Particle Accelerators

Introduction and Brief Outline of History

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Overview

- Useful definitions;
- Short history of accelerators - limitations;
- Maxwell's equations – recap;
- Waveguides – a visual guide;
- Radiofrequency accelerators, incl. the Rfq;
- Rf/laser cavitity modes;
- Simple beam optics
  - Hill's equation,
  - Different multipole fields and their use;
  - Diagnostics needs

Literature

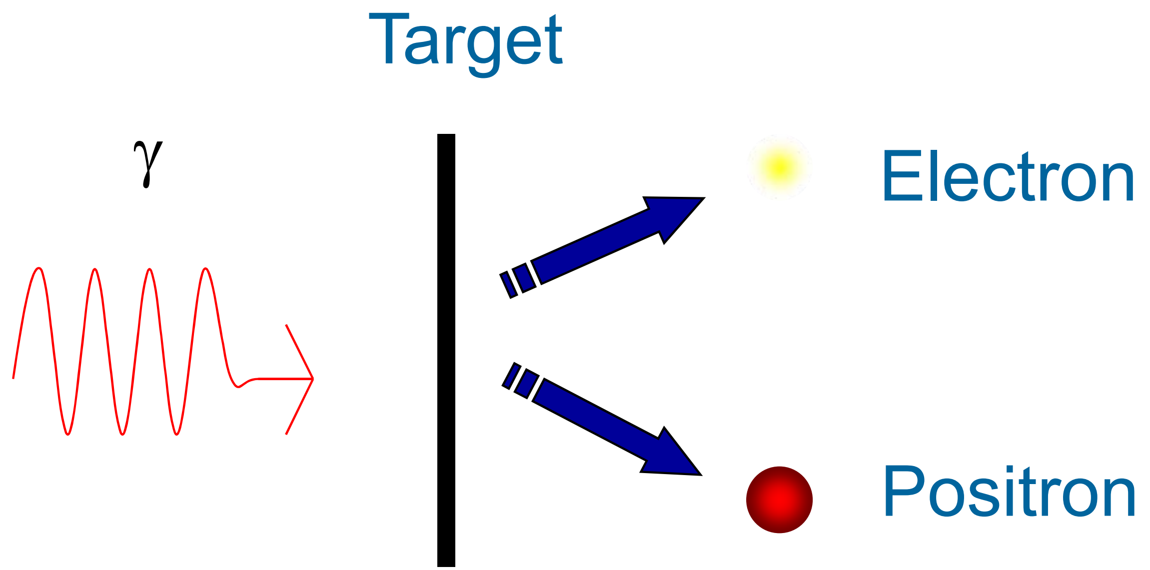
- J.D. Jackson, *Classical Electrodynamics*
- H. Wiedemann, *Particle Accelerator Physics I & II*
- K.G. Steffen, *High Energy Beam Optics*
- M. Livingston, J. Blewett, *Particle Accelerators*
- CERN *Yellow Reports*
- http://www.cern.ch/cas
- http://www.jacow.org

Where are Accelerators used ?

- High energy physics
- Medical applications (therapy, diagnostics, etc.)
- Light sources
- Nuclear, plasma, biophysics, material sciences, Archaeology, food sciences, chemistry,...

Today: > 20,000 (!) accelerators in operation.

How are New Particles Created ?

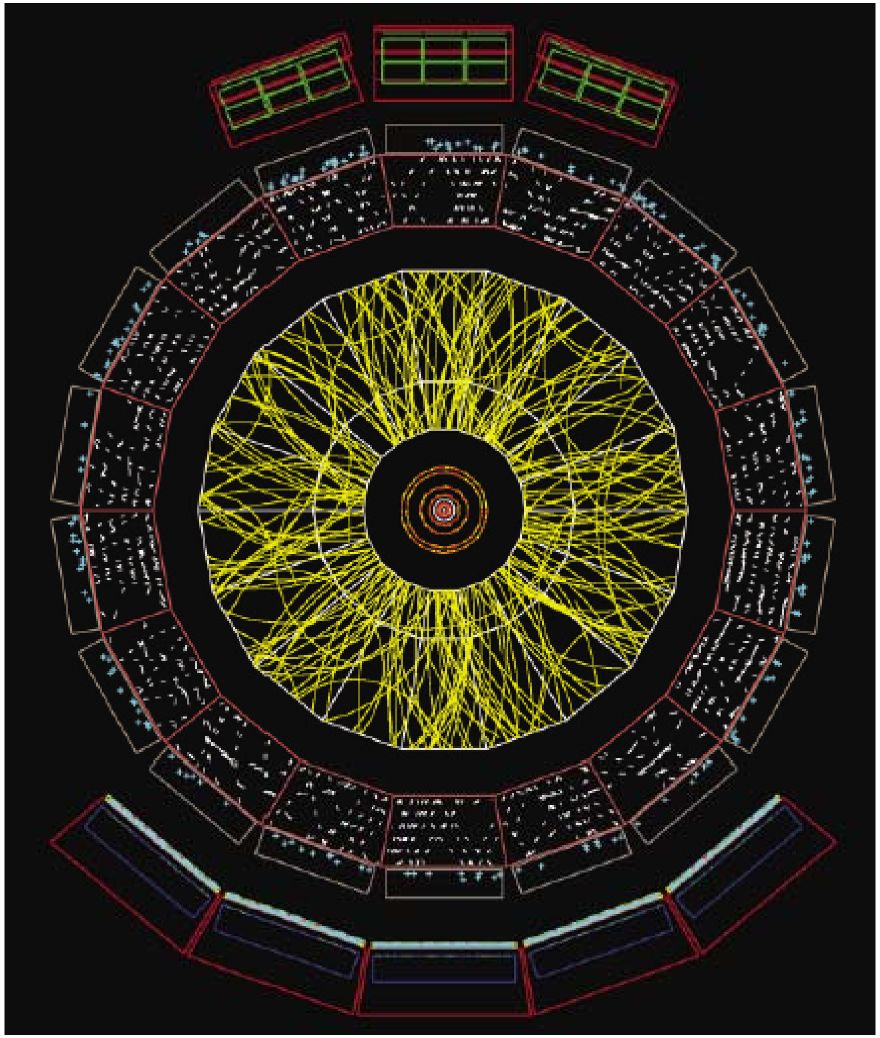


Charge conservation: Always in pairs! $E=mc^2$

Electron	511 keV
Proton	938 MeV
Bottom-Quark	4.735 MeV
Top-Quark	174.000 MeV

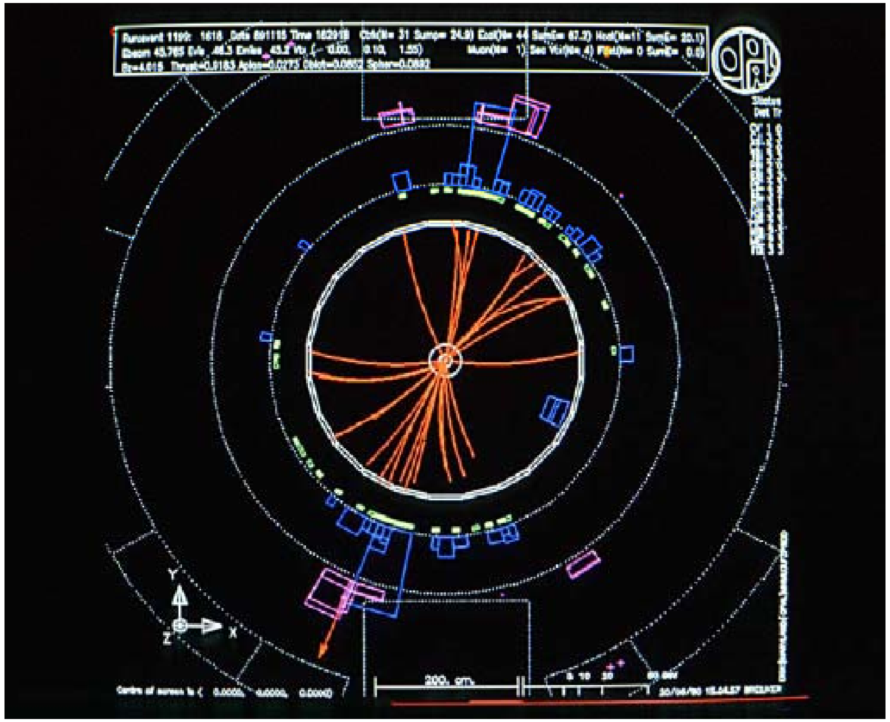
Differences

Hadron collision



Simulation of a lead-lead collision in the ALICE detector

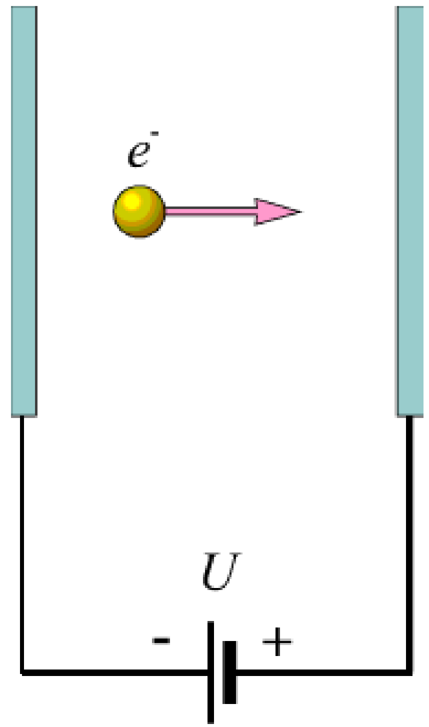
Lepton collision



Display from OPAL showing the decay of a Z into two jets of particles, originating from a quark-antiquark pair

Particle Energies

Definition of 1 eV:



$$E = eU = 1.602 \cdot 10^{-19}\,\mathrm{J}$$

$$\Leftrightarrow \quad E = 1\mathrm{eV}$$

Common units

$$1\ \mathrm{keV} = 10^3\ \mathrm{eV},$$
$$1\ \mathrm{MeV} = 10^6\ \mathrm{eV}$$
$$1\ \mathrm{GeV} = 10^9\ \mathrm{eV}$$
$$1\ \mathrm{TeV} = 10^{12}\ \mathrm{eV}$$

