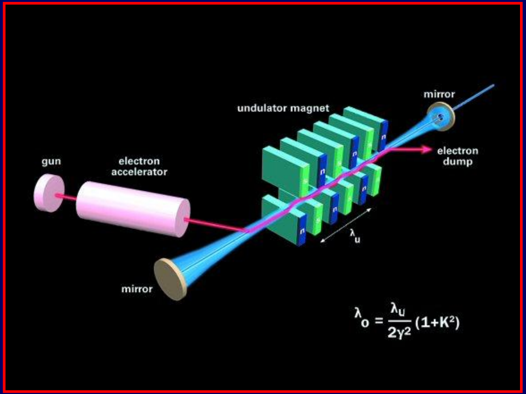
# Applications of (Electron) Accelerators



## Introduction to

# Free-Electron Lasers

Allan Gillespie

*Carnegie Laboratory of Physics*

*University of Dundee*

LA$^3$NET, 1st Intl School on Laser Applications, GANIL, France,15 October 2012

# Outline of Talk

- ★ Introduction: What is a Free-Electron Laser?
- ★ How does an FEL work?
- ★ Choosing the required parameters
- ★ FEL Output Characteristics
- ★ FEL vs Conventional Laser
- ★ Current Trends

# Acknowledgement

- Neil Thompson of Daresbury Laboratory, whose notes I plagiarised and modified ….

# Some Sources of information

- Brian McNeil - Cockcroft Institute lecture notes on FELs: http://www.cockcroft.ac.uk/education/academic0708.html
- J.B. Murphy & C. Pelligrini, “Introduction to the Physics of the Free Electron Laser”, Laser Handbook, vol. 6, p 9-69 (1990).
- R. Bonifacio et al, “Physics of the High-Gain Free Electron Laser & Superradiance”, Rivista del Nuovo Cimento, Vol. 13, no. 9 p1-69 (1990) [see also Rivista del Nuovo Cimento, Vol. 15, no. 11 p1-52 (1992) ]
- The World Wide Web Virtual Library: Free Electron Laser research and applications http://sbfel3.ucsb.edu/www/vl_fel.html
- Saldin E.L., Schneidmiller E.A., Yurkov M.V. The physics of free electron lasers, Springer, Berlin 2000 (Advanced texts in physics, ISSN 1439-2674).
- Charles Brau, Free Electron Lasers (Academic Press, 1990), slightly outdated but good basics
- European XFEL TDR, http://xfel.desy.de/tdr/index_eng.html
- Many other useful sources on web: e.g. www.4gls.ac.uk

# What is a Free-Electron Laser ?

**A beam of relativistic electrons**
co-propagating with
**an optical field**
through
**a spatially periodic magnetic field**

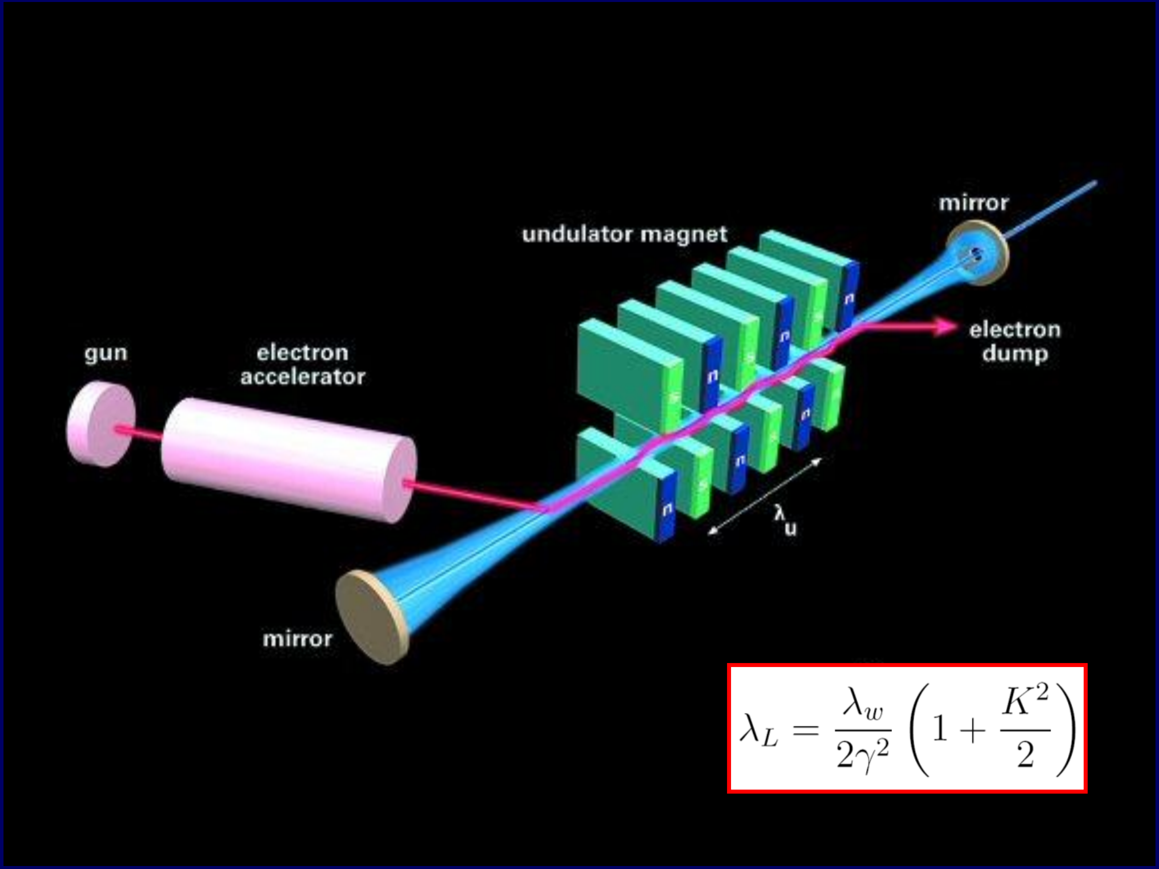
- ★ Undulator causes transverse electron oscillations
- ★ Transverse electron velocity couples to E-component (transverse) of optical field giving *energy transfer*
- ★ Interaction between electron beam and optical field causes *microbunching* of electron beam on scale of radiation wavelength, leading to *coherent emission of radiation*

# What is an FEL?

FEL output is radiation that is

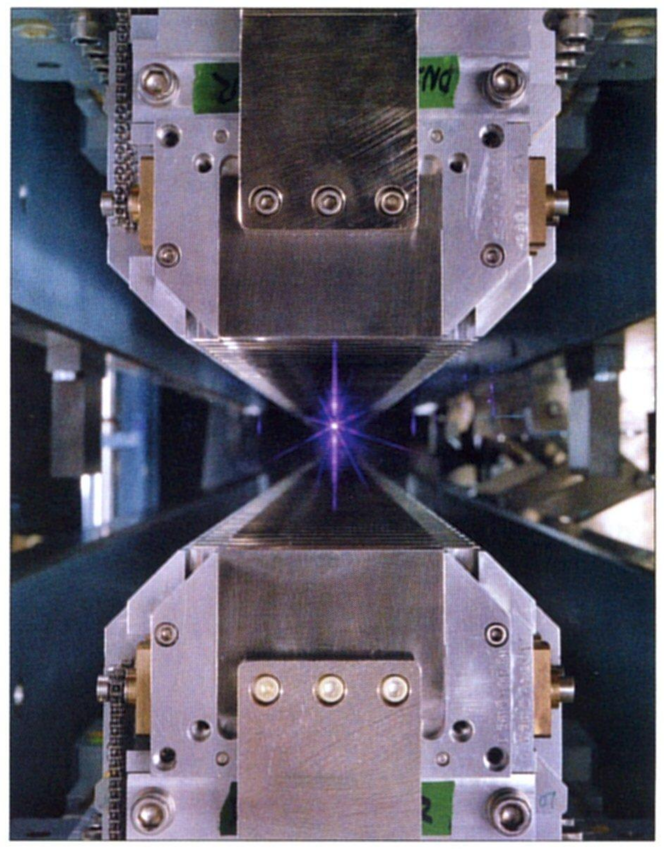
- ★ **tunable (over a wide range)**
- ★ **powerful**
- ★ **coherent**

# FEL Basic Components



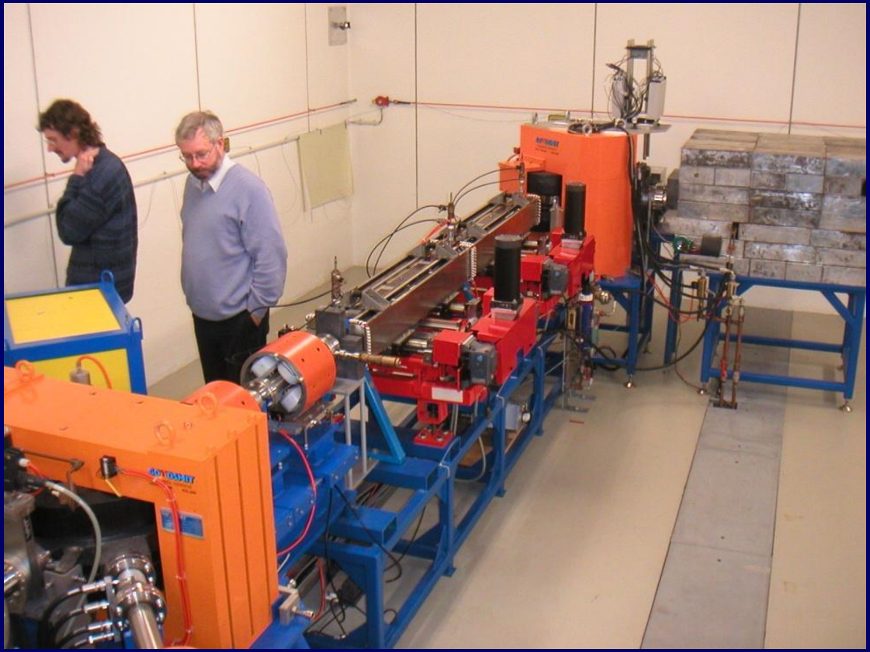
$$\lambda_L = \frac{\lambda_w}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

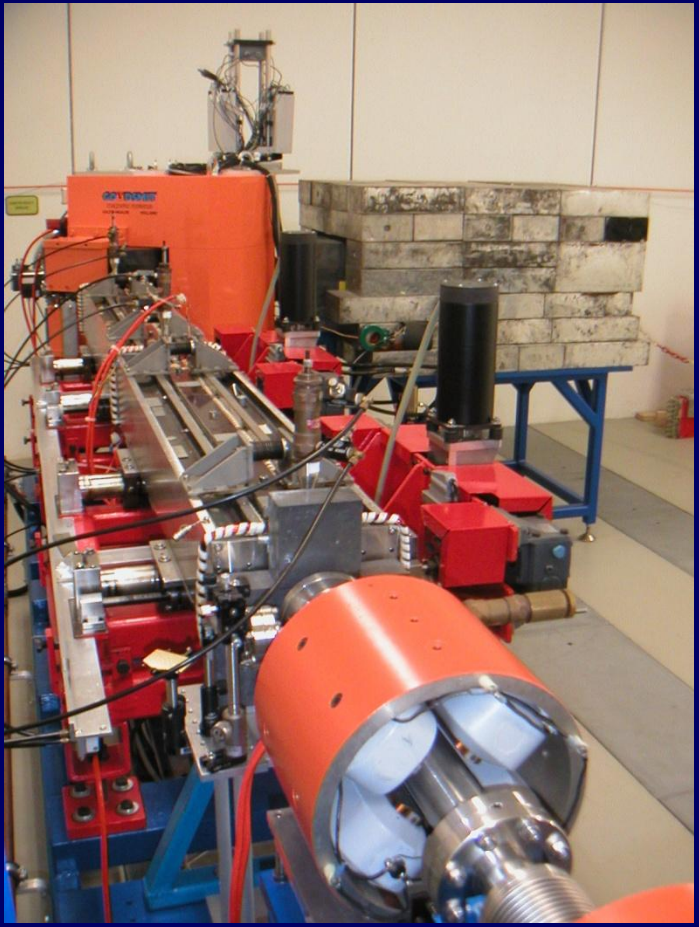
Principle of FEL oscillator



Typical Undulator

# FELs can be small ….





**FELIX Facility,**

**Rijnhuizen, The Netherlands**

# Or big …..


Stanford, CA, USA
Google Campus, US
Image © 2008 TerraMetrics
©2008 Google
6.55 km
37°24'54.37" N 122°08'40.77" W
Jul 2007
Eye alt 22.65 km


I280
3 km
LCLS


LINAC COHERENT LIGHT SOURCE
Injector
Beam Compressor #1
Linac
X-ray Optics
Beam Compressor
Experimental Hall
A
B
Undulator

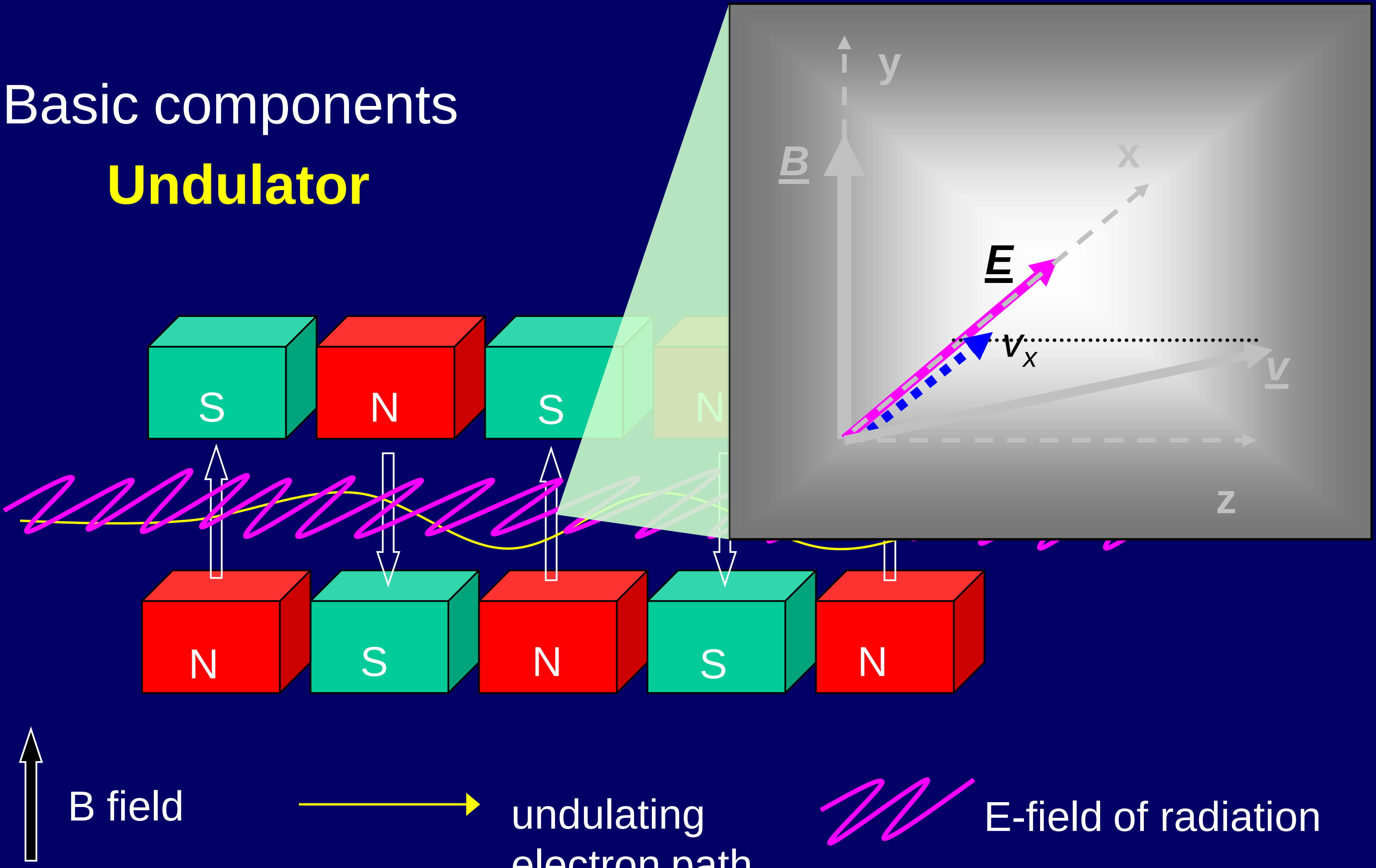
# Two basic types of FEL:

