The BESS model revisited as a Higgsless Linear Moose @ LHC

Stefania De Curtis

INFN and Dept. of Physics, Florence

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Outline of the talk

based on papers by: Casalbuoni, DC, Dolce, Dominici, Gatto

recent paper: Accomando, DC, Dominici, Fedeli, arXiv:0807.5051

- Motivations for Higgsless models
- Example of breaking the EW symmetry without Higgs (BESS)
- Linear moose: effective description for extra gauge bosons
- Unitarity bounds and EW constraints
- Direct couplings to fermions
- The four-site model, new vector and axial-vector resonances
- Drell-Yan processes @ the LHC

Problems of the Higgs sector

The evolution of the Higgs self-coupling (neglecting gauge fields and fermion contributions) shows up a Landau pole

$$\frac{1}{\lambda(M)} = \frac{1}{\lambda(m_H)} - \frac{3}{4\pi^2}\log\frac{M^2}{m_H^2} \qquad M_{Lp} = m_H e^{4\pi^2 v^2/3m_H^2}$$

- **or M_{Lp} pushed to infinity, but then λ goes to 0, triviality!**
- **or there is a physical cutoff at a scale $M < M_{Lp}$.**

If the cutoff is big ($M \sim M_{Planck}$, or M_{GUT}), λ is small. The theory is perturbative, but the Higgs mass acquires big radiative corrections:

naturalness problem - to avoid it the quadratic divergence should cancel (SUSY)

$$\delta m_H^2 = \frac{\lambda}{8\pi^2} M^2$$

If we keep the cutoff ~ 1 TeV, λ is large, m_H is O(TeV). The theory is non perturbative

1) $\lambda << 1 \Rightarrow$ new particles lighter than 1 TeV

2) $\lambda >> 1 \Rightarrow$ new particles around 1 TeV

In the following: <u>NEW STRONG PHYSICS at the TeV SCALE and NO HIGGS</u>

Symmetry Breaking without the Higgs

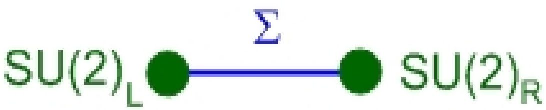
- A strongly interacting theory can only rely on an **effective description.** For the SB sector use a general σ model of the type G/H

- For $SU(2)_L x SU(2)_R / SU(2)_V$ the σ model can be obtained as the formal limit M_H to infinity of the SM and is described in terms of a field Σ in SU(2)

$$\Sigma \rightarrow g_L \Sigma g_R^{\dagger}, \quad g_L \in SU(2)_L, \quad g_R \in SU(2)_R$$

- The strong dynamics is completely characterized by the transformation properties of the field Σ summarized in the **moose diagram**

$$L = \frac{v^2}{4}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right), \quad \Sigma = e^{i\vec{\pi}\cdot\vec{\tau}/v}$$



- The breaking is produced by $\langle \Sigma \rangle = 1$

- Introduce covariant derivatives to **gauge the $SU(2)_L xU(1)_Y$**

$$D_\mu \Sigma = \partial_\mu \Sigma + igW_\mu \Sigma - ig' \Sigma Y_\mu$$

The interactions with W and Y are to be considered as **perturbations with respect to the strong dynamics** described by the σ model

- Due to **unitarity violation**, the validity of this description is up to

$$|a_0| = \frac{1}{16\pi}\frac{s}{v^2} \leq 1 \Rightarrow E \leq 4\sqrt{\pi}v \approx 1.7\ \text{TeV}$$

The BESS model

The simplest enlargement of the non-linear model is the BESS (Breaking Electroweak Symmetry Strongly) model (Casalbuoni, DC, Dominici ,Gatto, 1985) based on $SU(2)_L x SU(2)_R/SU(2)$ with an additional local group G_1=SU(2)

New vector resonances as the gauge fields of G_1

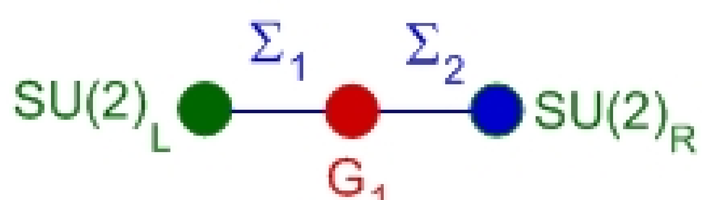
$$\boxed{L = f_1^2 \mathrm{Tr}\left[D_\mu \Sigma_1^\dagger D^\mu \Sigma_1\right] + f_2^2 \mathrm{Tr}\left[D_\mu \Sigma_2^\dagger D^\mu \Sigma_2\right] - \frac{1}{2}\mathrm{Tr}\left[F_{\mu\nu}(V) F^{\mu\nu}(V)\right]}$$

$$(D_\mu \Sigma_1 = \partial_\mu \Sigma_1 + i g_1 \Sigma_1 V_\mu, \quad D_\mu \Sigma_2 = \partial_\mu \Sigma_2 - i g_1 V_\mu \Sigma_2)$$

This model describes 6 scalar fields and 3 gauge bosons.

After the breaking $SU(2)_L x SU(2)_R x SU(2)_{local} \rightarrow$ SU(2) , we get 3 Goldstone bosons (necessary to give mass to W and Z after gauging the EW group) and 3 massive vector bosons with mass

$$M_V^2 = (f_1^2 + f_2^2) g_1^2$$ **(g_1=gauge coupling of V)**



Linear Moose model

(Son,Stephanov; Foadi et al; Casalbuoni et al; Chivukula et al; Georgi; Hirn,Stern)

- Generalize the moose construction: many copies of the gauge group G intertwined by link variables Σ
- Simplest example: G_i = SU(2). Each Σ_i describes 3 scalar fields.

- The model has two global symmetries related to the beginning and to the end of the moose, $G_L = SU(2)_L$ and $G_R = SU(2)_R$ which can be gauged to the standard $SU(2)_L x U(1)_Y$
- Particle content: 3 massive gauge bosons, W and Z, the massless photon and 3K massive vectors. $SU(2)_{diag}$ is a custodial symmetry
- The BESS model can be recast in a 3-site model (K=1), and its generalization (Casalbuoni, DC, Dominici, Gatto, Feruglio, 1989) can be recast in a 4-site model (K=2) (see also Foadi,Frandsen,Ryttov,Sannino, 2007)

The continuum limit

- The moose picture for large values of K can be interpreted as the discretization of a continuum gauge theory in 5D along a fifth dimension. The continuum limit is defined by

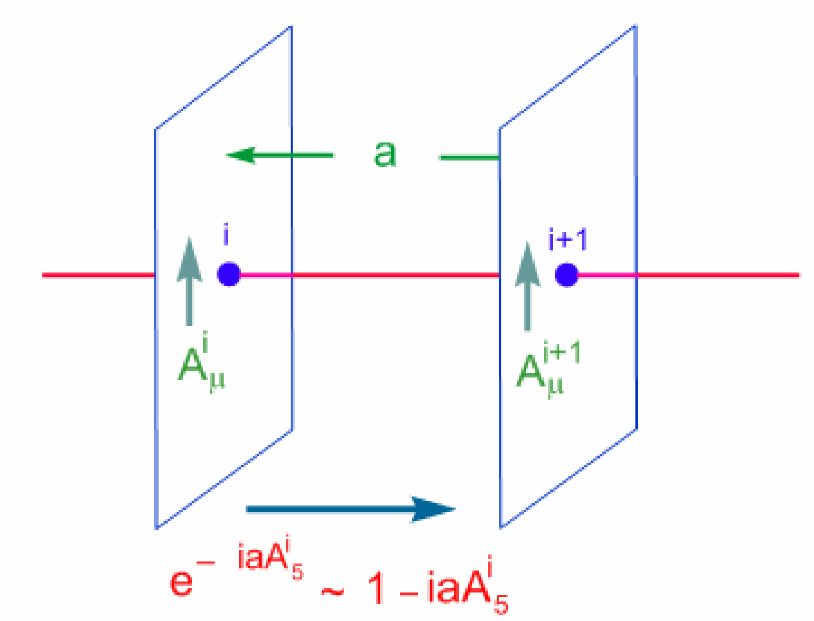
$$K \to \infty, \quad a \to 0, \quad Ka \to \pi R$$

$$\lim_{a\to 0} a g_i^2 = g_5^2, \quad \lim_{a\to 0} a f_i^2 = f^2(z)$$

a = lattice spacing, R= compactification radius, g_5= bulk gauge coupling

- The link couplings f_i and the gauge couplings g_i can be simulated in the continuum by non-flat 5-dim metrics.
- Flat metric corresponds to equal f's and g's
- In the continuum limit, the structure of the moose has an interpretation in terms of a **geometrical Higgs mechanism** in a pure 5D gauge theory

- **A gauge field is a connection: a way of relating the phases of the fields at nearby points. After discretizing the 5th dim, the field A_5 is naturally substituted by a link variable Σ realizing the parallel transport between two lattice sites (A_μ^i = KK modes)**



$$\Sigma_i \approx 1 - iaA^i{}_5 \approx e^{-iaA^i{}_5}$$

$$\Sigma\Sigma^\dagger = 1$$

$$D_\mu \Sigma_i = -iaF^{i-1}_{\mu 5}$$

$$F^i_{\mu 5} = \partial_\mu A^i_5 - \partial_5 A^i_\mu - i[A^i_\mu, A^i_5]$$

- **The action for the deconstructed gauge theory is (Hill, Pokorski, Wang, 2001)**

$$S = \int d^4x \frac{a}{g_5^2}\left(-\frac{1}{2}\sum_i \mathrm{Tr}\left[F^i_{\mu\nu}F^{\mu\nu i}\right] + \frac{1}{a^2}\mathrm{Tr}\left[(D_\mu\Sigma_i)(D_\mu\Sigma_i)^\dagger\right]\right), \quad A^i_\mu = \mathrm{KK\ modes}$$

- **Sintetically described by a moose diagram (Georgi, 1986 –Arkani-Hamed, Cohen, Georgi, 2001)**

Unitarity bounds for the Linear Moose

(Chivukula, He; Muck, Nilse, Pilaftis, Ruckl; Csaki, Grojean, Murayama, Pilo, Terning)

- Spin-one resonances generally delay the perturbative unitarity bound

- The worst high-energy behavior comes from the scattering of longitudinal vector bosons. For $s >> M_W^2$ these amplitudes can be evaluated using the equivalence theorem. Introduce the GB's, $\left(\Sigma_i = e^{i\vec{\pi}_i\cdot\vec{\tau}/2f_i}\right)$, in the high-energy limit

$$A_{\pi_i^+\pi_i^-\to\pi_i^+\pi_i^-} \to -\frac{u}{4f_i^2}$$

- The unitarity limit is determined by the smallest link coupling

by taking $f_i = f_c$: $A \to -\dfrac{u}{(K+1)v^2}$

$$\boxed{\Lambda_{\mathrm{moose}} = (K+1)^{1/2}\Lambda_{\mathrm{HSM}} \approx 1.7(K+1)^{1/2}\,\mathrm{TeV}}$$

$$M_V^{\max} < \Lambda_{\mathrm{moose}}, \qquad M_V^{\max} \approx 2\sqrt{K+1}\,\frac{g_c}{g}M_W$$

$$\Downarrow$$

$$2\sqrt{K+1}\,\frac{g_c}{g}M_W < 1.7\sqrt{K+1}\,\mathrm{TeV} \quad\Rightarrow\quad \frac{g_c}{g} < 10$$

too big EW corrections

Constraints from EW data

- Assuming universality among different generations, the EW corrections are coded in 3 parameters ε_i, i=1,2,3 (Altarelli, Barbieri, 1991), or S,T,U (Peskin, Takeuchi, 1990).