➡ Maxwell equations, Theory of relativity

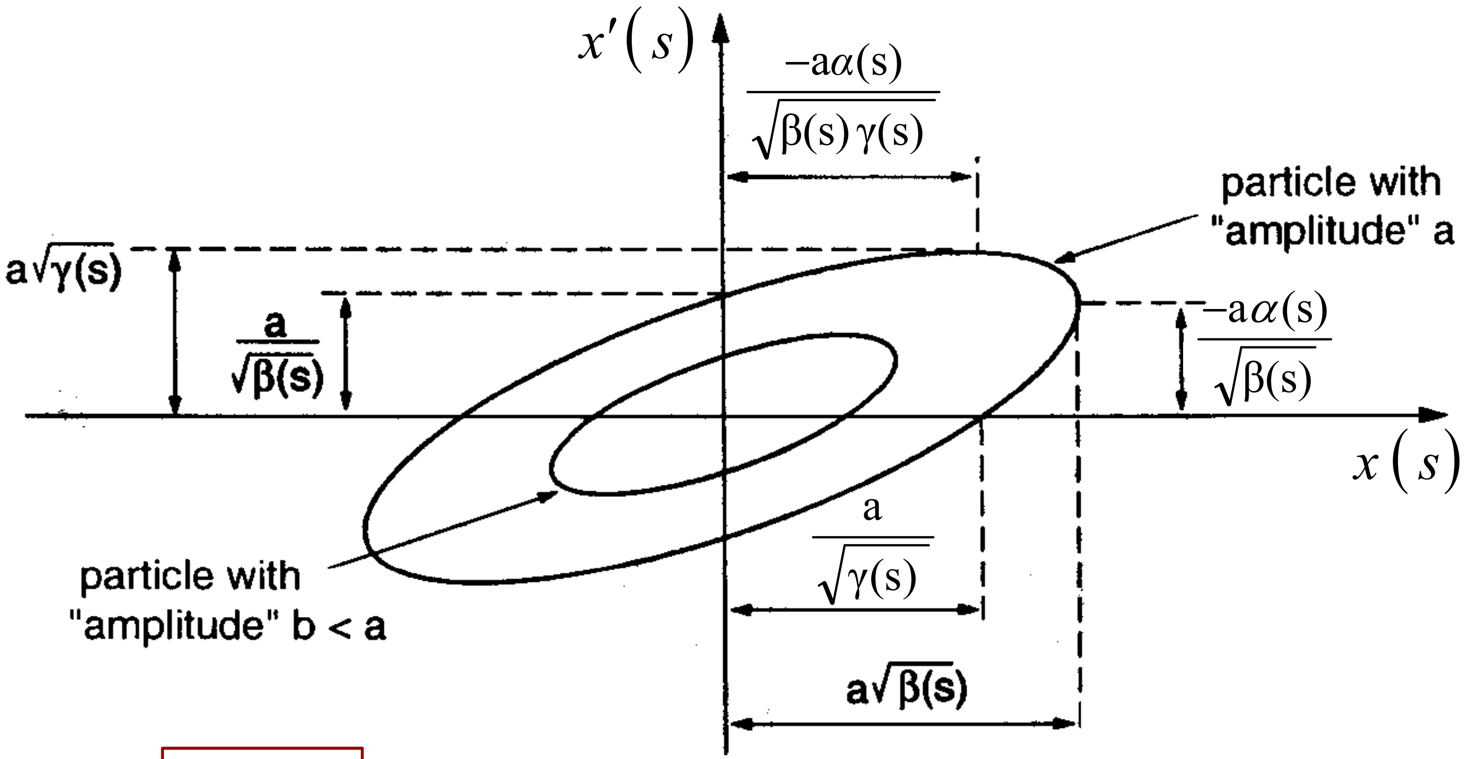
Other Important Units

- Mass [eV/c^2]
  - 1 eV/c^2 = 1.78 × 10^{-36} kg
  - Electron mass = 0.511 MeV/c^2
  - Proton mass = 938 MeV/c^2
  - Carsten's mass ≈ 4 × 10^{37} eV/c^2

- Momentum [eV/c]
  - 1 eV/c = 5.3 × 10^{-28} kg m/s
  - Momentum of football at 70 km/h

    ≈ 10 kg m/s ≈ 2 × 10^{28} eV/c

Definition



$\alpha(s)$ $\beta(s)$ $\gamma(s)$ are called TWISS Parameters.

Shape and orientation evolve along the machine.

Area stays constant *(Liouville's theorem)*.

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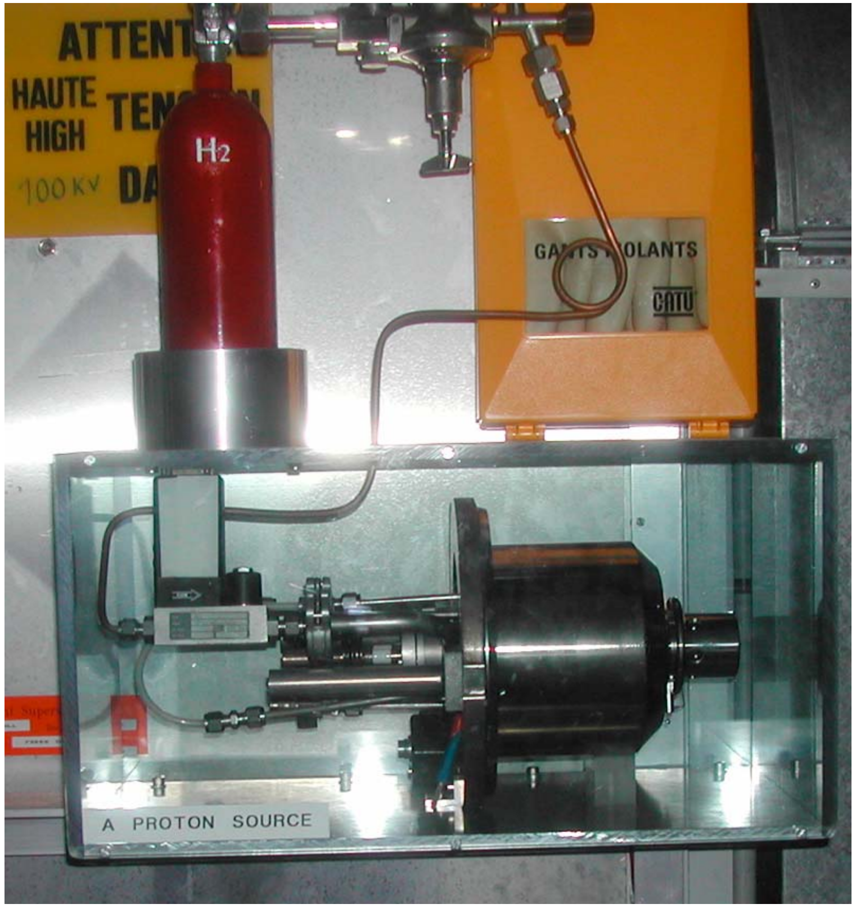
Remarks

- Unit of emittance is [mm · mrad]
  - Possible to confuse with ellipse area
- Area often given in publications
  - explicitly contained in [π · mm · mrad]
- No standard for percentage of particles in ellipse !
- Also: Statistical definitions available

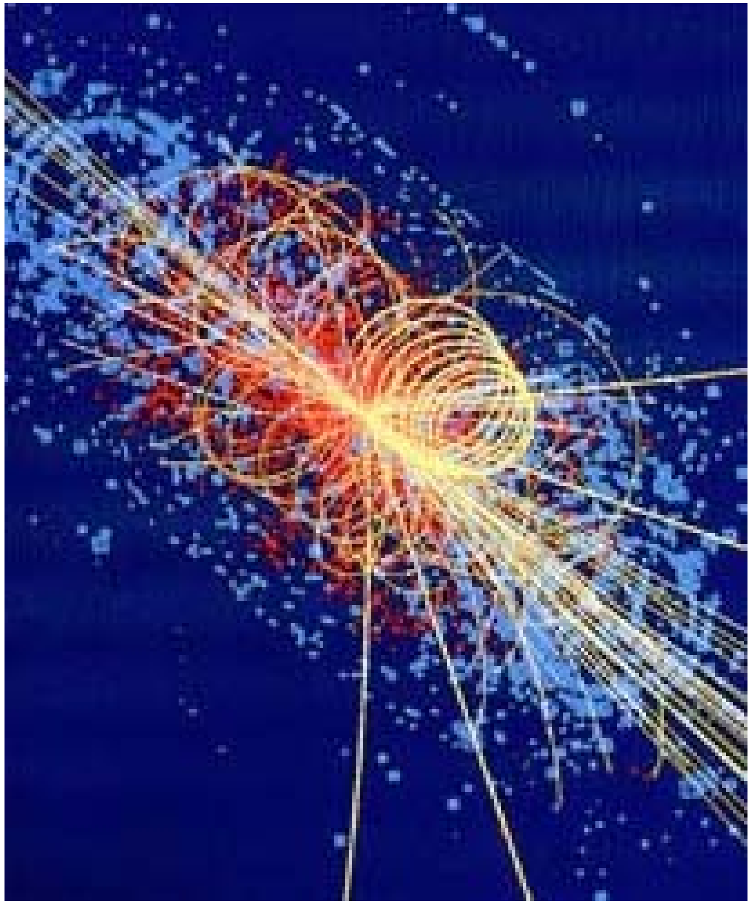
⇨ Look carefully at specific use/definition !

How to get…

From here:



To here ?



How to Accelerate Particles ?!?

Force	Rel. Strength	Reach [m]	Concerned particles
Gravitation	$6 \cdot 10^{-39}$	∞	all
Electro-Magnetism	1/137	∞	charged particles
Strong Force	~ 1	10^{-15}-10^{-16}	Hadrons
Weak Force	10^{-5}	$<<10^{-16}$	Hadrons and Leptons

