**Proposed questions for GANIL School – Free-Electron Lasers**

1. The equation relating the wavelength $\lambda_L$ emitted by a free-electron laser is

$$\lambda_L = \frac{\lambda_w}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

where $\lambda_w$ is the undulator (or wiggler) wavelength, $\gamma$ is the electron relativistic factor and K is the undulator parameter.

Calculate the range of electron energies required at SLAC LCLS to generate X-radiation within the range 1.5 nm – 1.5 Å.

2. Because the total energy is preserved the transverse oscillation in an undulator also affects the longitudinal motion. The average longitudinal velocity $\beta_z$ is:

$$\beta_z \approx 1 - \frac{1}{2\gamma^2} - \frac{1}{2}\beta_x^2 = 1 - \frac{1 + K^2/2}{2\gamma^2}$$

Calculate $\beta_z$ for $\gamma$ =100, and K = 1 and 3.

3. Explain physically why the variation of $\beta_x$ in the undulator implies a corresponding variation in $\beta_z$. What is the consequence of a change in $\beta_x$ as the K increases?

4. Why does the transition between and undulator (low K) and a wiggler (high K, usually few periods) result in a change from monochromatic radiation to broadband radiation – which resembles synchrotron radiation from a circular accelerator?

**Free-Electron Laser Solutions**

1. FEL wavelength equation

$$\lambda_L = \frac{\lambda_w}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

$$\gamma = \left(\frac{\lambda_w}{2\lambda_L}(1+\tfrac{1}{2}K^2)\right)^{\frac{1}{2}}$$ with K = 3.5, $(1+.5K^2) = 7.125$, and $\lambda_w = 3.0$ cm

So for $\lambda_L = 1.5$ nm, $\gamma = 8441.0$, and E = 4.31 GeV

And for $\lambda_L = 1.5$ Å, $\gamma = 26692.7$, and E = 13.64 GeV

So LCLS needs to span energies from 4.3 to 13.6 GeV.

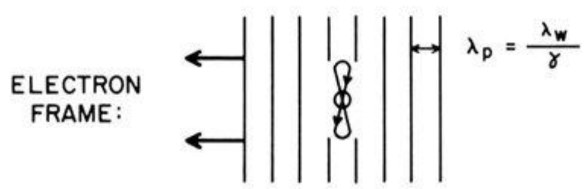
2. Longitudinal velocity in undulator is

$$\beta_z \approx 1 - \frac{1}{2\gamma^2} - \frac{1}{2}\beta_x^2 = 1 - \frac{1+K^2/2}{2\gamma^2}$$

e.g. for K = 1, $\gamma = 100$, $\beta_z = 0.999925$

and for K = 3, $\gamma = 100$, $\beta_z = 0.999725$

3. Figure-of-eight oscillation in electron rest frame



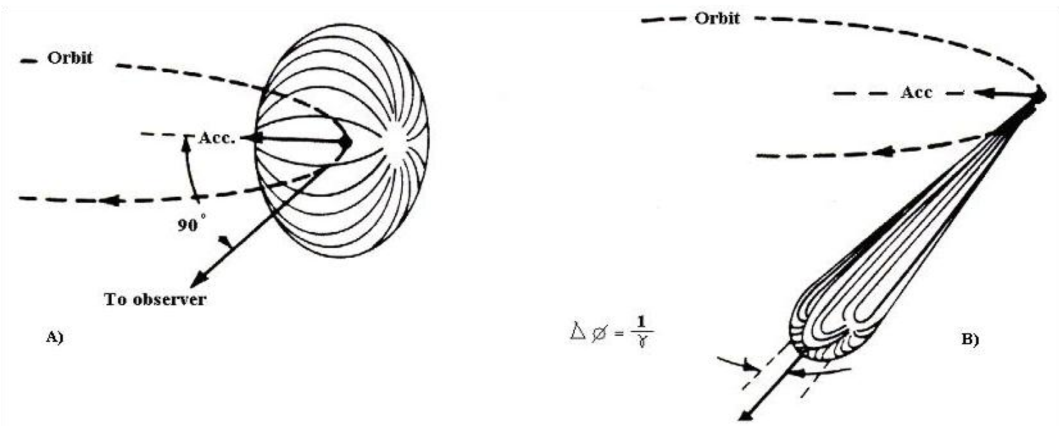

In a static magnetic induction (field) the **total** velocity must remain constant (since **v x B** always transverse). Therefore a change in $\beta_x$ implies a corresponding change in $\beta_z$.

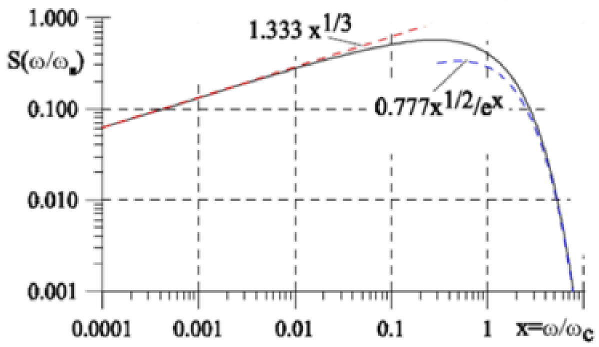
As the undulator K increases, the figure-of eight generates *anharmonic* oscillations, leading to radiation of **harmonics** of the fundamental wavelength $\lambda_L$.

4. Transition from <u>undulator</u> (low K, many periods) to <u>wiggler</u> (high K, usually few periods).

   Undulator radiation is emitted into a narrow cone of half-angle $\gamma^{-1}$ around the electron direction. At low K this angle is modest, and a downstream photon detector is always illuminated by the cone, so it "sees" continuous illumination and – by the Fourier theorem – few harmonics are produced.

   At high K values, this cone 'swings' well outside the detector angle, and the emitted power behaves like a 'searchlight' when viewed at the end of the undulator. The radiation detected is then pulsed in nature, and resembles synchrotron radiation from a circular accelerator (electron synchrotron). It therefore has a corresponding wide spectral range.




K ~ 1 $\rightarrow$ undulator K >> 1 $\rightarrow$ wiggler