- **AMPLIFIER (HIGH GAIN) FEL**
  - Long undulator (no optical cavity *)
  - Spontaneous emission from start of undulator interacts with electron beam.
  - Interaction between light and electrons grows, producing microbunching
  - Increasing intensity gives stronger bunching, yielding stronger emission
  - >>> High optical intensity achieved in **single pass** **(SASE)**
- **OSCILLATOR (LOW GAIN) FEL**
  - Short undulator
  - Spontaneous emission trapped in an optical cavity
  - Trapped light interacts with successive electron bunches leading to microbunching and coherent emission
  - >>> High optical intensity achieved over **many passes**

# How does a free-electron laser work?

- ★ Basic components

**Undulator**



B field

undulating electron path

E-field of radiation


UPPER
LOWER


ACHTUNG
STARKES MAGNETFELD
UND-1
V2
H1

# How does an FEL work?

- **Basic physics: Work = Force x Distance**
  - **Sufficient to understand basic FEL mechanism**
  - **Electric field of light wave produces a force on electron and work is done!**

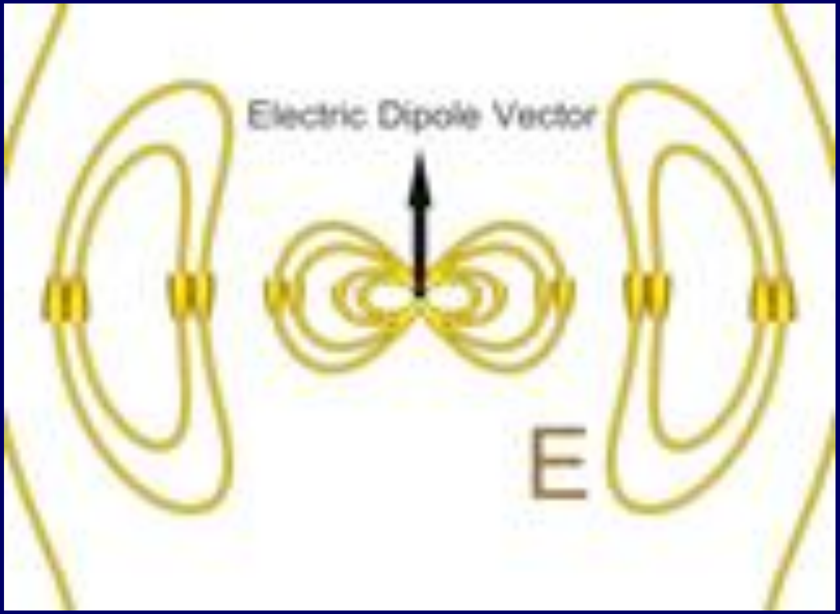
$$\boxed{\triangle W = -e \int \mathbf{E} \cdot \mathrm{ds} = -e \int \mathbf{v} \cdot \mathbf{E} \mathrm{d}t}$$

- **No undulator = No energy transfer**

  i.e. If electron velocity is entirely longitudinal then v.E = 0

- Basic mechanism very simple !!

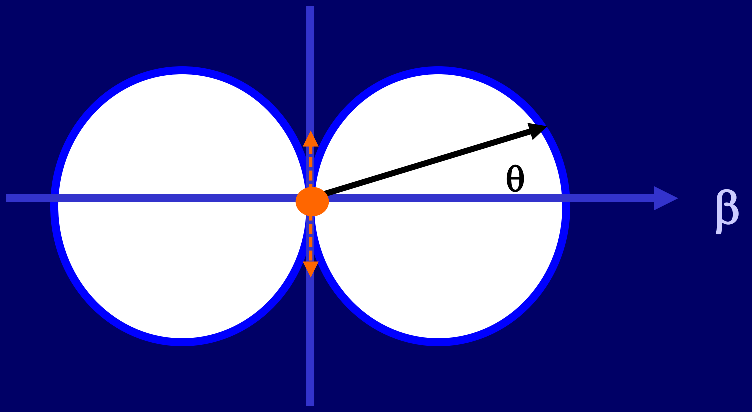
# Generation of EM Radiation



non-relativistic charge source

# Relativistic Emission

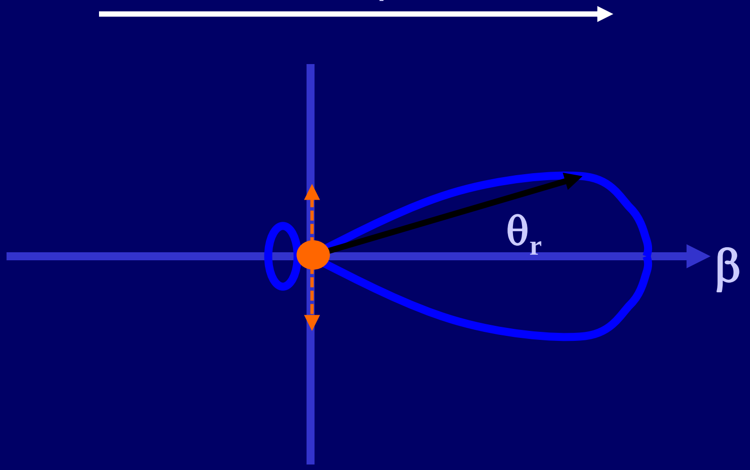
stationary electron



Energy emission confined to directions perpendicular to axis of oscillation

relativistic electron

$v \sim\leq c \qquad \beta = v/c$



Most energy confined to the relativistic emission cone

$$\theta_r = \gamma^{-1}$$

1. Relativistic electrons “see” Lorentz-contracted undulator (by a factor $\gamma$).

   → emit radiation due to large transverse oscillations

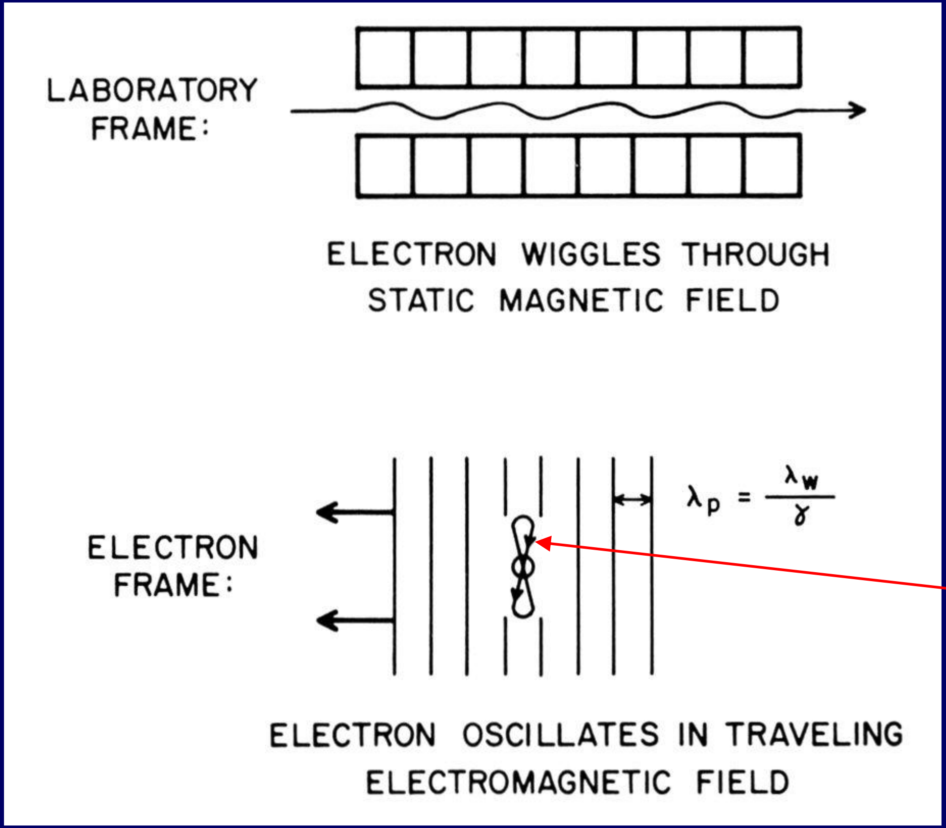
2. Radiation emitted in *rest frame* of electrons.

   Transforming back to *LAB frame upshifts* frequency by *another factor* $\gamma$

So emitted wavelength is:

$$\boxed{\lambda_L = \frac{\lambda_w}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)}$$

$$K = \frac{e}{2\pi mc} B_0 \lambda_w$$

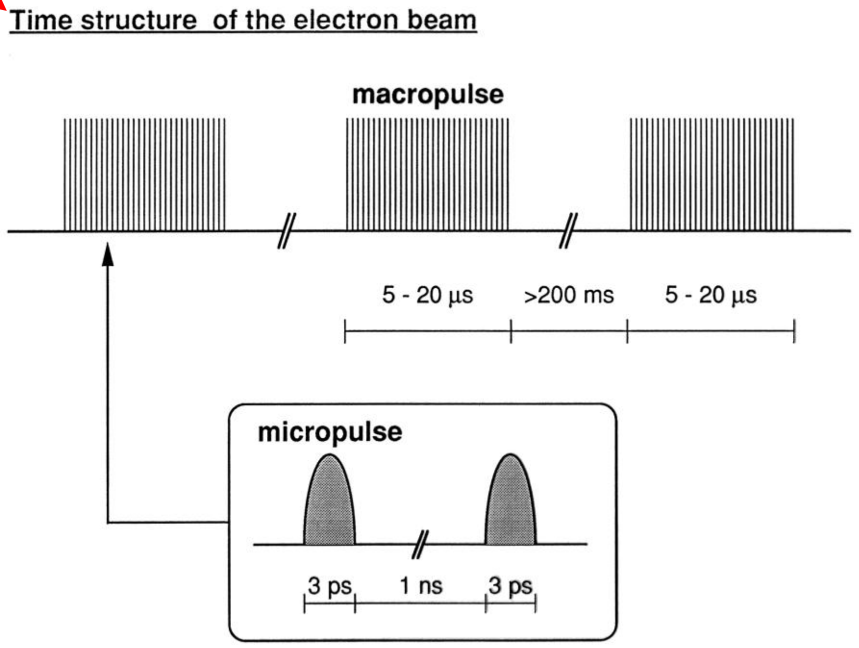


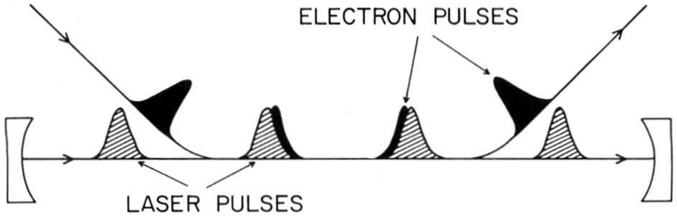
magnetic field gives both transverse and longitudinal velocity additions – hence “figure-of-eight”

High K ⇒ copious harmonics !

Electron beam from radiofrequency accelerator comes in short bunches

… so must synchronise e-bunches with laser pulses circulating in optical cavity





*Schematic diagram of a free-electron laser oscillator operating on the pulsed electron beam from an rf accelerator*

FEL *wavelength* can therefore be varied using 2 parameters: $\lambda_u$ and $\gamma$

$\lambda_u$ → undulator technology

$\gamma$ → electron energy

# How does an FEL work?

- ★ Basic mechanism described explains energy transfer between SINGLE electron and an optical field.

But in practice need to create the right conditions for :

**CONTINUOUS** energy transfer

in the **RIGHT DIRECTION**

with a **REAL ELECTRON BEAM**

★ **Q.** How do we ensure continuous energy transfer over length of undulator ?

★ **A.** Inject at **RESONANT ENERGY**:

$$\gamma_r = \sqrt{\frac{\lambda_w}{2\lambda_L}\left(1+\frac{K^2}{2}\right)}$$

$\lambda_w$ = wiggler(undulator) wavelength

$\lambda_L$ = FEL output wavelength

K is pptl to magnetic field strength

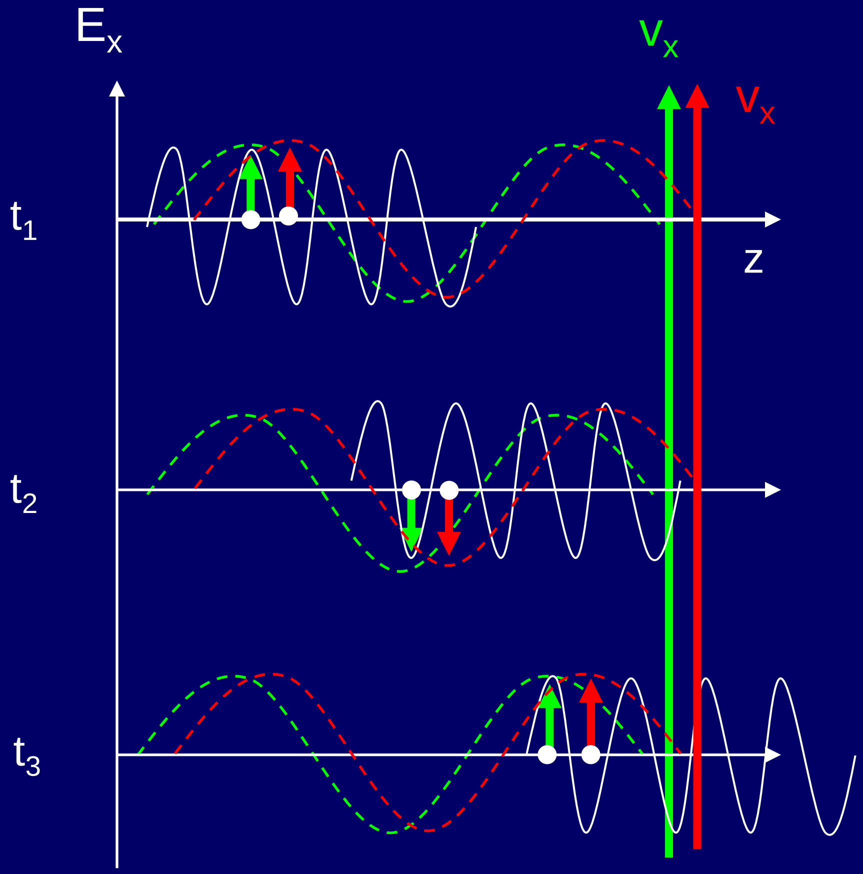
- ★ This is the energy at which the electron slips back 1 radiation wavelength per undulator period: relative phase between electron transverse velocity and optical field **REMAINS CONSTANT**
- ★ *Why does it slip back?* Because electron longitudinal velocity < c :
  - ★ Electrons not 100% relativistic
  - ★ Path length increased by transverse oscillations (undulations)

# How does an FEL work?

- **Q**. Which way does the energy flow ?
- **A**. Depends on the electron phase

Depending on phase, electron either:

- loses energy to optical field and decelerates: **GAIN**
- takes energy from optical field and accelerates: **ABSORPTION**



★ **Q.** What about the situation with a real e-beam?

