(1) Once fully commissioned, the Large Hadron Collider (LHC) will accelerate 2,808 bunches with up to 1.15×10^{11} protons per bunch to 7 TeV. The layout of the LHC is approximately circular, with a circumference of 27 km.

i. What is the total energy stored in the beam?

ii. At what speed would a car of mass m=1,500 kg drive if it had a similar (kinetic) energy?

iii. Calculate the strength of the dipole magnets in LHC, assuming the machine is a perfect circle. What would be the maximum energy of a proton beam in an accelerator going around the Earth if magnets of the same strength were used? (Circumference of the Earth: ~ 40,000 km.)

(i) 2808 bunches $\cdot$ $1.15\cdot10^{11}$ protons @ 7 TeV each =

$2808\cdot1.15\cdot10^{11}\cdot7\cdot10^{12}\cdot1.602\cdot10^{-19}$ Joules = 362 MJ per beam

(ii)

$$v = \sqrt{\frac{2\cdot E}{m}}$$

$$= \sqrt{\frac{2\cdot 362\cdot 10^6}{1500}} \sim 700\, m/s\cdot 3.6 \sim 2500\, km/h$$

(iii) Dipole strength in a synchrotron:

$$B = \frac{p[GeV]}{0.2998\cdot\rho[m]}$$

$$= \frac{7\cdot10^3}{0.2998\cdot 27\cdot 10^3/(2\cdot\pi)} = 5.5T$$

The Earth's radius: 40,000,000 m / ($2\cdot\pi$) = 6,366,000 m

$E_{max} \sim p_{max}$ = 0.2998 $\cdot$ 6,366,000 m $\cdot$ 5.5 T ~ 10.5 PeV (10^{15} eV).

(d) A quadrupole magnet is designed to focus a parallel beam with energy 3 GeV to a focal point 4 m away from the centre of the quadrupole. Calculate the magnetic field gradient required for a magnet of length l=0.2 m.

Focal length: $1/f=kl=0.25\ m^{-1}$

$l=0.2\ m$, $k=1.25\ m^{-2}$

$k=0.2998 \cdot g[T/m] / p[GeV]$

solving for $g=12.509\ T/m$

(e) What limits the maximum beam energy that can be achieved in:

i. a circular proton accelerator;

ii. a circular electron accelerator.

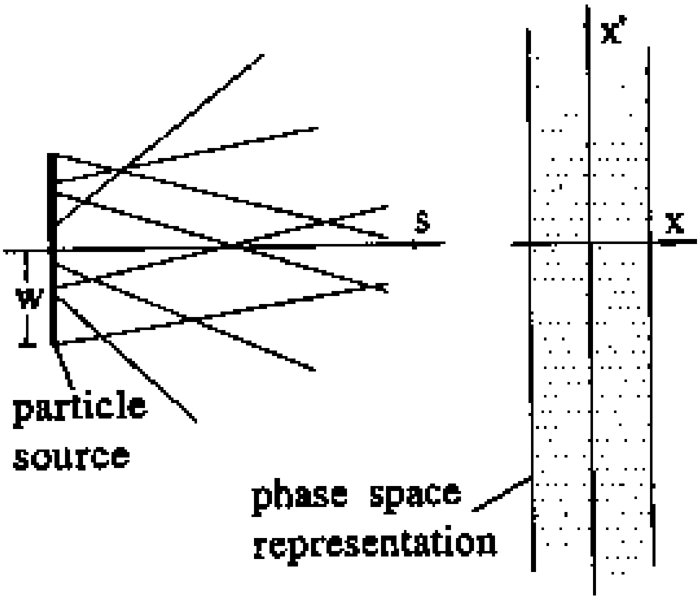
(i) Technical limit of SC magnet field strength reached (8.3 T).

(ii) Synchrotron radiation (SR).

(2)

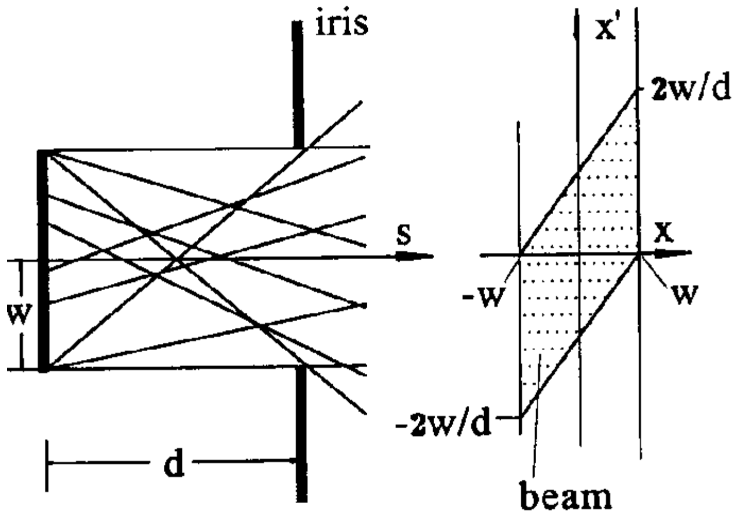
A. Consider a source at s_0 as a disc with radius w emitting particles. Make a drawing of this setup in configuration space and in phase space. Which part of phase space can be occupied by the emitted particles?

Particles are emitted from the entire source surface $x \in [-w; +w]$ with a trajectory slope $\varphi \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$, i.e. the particles can have any $x' \in \mathbb{R}$. The occupied phase space area is infinite.



B. Any real beam emerging from a source like the one above will be clipped by aperture limitations of the vacuum chamber. This can be modelled by assuming that a distance *d* away from the source there is an iris with an opening with radius $R = w$. Make a drawing of this setup in configuration and phase space. Which part of phase space is occupied by the beam at a location after the iris?

Particles with angle x' = 0 are emitted from the entire source surface $x \in [-w; +w]$ *and arrive behind the iris opening. For x = ± w there is a maximum angle x' = ± 2w/d that will still be accepted by the iris. This leads to a parallelogram in phase space. Such a beam has a specific emittance given by the occupied phase space area.*



(3) What limits the maximum beam energy that can be achieved in:

i. a circular proton accelerator;
Strength of bending magnets

ii. a circular electron accelerator.
Synchrotron radiation losses/max power that can be provided to the beam by rf cavities per turn.