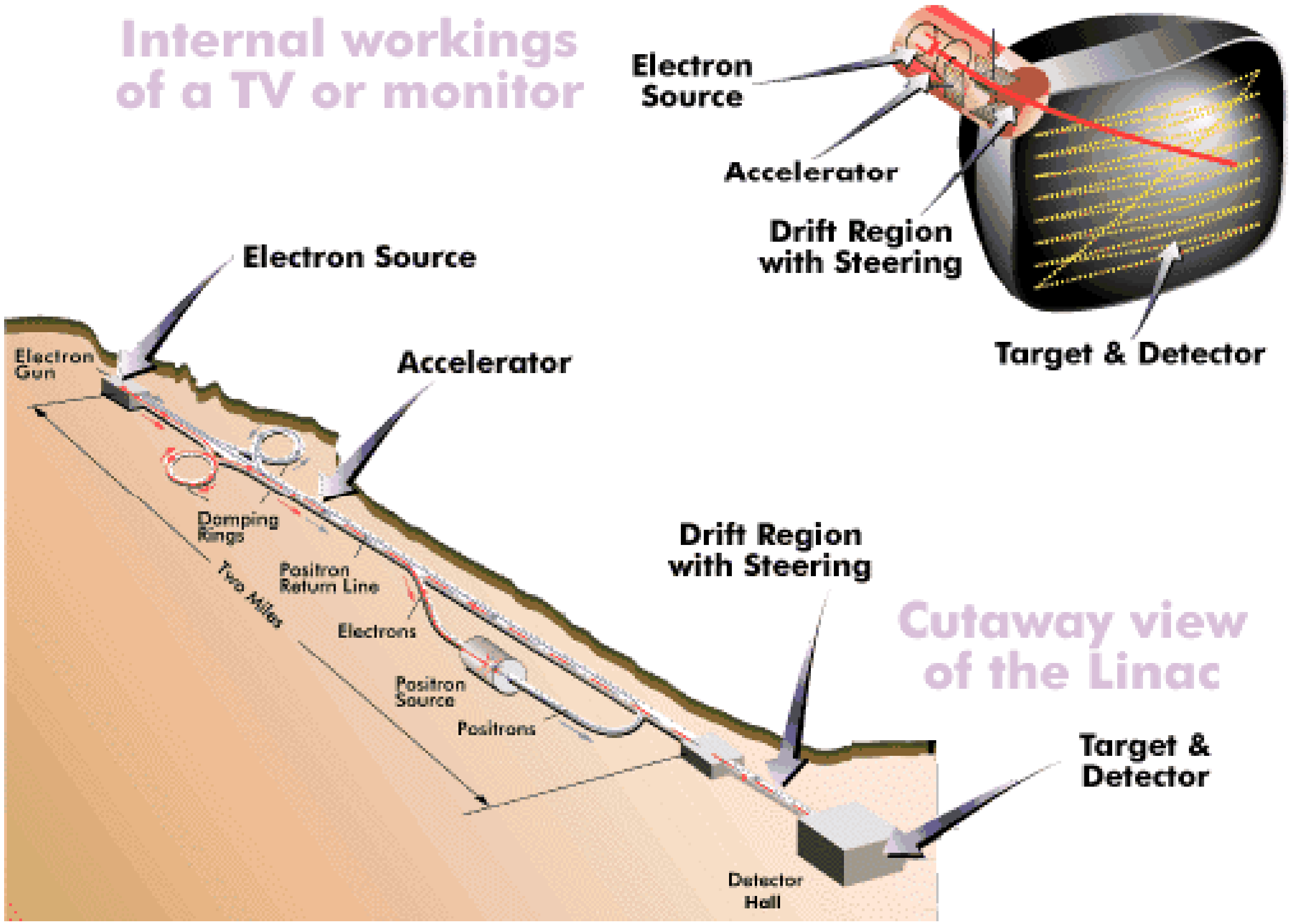
Do we really accelerate ?

- Energy 1 MeV => 1 GeV

$\beta = v/c$	0.95	0.99	0.999	0.999 999 9
$\gamma = m/m_0$	3	7	22	2000

➡ Velocity hardly changes.
Rather: *"Mass increaser"*

Illustration of an Accelerator



Accelerator History

- < 100 keV
  - **1895** Lenard: e^- scattering in gases
  - **1913** Frank/Hertz: e^- excitation through e^- bombardment
- Some MeV (α particles)
  - **1906** Rutherford targets thin foils
  - **1919** Rutherford induces nuclear reactions (N)

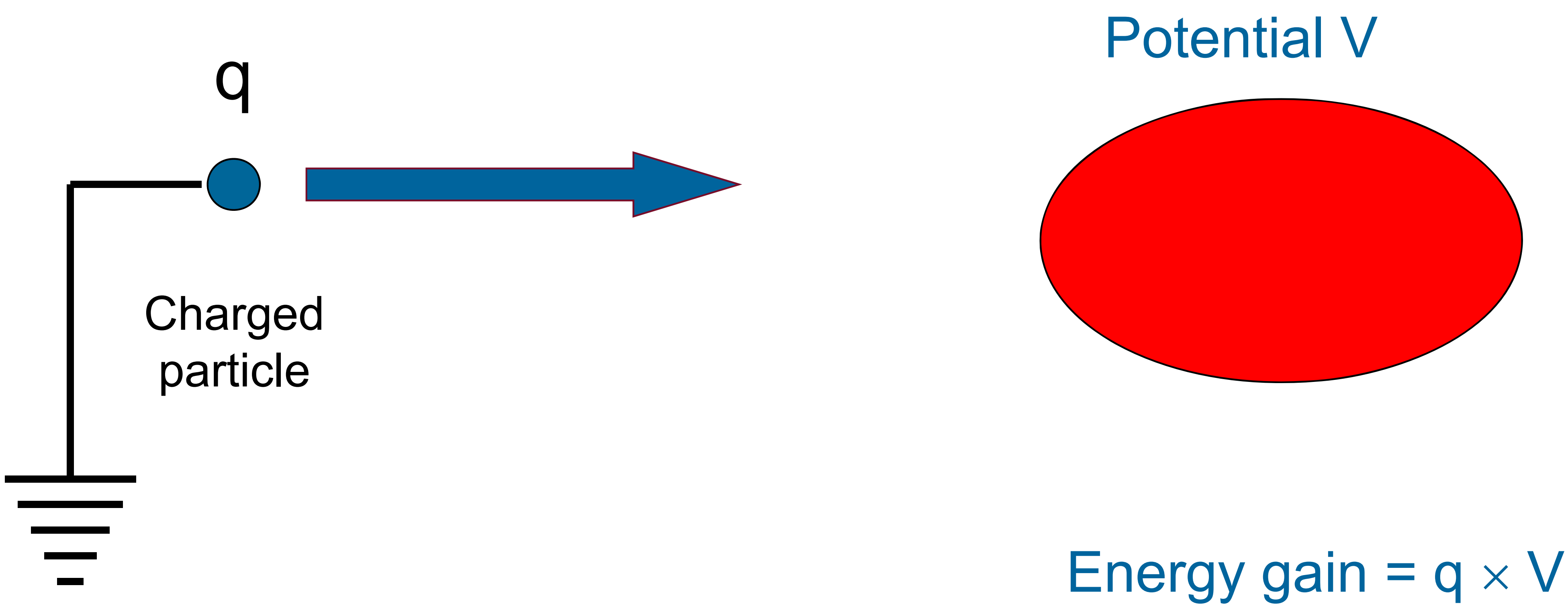
"How it all started…"

1927: Lord Rutherford demands a "copious supply" of projectiles with higher energies, as natural α and β-particles can provide.

At the opening of the *High Tension Laboratory* he demands:

What we require is an apparatus to give us a potential of the order of 10 million volts which can be safely accommodated in a reasonably sized room and operated by a few kilowatts of power. We require too an exhausted tube capable of withstanding this voltage. (…) I see no reason why such a requirement cannot be made practical.

DC Accelerators



Standard: Acceleration **towards** ground potential.

Unsuccessful Attempts

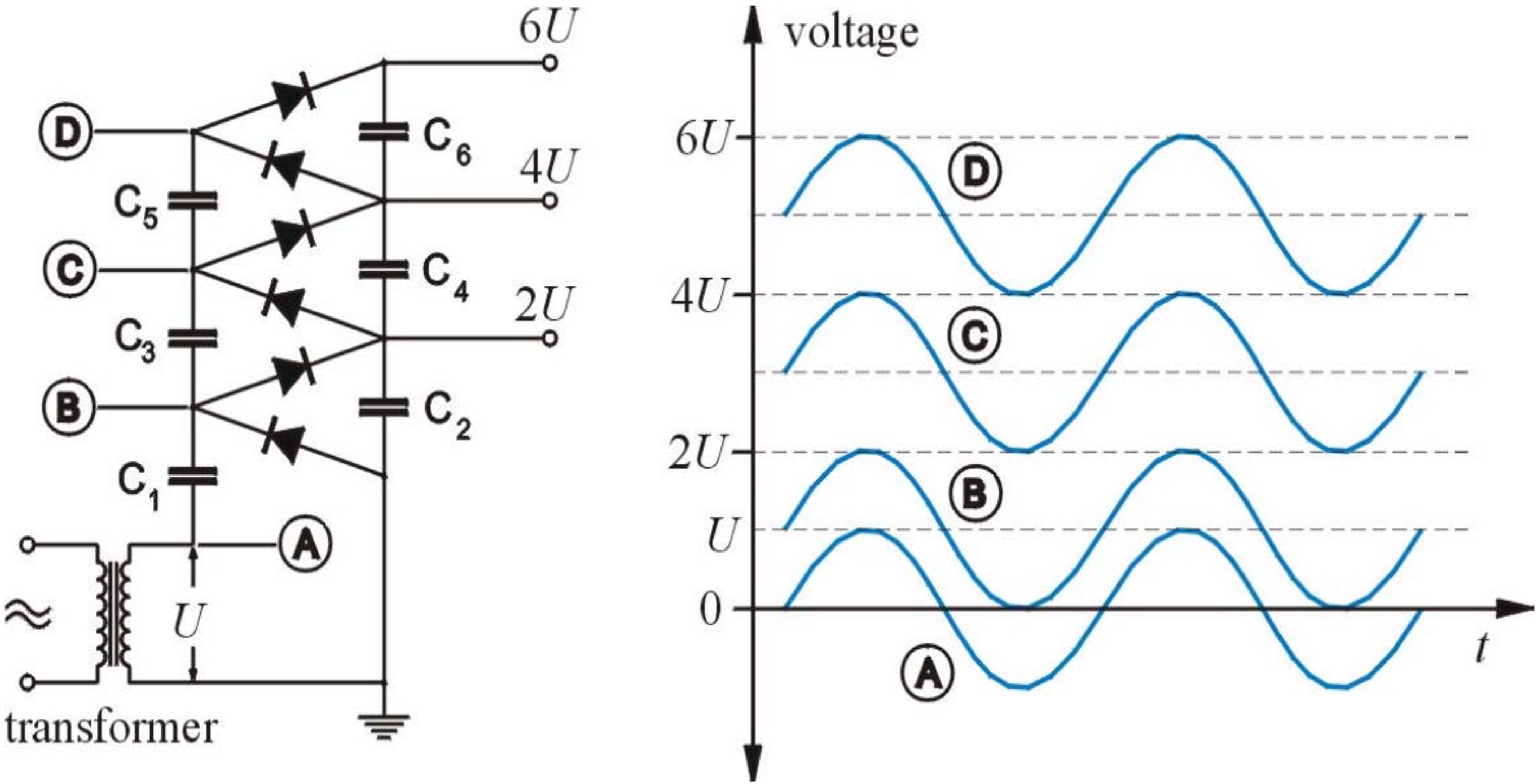
1928: Curt Urban, Arno Brasch and Fritz Lange reached 15 MV by harnessing lightning in the Italian Alps.

The two who survived the experiment then designed a drift tube able to withstand such voltages.