★ **A.** Electrons distributed evenly in phase:

- ★ For every electron with phase corresponding to **gain** there is another with phase corresponding to **absorption**
- ★ **So at resonant energy Net gain is zero**

**So is this the end of the story??**

**No! There is a way to proceed:**

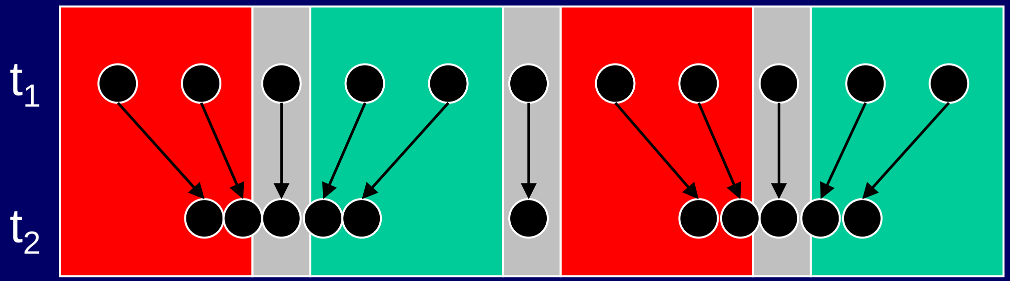
- **Energy modulation gives bunching.**
- **At resonant energy, bunching is around phase for zero net gain.**
- **By giving the electrons a bit of an energy *kick* we can shift them along in phase a bit and get bunching around a phase corresponding to positive net gain.**

# How does an FEL work?

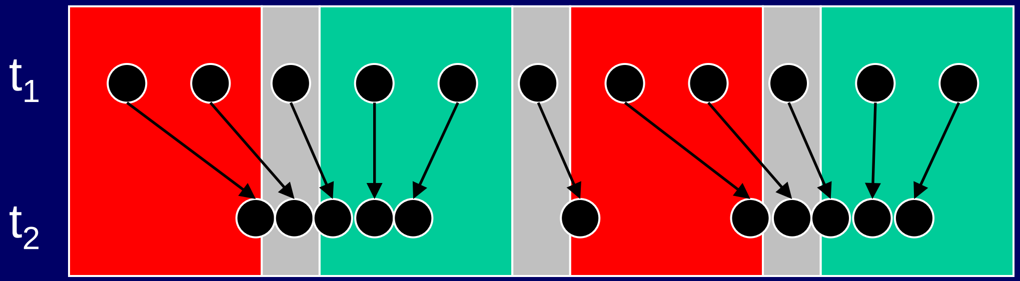
- Bunching

🟩 Phase corresponding to GAIN

🟥 Phase corresponding to ABSORPTION



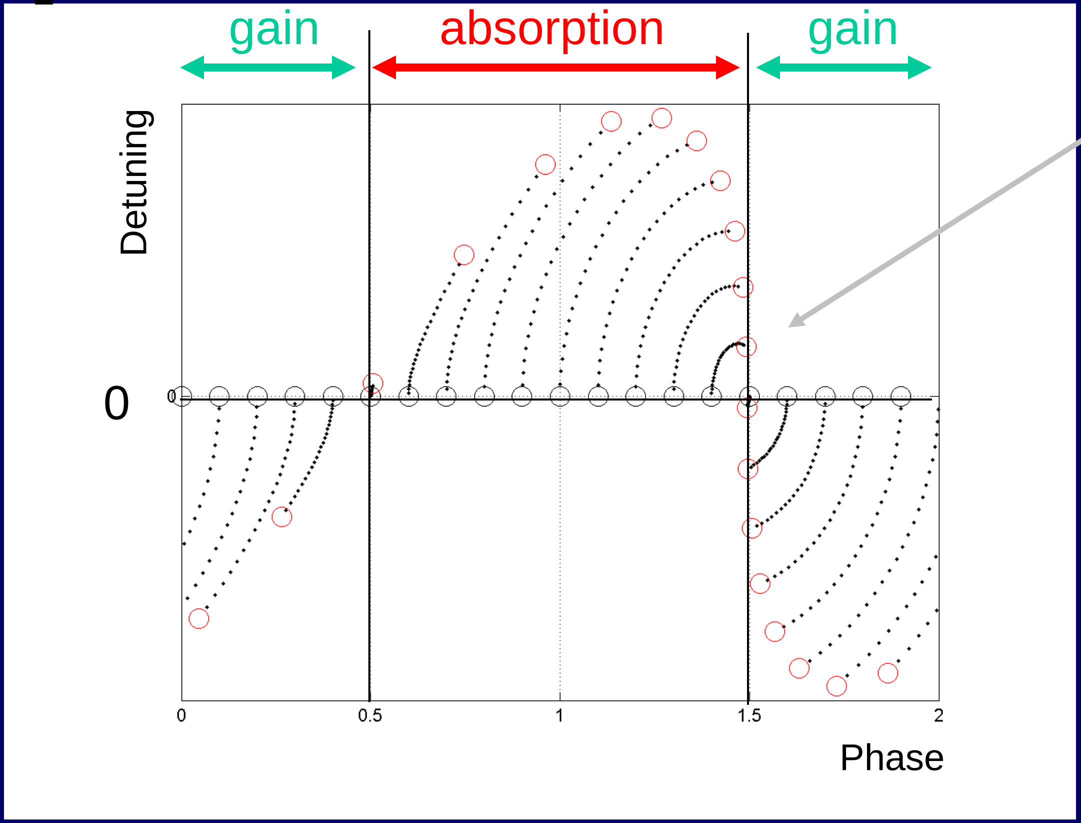
**E = RESONANT ENERGY**
**Bunching around phase corresponding to**
**ZERO NET GAIN**



**E > RESONANT ENERGY**
**Bunching around phase corresponding to**
**POSITIVE NET GAIN**

# simulation example

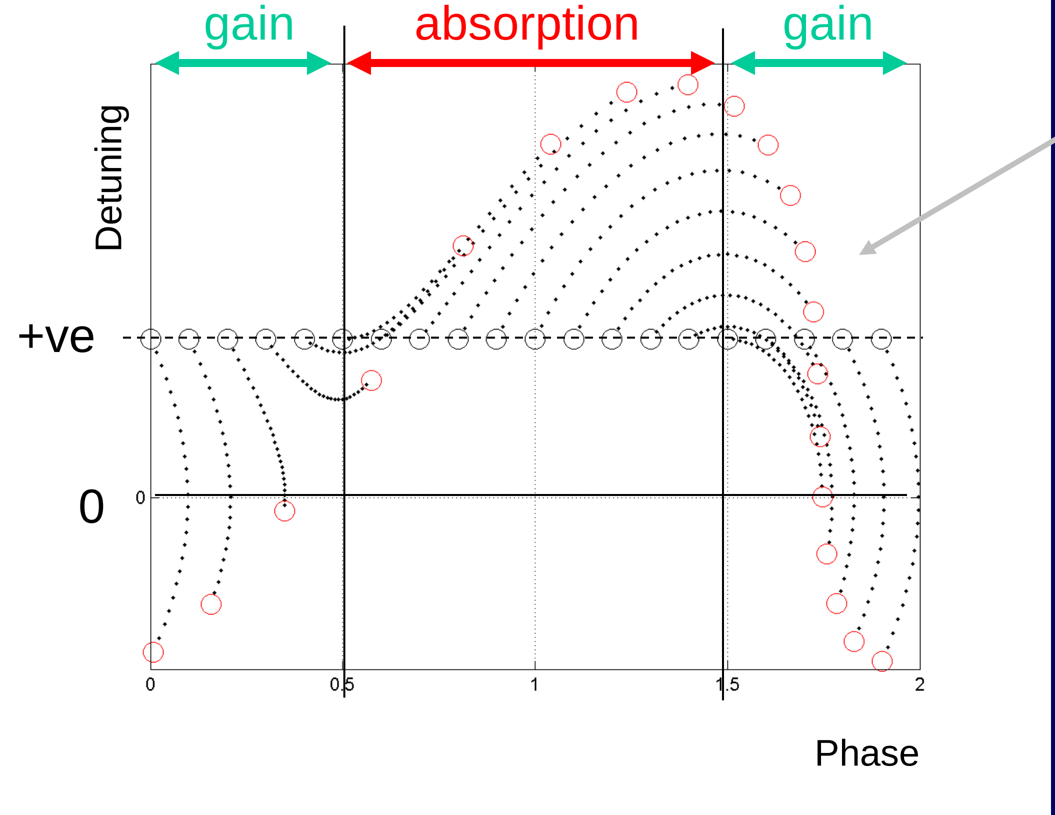
★ Inject 20 electrons at resonance energy: zero detuning



- Bunching around phase correponding to zero gain
- 10 electrons lose energy
- 10 electrons gain energy

# simulation example

★ Inject 20 electrons above resonance energy: +ve detuning



- Bunching around phase correponding to +ve gain
- 11 electrons lose energy
- 9 electrons gain energy

# How does an FEL work?

For **GAIN > ABSORPTION** inject electrons at energy *slightly higher than resonant energy*

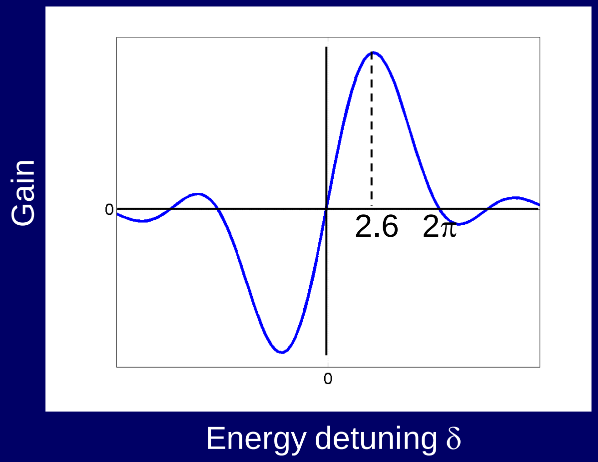
***'POSITIVE DETUNING'***

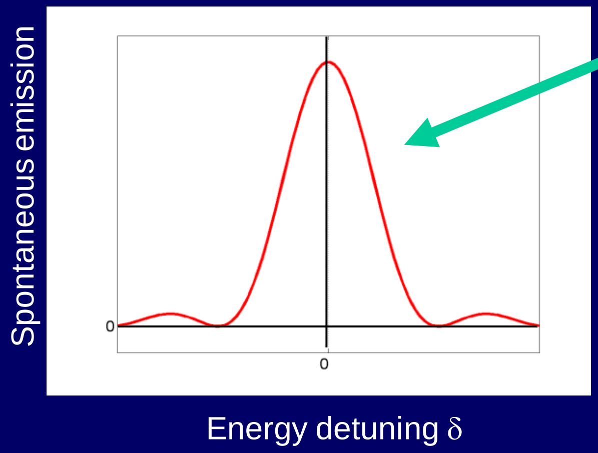
\>>>> NET TRANSFER of energy to optical field

*NB: **This is only true for LOW GAIN (OSCILLATOR) FELs.***

*For a HIGH GAIN FEL the maximum gain occurs at a positive detuning much closer to zero. But that's another story….*

# Small-Signal Gain Curve

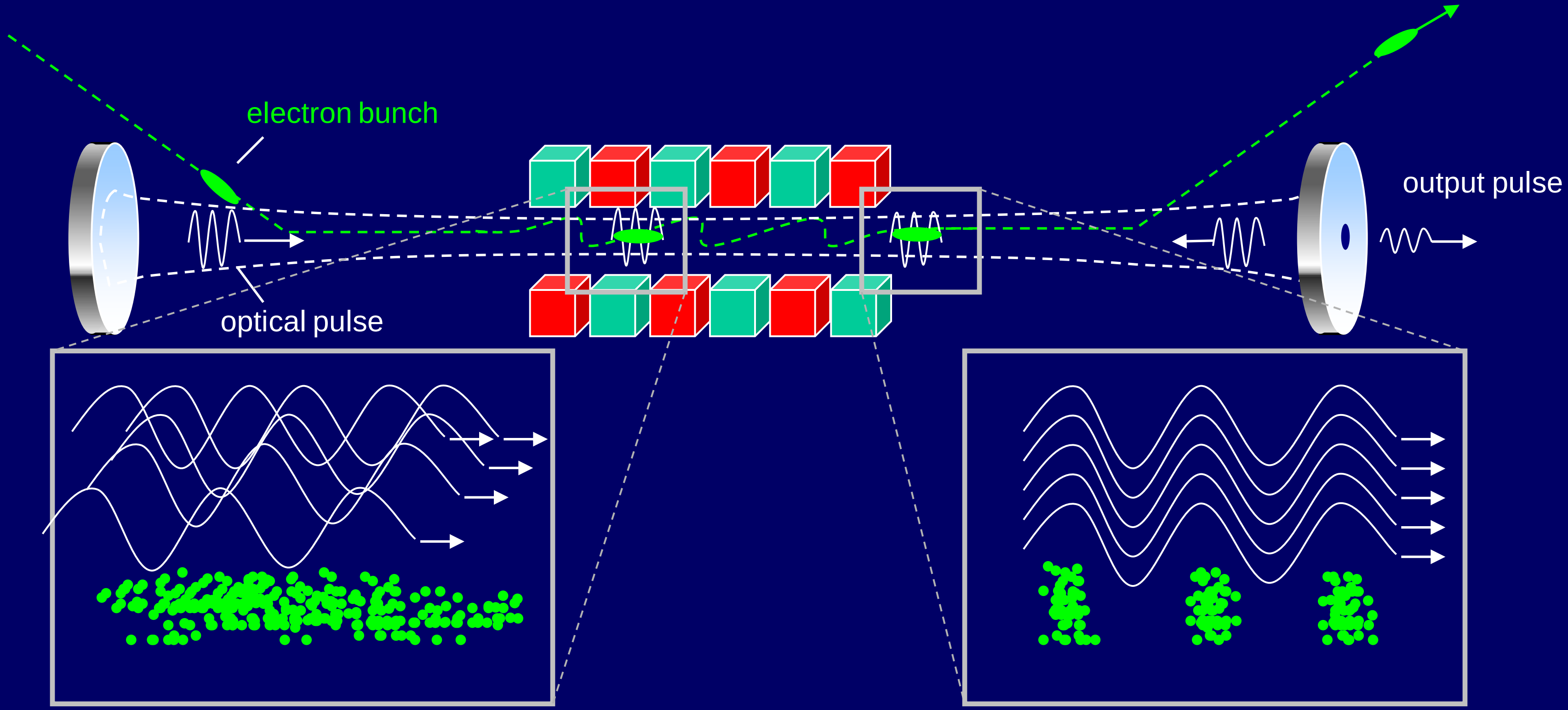




- **Shows how gain varies as a function of detuning**
- **Detuning parameter**
  - $\delta = 4\pi N(\Delta E/E)$
  - Maximum gain for $\delta = 2.6$
- **Madey Theorem**
  - *"Gain curve is proportional to negative derivative of spontaneous emission spectrum"*

**Broadening of natural linewidth causes gain degradation**

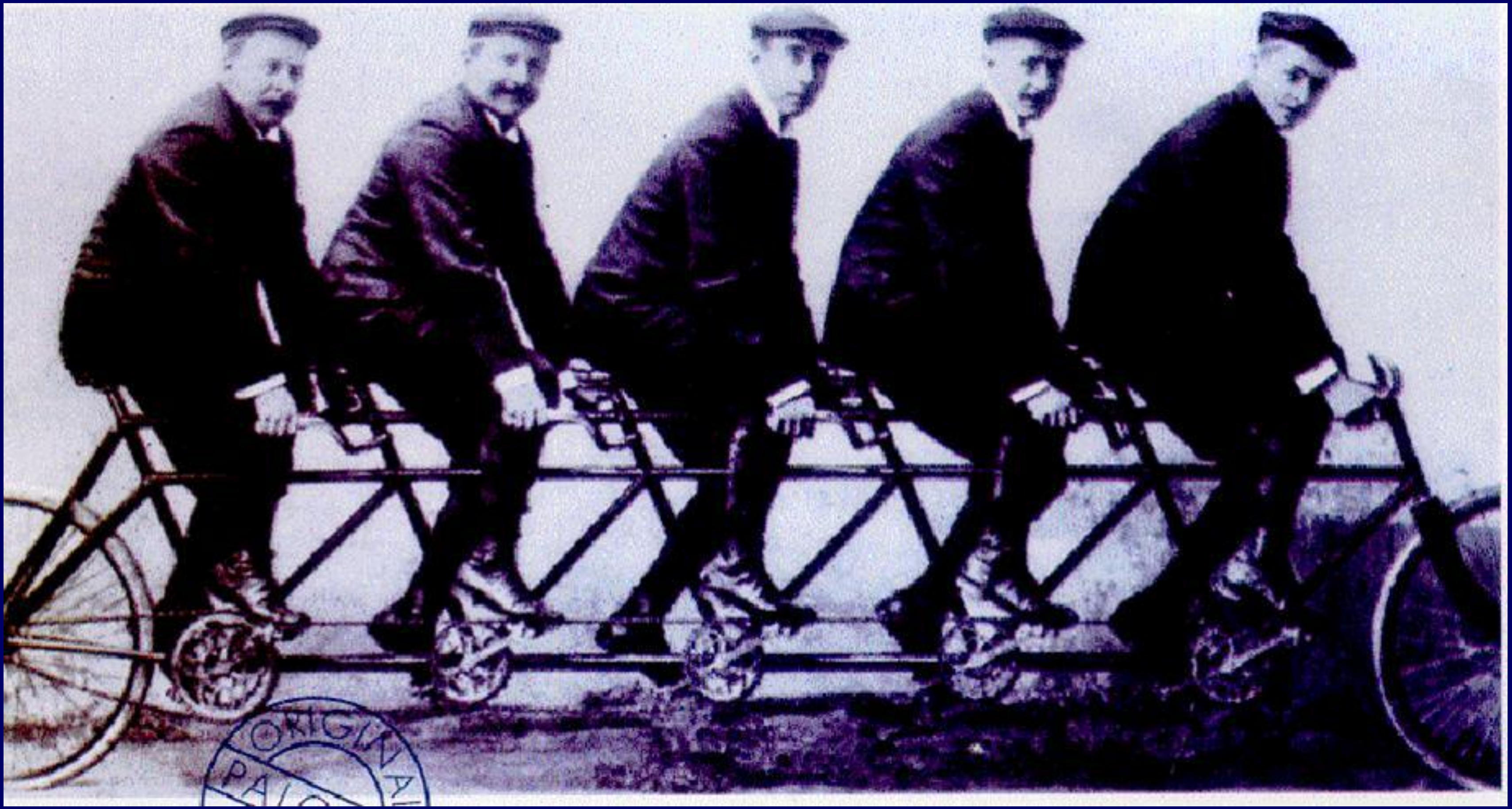
# The Oscillator FEL



Random electron phase:
**incoherent** emission

Electrons bunched at
radiation wavelength:
**coherent** emission

# Coherent action is what counts …



# The Oscillator FEL

- For oscillator FEL the single pass gain is small
- The emitted radiation is contained in a resonator to produce a FEEDBACK system
- In each pass the radiation is further amplified
- Some radiation is extracted, most radiation reflected
- **Increasing** cavity intensity **strengthens interaction** leading to **exponential growth**:

$$\triangle W = -e \int \mathbf{E} \cdot \mathrm{d}\mathbf{s} = -e \int \mathbf{v} \cdot \mathbf{E} \mathrm{d}t$$