- To the lowest order the new physics contribution to ε_1 and ε_2 vanishes due to the SU(2) custodial symmetry of the SB sector. At the same order ε_3 has a dispersive representation (for oblique corrections). Neglecting loop corrections (for loop see Dawson et al, Chivukula et al, Barbieri et al):

$$\varepsilon_3 = \frac{g^2}{4}\sum_{\mathbf{i}}\left(\frac{g_{iV}^2}{m_{\mathbf{i}}^4} - \frac{g_{iA}^2}{m_{\mathbf{i}}^4}\right) = g^2\sum_{i=1}^{K}\frac{(1-y_i)y_i}{g_i^2} \qquad \left(y_i = \sum_{j=1}^{i}\frac{f^2}{f_{\mathbf{j}}^2},\quad \frac{1}{f^2} = \sum_{i=1}^{K+1}\frac{1}{f_i^2}\right)$$

- Since $\boxed{0 \leq y_i \leq 1 \quad \Rightarrow \quad \varepsilon_3 \geq 0}$

- Example: $f_i = f_c, \quad g_i = g_c \quad \Rightarrow \quad \varepsilon_3 = \frac{1}{6}\frac{g^2}{g_c^2}\frac{K(K+2)}{K+1}$

- $\epsilon_3^{\exp} \sim 10^{-3}$, for K=1, g_c~(16 g)~10, for large K, g_c~$10\sqrt{K}$ ⟶ strongly interacting gauge bosons, UNITARITY VIOLATION

Direct fermionic couplings

(Csaki et al, Foadi et al, Casalbuoni et al, Chivukula et al)

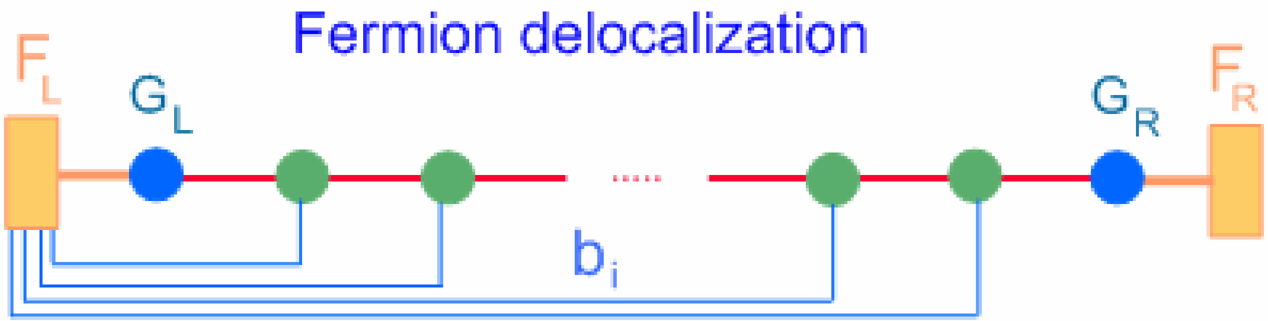
- Left- and right-handed fermions, $\psi_{L\,(R)}$ are coupled to the ends of the moose, but they can couple to any site by using a Wilson line

$$\chi_L^i = \Sigma_i^\dagger \Sigma_{i-1}^\dagger \cdots \Sigma_1^\dagger \psi_L, \qquad \chi_L^i \to U_i \chi_L^i$$

$$\Downarrow$$

$$b_i \overline{\chi}_L^i \gamma^\mu \left(\partial_\mu + i g_i V_\mu^i + \frac{i}{2} g'(B-L) Y_\mu \right) \chi_L^i$$

no delocalization of the right-handed fermions.

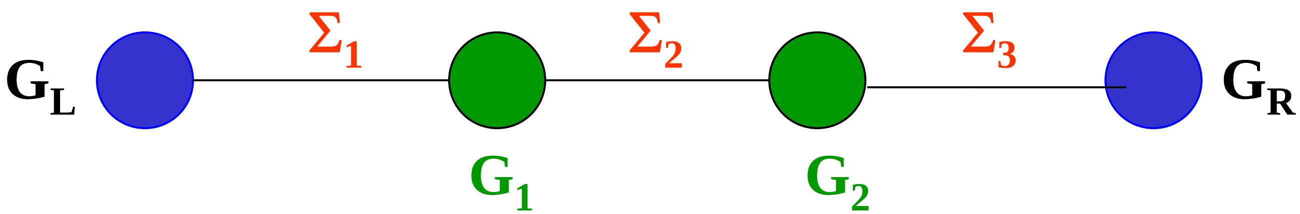


Small terms since they could contribute to right-handed currents constrained by the K_L-K_S mass difference

The Higgsless 4-site Linear Moose model

(Accomando, DC, Dominici,Fedeli)

- 2 extra gauge groups G_i=SU(2) with global symmetry $SU(2)_L \otimes SU(2)_R$ plus LR symmetry: $g_2=g_1$, $f_3=f_1$ (specific choice of BESS with vector and axial vector resonances);
- 6 extra gauge bosons $W`_{1,2}$ and $Z`_{1,2}$ (have definite parity when g=g`=0)



(K=2)

- 5 new parameters {f_1, f_2, b_1, b_2, g_1} related to their masses and couplings to bosons and fermions (one is fixed to reproduce M_Z)

$$f_1, f_2 \rightarrow M_1, M_2$$

$$M_1 = f_1 g_1$$

$$M_2 = \frac{M_1}{z} > M_1$$

$$z = \frac{f_1}{\sqrt{f_1^2 + 2f_2^2}} < 1$$

charged and neutral extra gauge bosons almost degenerate

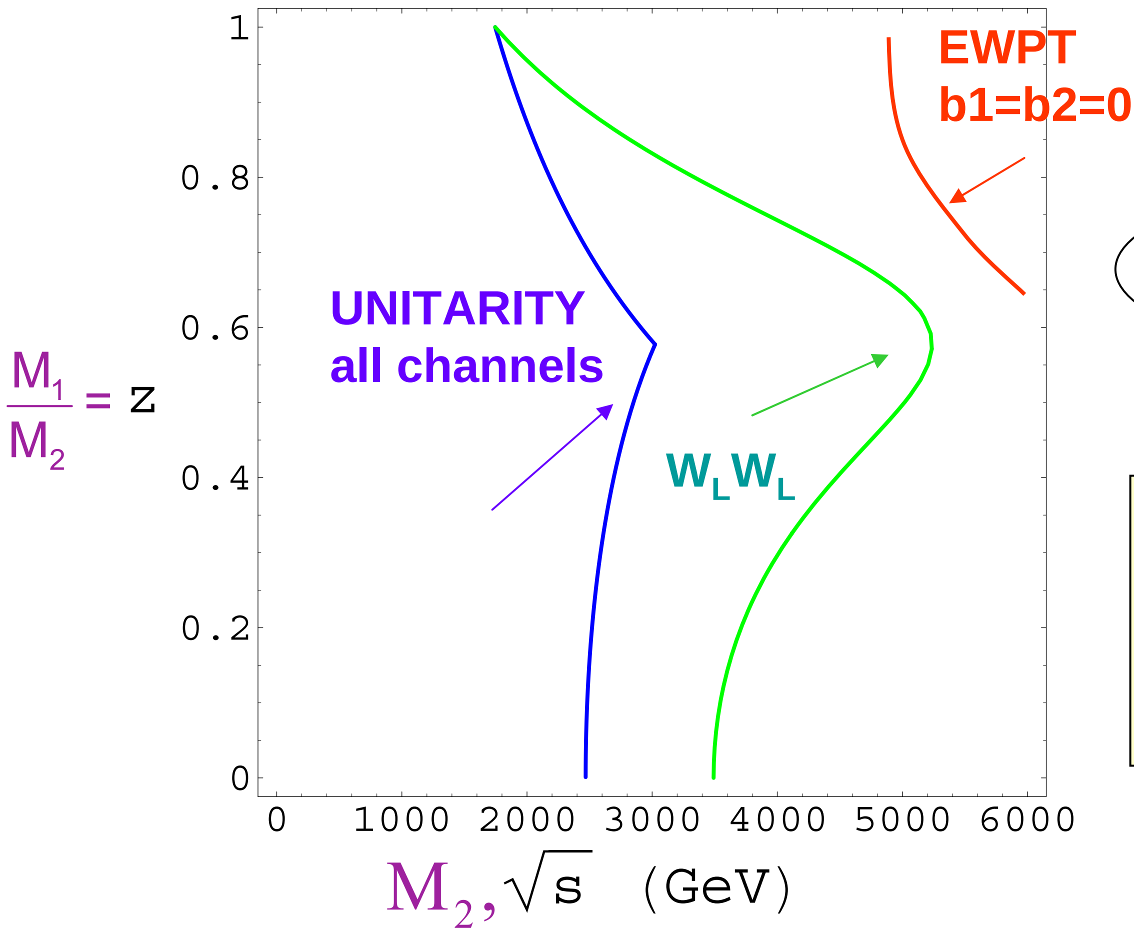
$$M_{1,2}^{c,n} \sim M_{1,2} + O\left(\frac{e^2}{g_1^2}\right)$$

The Higgsless 4-site Linear Moose model

Unitarity and EW precision tests

$$\varepsilon_1 \approx 0 \quad \varepsilon_2 \approx 0, \quad \varepsilon_3 \approx \left(\frac{g^2}{2g_1^2}(1 - z^4) \right)$$

$O(e^2/g_1^2)$, $b_1=b_2=0$



Best unitarity limit for $f_1=f_2$ or $z=1/\sqrt{3}$

Unitarity and EWPT are hardly compatible !

A direct coupling of the new gauge bosons to ordinary matter must be included: $b_{1,2} \neq 0$

The Higgsless 4-site Linear Moose model

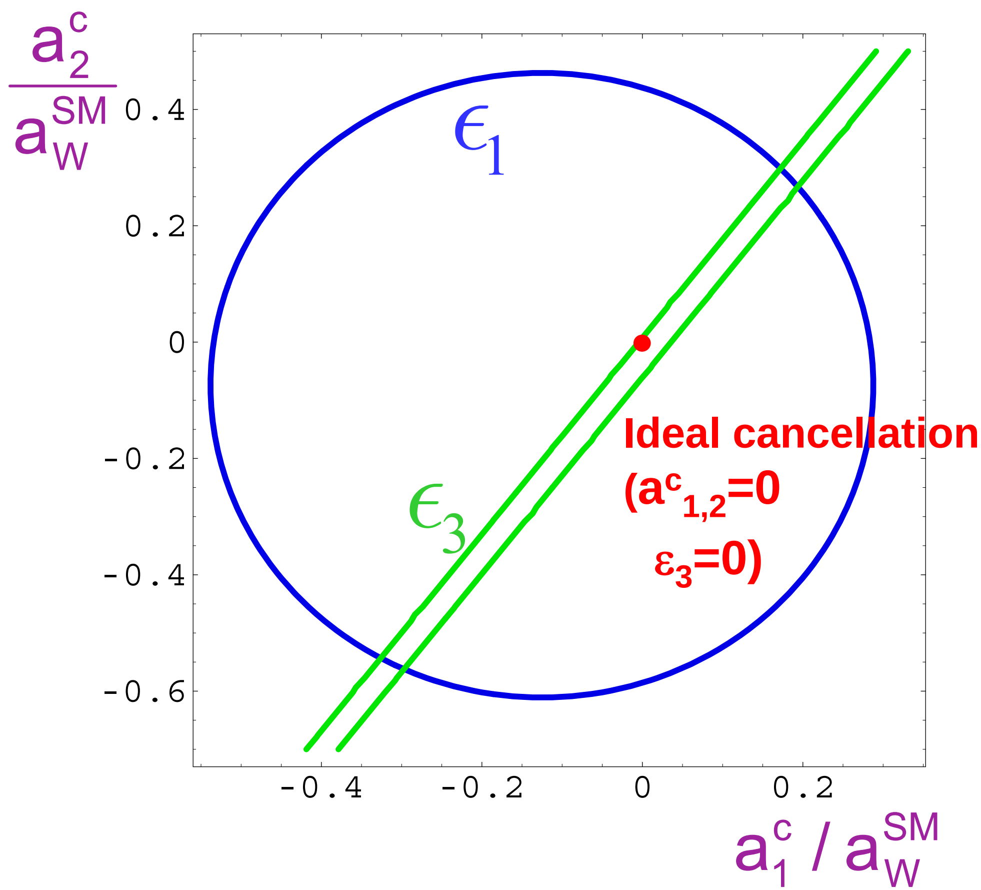
EW precision tests

Calculations $O(e^2/g_1^2)$, exact in b1, b2

$$\varepsilon_{1,2} \approx O(b^2), \quad \varepsilon_3 \approx \left(\frac{g^2}{2g_1^2}(1 - z^4) - \frac{b}{2} \right)$$

$$b = \frac{b_1 + b_2 - (b_1 - b_2) z^2}{1 + b_1 + b_2}$$

M_1=1000 GeV and M_2=1300 GeV



Bounds on <u>charged couplings</u> (and masses) from low energy precision measurements ε_i

$$\epsilon_3 \sim \frac{a_1^c}{g_1} - z^2 \frac{a_2^c}{g_1}$$

ε_3 bounds favour $a^c_2 > a^c_1$

$-0.1 < a^c_{1,2}(W`_{1,2}\, ff) < 0.25$

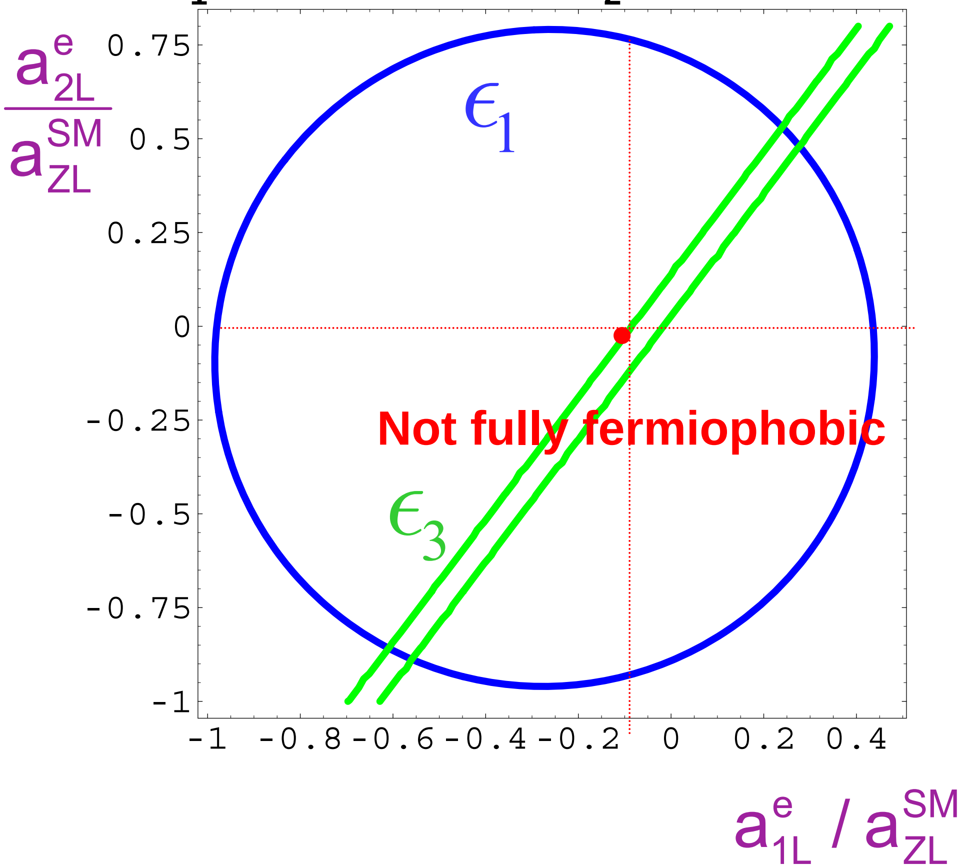
for larger $M_{1,2}$ the bounds from ε_1 are less stringent