Birth of Particle Physics

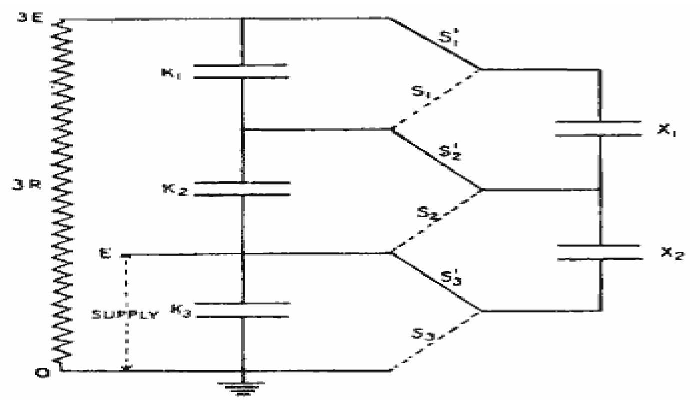
- 1919 Rutherford splits Nitrogen
- 1927 Rutherford demands particle accelerators
  Studies were started
- 1929 Cockcroft and Walton start high voltage exeriments
- 1932 Nobel prize: Cockcroft and Walton split Litium !!!!

Cockcroft-Walton Generator



Cockcroft-Walton Generator

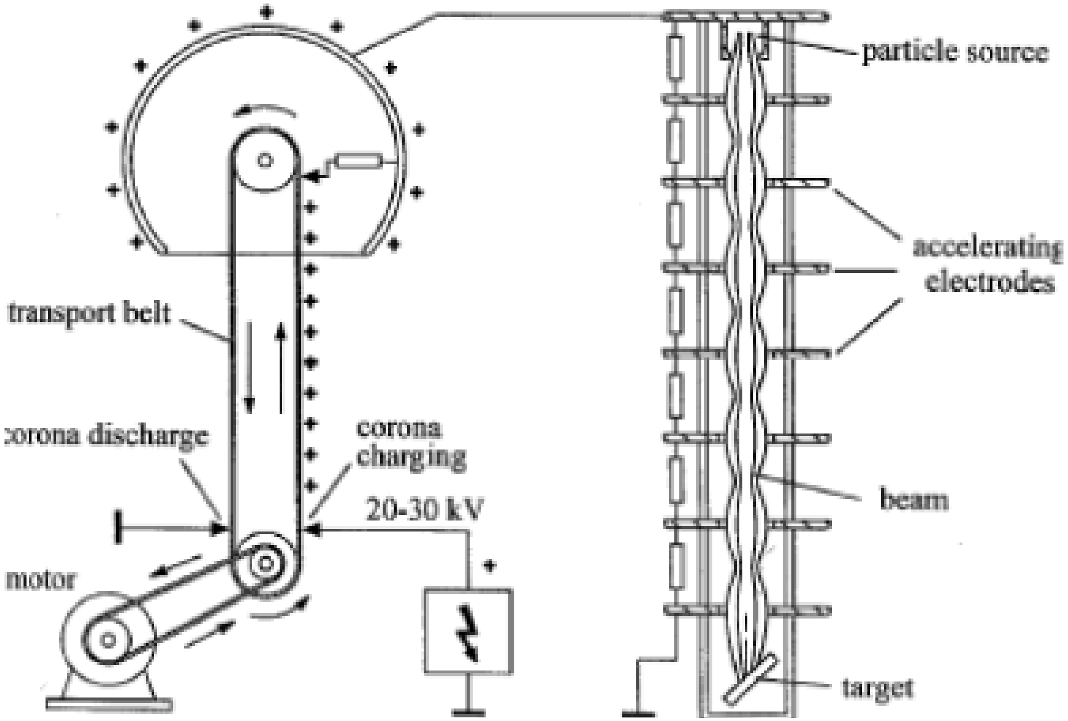
ISIS **665 kV**

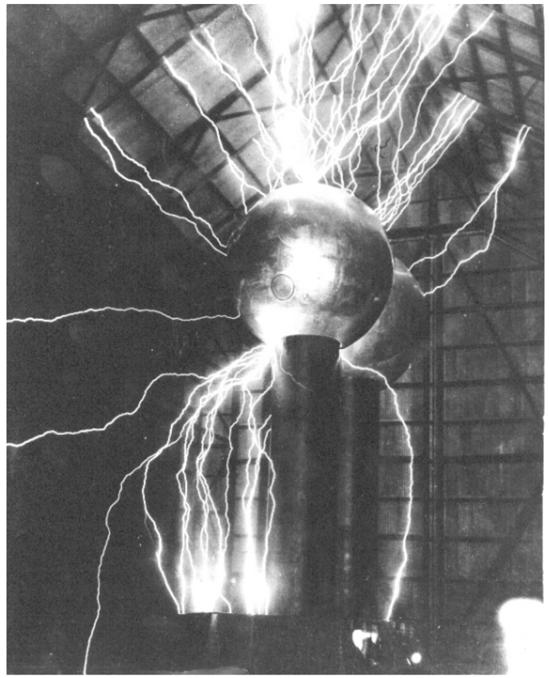


Greinacher 1921

Alternative Solution

1930: Van de Graaff builds first 1.5 MV accelerator



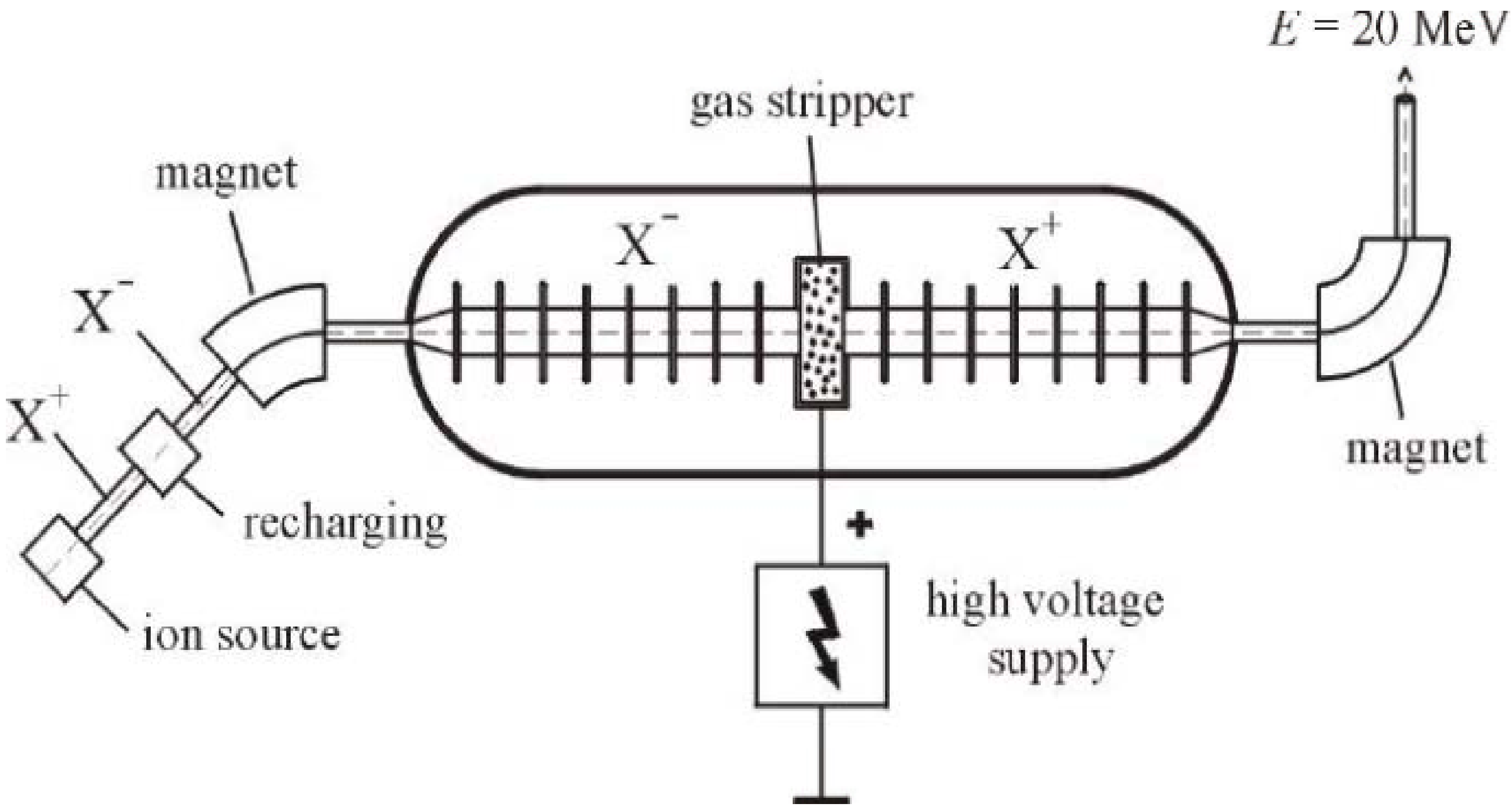


AT ROUND HILL SPARKING TO HANGAR (LONG EXPOSURE)

➡ Up to 17.5 MV with insulation gas (1MPa SF_6)