**Energy transfer** depends on **cavity intensity**

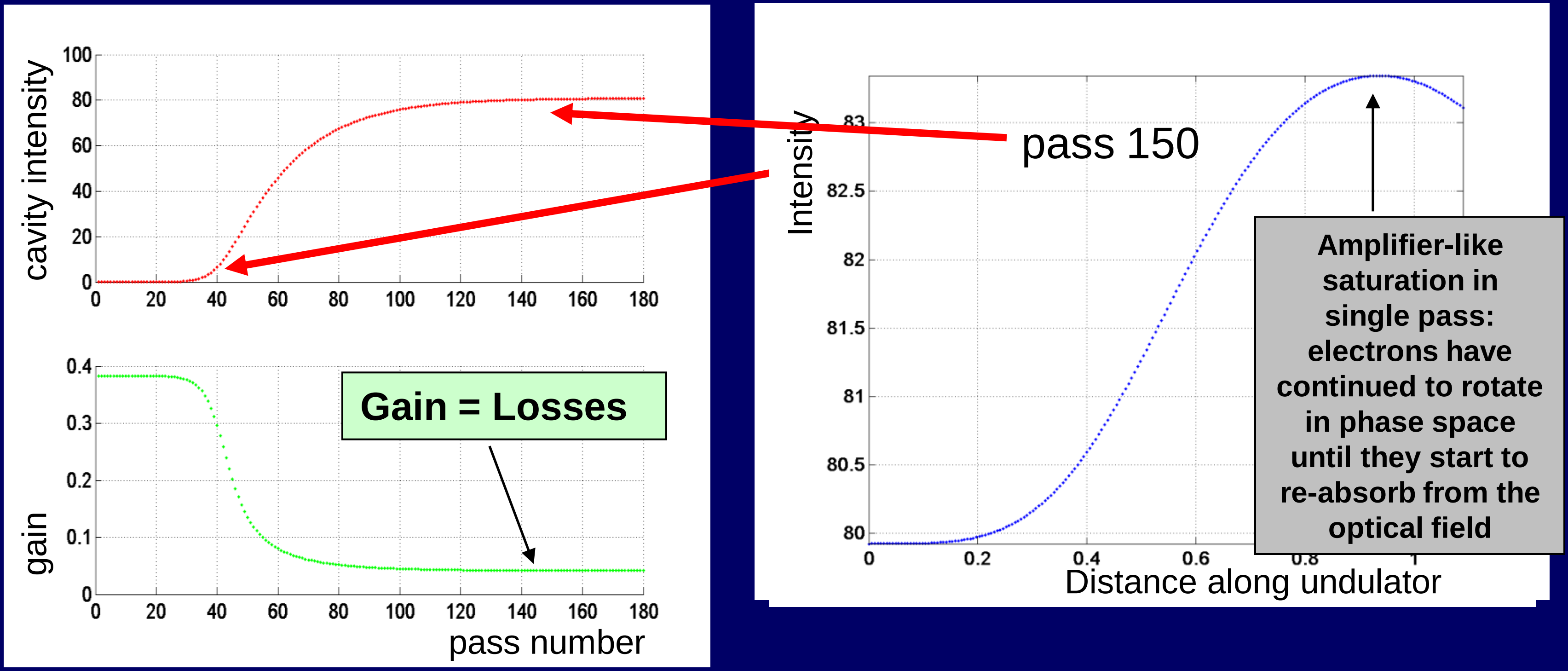
# Saturation: Oscillator

- For **ZERO** cavity loss intensity would increase indefinitely!
- *In practice* we have **passive loss** (diffraction, absorption) and **active loss** (outcoupling)

- Power lost is proportional to cavity intensity
- As intensity rises so does power loss
- Finite extraction efficiency
- Eventually power lost = power extracted from electrons
- No more growth. **SATURATION.**
- (Equivalently gain falls until it equals cavity losses)

# Saturation: Oscillator

- ★ Parameters: cavity loss = 4%



pass 150

**Gain = Losses**

**Amplifier-like saturation in single pass: electrons have continued to rotate in phase space until they start to re-absorb from the optical field**

# Choosing the required parameters

To achieve lasing with our Oscillator FEL the following parameters must be optimised :

- ★ Electron beam parameters
  - ★ Energy, Peak Current, Emittance, Energy spread
- ★ Undulator parameters
  - ★ K, Period, Number of periods (“wavelengths”)
- ★ Resonator parameters
  - ★ Length, mirror radii of curvature

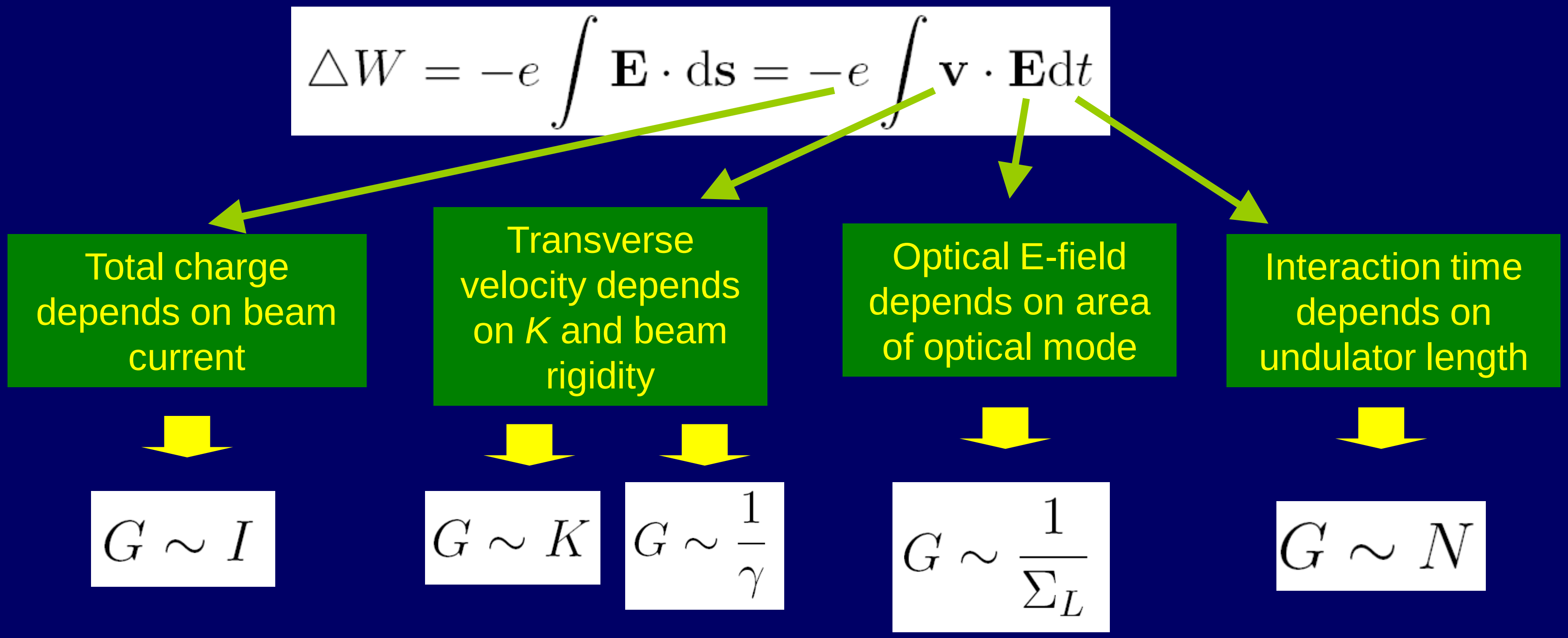
**For lasing must have**
**GAIN > LOSSES**

# Required parameters

- To ‘first order’
  - Select base parameters to optimise gain
- To ‘second order’
  - Optimise other parameters to minimise gain degradation

# Gain Scaling

- We can get an idea of how the gain should scale with various parameters by looking at our familiar equation



$$\triangle W = -e\int \mathbf{E}\cdot \mathrm{d}\mathbf{s} = -e\int \mathbf{v}\cdot \mathbf{E}\,\mathrm{d}t$$

Total charge depends on beam current

$$G \sim I$$

Transverse velocity depends on *K* and beam rigidity

$$G \sim K$$

$$G \sim \frac{1}{\gamma}$$

Optical E-field depends on area of optical mode

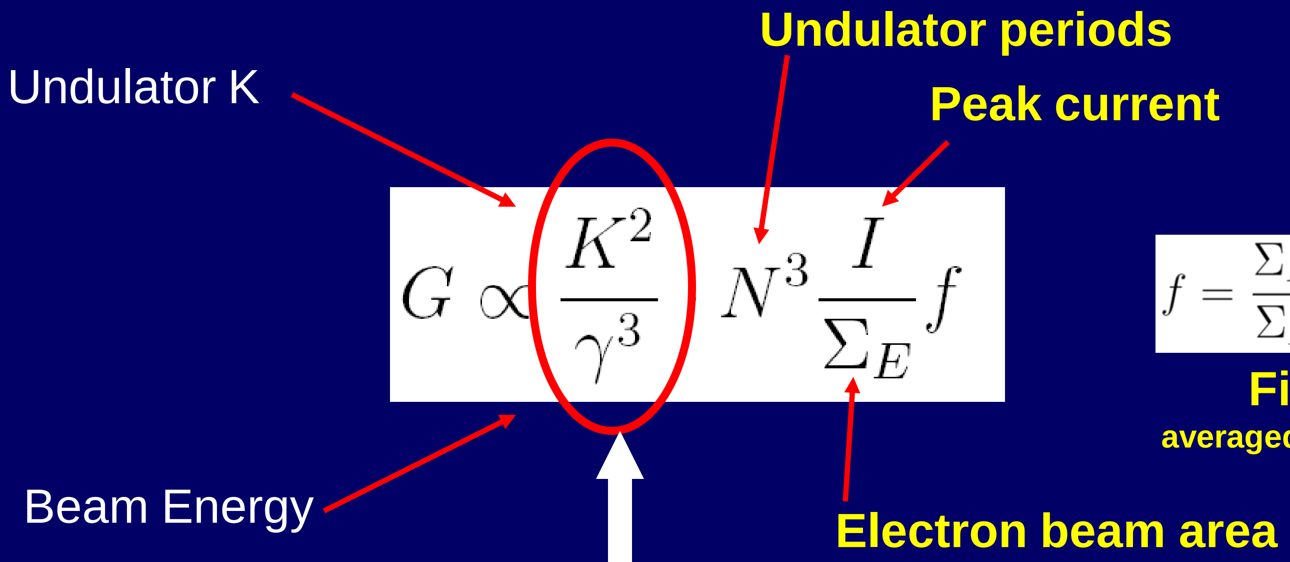
$$G \sim \frac{1}{\Sigma_L}$$

Interaction time depends on undulator length

$$G \sim N$$

# Small Signal Gain

- In fact, small-signal, single-pass maximum gain is given by:



**Undulator periods**

Undulator K

**Peak current**

$$G \propto \frac{K^2}{\gamma^3} N^3 \frac{I}{\Sigma_E} f$$

Beam Energy

**Electron beam area**

$$f = \frac{\Sigma_E}{\Sigma_L} \quad (\Sigma_E < \Sigma_L)$$

**Filling factor:**

**averaged over undulator length**

These parameters are varied to tune FEL: $\lambda_L = \frac{\lambda_w}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$

# Small Signal Gain

- To summarise, at a given wavelength we need:

| | |
|---|---|
| A long enough undulator | To allow sufficient interaction time |
| A good peak current and a tightly focussed electron beam | To provide high charge density |
| A small optical cross section | To provide high E field |

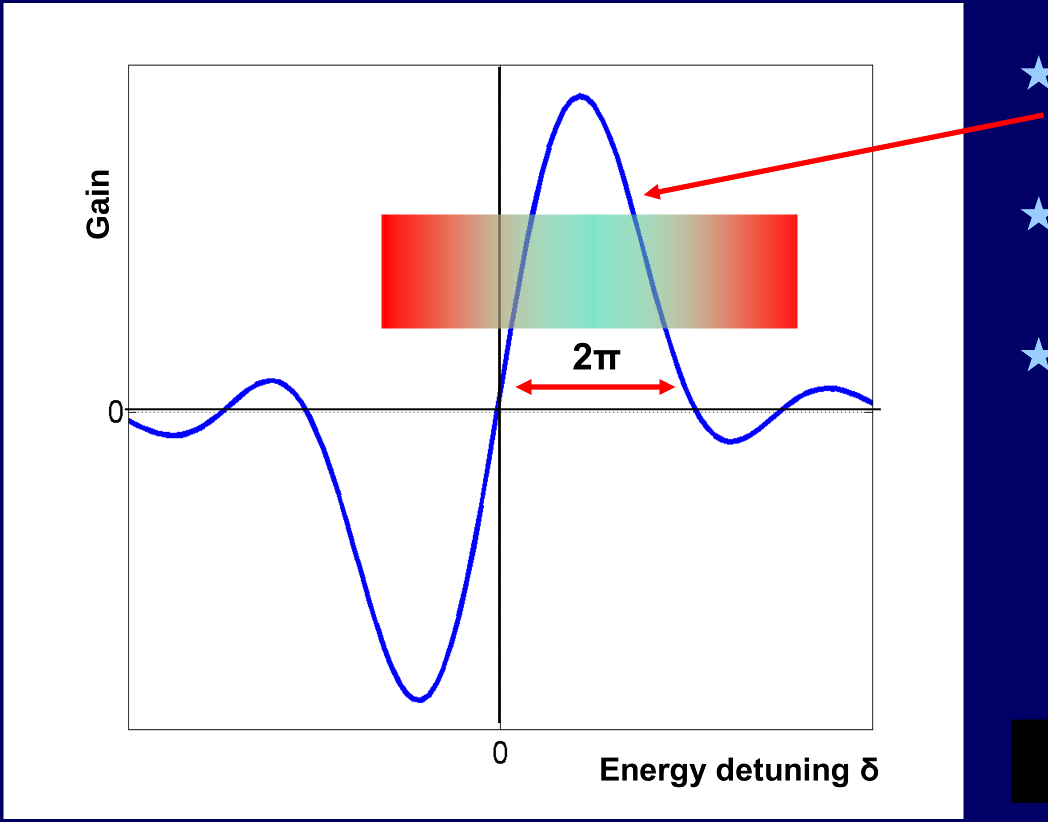
# Electron beam quality

- Now we’ve selected parameters to optimise the gain, we need to minimise the gain degradation

