrow Z/\gamma^{(*)} \rightarrow ZZ : \boxed{ZZ \gamma, ZZZ}
 \end{aligned}$$

Not permitted in SM

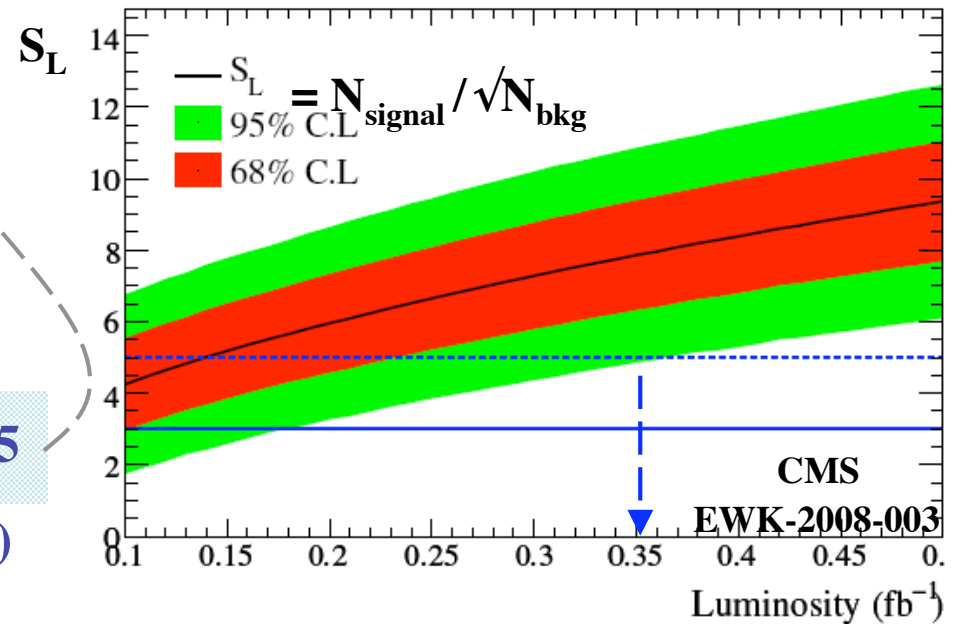
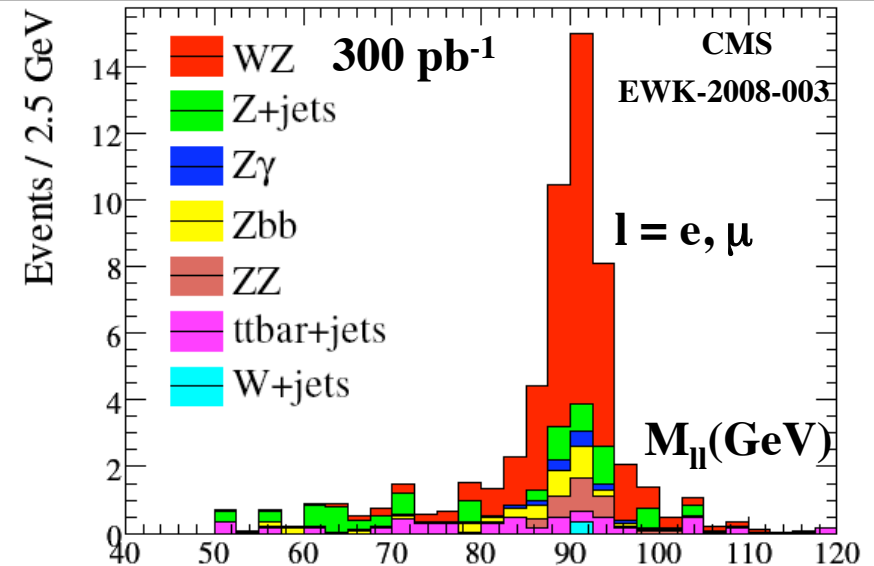
- ‘Anomalies’ appear as enhanced rates at high p_T^V or $M_T(VV)$ & changes in angular distributions
- All diboson processes already measured @ TeVatron (in leptonic channels), improvements expected @ LHC:
 - 1) cross-sections a factor ~ 10 higher
 - 2) higher energy allows to explore the most favorable kinematic region



Associated production of Gauge Bosons : WZ



- $\epsilon_{\text{trigger}} = 98\%$
- ≥ 3 high p_T isolated leptons (e, μ) + cuts on M_Z candidate and on M_{TW} candidate
- $\rightarrow 300 \text{ pb}^{-1}$: $N_{\text{signal}}^{\text{MC}} = 34.9 \pm 0.5$
 $N_{\text{signal}} / N_{\text{bkg}} = 2.6$
- $\left. \begin{array}{l} \text{ZZ\&Z}\gamma \text{ bkg (31\% of bkg)} \\ \text{tt, W+jets bkg (20\% of bkg)} \end{array} \right\} \text{from MC}$
- Data driven method to estimate Z+jet (main bkg) $N_{\text{signal}}^{\text{Pseudo-data}} = 33.0 \pm 3.5$
- Main syst.: lumi (10%), M_{TW} cut (10%)