The Higgsless 4-site Linear Moose model

EW precision tests

$$\epsilon_3 \sim \sqrt{2}\left(\frac{a^{(e)}_{1L}}{g_1} - z^2\frac{a^{(e)}_{2L}}{g_1}\right) - \frac{e^2}{g_1^2}\frac{(1+z^4)}{\cos^2\theta_W}$$

M_1=1000 GeV and M_2=1300 GeV



Bounds on <u>neutral couplings</u> (and masses) from low energy precision measurements ε_i

$$-0.15 < a^L_{1,2}(Z`_{1,2}\ ee) < 0.1$$

Ideal cancellation $a^c_2 = a^c_1 = 0$
BUT not fully fermiophobic

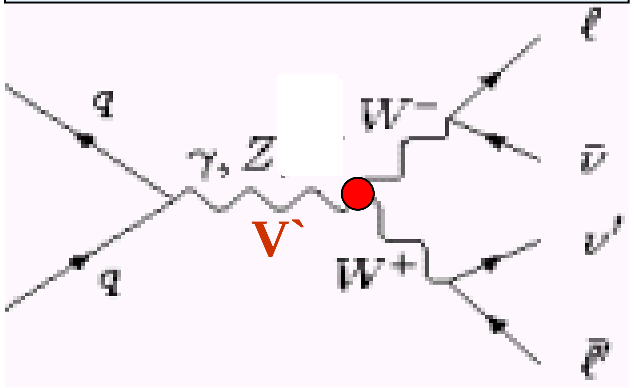
$$(a^{c,n}_{1,2} \sim \alpha^{c,n}_{1,2}\, g_1\left(\frac{b_1 \pm b_2}{2}\right) + \beta^{c,n}_{1,2}\frac{e^2}{g_1^2})$$

New spin-1 resonances @ the LHC

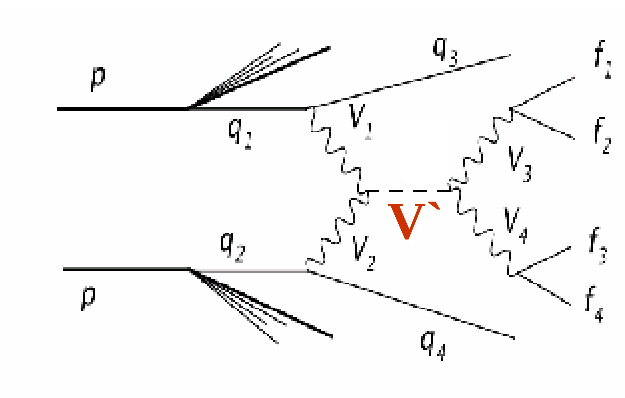
where do we get clues?

Drell-Yan

Di-boson production



Vector boson scattering

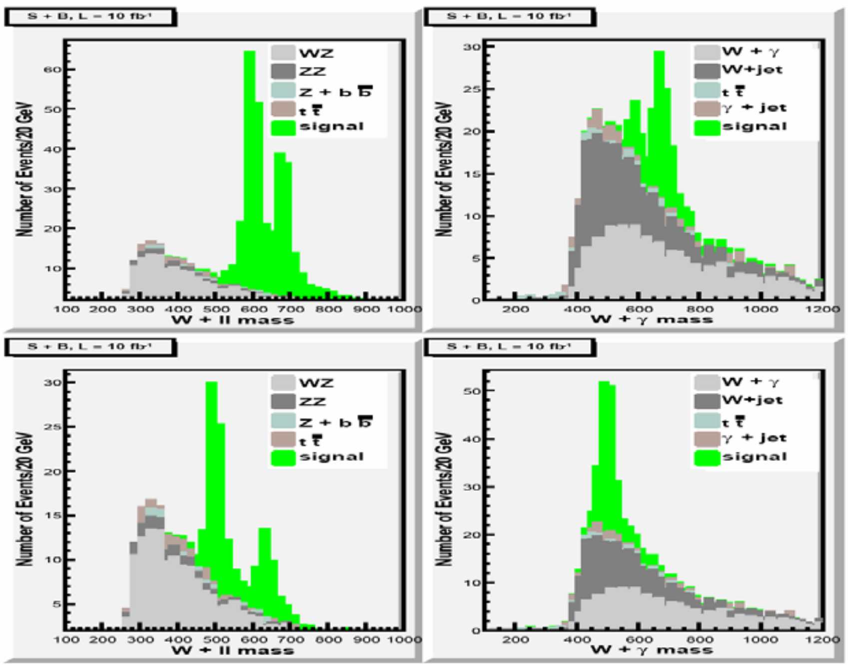


.... triple boson production, and ..... even more complicated processes where (extra) gauge bosons can be produced

Owing to the tension between unitarity and EW precision tests, the extra gauge-boson couplings to SM matter must be small

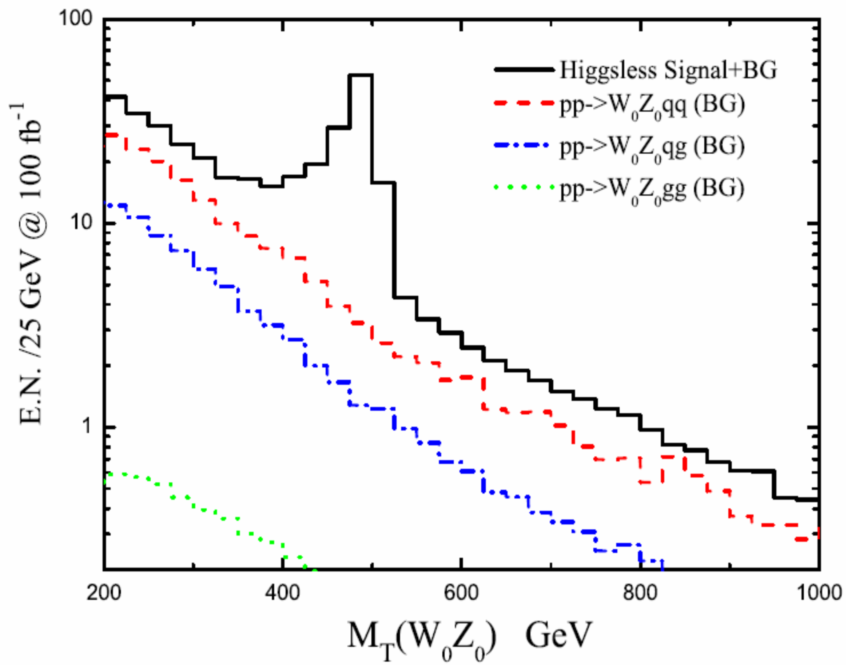
In literature main focus is on complex processes

(Hirn, Martin, Sanz `07)



WZ, Wγ di-boson production

(Belyaev,Chivukula, et al. `08)



Vector boson scattering

For the Higgsless 4-site Linear Moose Model DY processes can be as well a good discovery channel

Event Generator FAST_2f

(Accomando)

FAST_2f is an upgrade of PHASE [Accomando, Ballestrero, Maina], **a MCEG for multi-particle processes at the LHC. It is dedicated to Drell-Yan processes at the Leading-Order and interfaced with PYTHIA**

Processes

We consider charged and neutral Drell-Yan leptonic channels

- **pp $\rightarrow$ *ll* with *l*=e,μ**
- **pp $\rightarrow$ $l\nu$ with *l*=e,μ and $l\nu = l^-\nu + l^+\nu$**

CTEQ6L PDF

Kinematical cuts

Acceptance cuts:

$\eta(l)<2.5$, $P_t(l)>20$ GeV, $P_t^{miss}>20$ GeV

Selection cuts:

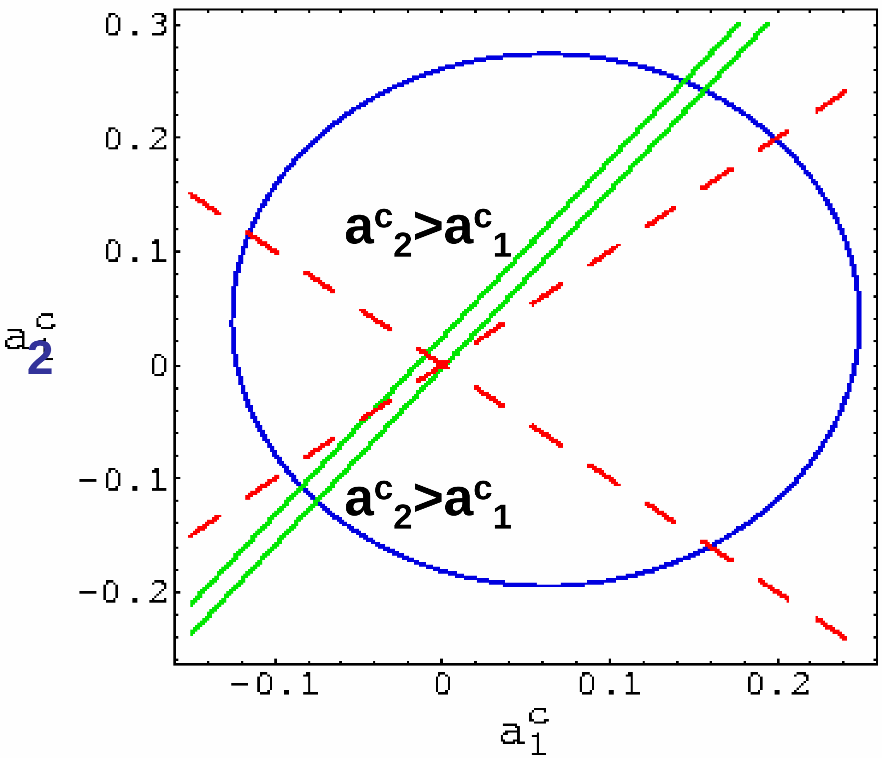
$M_{inv}(ll) > 500$ GeV for pp $\rightarrow$ *ll*