- Gain is degraded due to
  - energy spread
  - emittance

**For optimum FEL performance a high quality electron beam is required.**

# Electron Energy Spread



- ★ **SMALL SIGNAL GAIN CURVE**:
- ★ Small energy range for positive gain
- ★ If energy spread is too large electrons fall outside detuning for positive gain

**GAIN DEGRADATION**

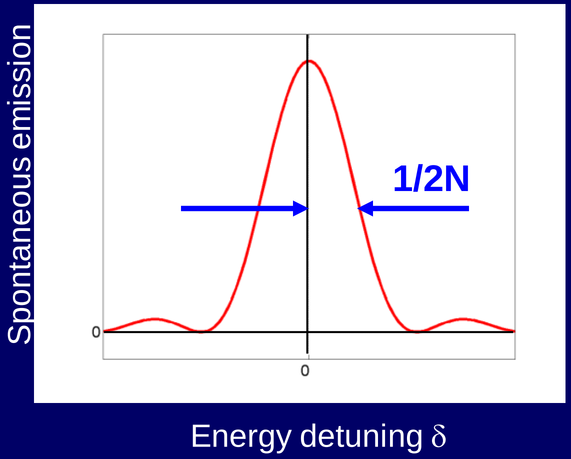
# Electron Energy Spread

- ★ Derivation of Limit on Energy Spread:
  - ★ FEL wavelength given by:

$$\lambda_L = \frac{\lambda_w}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

- Expand this in Taylor se
  giving linewidth spread
  to energy perturbation

**Typical IRFEL**:
N=42
Gives rms energy spread
of < 0.1% for negligible
gain degradation



- Require that spread is less than the natural line halfwidth **so that no broadening occurs and gain is not reduced**

$$\left|\frac{\Delta\lambda}{\lambda}\right| < \frac{1}{2N}$$

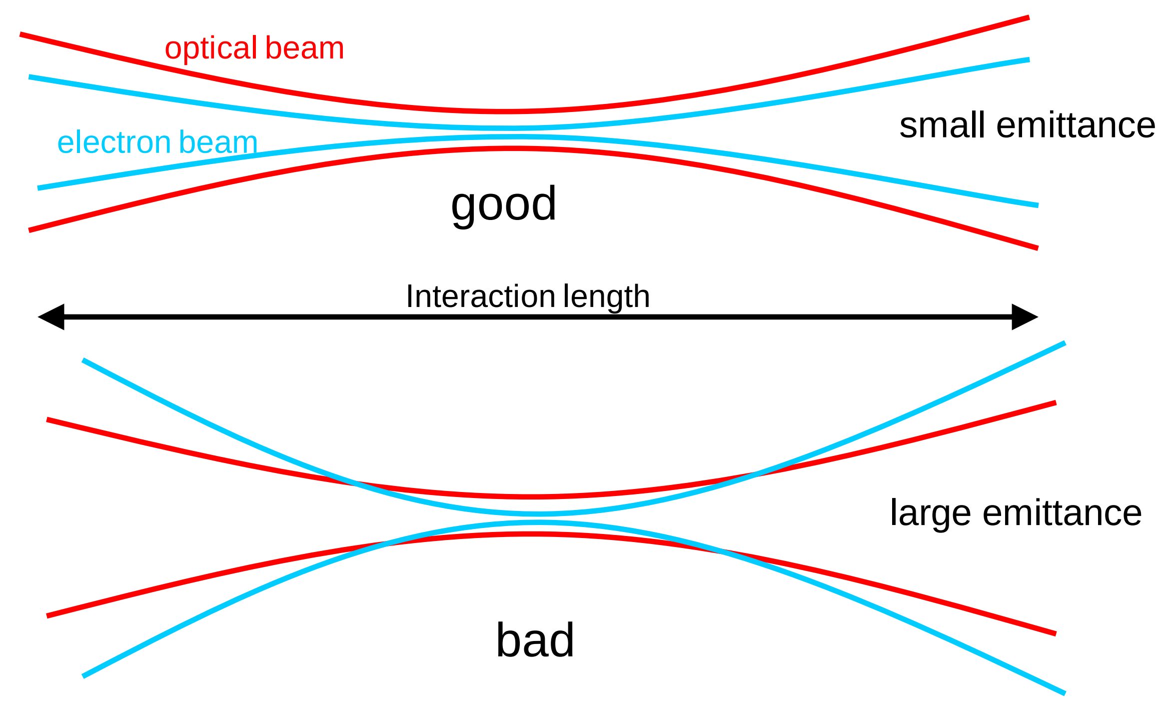
$$\boxed{\left|\frac{\Delta\gamma}{\gamma}\right|_{\mathrm{FULL}} < \frac{1}{4N}}$$

# Electron Beam Emittance

- The emittance controls :
  - 1. **BEAM DIVERGENCE**: this effects overlap between electron beam and optical mode
  - 2. **BEAM SIZE**: this affects quality of undulator field seen by electrons
  - We can do a separate analysis for each case to see how small an emittance we need to avoid gain degradation

# Emittance 1: overlap

- We need the electron beam contained within the optical beam over the whole interaction length



# Emittance 1: overlap

- Electron beam envelope equation at a waist gives electron beam Rayleigh length:

$$z_E = \frac{\gamma r_0^2}{\varepsilon_n}$$

- Similarly, Gaussian beam equations in laser resonator give optical Rayleigh length:

$$z_R = \frac{\pi w_0^2}{\lambda_L}$$

- For electron beam confinement need:

$$z_E \geq z_R$$

- So:

$$\boxed{\varepsilon_n < \frac{\gamma \lambda_L}{\pi} \left( \frac{r_0}{w_o} \right)^2}$$

**Typical IRFEL**:

Gives normalised emittance of < 10 mm-mrad

# Emittance 2 : broadening

- As emittance increases beam size increases, so electrons move more off-axis

- Undulator field has a sinusoidal *z*-dependence on axis, so off-axis electrons experience a different field (because curl B = 0) and thus a different *K.*

wavelength shift → linewidth broadening → gain degradation

- By requiring that linewidth broadening is within the natural linewidth, the following restriction can be derived:

$$\boxed{\varepsilon_n < \frac{\gamma^2 \lambda_L}{N K}}$$

→

**Typical IRFEL**:

Gives normalised emittance of < 500 mm-mrad

# Longitudinal effects

- So far we've only considered an 'infinitely long' electron beam: we haven't worried about the ends.
- Known as the *STEADY STATE* solution
- In reality we have finite electron bunches
- Need to extend model accordingly to include effects of *PULSE PROPAGATION*

**2D MODEL**
**(transverse effects)**

→

**3D MODEL**
**(transverse + longitudinal effects )**

# Longitudinal effects

- Slippage
  - Resonance condition: Electrons slip back by one radiation wavelength per undulator period
  - Slippage per complete undulator traverse = N x wavelength. This is known as the **slippage length**.
  - For short electron bunch and/or long wavelength we have

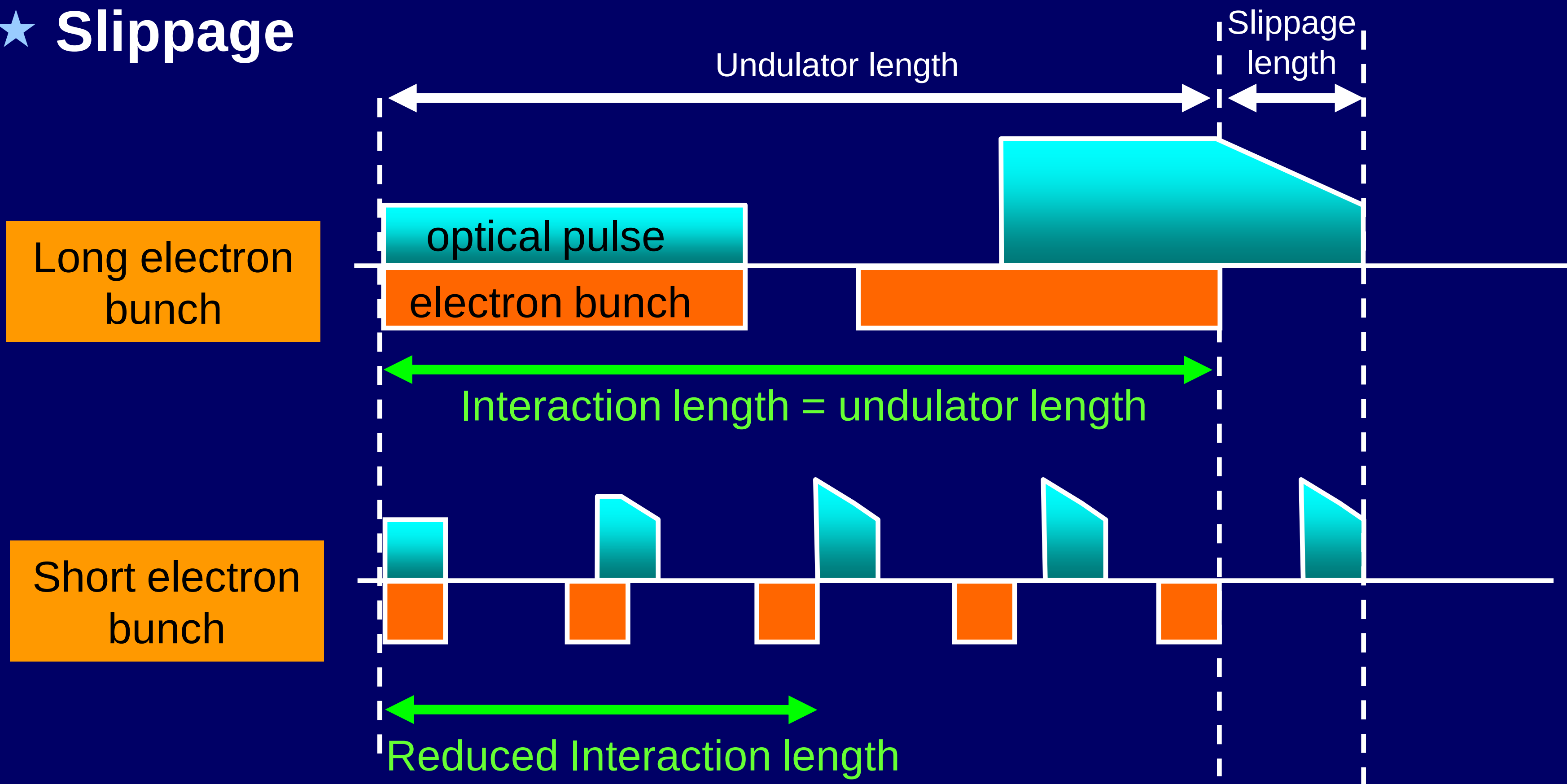
**slippage length ≈ bunch length**

⬇

**Effective interaction length reduced: GAIN DEGRADED.**

# Pulse effects: Slippage

- **Slippage**

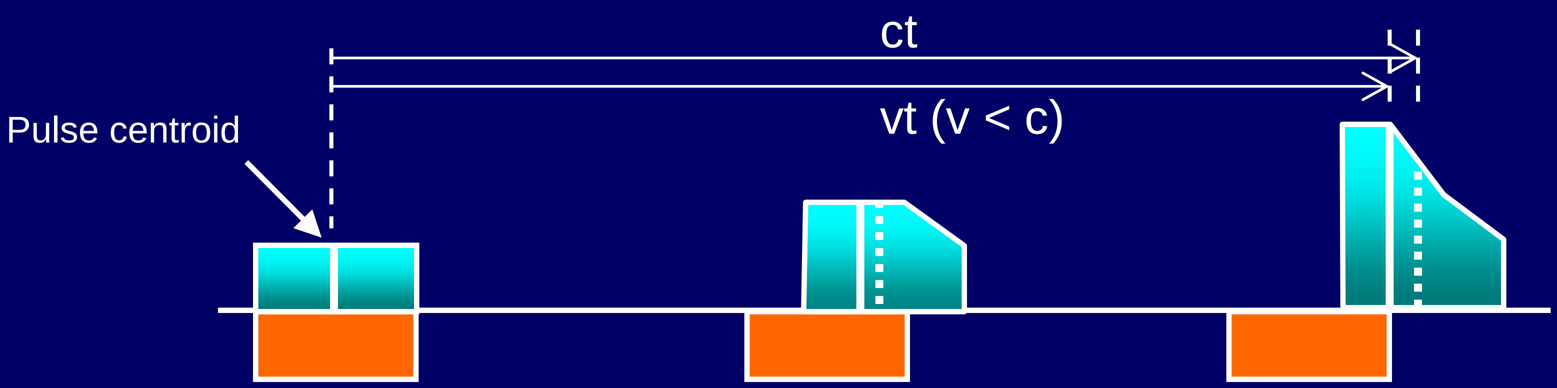


# Conventional laser vs FEL pulses

Conventional laser pulse interacts with all of the active medium

# Pulse effects: Lethargy

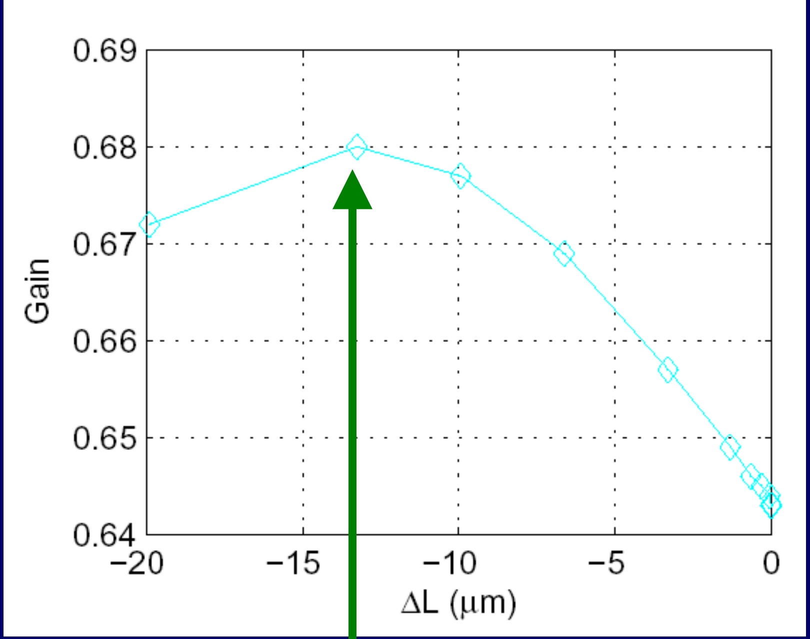
- ★ The electrons slip back over the optical pulse:
  - ★ bunching increases, and maximum emission occurs at end of undulator where bunching is strongest
  - ★ **RESULT**: optical pulse peaks at rear and centroid of pulse has velocity < c.
  - ★ Synchronism between pulse and electron bunch on next pass is not perfect
- ★ Known as laser lethargy