5 MeV Van-de-Graaff @ HMI *(Berlin)*

Higher Energies: Tandem VdG



Energy: $[q(X^+)+1]*V$ $\xrightarrow{C^{6+}}$ 70 MeV

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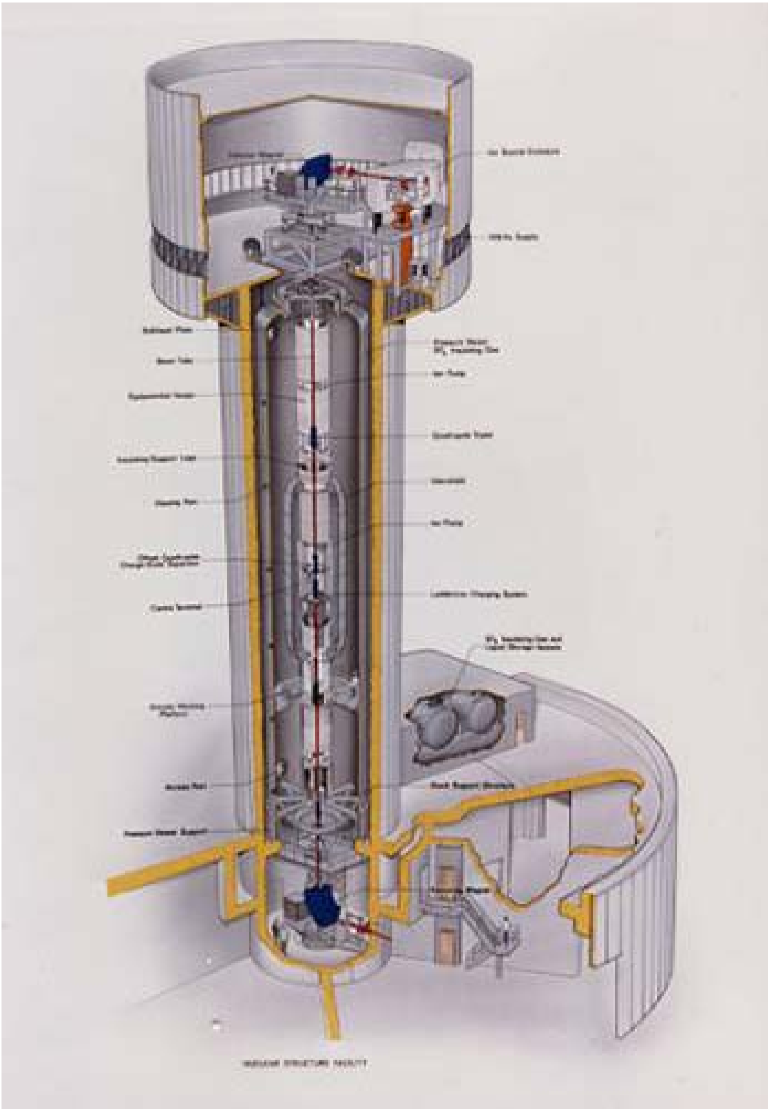
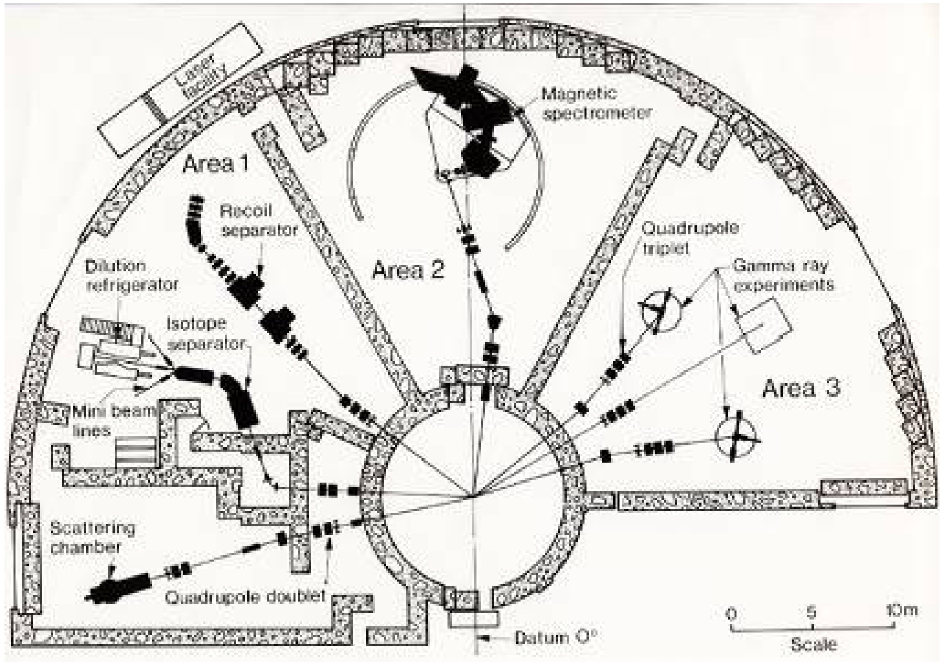
Daresbury Nuclear Structure Facility

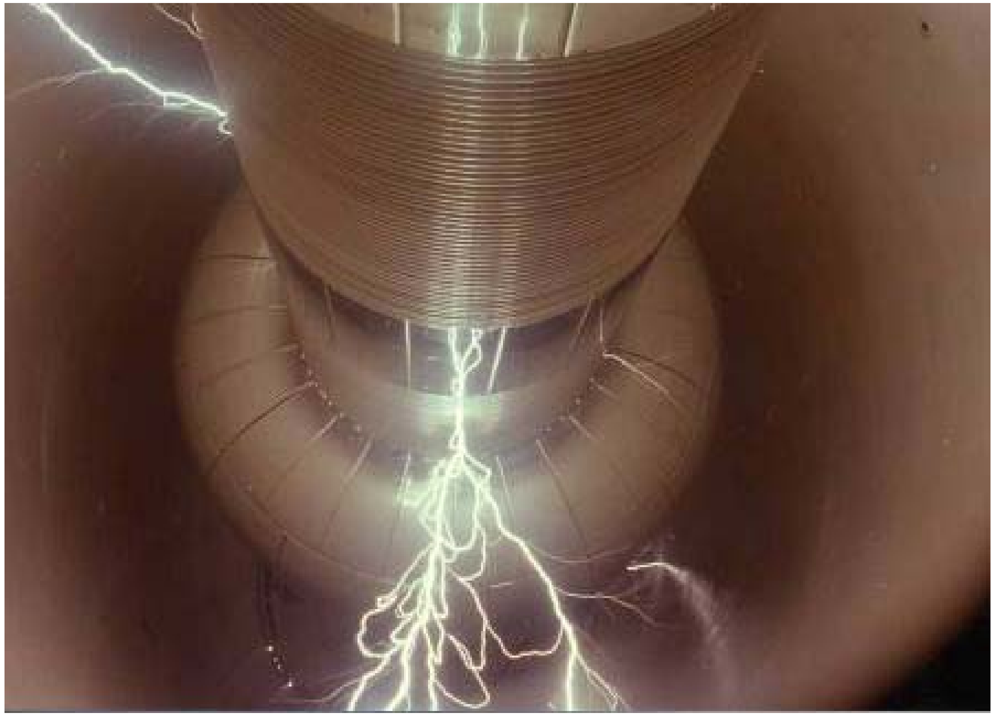


Van de Graaf 20+ MV



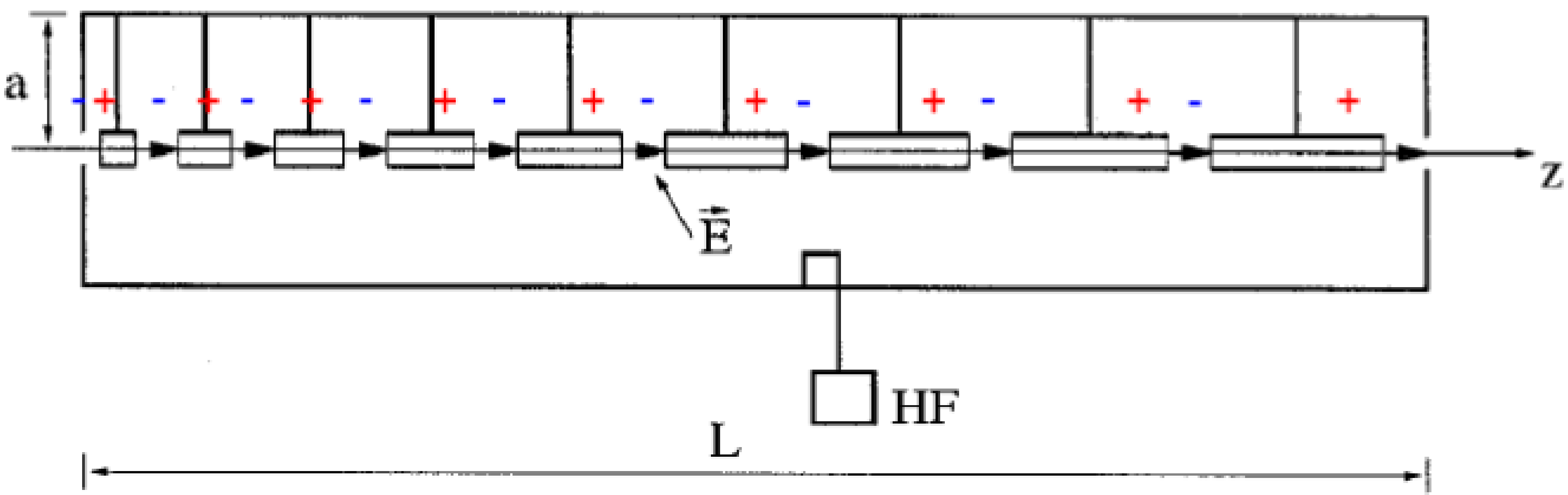
Last of its kind.

REALLY high voltages !



Target Energy: Some 10 GeV

- Problem: Not reachable with static fields.
- Solution:
  - Go for rf accelerators,
  - Re-use accelerating voltage.



Alvarez Linac: Working Principle

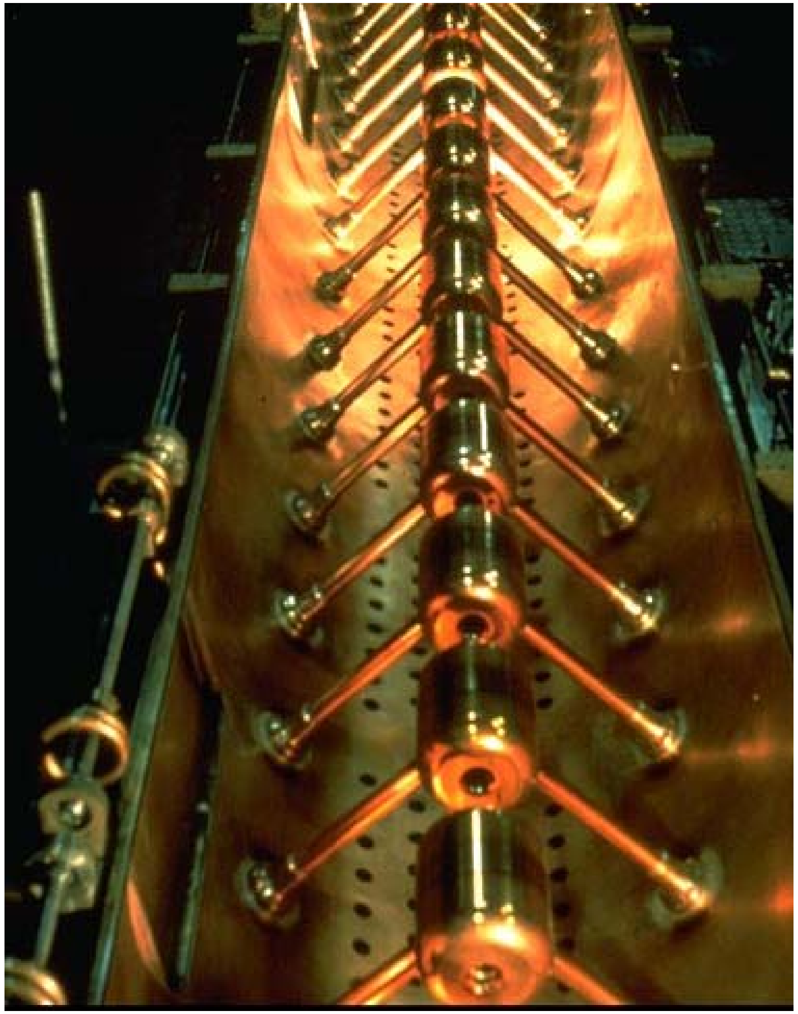
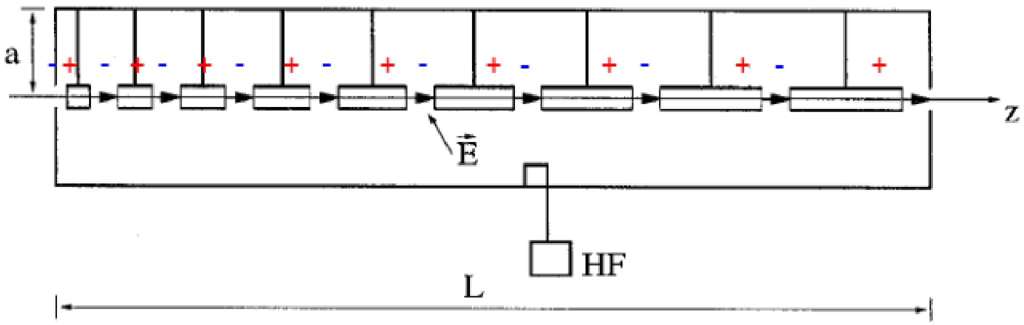