- 14 possible **WWZ** and **WW γ** couplings; usually use 5 independent, CP conserving, EM gauge invariance preserving couplings: $g_1^Z, k_\gamma, k_Z, \lambda_\gamma, \lambda_Z$

- 5 ‘Anomalous Couplings’ (AC) :

$$\Delta g_1^Z \equiv g_1^Z - 1, \quad \Delta \kappa_\gamma \equiv \kappa_\gamma - 1, \quad \Delta \kappa_Z \equiv \kappa_Z - 1, \quad \lambda_\gamma \quad \text{and} \quad \lambda_Z.$$

in SM : $\Delta g_1^Z = 0, \Delta \kappa_\gamma = \Delta \kappa_Z = 0, \lambda_\gamma = \lambda_Z = 0$

λ_V grow as \hat{s} (= invariant diboson mass) --> enhanced sensitivity @ LHC

WW more sensitive to Δk_V (grows as \hat{s}) than **WZ** & **W γ** (grows as $\sqrt{\hat{s}}$)

WZ more sensitive to Δg_1^Z than **WW** --> complementarity

- To avoid unitarity violation @ high energy --> introduce a cutoff scale Λ
replacing $\alpha \rightarrow \frac{\alpha}{(1 + \hat{s}/\Lambda^2)^n}$ $n = 2$ ($\alpha \equiv \Delta g_1^Z, \Delta k_{\gamma/Z}, \lambda_{\gamma/Z}$)

- To extract AC :

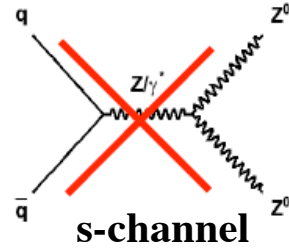
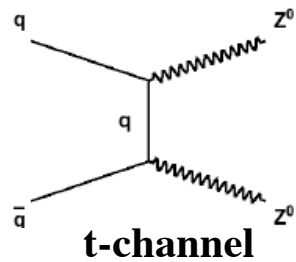
fits to total cross-sections and differential distributions (i.e. $E_T^\gamma, p_T^Z, M_T^{VV}$, sensitivity at high values)



Angular distribution have additional resolving power - not used here



Neutral TGC



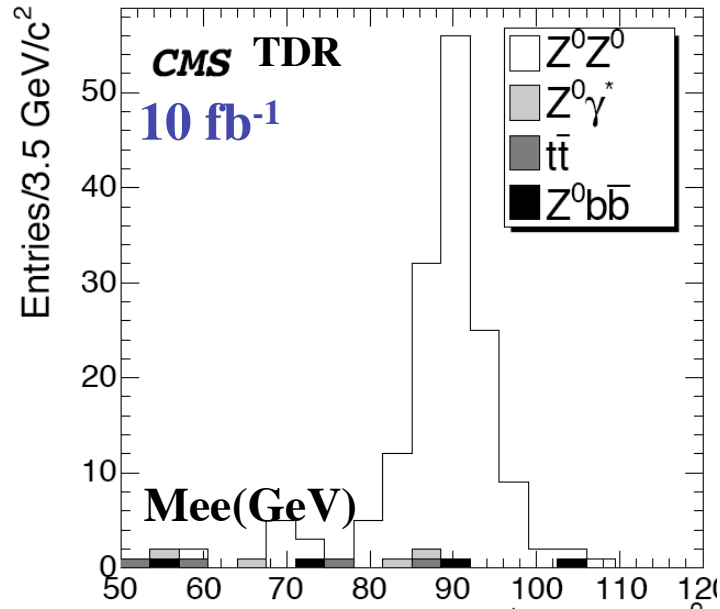
$$V = Z, \gamma$$

$$L = -\frac{e}{M_Z^2} [f_4^V (\partial_\mu V^{\mu\beta}) Z_\alpha (\partial^\alpha Z_\beta) + f_5^V (\partial^\sigma V_{\sigma\mu}) \tilde{Z}^{\mu\beta} Z_\beta],$$

• Unitarity :

$$f_i^V \rightarrow \frac{f_i^V(\hat{s})}{(1 + \hat{s}/\Lambda^2)^3} \quad i=4,5$$

ZZ : $\sigma^{\text{NLO}}(\text{SM}) \sim 20 \text{ pb}$ [t-channel, s-channel suppressed $O(10^{-4})$]