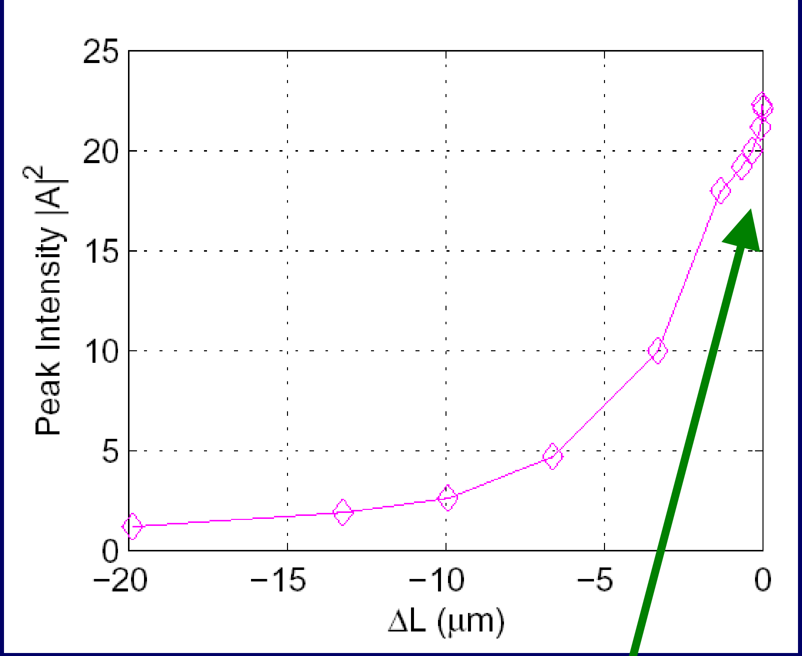
# Pulse effects: Lethargy

- Lethargy can be offset by slightly reducing cavity length : **CAVITY LENGTH DETUNING**
  - Centroid of optical pulse is then synchronised with e-bunch on successive passes
  - BUT: as intensity increases
    - back of pulse saturates first
    - **then** rest of pulse saturates, returning centroid velocity to vacuum value c.
  - So a single detuning can’t compensate for lethargy in both growth and saturation phases.
  - Different detunings exist for gain optimisation and power optimisation

# Cavity length detuning

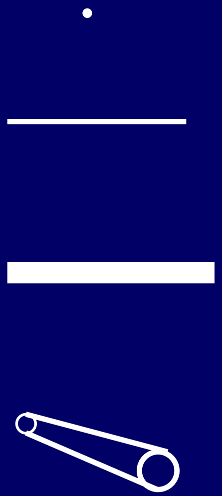


**one cavity length**
for maximum gain



**another cavity length**
for maximum power

# Summary



| | |
|---|---|
| **0D** beam:<br>(single electron) | (Resonance condition) |
| **1D** beam: | (Injection above resonance for net gain) |
| **2D** beam:<br>transverse effects | (Emittance – beam overlap and broadening<br>Energy spread) |
| **3D** beam:<br>longitudinal effects | (Pulse effects – slippage and lethargy) |

# FEL Output: power

Gain curve can be used to estimate maximum output power:

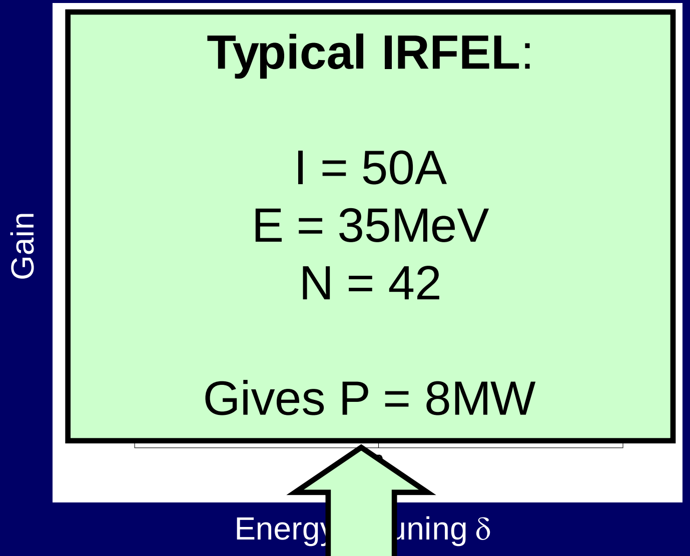
- δ = 4πN(ΔE/E)
- Maximum gain for δ = 2.6
- Maximum energy that can be lost by an electron injected at peak of gain curve is δ = 2.6: no more gain after that.

$$\Delta\delta = 4\pi N \frac{\Delta E}{E} = 2.6$$

$$\Delta E = \frac{2.6}{4\pi N} E.$$

Dividing by time, recognising that at equilibrium
**power extracted from electron beam = output power**

$$P_{\mathrm{out}} = \frac{2.6}{4\pi N} P_{\mathrm{beam}} \simeq \frac{1}{5N} P_{\mathrm{beam}}$$



**Typical IRFEL**:

I = 50A
E = 35MeV
N = 42

Gives P = 8MW

# FEL Output

- At saturation, pulse length matches electron bunch length

- Linewidth depends on pulse length (Fourier):

$$\left(\frac{\Delta\omega}{\omega}\right)_L \sim \frac{\lambda_r}{2\pi\sigma_z}$$

- Brightness: output is coherent, so assuming a diffraction-limited beam:

$$\mathcal{B}_L = \frac{4\dot{N}}{\lambda_r^2} \qquad \dot{N} = \frac{P_{\text{out}}}{hc/\lambda_r}$$

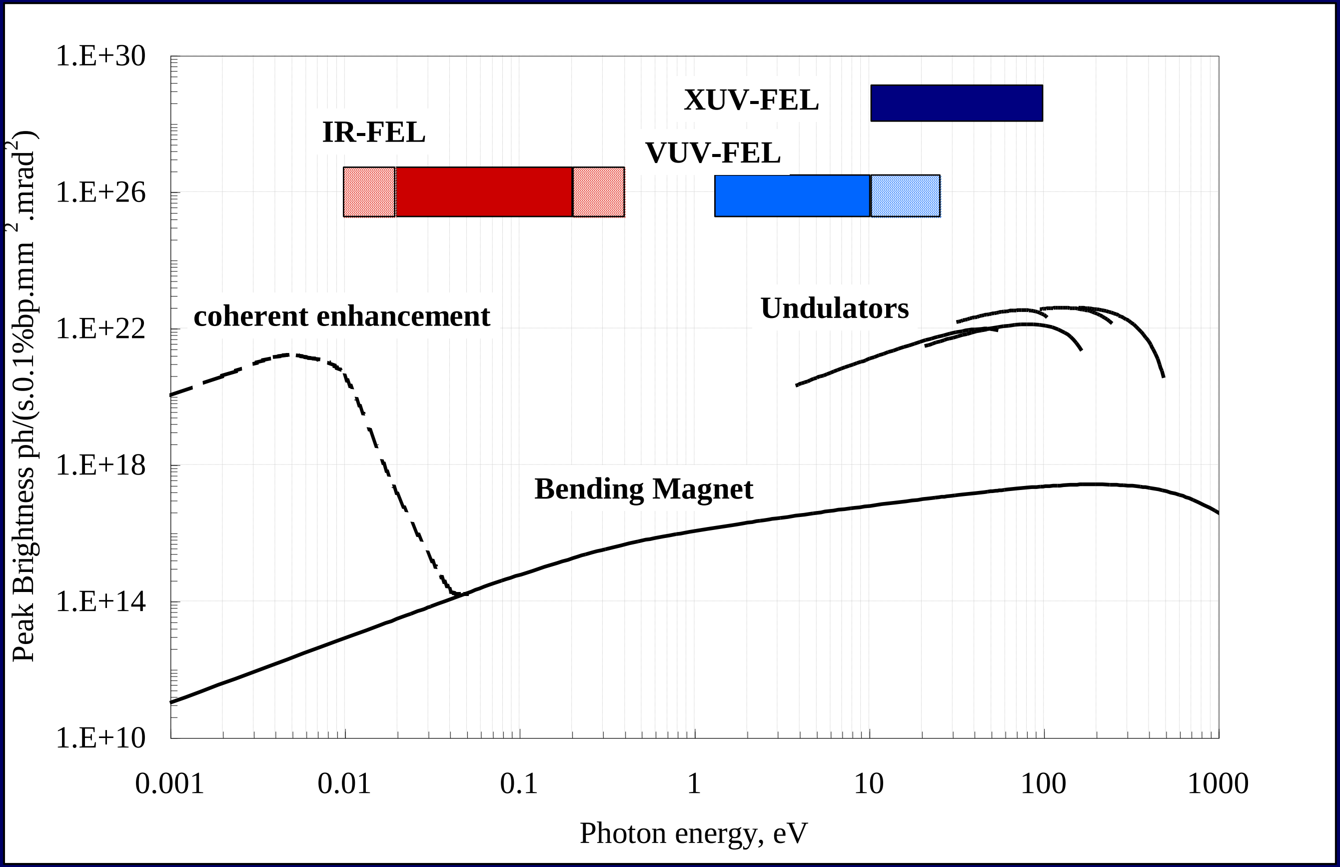
**Typical IRFEL**:

P = 8 MW
Wavelength = 4 micron

Gives B ~ $10^{26}$

- High power enables high brightness!

# Comparative Brilliances



# FEL Output

**TUNABLE OUTPUT !**

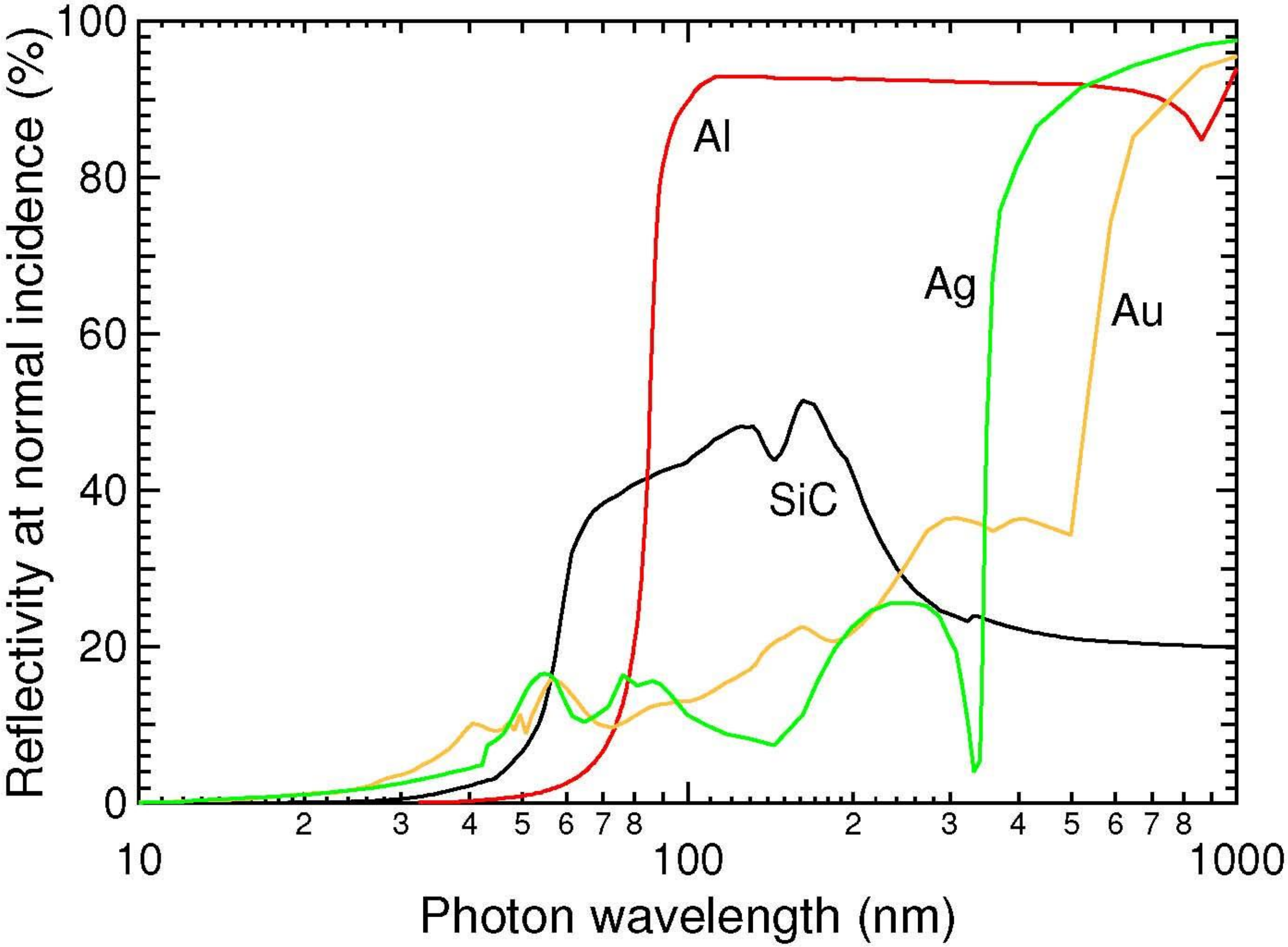
$$\lambda_L = \frac{\lambda_w}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

**A typical FEL facility operates at certain fixed beam energies then tunes over wavelength sub-ranges by varying undulator gap (and hence *K*)**

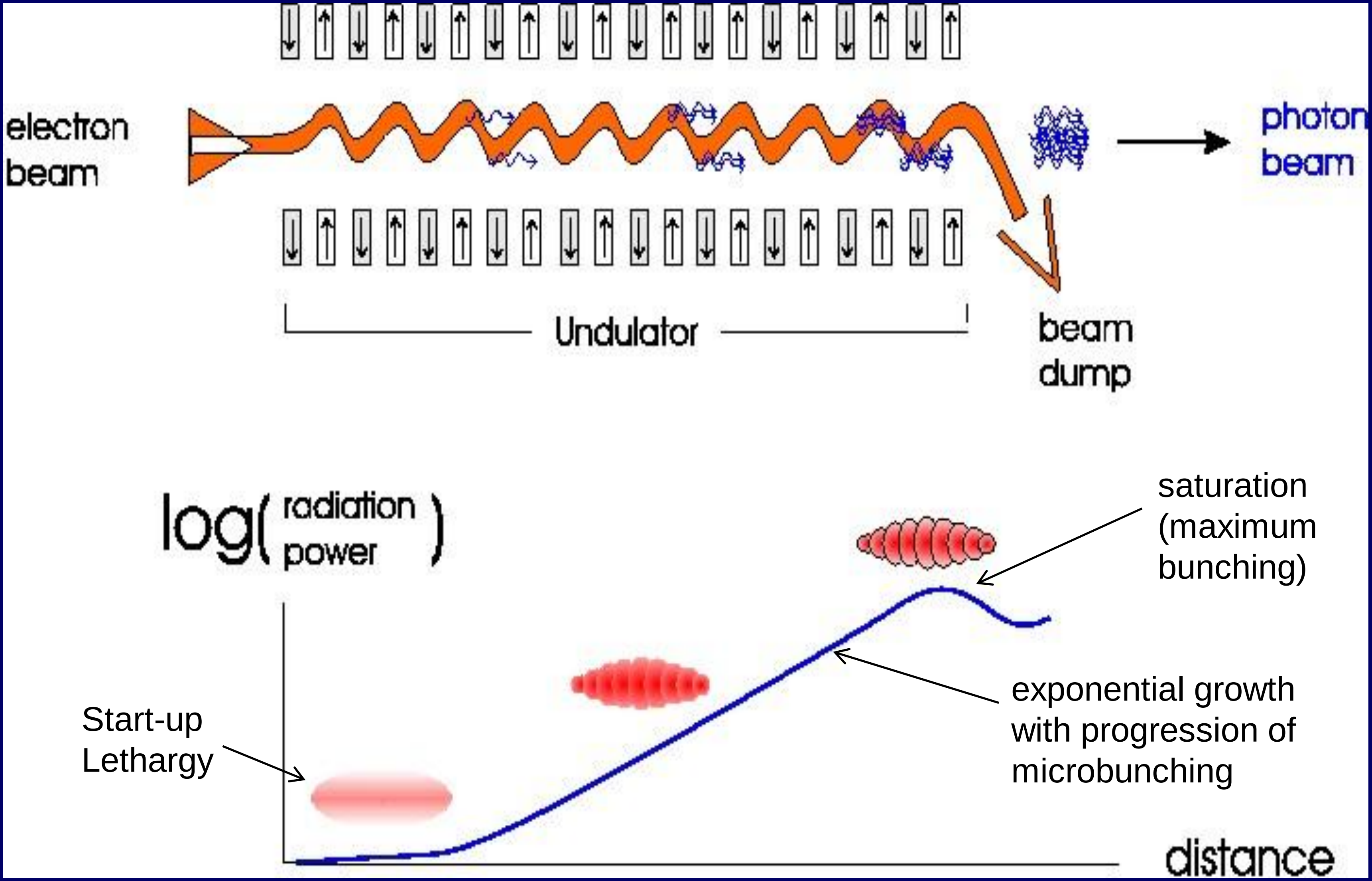
# High Gain (Amplifier) FEL

- No optical cavity / feedback
- Relies on growth of microbunching from shot noise
- Requires very long undulator(s)
- Essential for short-wavelength FELs (XFELs)
- Need to have ultra-precise control of electron beam emittance, size and position