Photo: Old Linac1 at CERN.



- Acceleration between cavities
- Cavity length increases to match speed

DC vs. RF Accelerators

DC accelerator

Rf accelerator

Maxwell Equations (in vacuum)

$$\nabla \times \vec{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E} = \mu_0 \vec{J} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \qquad \nabla \cdot \vec{E} = \mu_0 c^2 \rho$$

Why not DC ?

1) DC ($\frac{\partial}{\partial t} \equiv 0$): $\nabla \times \vec{E} = 0$ is solved by $\vec{E} = -\nabla \Phi$

Limit: To reach 1 MeV, one needs 1 MV !

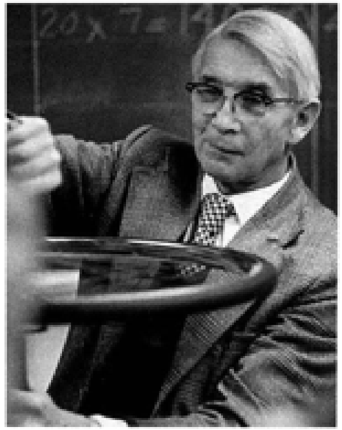
2) Circular accelerators: DC impossible, since $\oint \vec{E} \cdot \mathrm{d}\vec{s} = 0$

Time-varying fields:

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B} \qquad \oint \vec{E} \cdot \mathrm{d}\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot \mathrm{d}\vec{A}$$

Key Players

- Gustaf Ising
  Proposed rf concept
  - * 1881
  - † 1960



- Rolf Wideröe
  - * 11.7.1902, Oslo, Norway.
  - † 1996.

- Ernest Orlanda Lawrence
  - * 8.8.1901, South-Dacota, USA
  - † 27.8.1958

1939

TE or TM Modes

- TE: Electric field perpendicular to direction of propagation.

TE_{nml}

n:	azimuthal
m:	radial
l:	longitudinal component.

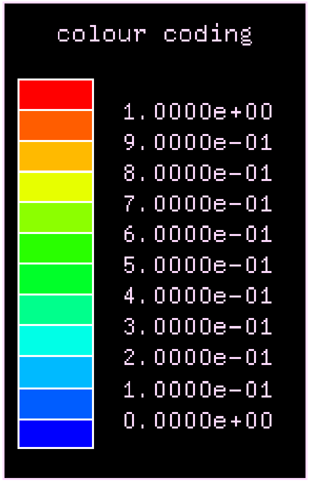
- TM: Magnetic field perpendicular to direction of propagation.

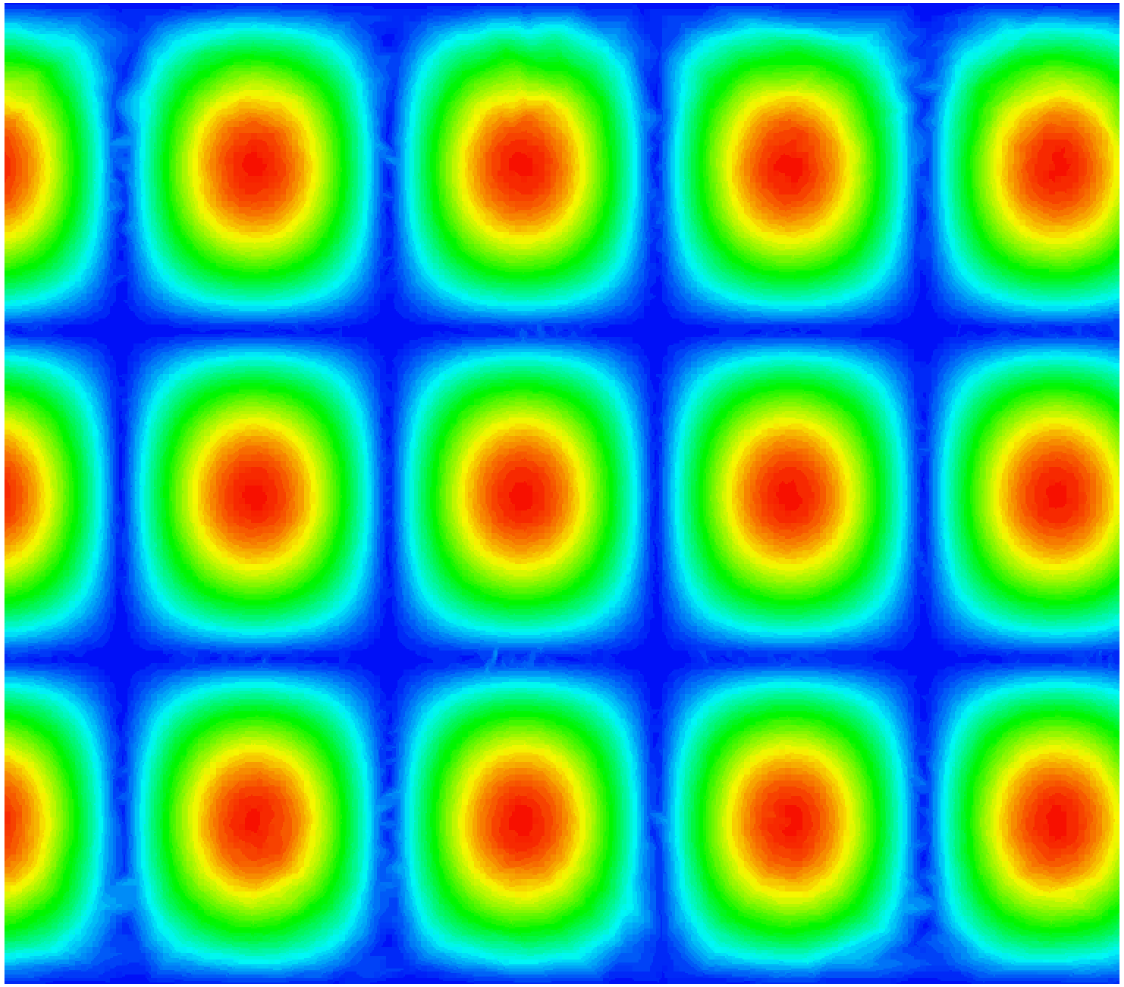
TM_{nml}

n:	azimuthal
m:	radial
l:	longitudinal component.

2 Superimposed Plane Waves

$$|\vec{E}|$$





Courtesy of Erk Jensen, CERN

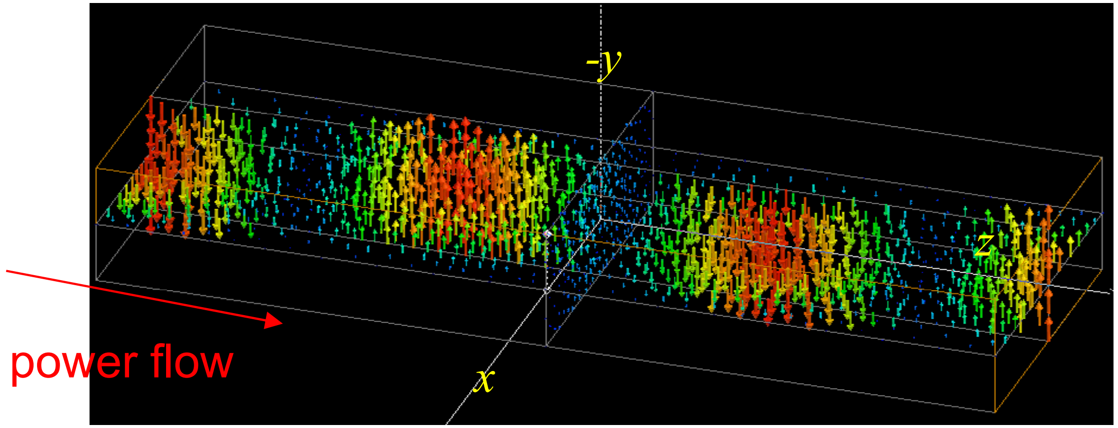
Waveguides

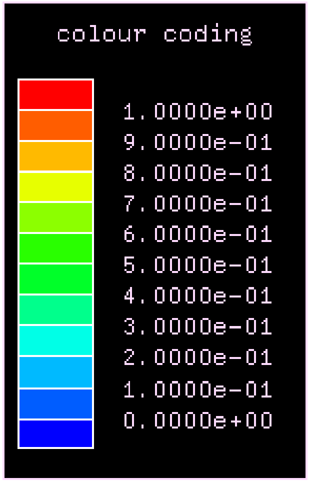
Fundamental (TE_{10} or H_{10}) mode
in a standard rectangular waveguide.

