

Problem 1: Image relay of a Gaussian beam to the cathode

The 4th harmonic of an Nd:YLF laser operating at 1047nm needs to be image relayed from the conversion crystal plane to the cathode, which is 11m away. 2 lenses are used on the optical table in the laser room, which is the first 3m of the transport. The first lens $f = 120\text{mm}$ is placed 167mm from the crystal. At the crystal for simplicity we take a waist with 0.25mm size and a perfect Gaussian profile.

a) Using simple lens makers equation, calculate the required position for the second, $f = 1500\text{mm}$ lens to get 12mm 4σ beamsize (magnification of 12) on the cathode.

b) How much can the beamsize be varied by moving the position of the second lens $\pm 10\text{cm}$ from the perfect imaging position?

Optional questions to think about:

At what energy/ pulse level will the focal spot sizes in this case can cause air-breakdown?

In the question b) how far does the object plane move, if the cathode is considered to be the image plane.

What diameter would you choose for the second lens to avoid diffraction due to beam clipping?

Where could you place a hard aperture in the lab to image onto the cathode?

$$\lambda := 0.262 \cdot 10^{-3} \quad \text{Wavelength}$$

Beam parameters assuming waist at the crystal (M2 and beamsize has to be experimentally defined)

$$w_{cx} := 0.25 \quad \text{waist size}$$

$$M_{x2} := 1 \quad \text{M square}$$

$$Z_{rx} := \frac{\pi \cdot w_{cx}^2}{\lambda} \quad \text{Rayleigh-range}$$

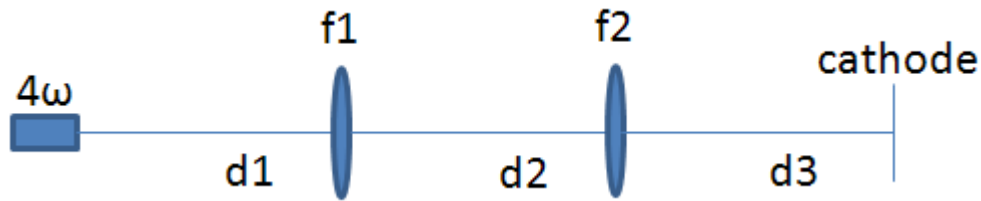
$$q_{cx} := \left(\frac{-i}{Z_{rx}} \right)^{-1} \quad \text{complex beam parameter}$$

Propagation matrices

propagation in space $\mathcal{M}^{(d)} := \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

propagation through a lens $\mathcal{M}^{(f)} := \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

Optimisation of the beam transport



dtot := 11000 overall distance to the cathode

dlab := 3000 distance accessible in the lab

required beam size at the cathode

wcath := 12 beamsize required (4sigma)

Magn := $\frac{wcath}{wcx \cdot 4}$ Magn = 12 magnification required

Setting the conditions for image relay with two lenses in the lab and the required magnification

Guess d1g := 407 d2g := 2018 d3g := dtot - d1g - d2g f1g := 120 f2g := 1500

Given

$$(d1g + d2g) < dlab$$

$$d1g + d2g + d3g = dtot$$

$$\frac{1}{d2g - \left(d1g \cdot \frac{f1g}{d1g - f1g} \right)} + \frac{1}{d3g} = \frac{1}{f2g}$$

$$d3g \cdot \frac{f1g}{\left(d2g - d1g \cdot \frac{f1g}{d1g - f1g} \right) \cdot (d1g - f1g)} = \text{Magn}$$

\mathbb{M} F := Find(d1g, d2g, d3g)

$$d1 := F_0$$

$$f2 := f2g$$

$$d2 := F_1$$

$$f1 := f1g$$

$$d3 := F_2$$

$$\text{cathode}(d2) := s(d3) \cdot l(f2) \cdot s(d2) \cdot l(f1) \cdot s(d1)$$

$$F = \begin{pmatrix} 167.272 \\ 2.242 \times 10^3 \\ 8.591 \times 10^3 \end{pmatrix}$$

$$q_{outx}(d2) := \frac{\begin{pmatrix} \text{cathode}(d2)_{0,0} \cdot q_{cx} + \text{cathode}(d2)_{0,1} \\ \text{cathode}(d2)_{1,0} \cdot q_{cx} + \text{cathode}(d2)_{1,1} \end{pmatrix}}$$

$$w_{outx}(d2) := \sqrt{\left(\text{Im} \left(\frac{1}{q_{outx}(d2)} \right) \cdot -1 \cdot \frac{\pi}{\lambda \cdot Mx2} \right)^{-1}}$$

beam size at the cathode

$$w_{outx}(F_1) = 3$$

$$ls(z, f) := s(z) \cdot l(f)$$

full propagation matrix

$$\text{prop}(z, d1, d2, f1, f2) := \begin{cases} s(z) & \text{if } z \leq d1 \\ (ls(z - d1, f1) \cdot s(d1)) & \text{if } d1 < z \leq d1 + d2 \\ [ls[z - (d1 + d2), f2] \cdot ls(d2, f1) \cdot s(d1)] & \text{if } d1 + d2 < z \end{cases}$$

$$q_x(z, d1, d2, f1, f2) := \frac{\begin{pmatrix} \text{prop}(z, d1, d2, f1, f2)_{0,0} \cdot q_{cx} + \text{prop}(z, d1, d2, f1, f2)_{0,1} \\ \text{prop}(z, d1, d2, f1, f2)_{1,0} \cdot q_{cx} + \text{prop}(z, d1, d2, f1, f2)_{1,1} \end{pmatrix}}$$