$P_t(l)>250$ GeV for pp $\rightarrow$ $l\nu$

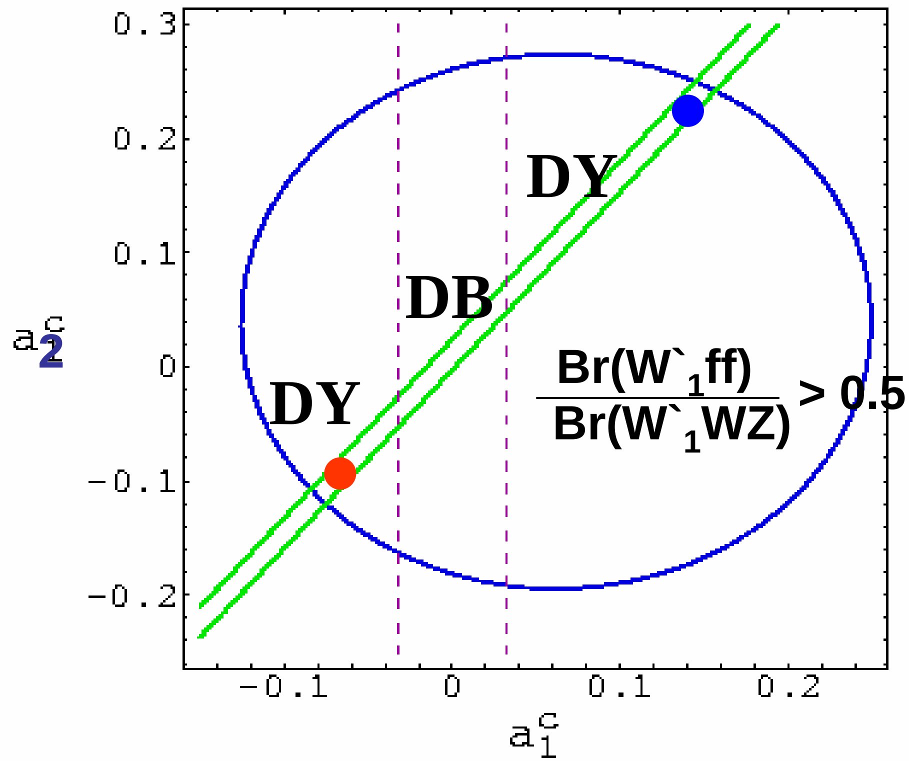
no detector simulation is included

The Higgsless 4-site Linear Moose model

Fermionic coupling features



Resonance hierarchy



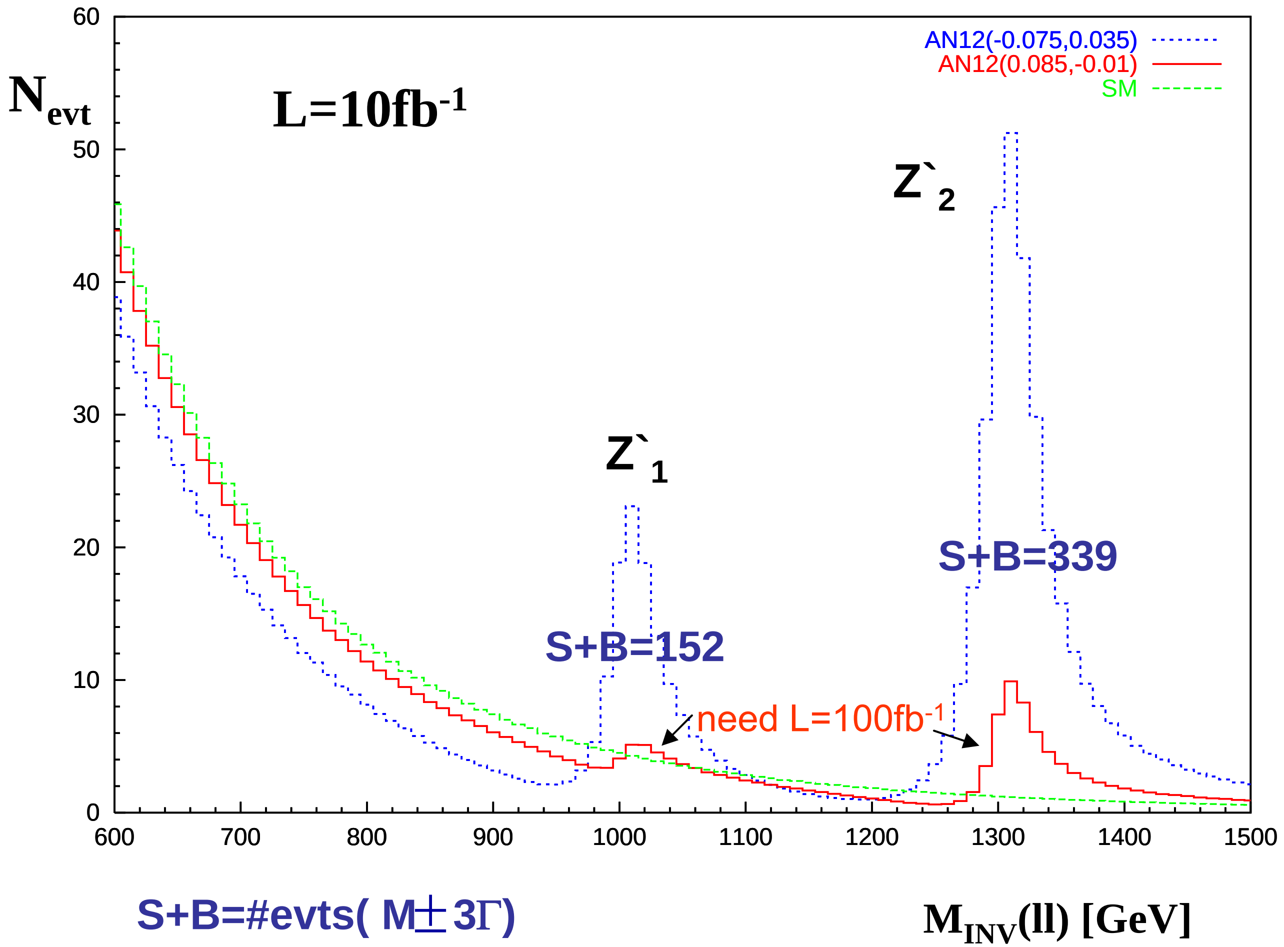
Drell-Yan vs Di-Boson

The Higgsless 4-site Linear Moose model

Drell-Yan processes

Z`$_1$ and Z`$_2$ production in the neutral channel

$$pp \to l^+ l^- \quad (l = e, \mu)$$



g_1= 3.7

(M_1,M_2)=(1000,1300)GeV

b_1 = (-0.075) (0.085)

b_2 = (0.035) (-0.01)

$M_{Z`1}$ =1010.2 GeV

$M_{Z`2}$ =1304.8 GeV

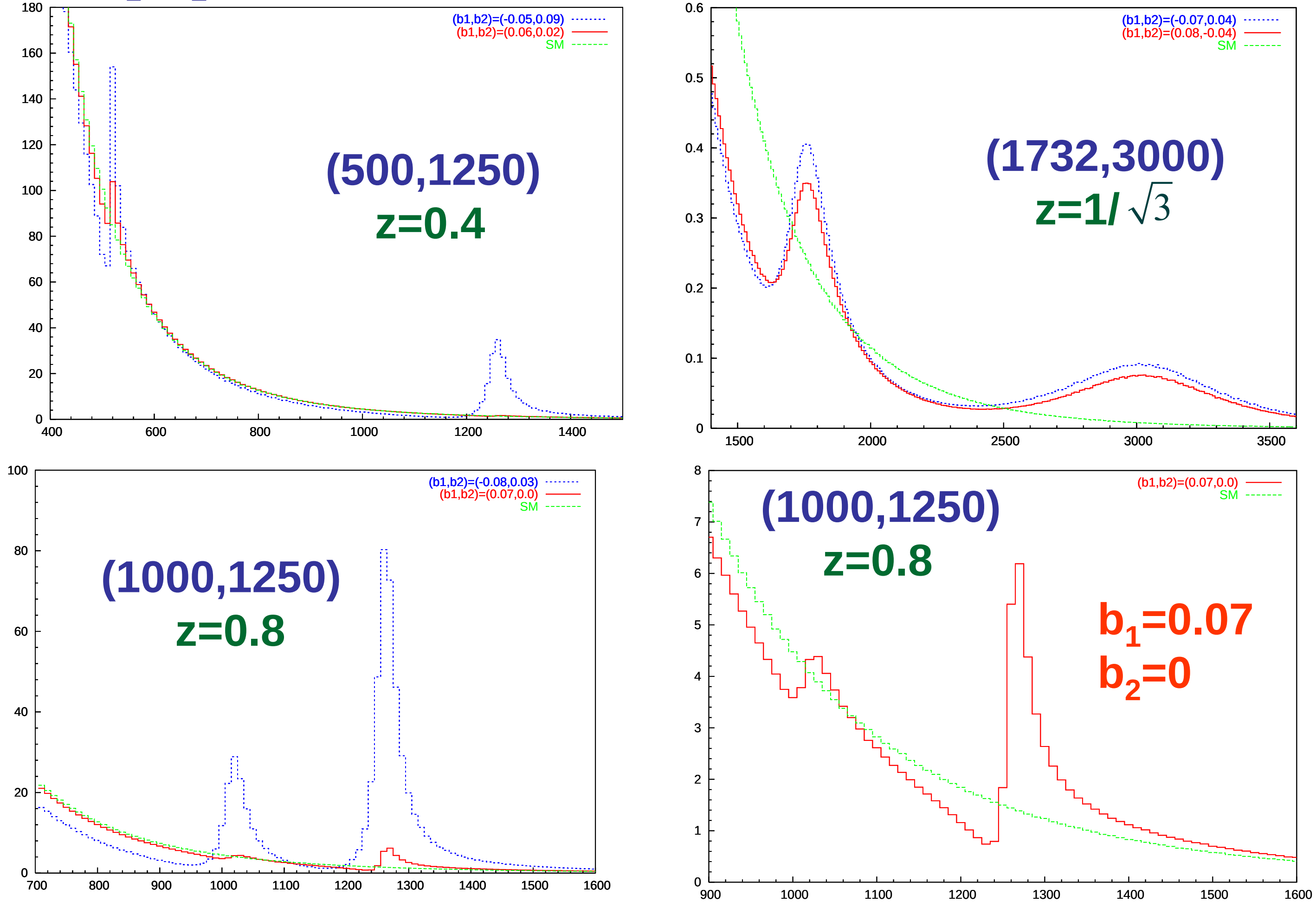
$\Gamma_{Z'1}$ = (37.6) (35.1) GeV

$\Gamma_{Z'2}$ = (44.6) (35.0) GeV

S+B=#evts(M$\pm$3Γ)

Z'_1, Z'_2 production

(M_1, M_2) GeV



Total # of evts in a 10GeV-bin versus $M_{inv}(l+l-)$ for $L=10fb^{-1}$. Sum over e,μ

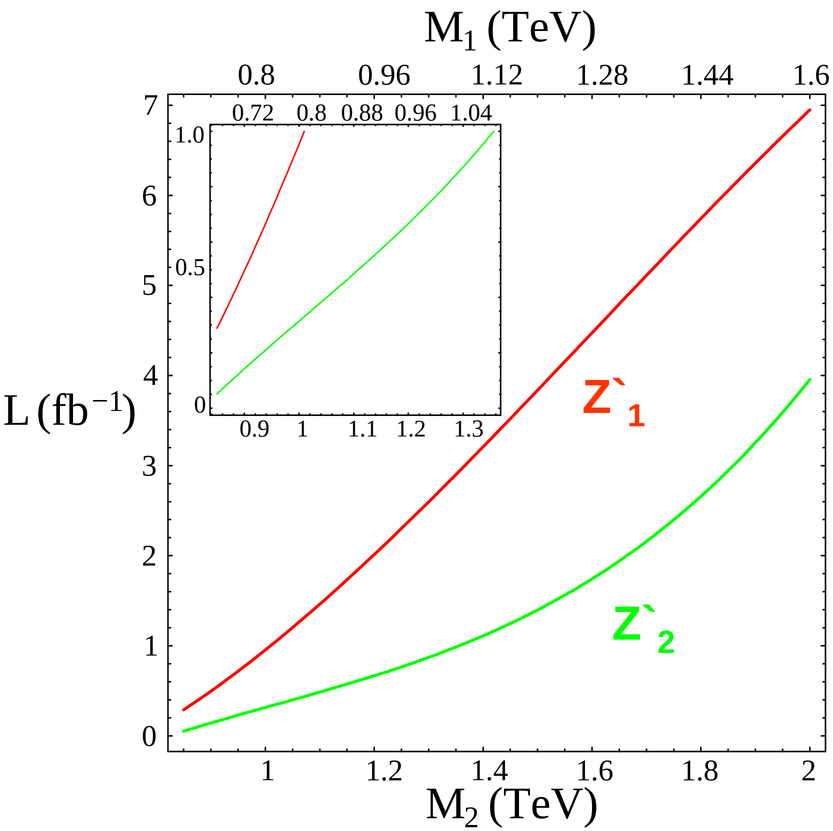
Z'_1, Z'_2 production

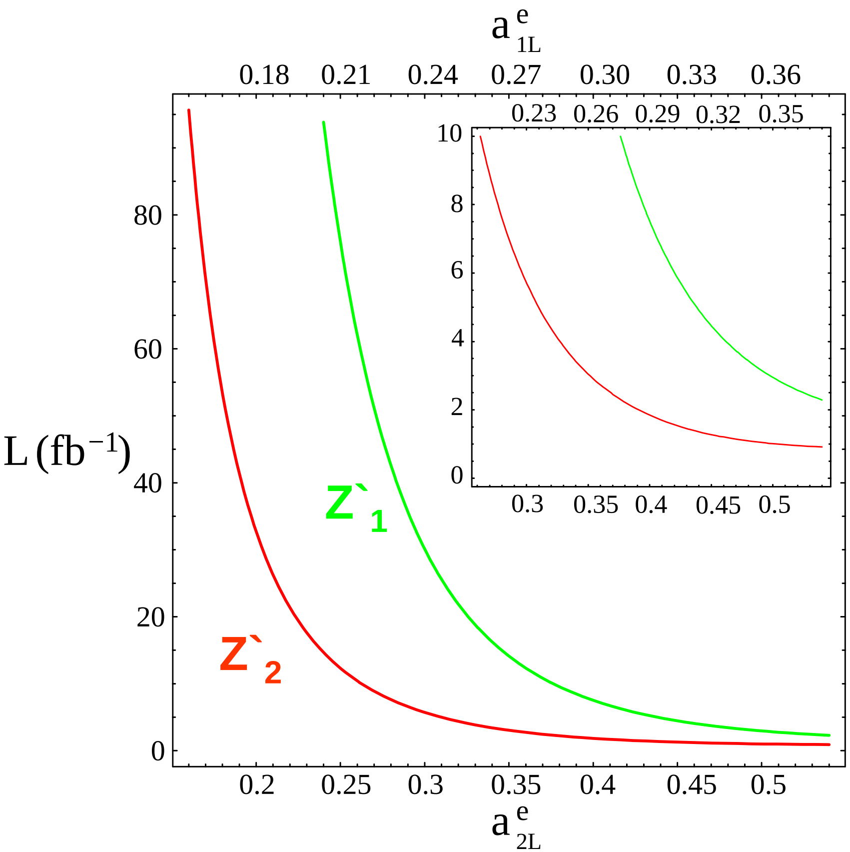
	$M_{1,2}$(GeV)	$b_{1,2}$	$N^{sig}_{evt}(Z_1)$	$N^{tot}_{evt}(Z_1)$	$\sigma(Z_1)$	$N^{sig}_{evt}(Z_2)$	$N^{tot}_{evt}(Z_2)$	$\sigma(Z_2)$
1	500,1250	-0.05,0.09	47	154	3.8	134	143	11.2
2	500,1250	0.06,0.02	11	123	1.0	0	9	0.0
3	1732,3000	-0.07,0.04	7	10	2.2	7	8	2.5
4	1732,3000	0.08,-0.04	5	9	1.7	6	6	2.4
5	1000,1250	-0.08,0.03	108	119	9.9	291	302	16.7
6	1000,1250	0.07,0.0	3	28	0.0	15	22	3.2