# An optical cavity is no longer possible for wavelengths below 100 nm



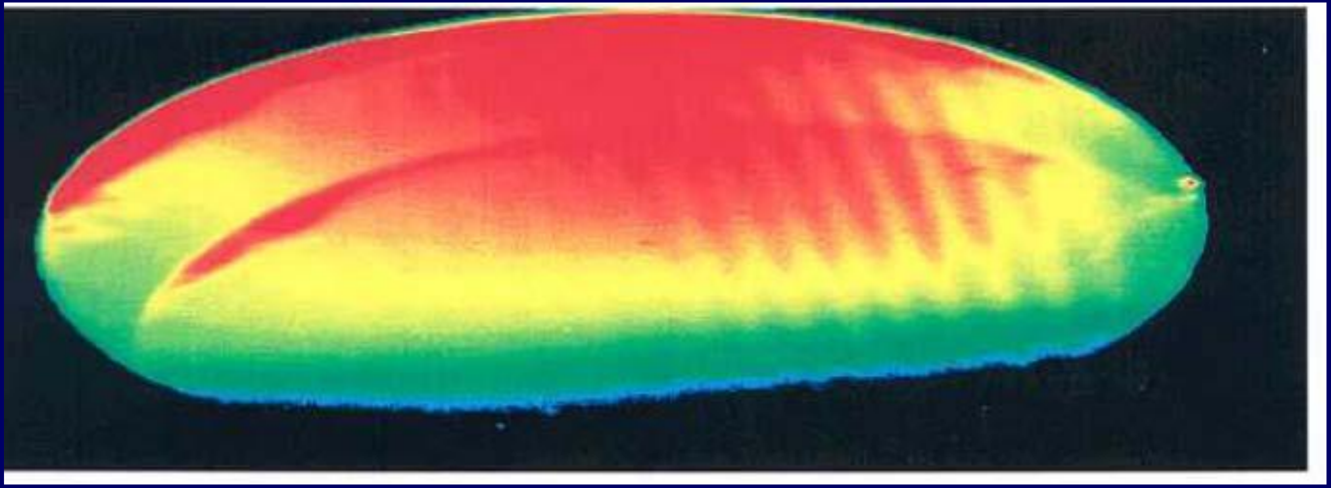
# Single pass high-gain amplifier (long undulator)



saturation (maximum bunching)

exponential growth with progression of microbunching

Start-up Lethargy

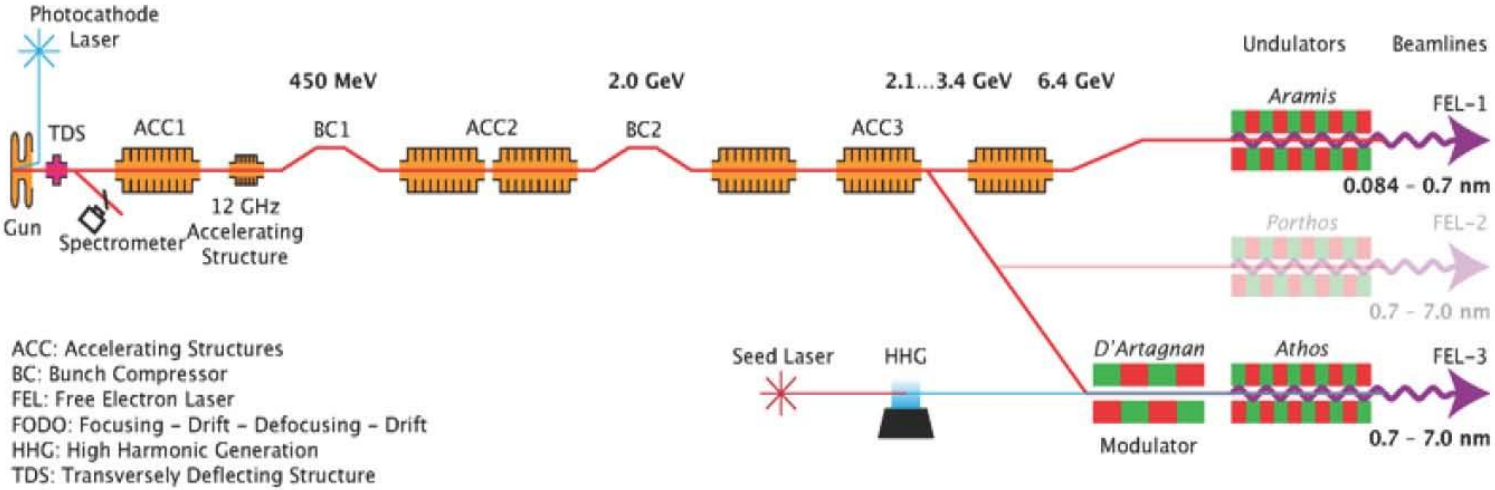


Early experimental observation of microbunching at the 60 µm FEL Firefly, Stanford University

**Examples of high-gain FELs:**

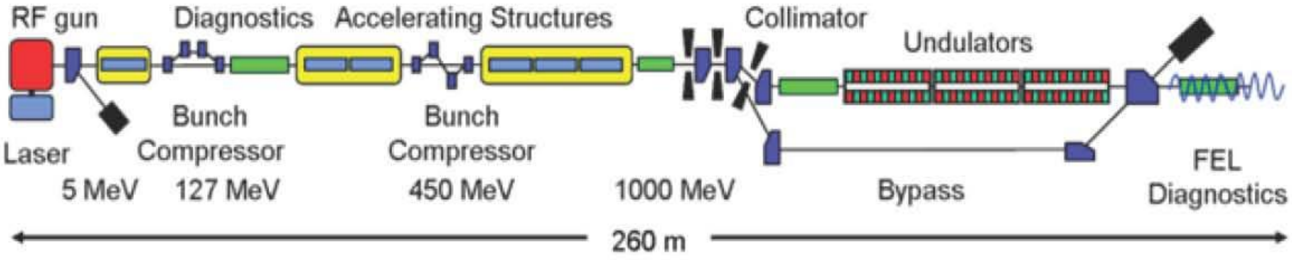
- LCLS at SLAC, California
- XFEL at DESY, Hamburg
- SPring8 at Hyogo, Japan
- SwissFEL at PSI, Villigen

- SwissFEL

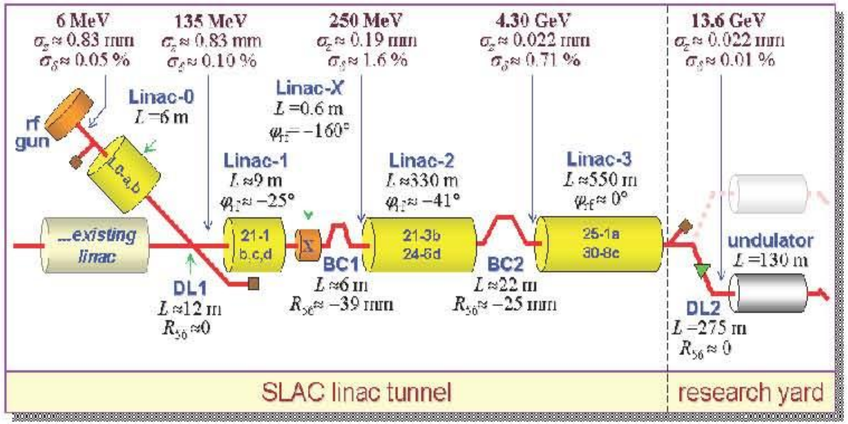


ACC: Accelerating Structures
BC: Bunch Compressor
FEL: Free Electron Laser
FODO: Focusing – Drift – Defocusing – Drift
HHG: High Harmonic Generation
TDS: Transversely Deflecting Structure

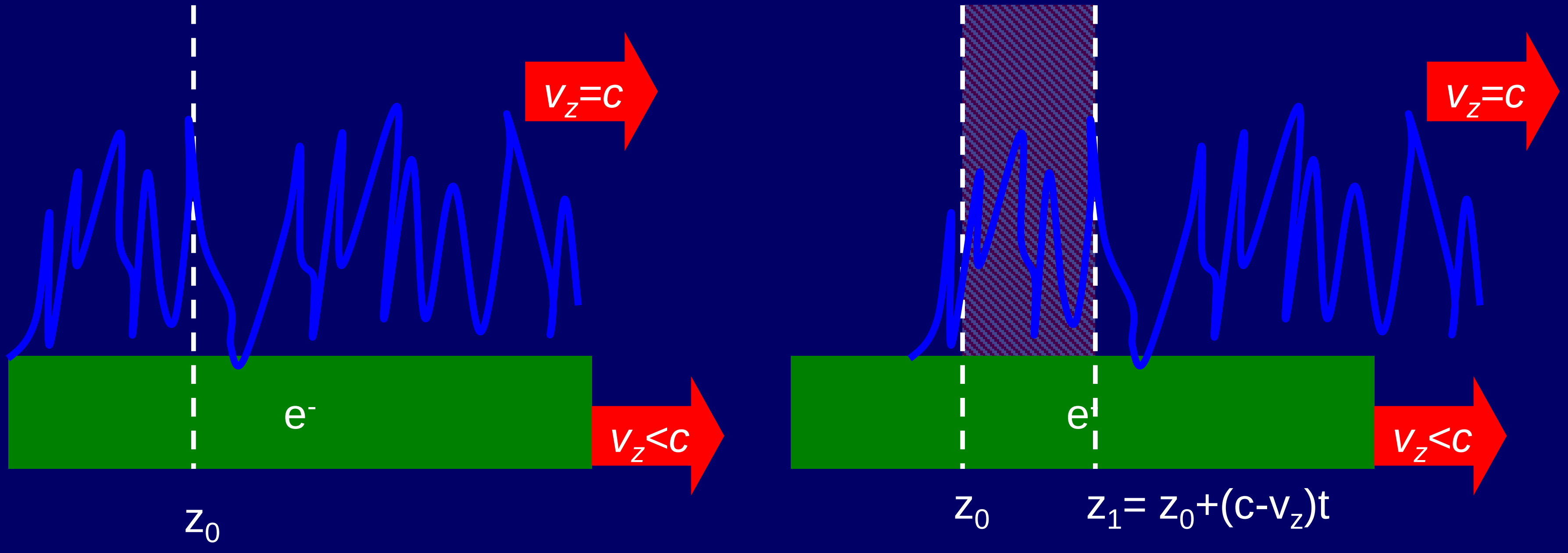
- FLASH



- LCLS



# FEL pulses starting from noise in a High-Gain amplifier (SASE)



Many regions of radiation pulse evolve independently from other regions

# Self Amplified Spontaneous Emission (SASE) in the X-ray regime

**SASE Power output**

**SASE spectrum**:

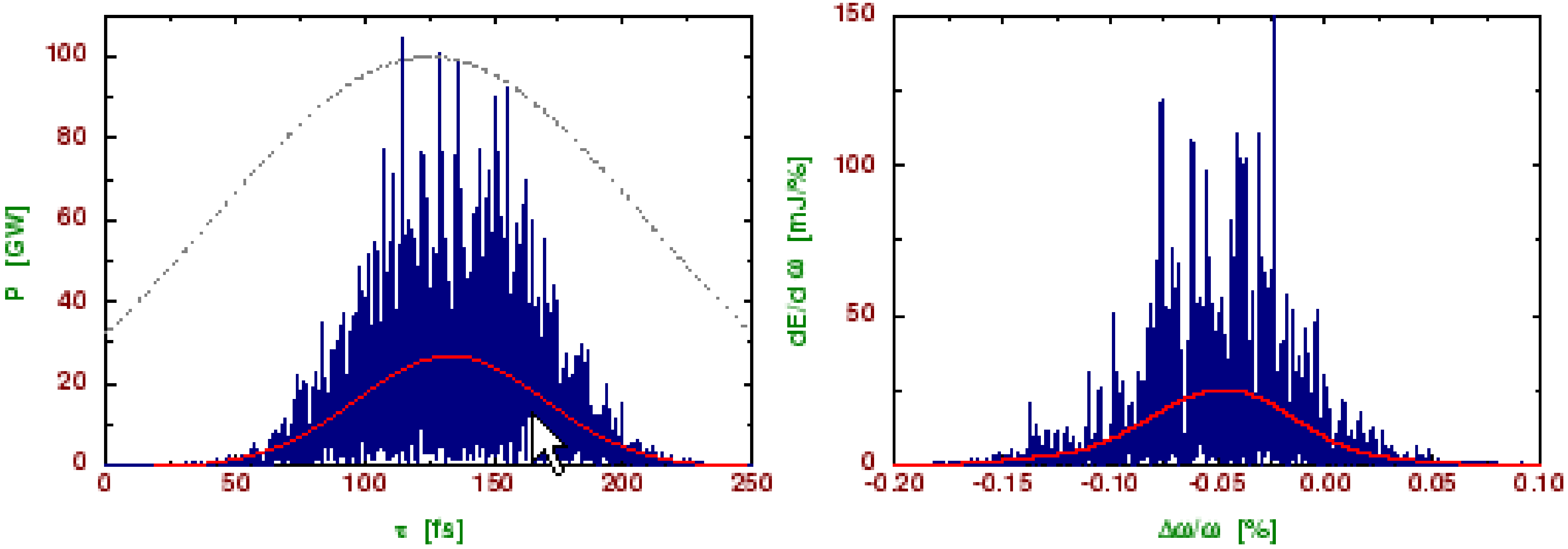
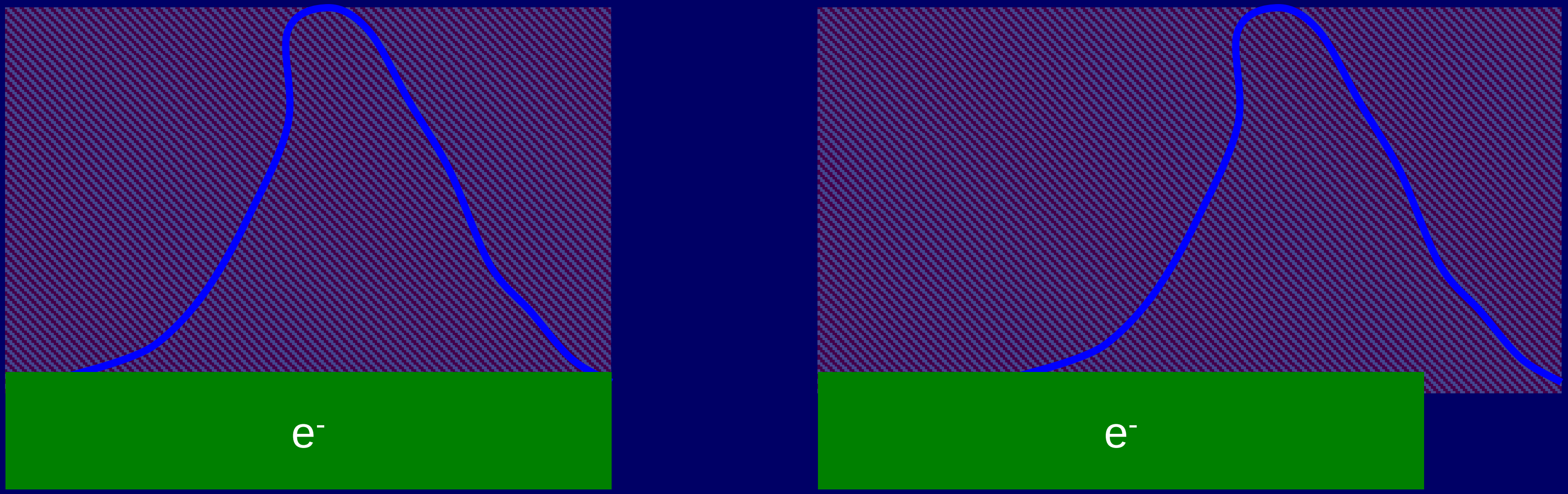


**Figure 5:** Typical temporal (left) and spectral (right) structure of the radiation pulse from a SASE XFEL at a wavelength of 1Å. The red lines correspond to averaged values. The dashed line represents the axial density profile of the electron bunch. Note that the growth rate in the electron bunch tail is reduced due to the reduced current. Therefore, the radiation pulse length of 100fs (FWHM) is about a factor of two shorter than the electron bunch.