ZZ : clean signal: 4 isolated leptons

ATLAS (1 fb⁻¹)

$$N_{\text{sig}} = 17 \pm 0.5$$

$$N_{\text{bkg}} = 2 \pm 0.2$$

$$\epsilon = 7.7\%$$

$$S = 6.8$$



'irreducible' bkg for H->4l



Summary: dibosons @ LHC (ATLAS)



- @LHC with **0.1 fb⁻¹** and **20%** systematic uncertainties, SM signal of WW, WZ, W γ , Z γ established with significance S (=N/ \sqrt{B}) better than **5 σ** (**1 fb⁻¹** for ZZ)
- Systematics (lumi, ϵ_{lept} , PDF, factorization scale) will start to dominate the cross-sections uncertainties from **5-30 fb⁻¹**
- **95% CL limit on AC ($\Lambda=2$ TeV) 10 fb⁻¹** (~ 10 x better than present CDF 2 fb⁻¹)

Diboson,	λ_Z	$\Delta\kappa_Z$	Δg_1^Z	$\Delta\kappa_\gamma$	λ_γ
WZ, (M_T)	[-0.015, 0.013]		[-0.011, 0.034]		[-0.05, 0.02]
W γ , (p_T^γ)					
WW, (M_T)		[-0.035, 0.073]		[-0.088, 0.089]	
WW, (LEP)			[-0.051, 0.034]	[-0.105, 0.069]	[-0.059, 0.026]

- **95% CL limit on AC ($\Lambda=2$ TeV) 10 fb⁻¹**

$ZZ \rightarrow llll$	f_4^Z	f_5^Z	f_4^γ	f_5^γ
$ZZ \rightarrow ll\nu\nu$ +	[-0.009, 0.009]	[-0.009, 0.009]	[-0.010, 0.010]	[-0.011, 0.010]
LEP Limit	[-0.30, 0.30]	[-0.34, 0.38]	[-0.17, 0.19]	[-0.32, 0.36]

LHC improves wrt TeVatron and LEP



Conclusions



- LHC will be a W, Z, top factory. LHC goals:
 - $\Delta M_W < 10 \text{ MeV}$
 - $\Delta M_{\text{top}} < 1 \text{ GeV}$
 - $\Delta \sin^2 \theta_{\text{eff}}^{\text{lept}} \approx 10^{-4}$SM M_H constraint to $< 15\%$
- **EW dibosons** signals are expected to be established @ ATLAS & CMS with $\sim 100 \text{ pb}^{-1}$ to 1 fb^{-1}
- **Anomalous Gauge boson Couplings** improved with $\approx 10 \text{ fb}^{-1}$ data

Main issues: understand detector response, measure soft QCD

- ❖ Even after finding a Higgs signal, (precision)EW measurements important:
 - * A Higgs is not necessarily a SM Higgs --> indirect constraints will help interpretation
 - ❖ Ultimately understanding **systematics** will be our main concern. This will come from **data driven method** and especially **from the use of independent analysis methods**
- > LHC will play a major role in establishing a coherent picture of the (EW) theory