# of evts for the Z`$_{1,2}$ DY production within $|M_{inv}(l^+l^-)-M_i| < \Gamma_i$

$\sigma = N^{sig}_{evt}/\sqrt{N^{tot}_{evt}}$ for an integrated luminosity L=10 fb^{-1}

Discovery @ LHC in the early stage low-luminosity run



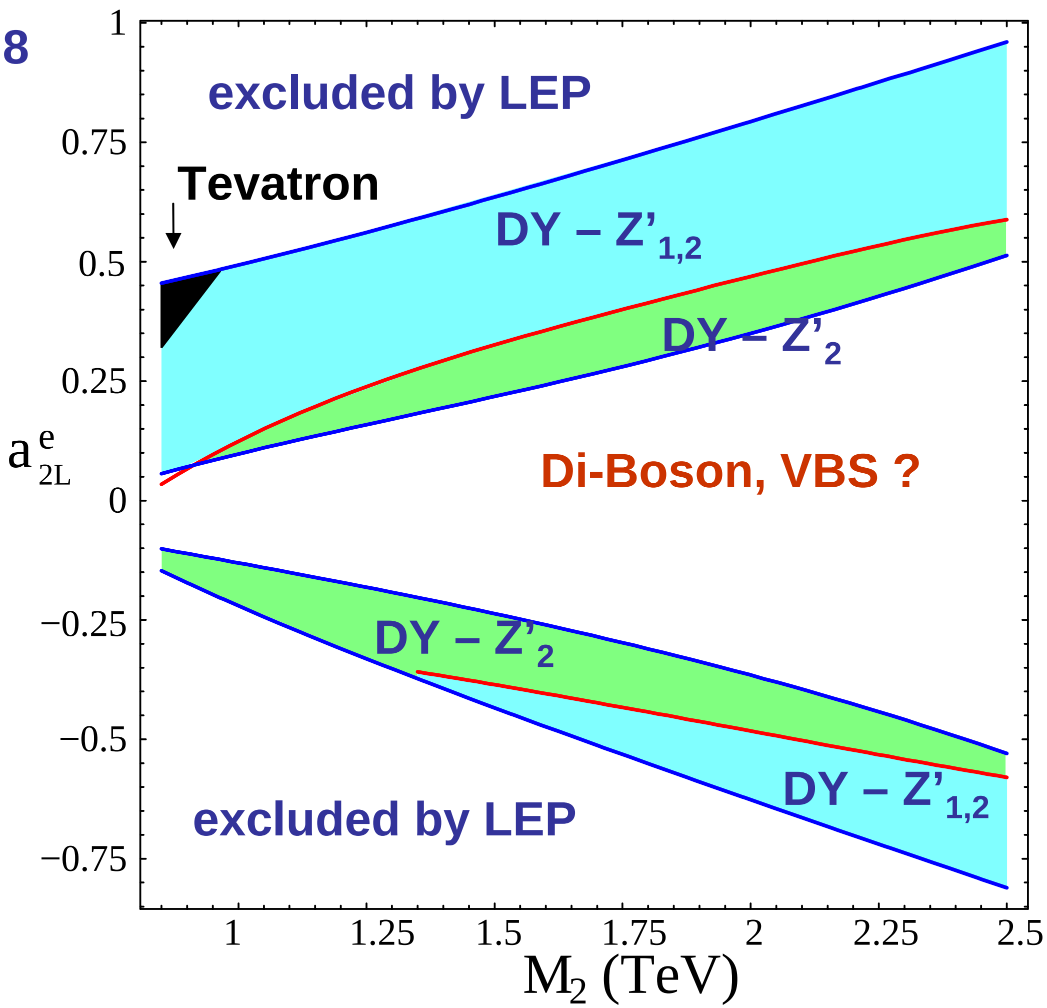


Luminosity needed for a 5σ-discovery for the maximum coupling allowed by EWPT (z=0.8)

Luminosity needed for a 5σ-discovery versus the electron-boson left handed coupling (z=0.8, M_1=1TeV, M_2=1.25TeV)

Discovery @ LHC
DY-processes in the neutral channel, Z'_1, Z'_2 exchange

Z=0.8



$L=100fb^{-1}$
acceptance cuts:
$\eta(l)<2.5$, $Pt(l)>20$ GeV

$$\frac{S}{\sqrt{S+B}} > 5$$

within $|M_{inv}(l+l-)-M_i| < \Gamma_i$ (i=1,2)

(in the coupling the electric charge –e is factorized)

Tevatron: direct limit from neutral DY leptonic channels for $L=4fb^{-1}$

$$p\bar{p} \to l^+l^- \quad (l = e, \mu)$$

Bounds from LEP2 not effective

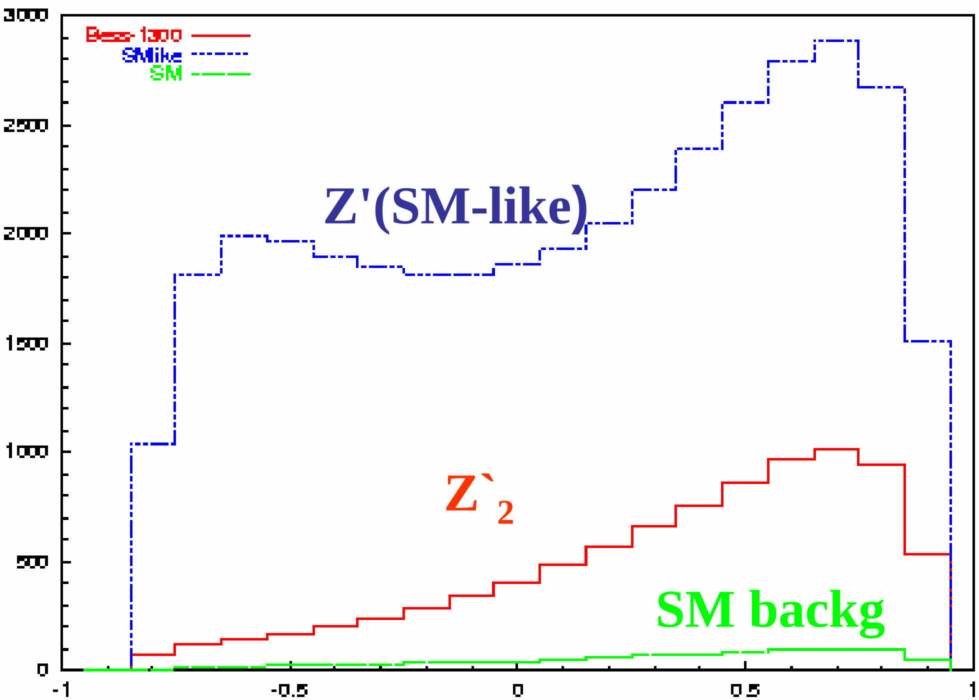
How to distinguish the various models?

Forward-backward asymmetry A_{FB} in $pp \rightarrow l^+ l^-$

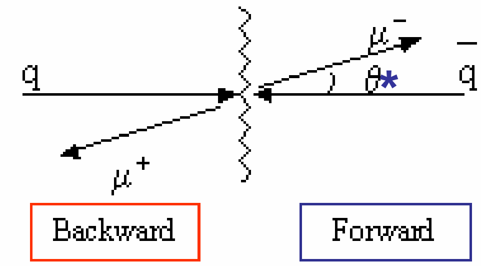
$$\frac{d\sigma}{d\cos\theta^*} \propto \frac{3}{8}(1+\cos^2\theta^*) + A^{\ell}_{FB}\cos\theta^*$$

L=100 fb^{-1}

$d\sigma\ L/d\cos\theta^*(l^-)$ ($l=e,\mu$)



$\cos\theta^*(l^-)$



θ* is the angle of the l^- with the incoming quark in the dilepton frame (Collins-Soper)

We assume the direction on the z-axis of the dilepton system to give the direction of the incoming quark

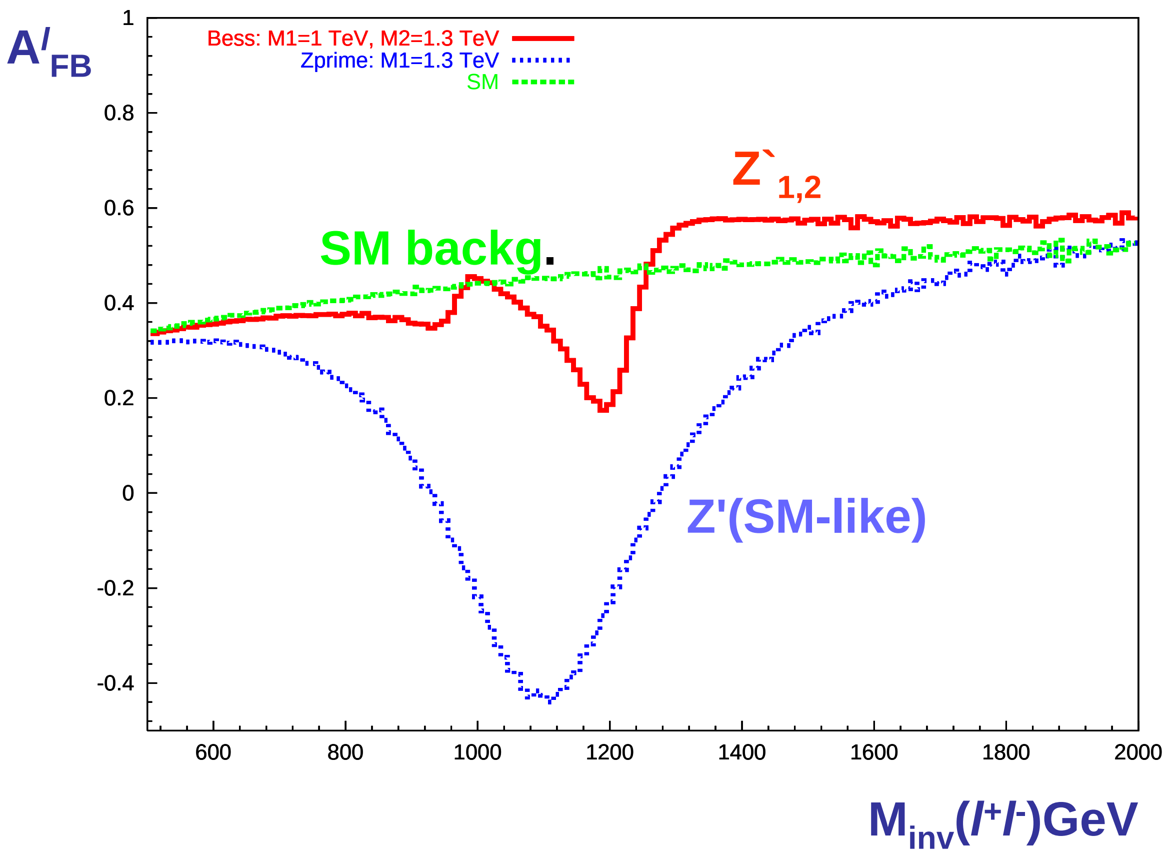
$M_{Z`2}=M_{Z`(SM\text{-}like)}$=1.3 TeV

we select the events within
$|M_{inv}(l^+l^-)-M_{Z`}| < 3\Gamma_{Z`}$.
Rapidity cut: $|y(l^+l^-)|>1$

 # evts for $Z`_2$ ~ 1000

Forward-backward asymmetry A_{FB} in $pp \to l^+ l^-$

(Dittmar,Nicollerat,Djouadi 03; Petriello,Quackenbush 08)