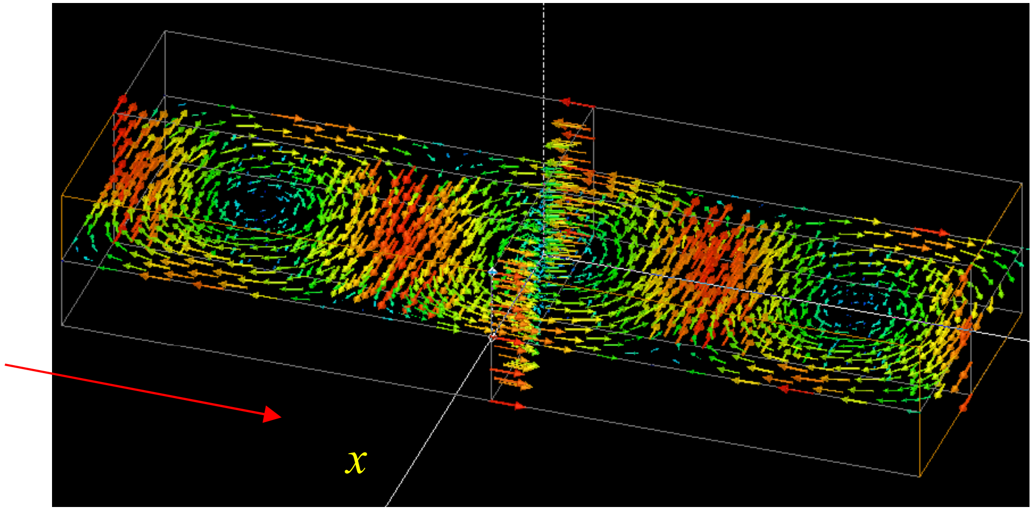
E.g. forward wave

power flow: $\frac{1}{2}\mathrm{Re}\left\{\iint\limits_{\substack{\text{cross}\\\text{section}}} \vec{E}\times\vec{H}^{*}\cdot \mathrm{d}\vec{A}\right\}$

Electric field







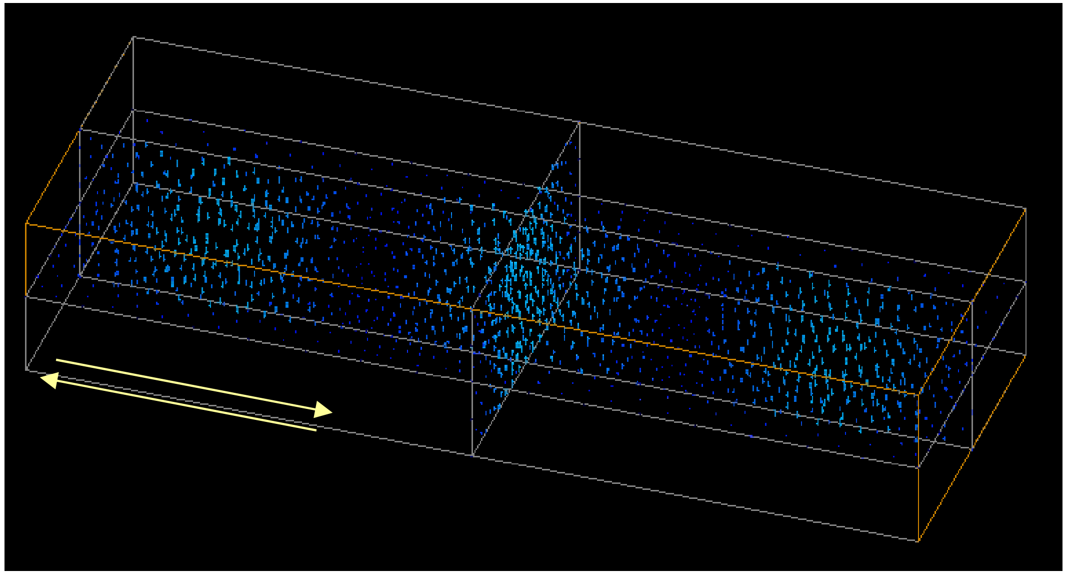
Magnetic field

Courtesy of Erk Jensen, CERN

Standing wave – resonator

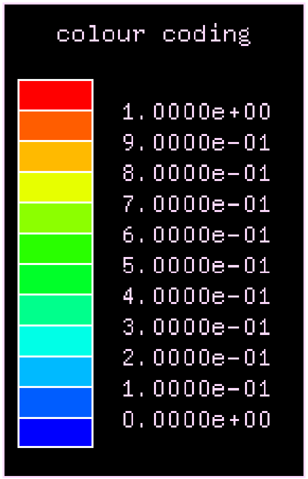
Two counter-running waves of identical amplitude.

electric field

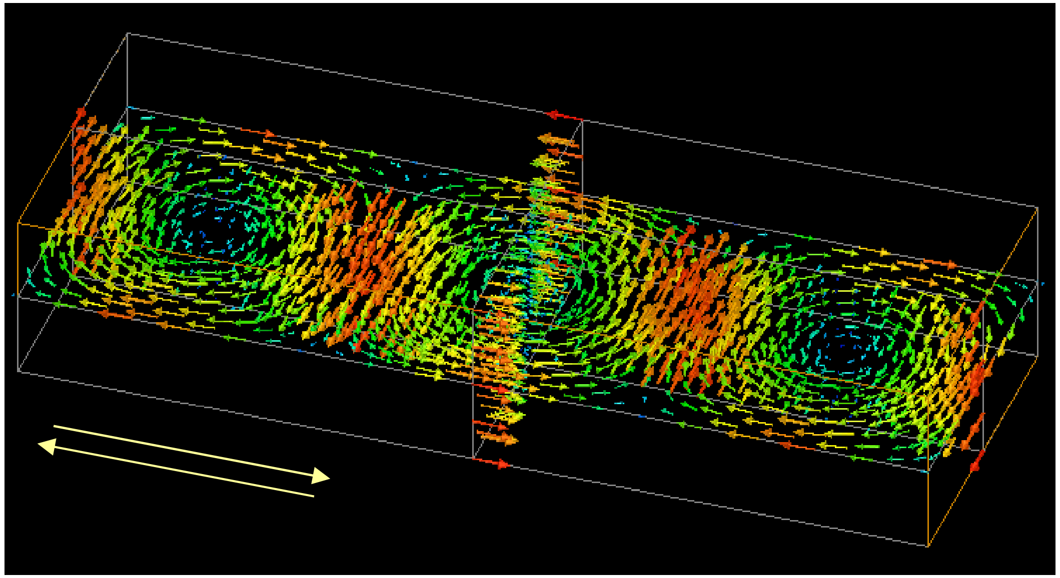


NO net power flow:

$$\frac{1}{2}\mathrm{Re}\left\{\iint\limits_{\substack{\text{cross}\\\text{section}}} \vec{E}\times\vec{H}^{*}\cdot \mathrm{d}\,\vec{A}\right\}=0$$



magnetic field
(90° out of phase)



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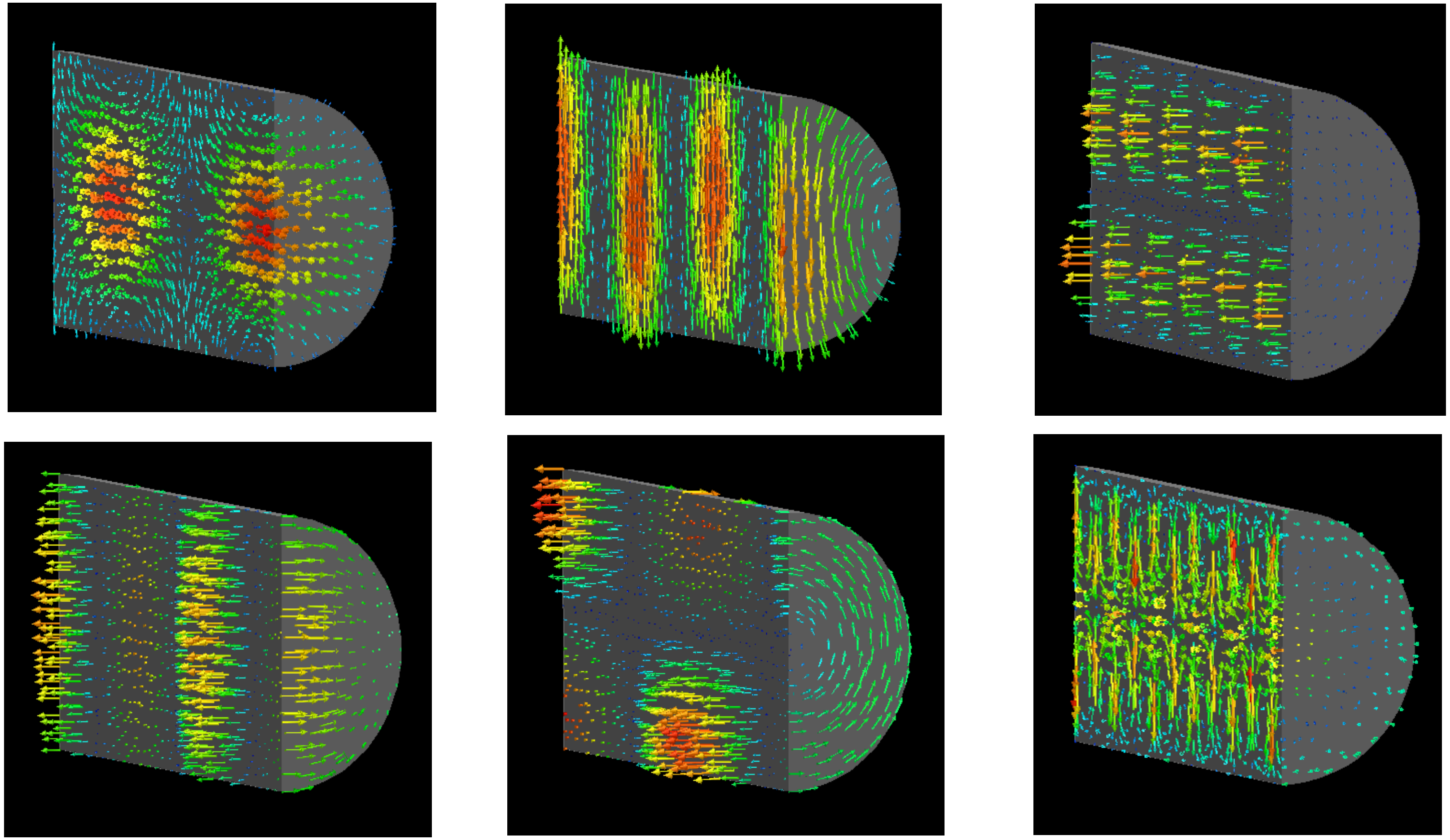
Round waveguide