$$w_{xprop}(z, d1, d2, f1, f2) := \sqrt{\left(\text{Im} \left(\frac{1}{q_x(z, d1, d2, f1, f2)} \right) \cdot -1 \cdot \frac{\pi}{\lambda \cdot Mx2} \right)^{-1}}$$

$$N := 10000$$

$$i := 0 .. N - 1$$

$$dz := \frac{(dtot + 200)}{N}$$

$$w_{x_i} := w_{xprop}(i \cdot dz, d1, d2, f1, f2)$$

beamsize along the propagation

lens positions

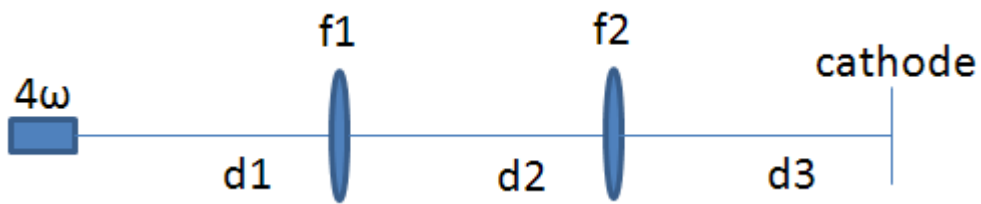
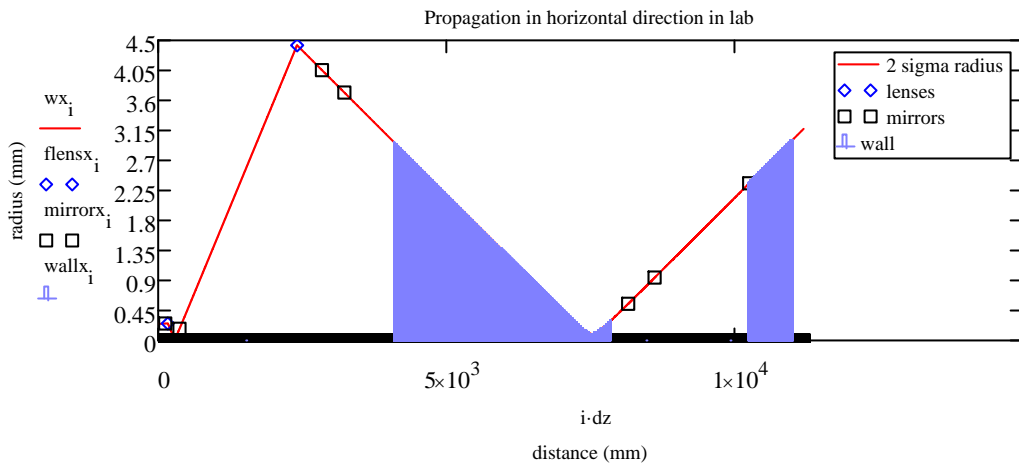
$$\text{flens}_{x_i} := \begin{cases} w_{x_i} & \text{if } i = \text{ceiling} \left(\frac{d1}{dz} \right) \\ w_{x_i} & \text{if } i = \text{ceiling} \left[\frac{(d1 + d2)}{dz} \right] \\ 0 & \text{otherwise} \end{cases}$$

mirror positions for this specific system

$$\text{mirrorx}_i := \begin{cases} \text{wx}_i & \text{if } i = \text{ceil}\left(\frac{120}{dz}\right) \\ \text{wx}_i & \text{if } i = \text{ceil}\left[\frac{(120 + 250)}{dz}\right] \\ \text{wx}_i & \text{if } i = \text{ceil}\left[\frac{(120 + 250 + 2470)}{dz}\right] \\ \text{wx}_i & \text{if } i = \text{ceil}\left[\frac{(120 + 250 + 2470 + 390)}{dz}\right] \\ \text{wx}_i & \text{if } i = \text{ceil}\left(\frac{120 + 250 + 2470 + 390 + 4070 + 855}{dz}\right) \\ \text{wx}_i & \text{if } i = \text{ceil}\left[\frac{(120 + 250 + 2470 + 390 + 4070 + 855 + 460)}{dz}\right] \\ \text{wx}_i & \text{if } i = \text{ceil}\left[\frac{(120 + 250 + 2470 + 390 + 4070 + 855 + 460 + 1470 + 170)}{dz}\right] \\ 0 & \text{otherwise} \end{cases}$$

inside walls, where there is no access to optics

$$\text{wallx}_i := \begin{cases} \text{wx}_i & \text{if } \text{ceil}\left(\frac{120 + 250 + 2470 + 390 + 3770 + 855}{dz}\right) > i \geq \text{ceil}\left(\frac{120 + 250 + 2470 + 390 + 855}{dz}\right) \\ \text{wx}_i & \text{if } \text{ceil}\left(\frac{dtot}{dz}\right) > i \geq \text{ceil}\left(\frac{120 + 250 + 2470 + 390 + 4070 + 855 + 460 + 1470 + 170}{dz}\right) \\ 0 & \text{otherwise} \end{cases}$$



$$d1 = 167.272 \quad d2 = 2.242 \times 10^3 \quad d3 = 8.591 \times 10^3 \quad f1 = 120 \quad f2 = 1.5 \times 10^3$$

$$wout(d2) = 3$$

beam sizes at the cathode

this is 2sigma radius