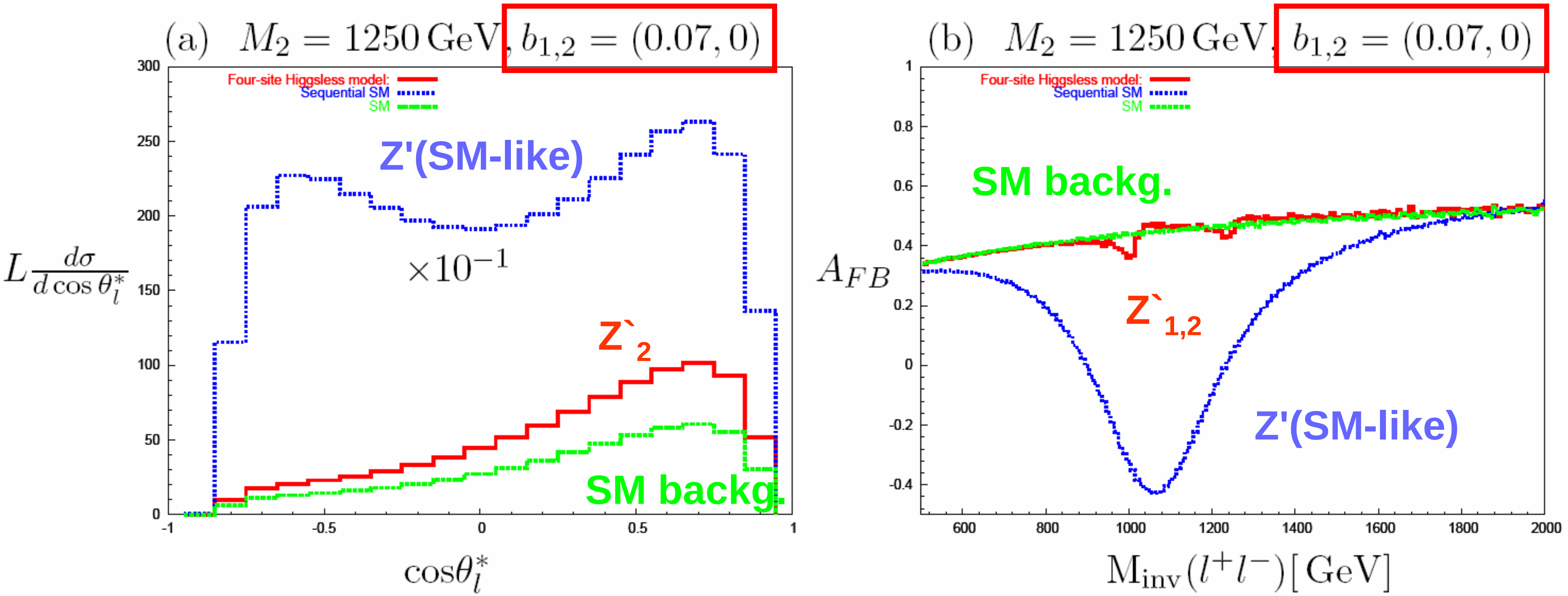
$M_{Z`1}$ = 1.0TeV

$M_{Z`2}$ = 1.3TeV

$M_{Z`(SM\text{-}like)}$ =1.3TeV

$$A_{FB} = \left[\frac{d\sigma^F}{dM_{\rm inv}} - \frac{d\sigma^B}{dM_{\rm inv}}\right] / \left[\frac{d\sigma^F}{dM_{\rm inv}} + \frac{d\sigma^B}{dM_{\rm inv}}\right]$$

On- and off-resonance A_{FB} for a single resonance scenario

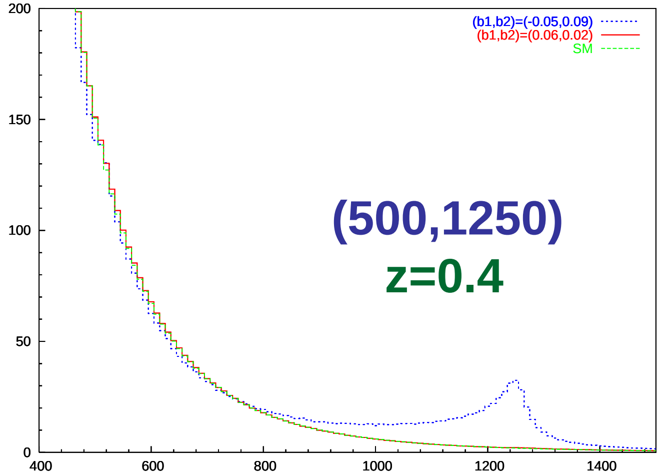


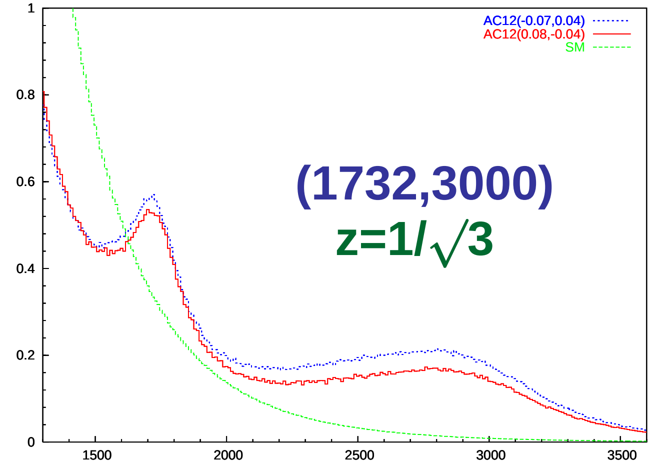
- The on-resonance A_{FB} is more pronounced in the 4-site model due to the difference between the left and the right-handed fermion-boson couplings
- The off-resonance A_{FB} could reveal the double-resonant structure not appreciable in the dilepton invariant mass distribution

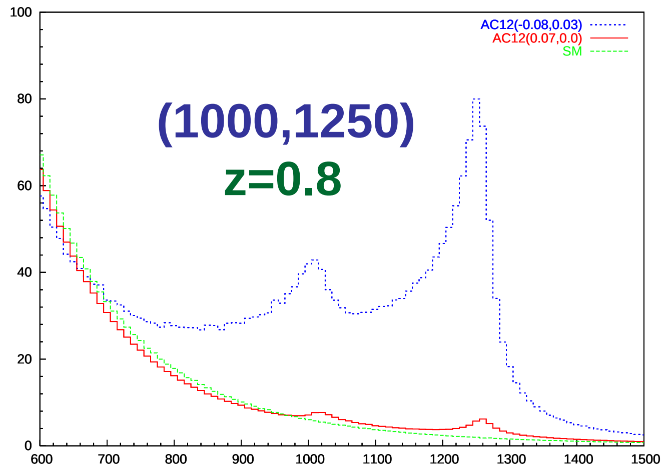
W'_1, W'_2 production

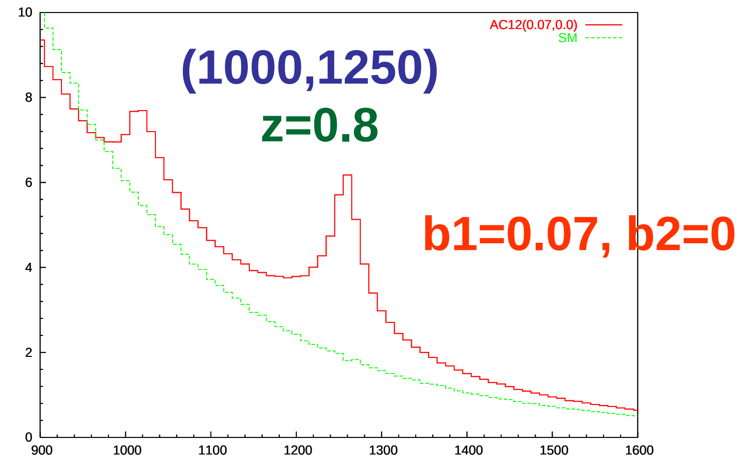
$Z`_i$ and $W`_i$ are nearly degenerate

(M_1,M_2) GeV









Total # of evts in a 10GeV-bin versus $M_T(l\nu)$ for $L=10fb^{-1}$. Sum over e,μ

W'_1, W'_2 production

	$M_{1,2}$(GeV)	$b_{1,2}$	M_t^{cut}(GeV)	$N_{\rm evt}^{\rm sig}(W_{1,2}^{\pm})$	$N_{\rm evt}^{\rm tot}(W_{1,2}^{\pm})$	σ
1	500,1250	-0.05,0.09	800	641	910	21.2
3	1732,3000	-0.07,0.04	1500	38	45	5.7
5	1000,1250	-0.08,0.03	700	1715	2323	35.6

# of evts for the $W`_{1,2}$ DY-production for $M_t(l\nu_l)$ **>** M_t^{cut}

$\sigma = N_{\rm evt}^{\rm sig}/\sqrt{N_{\rm evt}^{\rm tot}}$ for an integrated luminosity L=10 fb^{-1}

The statistical significance for the W`s production is ~ a factor 2 bigger than for the Z`s but it is less clean

Neutral and charged channel are complementary

All six extra gauge bosons could be investigated at the LHC start-up with L ~ 100 pb^{-1}

Conclusions

- Higher dimensional gauge theories naturally suggest the possibility of Higgsless theories
- Linear moose models provide an effective description of Higgsless theories. They are calculable and not excluded by the EW precision measurements (the BESS model is a 3-site moose)
- They describe new spin-1 gauge bosons which delay the unitarity violation to energy scales higher than those probed at the LHC
- Drell-Yan processes are a very good channel to discover these extra gauge bosons at the LHC
- A_{FB} for distinguishing among various models with Z\`

Di-boson production and VBS in progress

interesting because V_1=vector and V_2=axial vector (broken by weak ints)

Hard to compete with the Higgs boson but interesting mechanism with heavy spin-1 resonances

extra slides

The transformation properties of the fields are

$$\Sigma_1 \to L\Sigma_1 U_1^\dagger,$$
$$\Sigma_i \to U_{i-1}\Sigma_i U_i^\dagger\ , \qquad i = 2, \cdots, K,$$
$$\Sigma_{K+1} \to U_K \Sigma_{K+1} R^\dagger,$$

$$U_i \in G_i \equiv SU(2)_i \qquad A_\mu^i = A_\mu^{ia}\tau^a/2, \quad g_i, \quad i = 1, 2, \cdots, K,$$
$$L \in G_L \equiv SU(2)_L \qquad \tilde{W}_\mu = \tilde{W}_\mu^a \tau^a/2, \quad \tilde{g},$$
$$R \in G_R \equiv SU(2)_R \supset U(1)_Y \qquad \tilde{Y}_\mu = \tilde{\mathcal{Y}}_\mu \tau^3/2, \quad \tilde{g}'$$

$$\boxed{\mathcal{L} = \sum_{i=1}^{K+1} f_i^2 \mathrm{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2}\sum_{i=1}^{K} \mathrm{Tr}[(F_{\mu\nu}^i)^2] - \frac{1}{2}\mathrm{Tr}[(F_{\mu\nu}(\tilde{W}))^2] - \frac{1}{2}\mathrm{Tr}[(F_{\mu\nu}(\tilde{Y}))^2]}$$

Covariant derivatives

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 - i\tilde{g}\tilde{W}_\mu \Sigma_1 + i\Sigma_1 g_1 A_\mu^1,$$
$$D_\mu \Sigma_i = \partial_\mu \Sigma_i - i g_{i-1} A_\mu^{i-1}\Sigma_i + i\Sigma_i g_i A_\mu^i, \qquad i = 2, \cdots, K,$$
$$D_\mu \Sigma_{K+1} = \partial_\mu \Sigma_{K+1} - i g_K A_\mu^K \Sigma_{K+1} + i\tilde{g}'\Sigma_{K+1}\tilde{Y}_\mu$$

Mass spectrum (charged sector): $f_i = f_c$; $g_i = g_c$; $x = g/g_c$

$$M^2 = g_c^2 f_c^2 \begin{pmatrix} x^2 & -x & 0 & \dots & 0 & 0 \\ -x & 2 & -1 & \dots & 0 & 0 \\ 0 & -1 & 2 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2 & -1 \\ 0 & 0 & 0 & \dots & -1 & 2 \end{pmatrix}$$

To the leading order in x:

$$M_W^2 = \frac{g^2 f_c^2}{K+1} \longrightarrow f_c^2 = (K+1)\frac{v^2}{4}$$

$$M_n^2 = 4 g_c^2 f_c^2 \sin^2\left(\frac{\pi n}{2(K+1)}\right) \qquad \mathbf{n=1,..,K}$$

K=1 $M_1^2 = v^2 g_c^2$

K=2 $M_1^2 = \frac{3}{4} v^2 g_c^2, \quad M_2^2 = \frac{9}{4} v^2 g_c^2, \quad \left(z = \frac{1}{\sqrt{3}}\right)$

K=3 $M_1^2 \simeq 0.6 v^2 g_c^2, \quad M_2^2 = 2 v^2 g_c^2, \quad M_3^2 \simeq 3.4 v^2 g_c^2$

Ex: $g_c \sim 2 \div 2.5$, M_1=500 GeV, M_2=900 GeV, M_3=1200 GeV, …..
$g_c \sim 4 \div 5$, M_1=1000 GeV, M_2=1800 GeV, M_3=2400 GeV, …..