# Contrast → Seeded FEL



Longitudinal coherence of radiation pulse is inherited from that of seed if $P_{seed} >> P_{noise}$

# FEL vs Conventional Laser

- Conventional Laser
  - **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation
  - Electrons in bound states - discrete energy levels
  - Waste heat in medium ejected at speed of SOUND

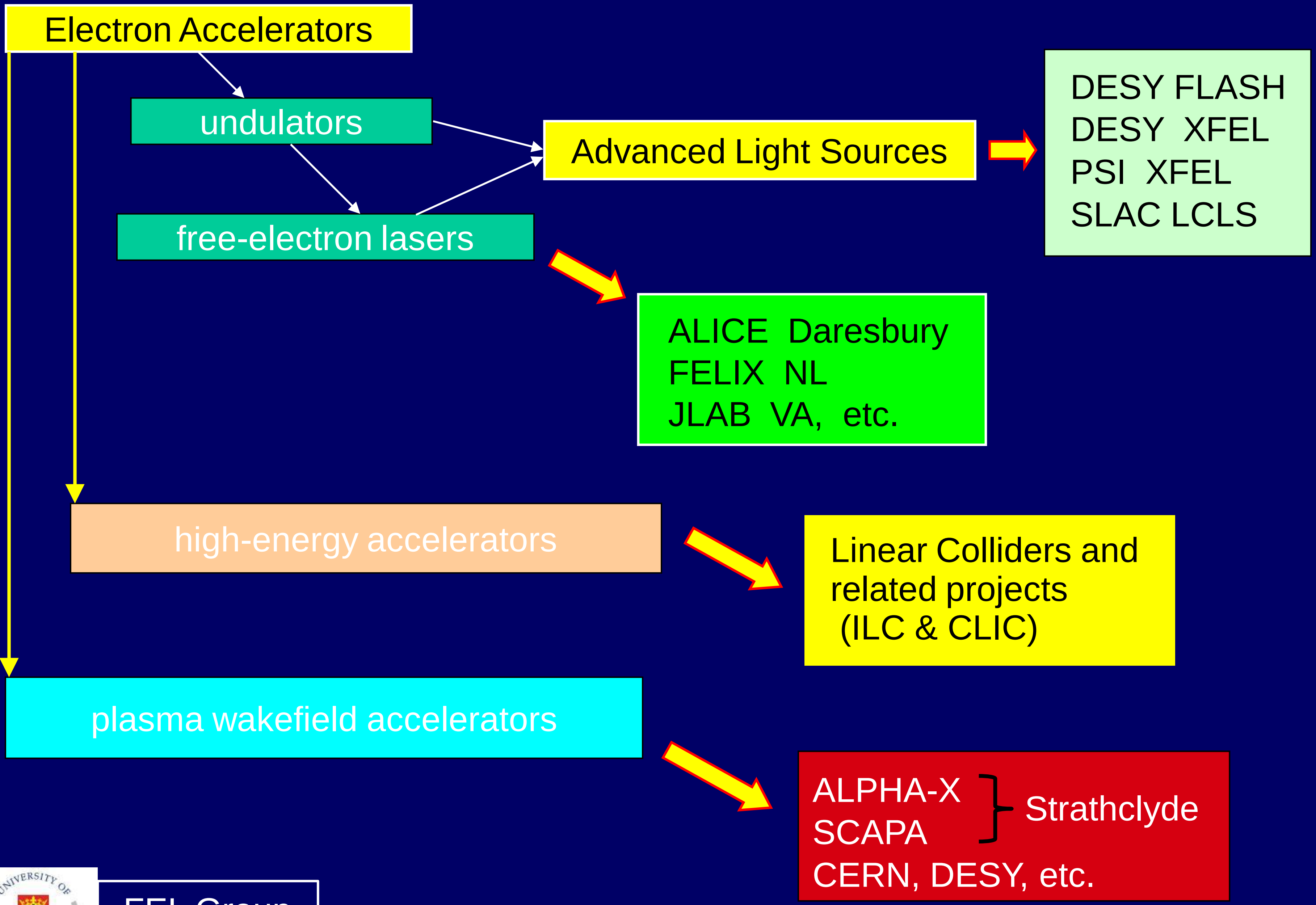
| | |
|---|---|
| Limited tunability | Limited power |
| Continuous tunability | High power |

- Free-Electron Laser
  - **L**ight **A**mplification by **S**ynchronised **E**lectron **R**etardation ??
  - Electrons free - continuum of energy levels
  - Waste heat in e-beam ejected at speed of LIGHT (and recovered in ERL)

# Some advantages of FELs

- Tuneable by varying electron energy $\gamma$ or undulator parameters ( $B_u$ and/or $\lambda_u$ )
- Spectral reach – THz, VUV to X-ray
- Cannot damage lasing medium ($e^-$-beam)
- High peak powers (>GW)
- Very bright [ $>10^{30}$ ph/(s $mm^2$ $mrad^2$ 0.1% B.W.)]
- High average powers – e.g. >10kW at Jefferson Lab
- Short pulses (<100*fs* → 100's of *as* ($10^{-18}$*s*) )

The next generation of FELs will ensure that these sources are at the forefront of light source provision for many years to come. Other sources are unable to meet all of the qualities of FELs by orders of magnitude in at least one respect.



Electron Accelerators

undulators

free-electron lasers

Advanced Light Sources

DESY FLASH
DESY XFEL
PSI XFEL
SLAC LCLS

ALICE Daresbury
FELIX NL
JLAB VA, etc.

high-energy accelerators

Linear Colliders and related projects (ILC & CLIC)

plasma wakefield accelerators

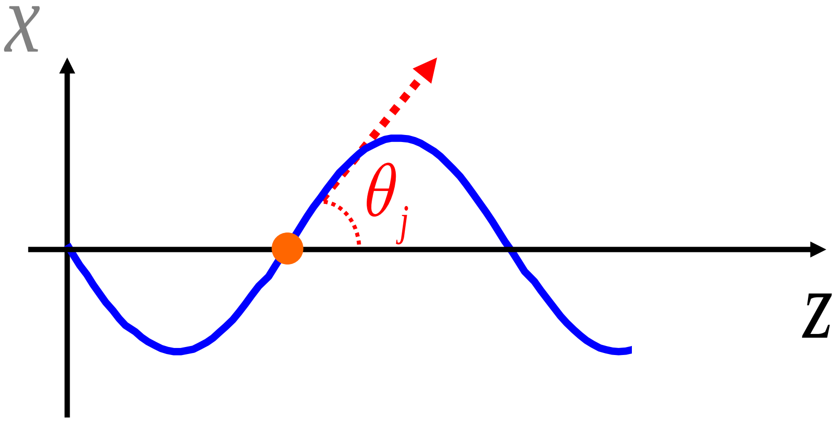
ALPHA-X
SCAPA
} Strathclyde
CERN, DESY, etc.

FEL Group

… or at least this was not your response.

The End

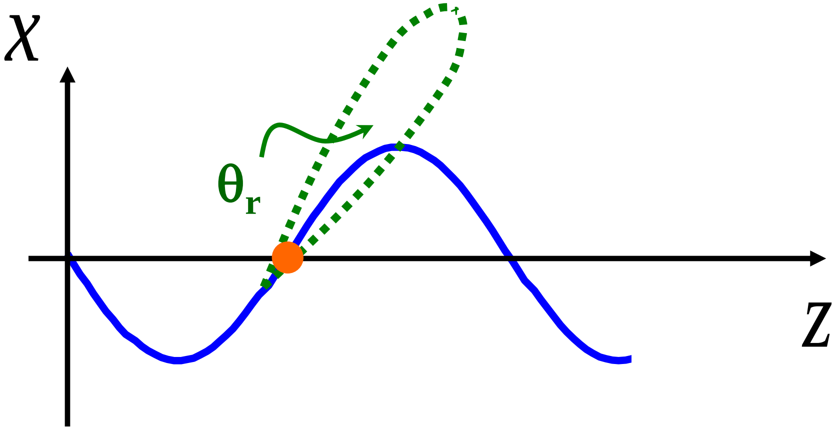
# An electron trajectory in an undulator



The angle the electron makes with respect to the undulator axis can be approximated as:

$$\tan\theta_j = \frac{dx_j}{dz} \approx -\frac{a_u}{\gamma_0}\cos\left(k_u z\right)$$

$$\Rightarrow \theta_j \propto \frac{a_u}{\gamma_0} \text{ for } \gamma_0 \square\ a_u$$



The radiated power is confined mainly to an angle $\theta_r \approx 1/\gamma_0$.

Hence if: $\theta_j >> \theta_r$ i.e. $a_u \square\ 1$,

The emitted power behaves like a ‘searchlight’ when viewed at end of the undulator.

| | |
|---|---|
| $a_u \square 1$ | - ‘Undulator’ |
| $a_u \square\ 1$ | - ‘Wiggler’ |

# Undulator Equation

Substituting for the average longitudinal velocity of the electron, $\bar{\beta}_z$ , for the earlier planar case:

Substitute $\bar{\beta}_z \approx 1-\frac{1}{2\gamma_0^2}\left(1+\frac{a_u^2}{2}\right)$ into $n\lambda_r = \frac{\lambda_u}{\bar{\beta}_z} - \lambda_u \cos\theta$

$$\Rightarrow \quad \lambda_r = \frac{\lambda_u}{2n\gamma_0^2}\left(1+\bar{a}_u^2 + \theta^2\gamma_0^2\right)$$

Including angular dependence

$$\bar{a}_u = \frac{e\lambda_u B_u^{RMS}}{2\pi mc}$$

is the RMS “wiggler/undulator parameter”
- In this form also valid for helical undulators

For a 3 GeV electron passing through a 5 cm period undulator with $\bar{a}_u$ = 3, the wavelength of the first harmonic ($n$ = 1) on axis ($\theta = 0$) is ~ 4 nm

The expression for the fundamental resonant wavelength shows us the origin of the FEL tunability:

$$\lambda_r = \left( \frac{1+\bar{a}_u{}^2}{2\gamma_0{}^2} \right) \lambda_u$$

As the beam energy is increased, the spontaneous emission peak moves to shorter wavelengths.

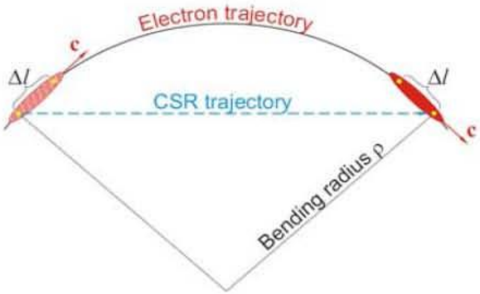
For an undulator parameter $\bar{a}_u \approx 1$ and $\lambda_u = 1$cm :

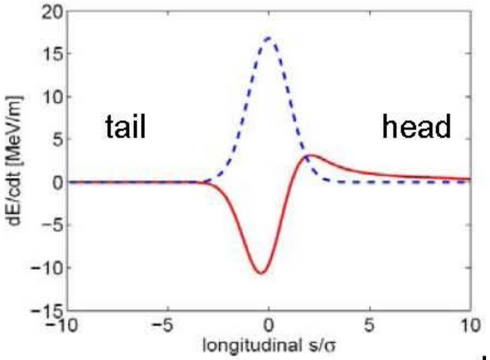
For mildly relativistic beams ($\gamma \approx 3$) : $\lambda_r \approx$ 1mm (microwaves)
more relativistic beams ($\gamma \approx 30$) : $\lambda_r \approx$ 10μm (infra-red)
ultra-relativistic beams ($\gamma \approx 30000$) : $\lambda_r \approx$ 0.1nm (X-ray)

Further tunability is possible through $B_u$ and $\lambda_u$ as $\bar{a}_u \propto B_u \lambda_u$

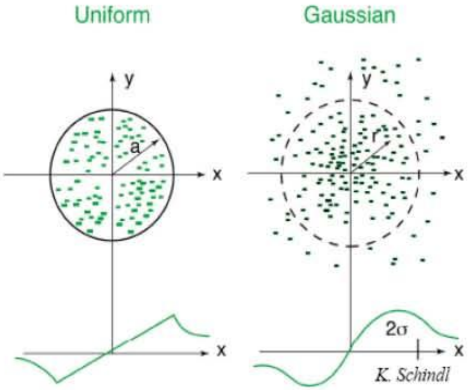
High charge densities give rise to strong electro-magnetic fields generated by the electron bunches. Electrons within the bunch experience these fields.

- Coherent Synchrotron Radiation

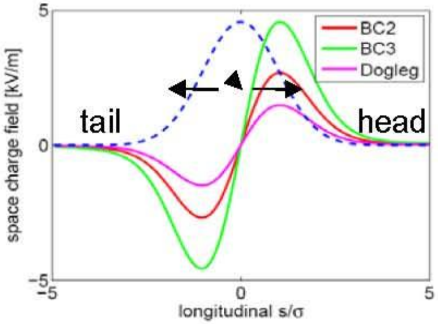




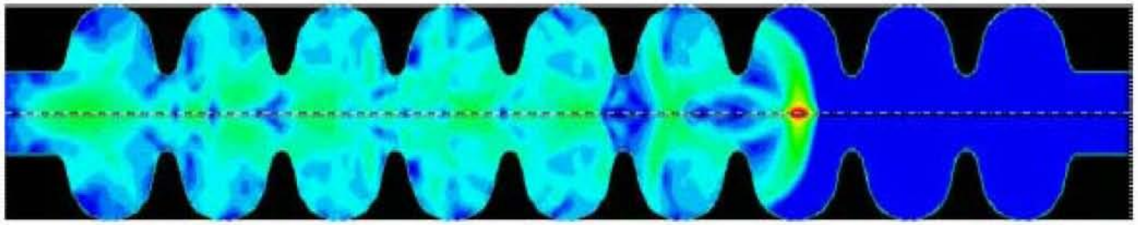
- Space Charge fields

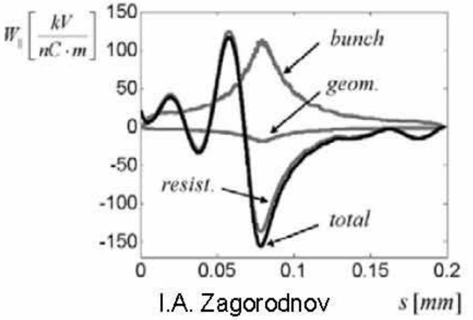


K. Schindl

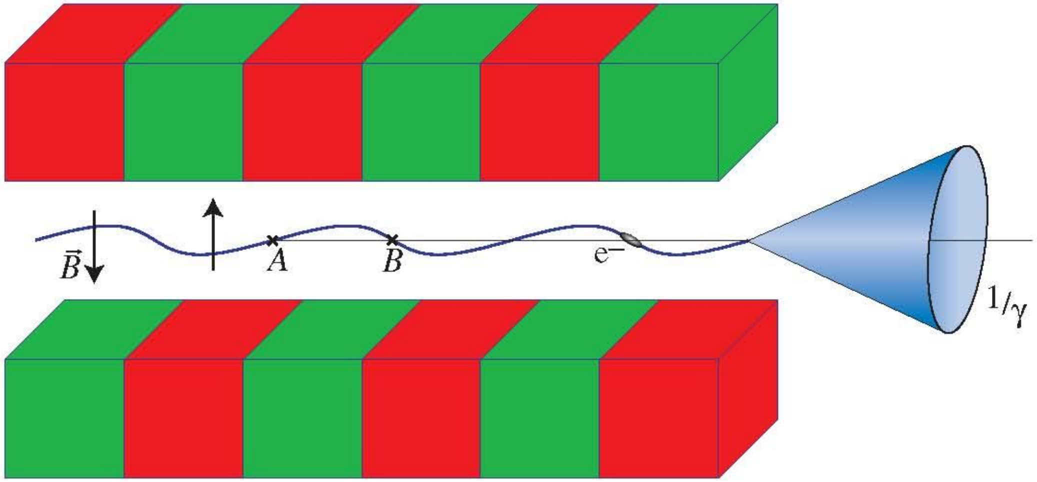


- Wake fields


M. Dohlus



I.A. Zagorodnov


$\vec{B}$
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