Laser Fundamentals
Some Problems

1.- Determine the time it takes light pulses to travel different distances in vacuum: The diameter of an atom; the diameter of one bacteria; one centimetre. Determine the time that a 700 nm photon needs to travel one cm inside a Ti:sapphire crystal. Is it the same as for a 900 nm photon?

Guide for the solution. Just use the speed of light c=3x10e8 m/s
It is important to use those numbers to get the idea that light travels one micron (more or less the size of a bacteria) in 3 fs.
For the crystal case you must use the refractive index, n, and the velocity is c/n. The point is that n depends with the wavelength. One can use this relationship to get the refractive index dependence with the wavelength. Sapphire is an anisotropic crystal and typically the Ti:sapphire laser works with the light polarized parallel to the crystal axis, so we just consider the extraordinary ray, n(700) =1,790 and n(900)=1,770.

2.- Some basic numbers related to the comparison between laser light and sun light. The intensity of the sun light on earth is 1.3 kW/m2.
- Evaluate the power within an area of 1 cm2.
- Estimate the intensity if the radiation incident on an area of 10 cm2 is focused to a surface of 100 micrometer in diameter (note that focusing to a smaller diameter is not possible because of the divergence -5 mrad- of the sun radiation).
- Estimate the power density of the radiation of a visible laser (power 1 mW, cross sectional area 1 cm2) focused to an area that has a diameter of one micrometer.

Guide for the solution. Just divide. No trick at all.

3.- Is it posible to make an attosecond pulse with visible light frequencies, say between 300 and 600 nanometers? (1000 attoseconds = 1 femtosecond). For example, with a laser as the titanium:sapphire that emits between 650 and 1100 nanometers, estimate what is the minimum pulse duration achievable.

Guide for the solution. Just note that for that wavelength range the laser periods are of a few femtoseconds (1,5 fs correspond to 500 nm –green- , 2,6 fs correspond to 800 nm, 3 fs to one micron). It is impossible to build up a pulse with less than one cycle, otherwise it would be a DC field instad than a wave. So for attosecond pulses a set of UV photons are needed.

4.- Calculate the photon density (number of photons per cubic centimetre) in a 1 micrometer wavelength laser with an energy density of 1 J/m3.

Guide for the solution. Just divide. No trick at all.
One micrometer wavelength corresponds to 1.24 eV per photon.
One Joule = 6.24 x 10e18 eV.

5.- Calculate the number of modes of a 1 cm long resonator (round trip 2 cm). How many of those modes belong to the visible spectral range?

Guide for the solution.

Trick … there are infinite modes … assuming that the mirrors would work for X-rays, … As the wavelength goes to zero, more and more modes fit in the cavity. In a real case the limit will be given by the frequency at which the mirrors become transparent. For a metal mirror this can be at 5 eV more or less.
For the number of modes within the visible (say between 350 and 700 nm approx.) :
Consider the order of the mode for 350 nm given by n1=2cm/350 nm
Consider the order of the mode for 700 nm given by n2=2cm/700 nm
The solution will be n1-n2.

6.- A Fabry Perot resonator with a distance of 10 cm from mirror to mirror, has loss due to output coupling because one of the mirrors is a partial mirror (Reflection factor 95 percent in intensity). Determine the lifetime of a photon in the resonator.

Guide for the solution.
The round trip is 20 cm, this takes 6,6 ns (c=3cm/ns). So the loss is gc=0.05/ns (gc means gamma of the cavity, assuming that we call gamma to the relaxation and loss parameters).
Lifetime (1/2) will be 0,69/gc   (0,69=ln2, is the conversion for exponential to half-life)

7.- A Ti:sapphire crystal of 1 cm long is optically pumped in a cylindrical volume of 0.2 mm diameter (Ti3+ concentration 0.1 wt-%).
- Estimate the pump power necessary to excite a tenth of the Ti3+ ions into the excited state. Determine the absolute number of excited Ti3+ ions.
- Determine the energy stored as excitation energy and the corresponding energy density per cubic centimeter.
- Estimate the pump power necessary to excite a tenth of the Ti3+ ions into the excited state.

Guide for the solution.
First estimate the amount of Ti ions. The density of sapphire is 4 gr/cm3. If 0.1 percent is Ti, it meand that the Ti density is about 4 miligrams per cubic centimetre. From here you can calculate the number of Ti ions.
To excite each Ti ion you need a green photon, typically 532 nm (2,33 eV) and we store about 1,55 eV (800 nm) the rest is the quantum defect that is lost by dissipation.
The rest is jut multiplications.

8.- Prove that the well known energy mass relation E=mc2, is equivalent to rho=I c3 . In other words, show that a laser beam of intensity I is equivalent to a mass density rho. (This is just a formal equivalence, you all know that a photon has not rest mass). What would be the intensity needed to arrive to an energy density equivalent to water density (1 grams/cm3).

Consider a pulse of duration T with an energy E and a cross section S
I=P/S   I=laser intensity, P laser power, S beam section.
P=E/T E= laser pulse energy, T pulse duration
Pulse duration can be associated to pulse length L, c=L/T, so
I=P/S=E/ST=Ec/SL
SL is the "volume" of the pulse, that contains an energy E, equivalent to a mass density mc2. m/SL is a mass density.
Observe that this is just an analogy, nothing else. Light is pure energy, not mass. But mass density is also an energy density and we can compare them.
Water density (1 gram/cm3) corresponds to 2.7 x 10e24 W/cm3

World record is now two orders of magnitude below, say 10 miligrams/cm3.
So lasers are now denser that air and soon will be denser than water.

9.- Are there lasers (or something similar) in the radiofrequency domain?

A radio-station is very much like a laser. It is not spontaneous emission, but it is coherent radiation. The non-directionality is jus due to the small size of the antenna compared to the wavelength. When a km long antenna is used, radiation is as directional as a laser beam.

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