Electroweak Corrections for the Linear Moose

(Burgess et al.; Anichini, Casalbuoni, DC)

LEP I puts very stringent bounds on models of new physics. These limits, assuming universality among different generations, are coded in 3 parameters (using G_F, m_Z and α as input parameters)

$$\Delta r_W : \quad \frac{m_W^2}{m_Z^2} = \left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha(m_Z)}{\sqrt{2} G_F m_Z^2 (1 - \Delta r_W)}} \right]^2$$

And from the modifications of the Z couplings to fermions:

$$L_{neutral} = -\frac{e}{s_\theta c_\theta}\left(1 + \frac{\Delta\rho}{2}\right)\bar{\psi}\left(g_V \gamma^\mu + g_A \gamma^\mu \gamma_5\right) Z_\mu$$

$$g_V = \frac{1}{2}(T_3)_L - \bar{s}_\theta^2 Q_{em}, \quad g_A = -\frac{1}{2}(T_3)_L$$

$$\bar{s}_\theta^2 = s_\theta^2\left(1 + \Delta k\right), \quad c_\theta^2 = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha(m_Z)}{\sqrt{2} G_F m_Z^2}}$$

It is usual to introduce another set of parameters ε_i, i=1,2,3 (Altarelli, Barbieri, 1991), or S,T,U (Peskin, Takeuchi, 1990), much more convenient on the theoretical side

$$\boxed{\epsilon_1 = \Delta\rho, \quad \epsilon_2 = c_\theta^2\,\Delta\rho + \frac{s_\theta^2}{c_{2\theta}}\Delta r_W - 2s_\theta^2\,\Delta k, \quad \epsilon_3 = c_\theta^2\,\Delta\rho + c_{2\theta}\,\Delta k}$$

At the lowest order in the EW corrections the parameters ε_1 and ε_2 vanish if the SB sector has a SU(2) custodial symmetry (as it is the case for the BESS model). At the same order, ε_3 has a convenient dispersive representation

$$\epsilon_3 = -\frac{g^2}{4\pi}\int_0^\infty \frac{ds}{s^2}\left[\mathrm{Im}\,\Pi_{VV}(s) - \mathrm{Im}\,\Pi_{AA}(s)\right], \quad \Pi_{VV(AA)} = \langle J_{V(A)} J_{V(A)}\rangle_0$$

Assuming vector dominance:

$$\mathrm{Im}\Pi_{VV(AA)}(s) = -\pi\, g_{iV(iA)}^2\,\delta\left(s - M_i^2\right), \quad \langle 0|\,J_{V(A)}^\mu\,|A_i(k)\rangle = g_{iV(iA)}\epsilon^\mu(k)$$

For example, in the BESS model the decay coupling constants of the vector meson are:

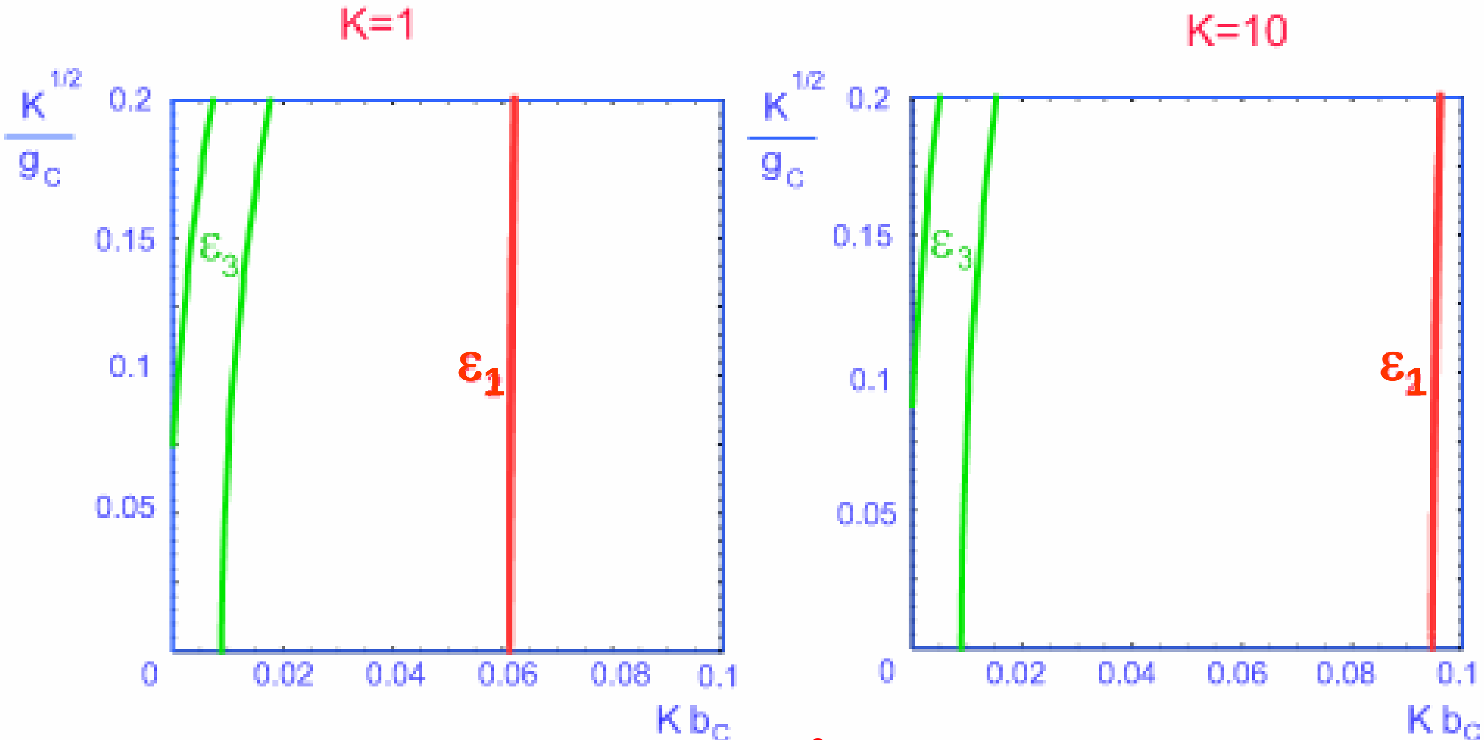
$$g_{V(A)} = (f_1^2 \pm f_2^2)g_1$$

$$\varepsilon_1 \approx O(b_i^2), \quad \varepsilon_2 \approx O(b_i^2), \quad \varepsilon_3 \approx \sum_{i=1}^{K} y_i \left(\frac{g^2}{g_i^2}(1-y_i) - b_i \right)$$

Neglecting: $O(g^4/g_i^4)$, $O(b_i g^2/g_i^2)$

Possibility of agreement with EW data with fine-tuning:

- **Simplest case:** $f_i = f_c, \quad g_i = g_c, \quad b_i = b_c$



(95% CL, with rad. corrs. as in the SM with 1 TeV Higgs)

Very loose bounds from ε_1 and ε_2

- **Ideal cancellation:** $b_i = \frac{g^2}{g_i^2}(1-y_i)$

Can the linear moose considered so far, be derived by discretizing a SU(2) gauge theory in 5D?

To describe the moose structure **including the breaking**, one needs kinetic terms on the branes plus BC's. In the case of a conformally flat metrics along the fifth direction the complete action for a SU(2)-moose would be

$$S = -\frac{1}{4}\int d^4x \int_0^{\pi R} dz\, e^{-A(z)} \frac{1}{g_5^2(z)}\left[(F^a_{\mu\nu})^2 - 2(F^a_{\mu 5})^2\right] +$$

$$-\frac{1}{4}\int d^4x \int_0^{\pi R} dz\, e^{-A(z)} \left[\frac{1}{\tilde{g}^2}(F^a_{\mu\nu})^2\delta(z) + \frac{1}{\tilde{g}'^2}(F^3_{\mu\nu})^2\delta(z-\pi R)\right]$$

$$\text{BC's}: \quad A^{1,2}_\mu \Big|_{z=\pi R} = 0, \quad \partial_z A^a_\mu \Big|_{z=0} = 0$$

- **Introducing the link variables** $\Sigma_i = e^{-iaA_5^i}$, **i=1,…,K+1**

$$S_{moose} = \int d^4x \left(-\sum_{i=1}^{K} \frac{1}{2g_i^2} \mathrm{Tr}\left[F^i_{\mu\nu}F^{\mu\nu i}\right] + \sum_{i=1}^{K+1} f_i^2 \mathrm{Tr}\left[(D_\mu\Sigma_i)(D_\mu\Sigma_i)^\dagger\right]\right)$$

$$a e^{-A_i}/g_{5i}^2 = 1/g_i^2, \quad e^{-A_i}/(a g_{5i}^2) = f_i^2$$

$$A^1_\mu = W_\mu{}^a\tau_a/2, \quad A^{K+1}_\mu = Y^\mu\tau_3/2$$

FLAT METRIC:

$$f_i = f_c, \quad g_i = g_c, \quad e^{-A_i} = 1, \quad g_{5i}^2 = a g_c^2$$

How can we get b_i from a 5D bulk?

(Foadi,Gopalakrishna,Schmidt; Csaki,Hubitsz,Meade; Bechi,Casalbuoni, DC, Dominici)

Consider fermions propagating in the warped 5D bulk with additional brane kinetic terms + BC's: $\psi_R|_0 = 0$, $\psi_L|_{\pi R} = 0$

$$S_{ferm.} = \int d^4x \int_0^{\pi R} dz \Big[e^{-4A(z)} [(\frac{i}{2}\bar{\psi}\Gamma^M D_M \psi + h.c.)] - e^{-A(z)} M\bar{\psi}$$

$$+ e^{-4A(0)} \frac{\delta(z)}{\hat{t}_L^2} i\bar{\psi}_L \gamma^\mu D_\mu \psi_L + \; e^{-4A(\pi R)} \delta(\pi R - z) i\bar{\psi}_R (\frac{1}{\hat{t}_R^2}) \gamma^\mu D_\mu \psi_R \Big]$$

where $D_M\psi = (\partial_M + iT^a A_M^a(z) + iY_L A_M^3(\pi R))\psi$ and $\hat{t}_{L,R}$ set the weight of the brane kinetic terms with respect to the bulk one.

• DISCRETIZE the fifth dimension $\longrightarrow$ the fermions on the j-site with $j = 0, .., K+1$, with a mass term $m_j = (aM_j + 1)/a$, $j = 1, .., K$, "hop" from one site to the near one due to ∂_z.

• Study the effects of ψ_i $(i = 1, .., K)$ in the low-energy limit that is neglect kinetic terms with respect to mass terms. DECOUPLE the heavy fermions with the solutions of their e.o.m. (consider only the quadratic interactions among fermions)

$$\alpha_j L_j - m_{j+1} L_{j+1} = 0, \qquad j = 0,..,K-1$$
$$\alpha_j R_{j+1} - m_j R_j = 0, \qquad j = 1,..,K$$

where $L_j = \psi_L^j$ e $R_j = \psi_R^j$ $(j = 1,..,K)$, L_0 and R_{K+1} are, up to mixing corrections, the left and right components of the SM fermions, and $\alpha_0 = \hat{t}_L/\sqrt{a}$, $\alpha_j = 1/a$ $(j = 1,..,K-1)$, $\alpha_K = \hat{t}_R/\sqrt{a}$, are the "hopping" strengths.

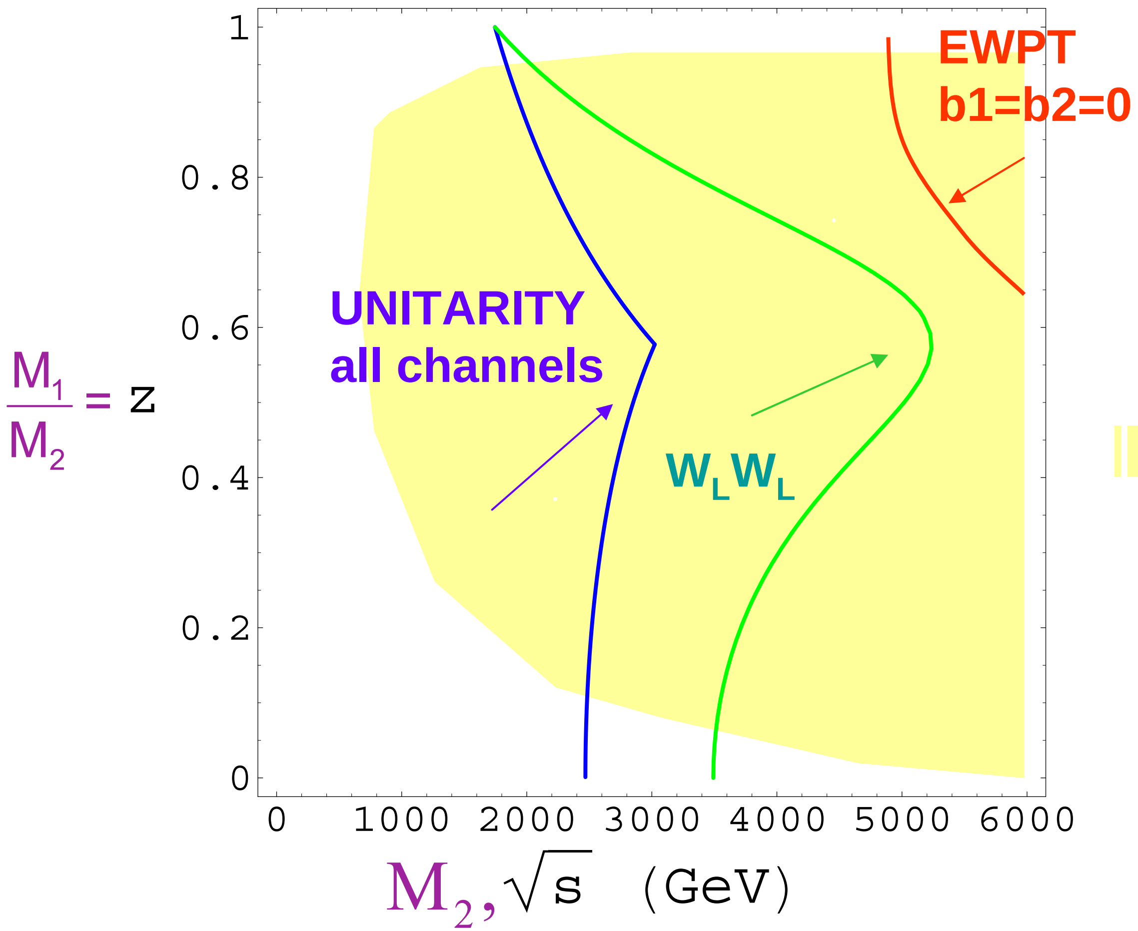
• PLUG the solutions in the gauge-fermion interaction, get direct SM fermion couplings to A_μ^i + SM fermion mass term (normalized fields):

$$S^b_{ferm} = \int d^4x \sum_{j=1}^{K} \frac{b_j^L}{1+\sum_{i=1}^K b_i^L} i\bar{L}_0 \gamma^\mu (\partial_\mu + i g_j T^a A_\mu^{aj} + i\tilde{g}' Y_L A_\mu^{K+1}) L_0$$
$$+ \sum_{j=1}^{K} \frac{b_j^R}{1+\sum_{i=1}^K b_i^R} \; i\bar{R}_{K+1} \gamma^\mu (\partial_\mu + i g_j T^3 A_\mu^{3j} + i\tilde{g}' Y_L A_\mu^{K+1}) R_{K+1}$$
$$+ \sum_{j=1}^{K} \frac{b_j^R}{1+\sum_{i=1}^K b_i^R} \frac{g_j}{\sqrt{2}} (\bar{R}_{K+1} \gamma^\mu A_\mu^{+j} R_{K+1} + h.c.) - m^f (\bar{L}_0^f R_{K+1}^f + h.c.)$$

with $\quad b_j^L = (\frac{\alpha_0}{m_j} \prod_{i=1}^{j-1} \frac{\alpha_i}{m_i})^2 \geqslant 0, \quad b_j^R = (\frac{\alpha_K}{m_K} \prod_{i=j}^{K-1} \frac{\alpha_i}{m_i})^2 \geqslant 0 \quad (\alpha_K \ll \alpha_0)$

$$m^f = m_j \sqrt{\frac{b_j^L}{(1+\sum_{i=1}^K b_i^L)}} \sqrt{\frac{b_j^R}{(1+\sum_{i=1}^K b_i^R)}} \qquad \forall j = 1,..,K$$