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Altarelli '2008

Fit results

Here only m_W and not m_t is used:
shows m_t from rad. corr.s

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only m_W 

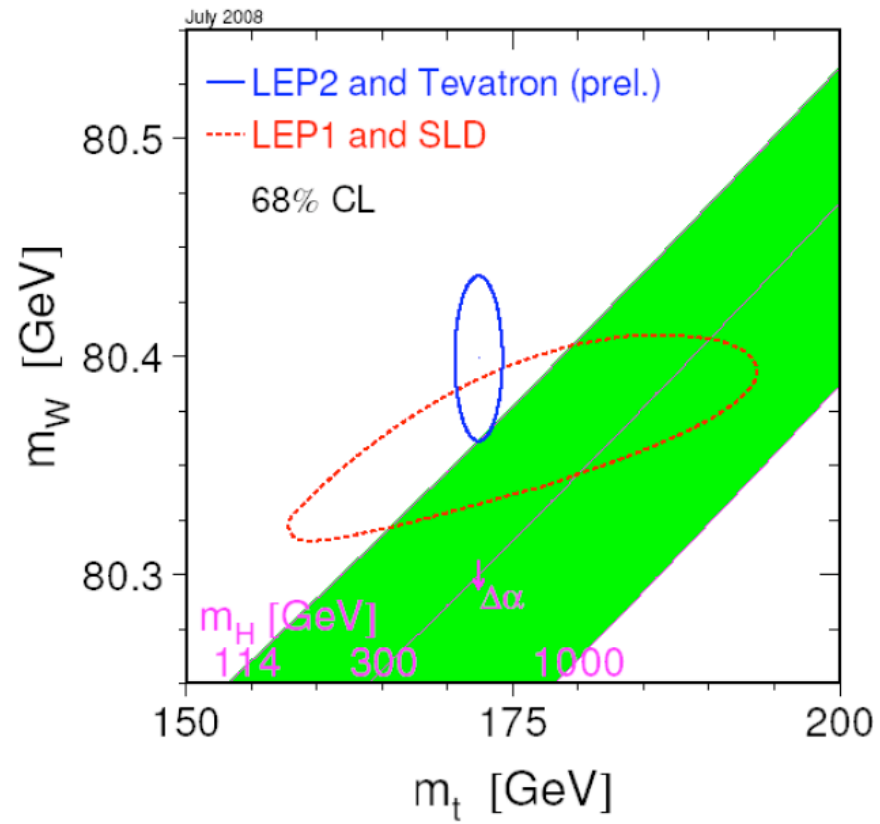
only m_t

m_W, m_t

$m_t(\text{GeV})$	178.7 ⁺¹² ₋₉	172.6 \pm 1.4	172.8 \pm 1.4
$m_H(\text{GeV})$	143 ⁺²³⁶ ₋₈₀	111 ⁺⁵⁶ ₋₃₉	87 ⁺³⁶ ₋₂₇
$\log[m_H(\text{GeV})]$	2.16 \pm 0.39	2.05 \pm 0.18	1.94 \pm 0.16
$\alpha_s(m_Z)$	0.1190(28)	0.1190 (27)	0.1185 (26)
χ^2/dof	16.8/12	16.0/11	17.2/13
$m_W(\text{MeV})$	80385(19)	80363(20)	80377(15)

WA: $m_W=80398(25)$

Rad. corr.'s predict m_t and m_W very well. May be also m_H !





Mass of W boson : longer term perspectives

10 fb⁻¹ : Systematics on the result of the method based on p_T^{lep} (1 experiment)

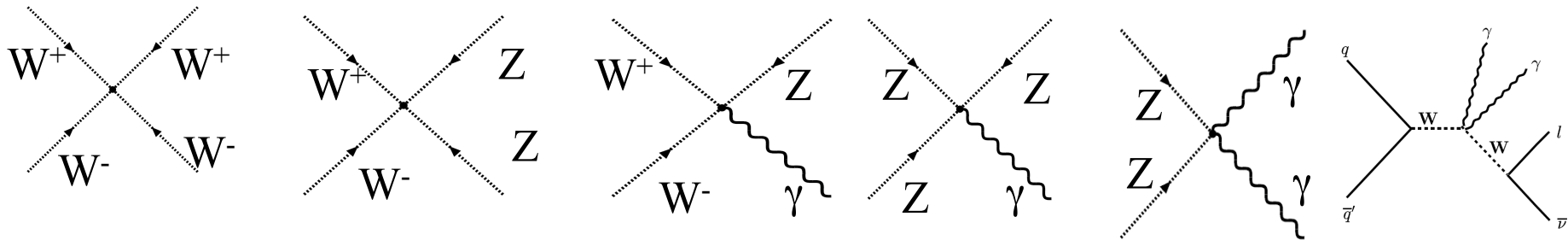
Source	effect	δm_W (MeV)
Theoretical model	Γ_W	0.5
	y_W	1
	p_{tW}	3
	QED radiation	<1
Lepton measurement	linearity and scale	4
	resolution	1
	efficiency	4.5 (e); <1 (μ)
Backgrounds	$W \rightarrow \tau \nu$	2.0
	$Z \rightarrow l (l)$	0.3
	$Z \rightarrow \tau \tau$	0.1
	jet events	0.5
Pile-up and UE		<1 (e); ~0 (μ)
Beam crossing angle		<0.1
total		~7(e); 6(μ)

N.Besson et al. : SN-ATLAS-2008-70

- For equal contribution to M_H (indirect) uncertainty :

$$\text{if } \Delta M_W < 10 \text{ MeV} \rightarrow \Delta M_{\text{top}} < 1.5 \text{ GeV}$$

- Signature : **three** bosons in the final state



- Small yields, not an early measurement :

SM: 100 fb⁻¹ in leptonic channels

(p_T > 20 GeV, |η| < 3) hep-ph/0003275

M_{Higgs} (GeV)	200	400	600	800
$W^+W^-W^-$	68	28	25	25
$W^+W^+W^-$	112	49	44	44
W^+W^-Z	32	17	15	15
W^-ZZ	1.0	0.51	0.46	0.45
W^+ZZ	1.7	0.88	0.79	0.79
ZZZ	0.62	0.18	0.13	0.12

--> limits on AQC probably difficult

- Useful cross-check:

if something new seen in the trilinears, one might need the quartics to sort things out.