The Higgsless 4-site Linear Moose model



region of validity of the perturbative expansion

$(g/g_1)^4 < 0.005$

The Higgsless 4-site Linear Moose model
charged gauge boson spectrum

$$\boxed{\begin{aligned} M_W^2 &\approx \tilde{M}_W^2\left(1 - \frac{\tilde{g}^2}{g_1^2} z_W\right) \\ M_{1,c}^2 &\approx M_1^2\left(1 + \frac{\tilde{g}^2}{2g_1^2}\right) \\ M_{2,c}^2 &\approx M_2^2\left(1 + \frac{\tilde{g}^2}{2g_1^2} z^4\right) \end{aligned}}$$

$$\tilde{M}_W^2 = \tilde{g}^2 \frac{f_1^2 f_2^2}{f_1^2 + 2f_2^2} \quad z_W = \frac{f_1^4 + 2f_1^2 f_2^2 + 2f_2^4}{(f_1^2 + 2f_2^2)^2} = \frac{1}{2}(1 + z^4)$$

$$M_1^2 = f_1^2 g_1^2 \quad M_2^2 = g_1^2(f_1^2 + 2f_2^2) \quad z = \frac{M_1}{M_2} = \frac{f_1}{\sqrt{f_1^2 + 2f_2^2}}$$

neglecting terms $\mathcal{O}(\tilde{g}^4/g_1^4)$

The Higgsless 4-site Linear Moose model neutral gauge boson spectrum

$$M_\gamma^2 = 0$$

$$M_Z^2 = \tilde{M}_Z^2 \left(1 - \frac{\tilde{g}^2}{g_1^2} z_Z\right)$$

$$M_{1,n}^2 = M_1^2 \left(1 + \frac{\tilde{g}^2}{g_1^2} \frac{\sec^2\tilde{\theta}}{2}\right)$$

$$M_{2,n}^2 = M_2^2 \left(1 + \frac{\tilde{g}^2}{g_1^2} \frac{z^4 \sec^2\tilde{\theta}}{2}\right)$$

$$\tan\tilde{\theta} = \frac{\tilde{g}'}{\tilde{g}} \qquad \tilde{M}_Z^2 = \frac{\tilde{M}_W^2}{\cos^2\tilde{\theta}}, \qquad z_Z = \frac{1}{2} \frac{(z^4 + \cos^2 2\tilde{\theta})}{\cos^2\tilde{\theta}}$$

neglecting terms $\mathcal{O}(\tilde{g}^4/g_1^4)$

The Higgsless 4-site Linear Moose model fermionic couplings charged sector

$$\mathcal{L}_{CC} = \bar{\psi}_L \gamma^\mu T^- \psi_L \left(a_W W^+_\mu + a_1^c W^+_{1\mu} + a_2^c W^+_{2\mu} \right) + h.c.$$

$$a_W = -\frac{\tilde{g}}{\sqrt{2}} \left(1 - \frac{b}{2}\right) \left(1 - \frac{\tilde{g}^2}{g_1^2} \frac{z_W}{2}\right)$$

$$a_1^c = -\frac{g_1}{2(1+b_+)} \left(b_+ - \frac{\tilde{g}^2}{g_1^2} \right)$$

$$a_2^c = -\frac{g_1}{2(1+b_+)} \left(b_- - \frac{\tilde{g}^2}{g_1^2} z^2 \right)$$

$$z_W = \frac{1}{2}(1+z^4) \qquad b = \frac{b_+ - b_- z^2}{(1+b_+)} \qquad b_\pm = b_1 \pm b_2 \qquad z = \frac{f_1}{\sqrt{f_1^2 + 2 f_2^2}}$$

neglecting terms $\mathcal{O}(\tilde{g}^4/g_1^4)$ **and** $\mathcal{O}(b_i \tilde{g}^2/g_1^2)$

The Higgsless 4-site Linear Moose model fermionic couplings neutral sector

$$\mathcal{L}_{NC} = \bar{\psi}\gamma^\mu(-a_F \mathbf{Q} A_\mu + a_1^n Z_{1\mu} + a_2^n Z_{2\mu} + a_Z Z_\mu)\psi$$

$$a_F = \tilde{g} s_{\tilde{\theta}} \left(1 - \frac{\tilde{g}^2}{g_1^2} z_\gamma\right) \equiv e$$

$$a_Z = -\frac{\tilde{g}}{c_{\tilde{\theta}}} \left(1 - \frac{b}{2}\right) \left(1 - \frac{\tilde{g}^2}{g_1^2}\frac{z_Z}{2}\right) \left[\mathbf{T^3} - \frac{s_{\tilde{\theta}}^2}{(1-\frac{b}{2})} \left(1 - \frac{\tilde{g}^2 c_{\tilde{\theta}}}{g_1^2 s_{\tilde{\theta}}} z_{Z\gamma}\right) \mathbf{Q}\right]$$

$$a_1^n = -\frac{g_1}{\sqrt{2}(1+b_+)} \left(b_+ - \frac{\tilde{g}^2}{g_1^2}\frac{c_{2\tilde{\theta}}}{c_{\tilde{\theta}}^2}\right) \mathbf{T^3} + \frac{\tilde{g}^2 \tan^2\tilde{\theta}}{\sqrt{2} g_1} \mathbf{Q}$$

$$a_2^n = -\frac{g_1}{\sqrt{2}(1+b_+)} \left(b_- - \frac{\tilde{g}^2}{g_1^2}\frac{z^2}{c_{\tilde{\theta}}^2}\right) \mathbf{T^3} - \frac{\tilde{g}^2 z^2 \tan^2\tilde{\theta}}{\sqrt{2} g_1} \mathbf{Q}$$

with

$$z_\gamma = s_{\tilde{\theta}}^2, \qquad z_Z = \frac{1}{2}\frac{(z^4 + c_{2\tilde{\theta}}^2)}{c_{\tilde{\theta}}^2}, \qquad z_{Z\gamma} = -\tan\tilde{\theta} c_{2\tilde{\theta}}$$

$$\mathbf{T^3} = \tau_L^3/2 \ (\tau_L^3 \psi_L = \pm\psi_L \text{ and } \tau_L^3 \psi_R = 0), \ \tan\tilde{\theta} = s_{\tilde{\theta}}/c_{\tilde{\theta}} = \tilde{g}'/\tilde{g}$$

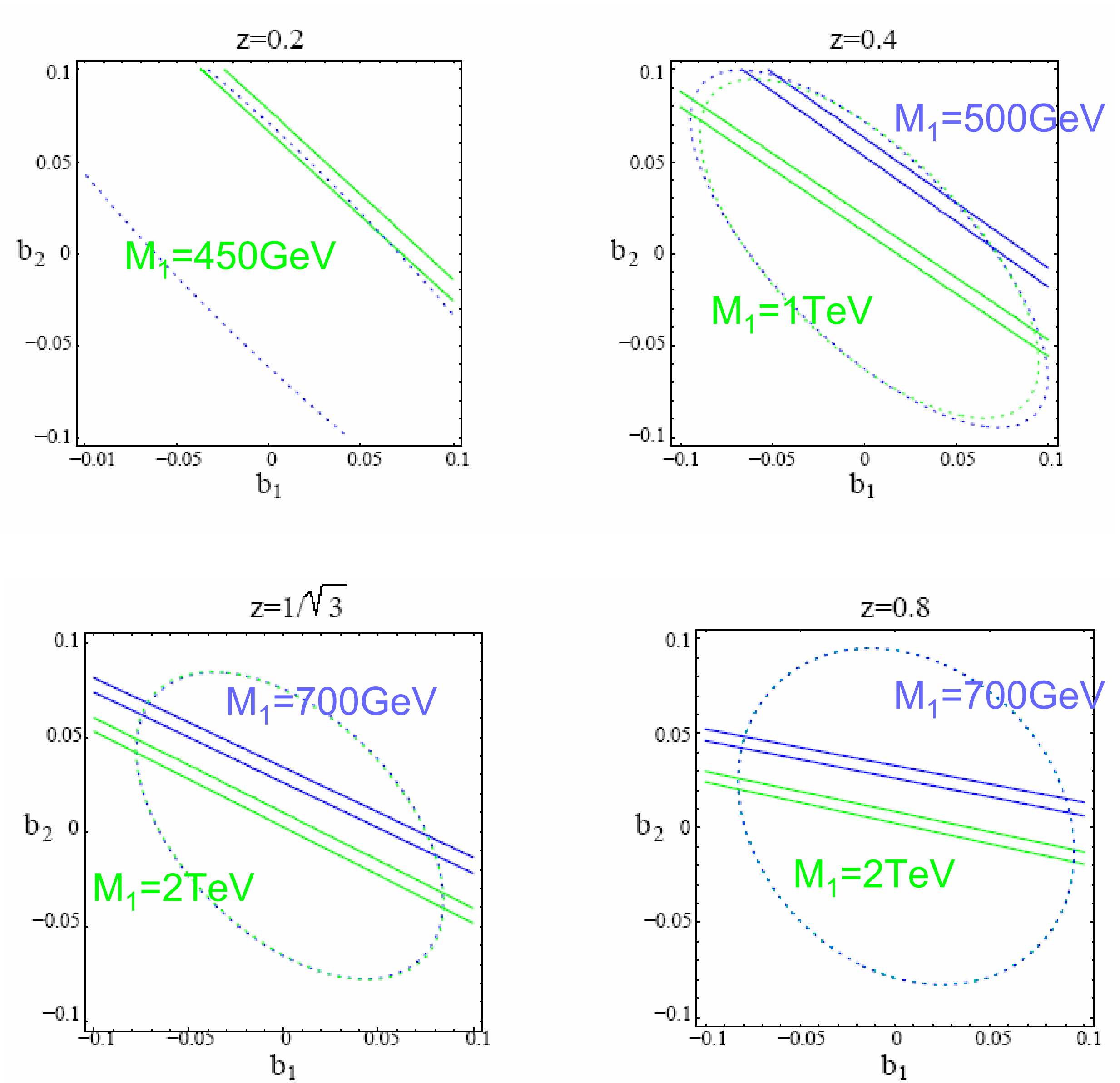
charged

	$M_{(1,2),c}$(GeV)	$\Gamma_{1,2}$(GeV)	a_1^c	a_2^c
1	508,1251	6.2,35.5	0.03	0.20
2	508,1251	6.2,28.9	-0.01	-0.03
3	1745,3001	183,746	0.15	0.47
4	1745,3001	183,734	-0.15	-0.44
5	1009,1255	35.3,30.5	0.14	0.24
6	1009,1255	33.1,22.2	-0.06	-0.09

	$M_{(1,2),n}$(GeV)	$\Gamma_{1,2}$(GeV)	a_{1L}^e	a_{1R}^e	a_{1L}^d	a_{1R}^d	a_{1L}^u	a_{1R}^u	a_{2L}^e	a_{2R}^e	a_{2L}^d	a_{2R}^d	a_{2L}^u	a_{2R}^u
1	510,1251	6.4,36.0	0.12	0.11	0.05	0.04	-0.09	-0.07	0.43	-0.02	0.46	-0.01	-0.44	0.01
2	510,1251	6.3,28.8	0.02	0.11	-0.05	0.04	0.01	-0.07	-0.08	-0.02	-0.07	-0.01	0.07	0.01
3	1736,3001	184,756	0.36	0.04	0.34	0.01	-0.35	-0.02	1.06	-0.01	1.07	0.0	-1.07	0.01
4	1736,3001	184,742	-0.32	0.04	-0.34	0.01	0.33	-0.02	-1.0	-0.01	-0.99	0.0	1.0	0.01
5	1012,1256	36.2,32.0	0.37	0.08	0.31	0.03	-0.34	-0.06	0.50	-0.05	0.54	-0.02	-0.52	0.04
6	1012,1256	33.7,22.9	-0.11	0.08	-0.16	0.03	0.14	-0.06	-0.24	-0.05	-0.20	-0.02	0.22	0.04

neutral

The Higgsless 4-site Linear Moose model



95% CL bounds on (b_1,b_2) from ε_1 (dash) and ε_3 (solid)

$M_2 = M_1 / z$