parameters used in calculation:
f = 1.43, 1.09, 1.13 f_c, a: radius

$\vec{E}$

$\vec{B}$



TE$_{11}$: fundamental mode

$$\frac{f_c}{\text{GHz}} = \frac{87.85}{a\,/\,\text{mm}}$$

TM$_{01}$: axial electric field

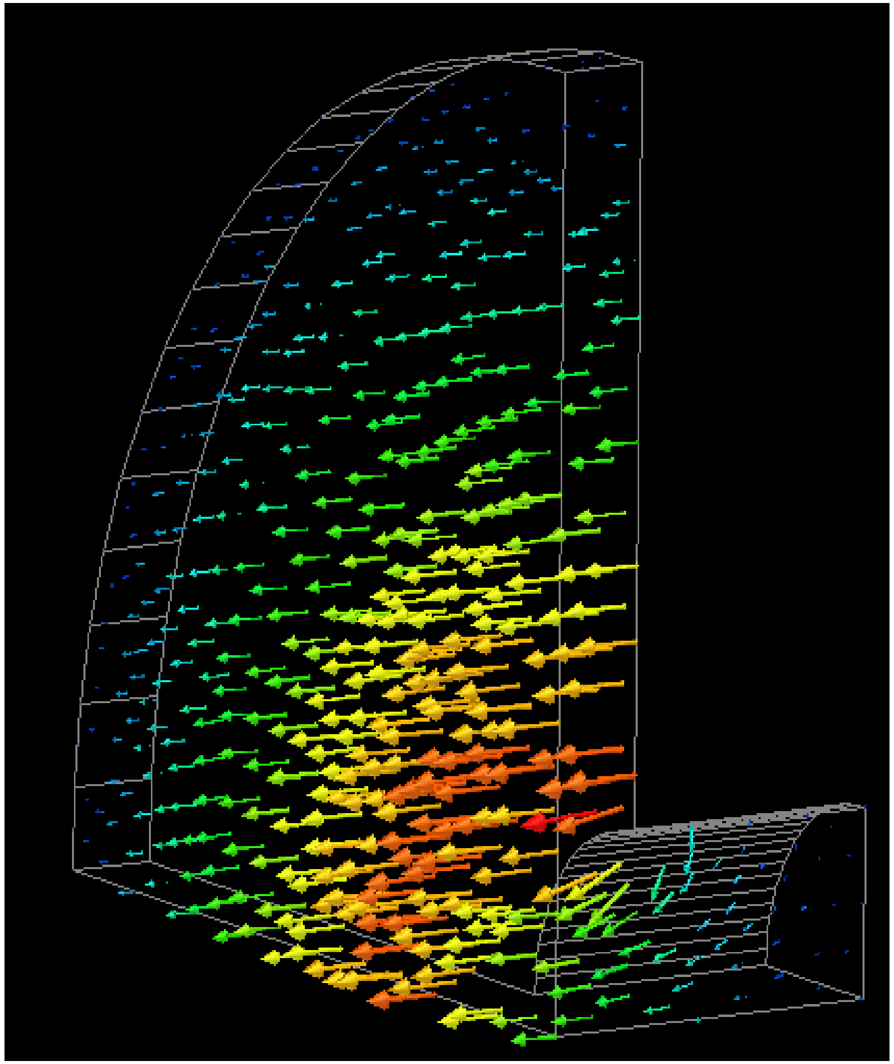
$$\frac{f_c}{\text{GHz}} = \frac{114.74}{a\,/\,\text{mm}}$$

TE$_{01}$: lowest losses!

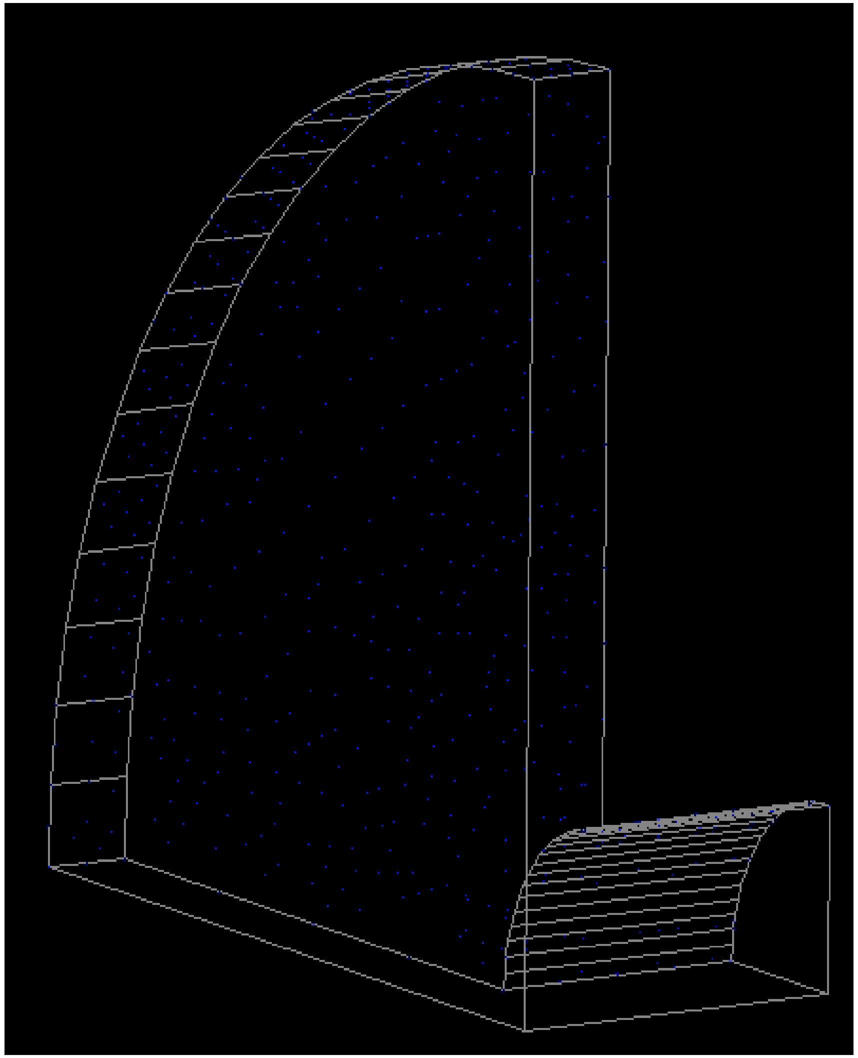
$$\frac{f_c}{\text{GHz}} = \frac{334.74}{a\,/\,\text{mm}}$$

Pillbox cavity

TM$_{010}$-mode (only 1/8 shown)



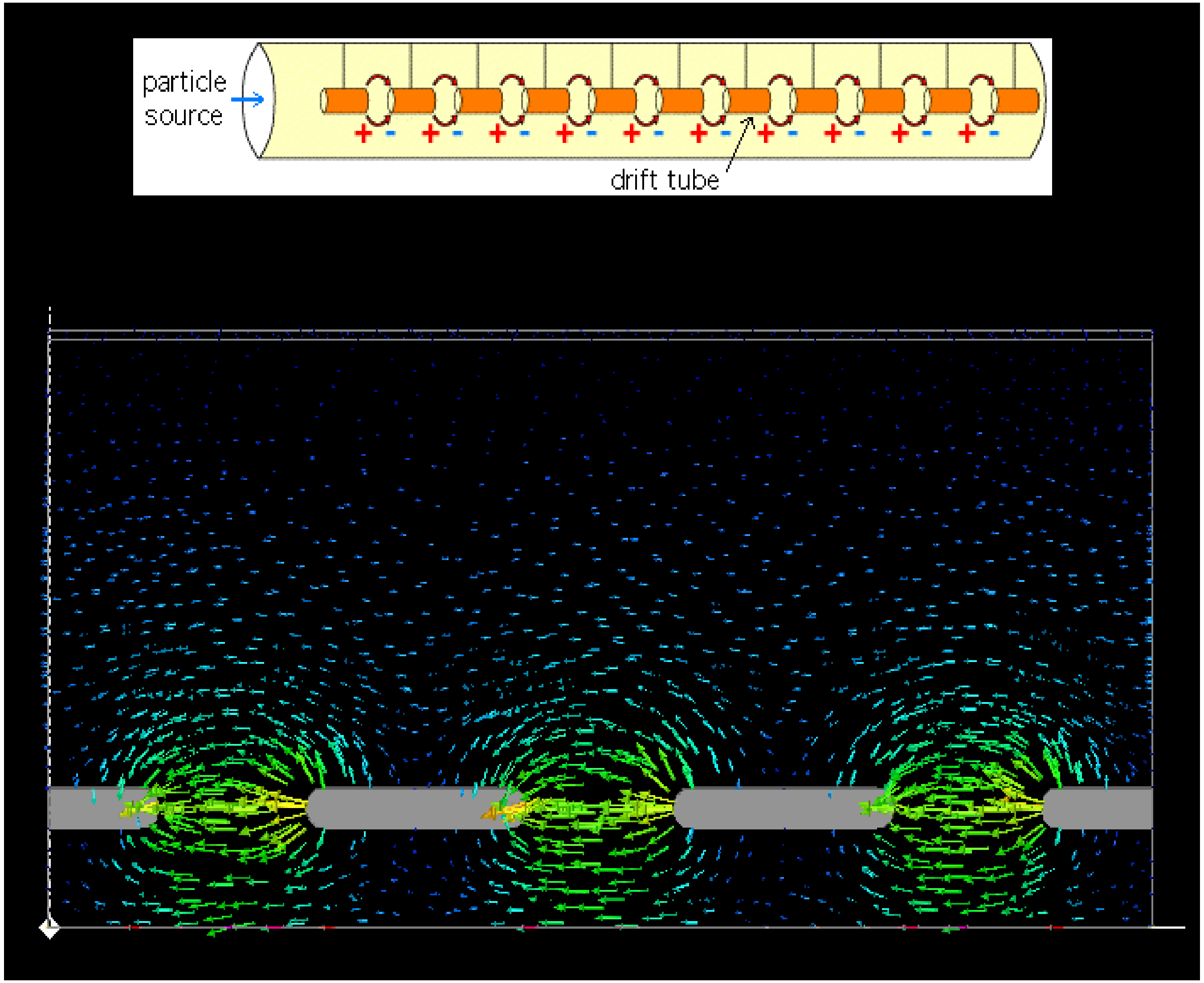
Electric field



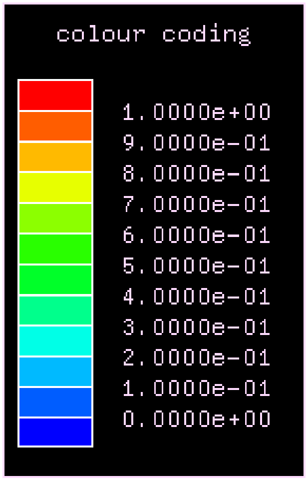
Magnetic field

Drift Tube Linac (DTL) – how it works

For slow particles the drift tube lengths can be easily adapted.



Electric field

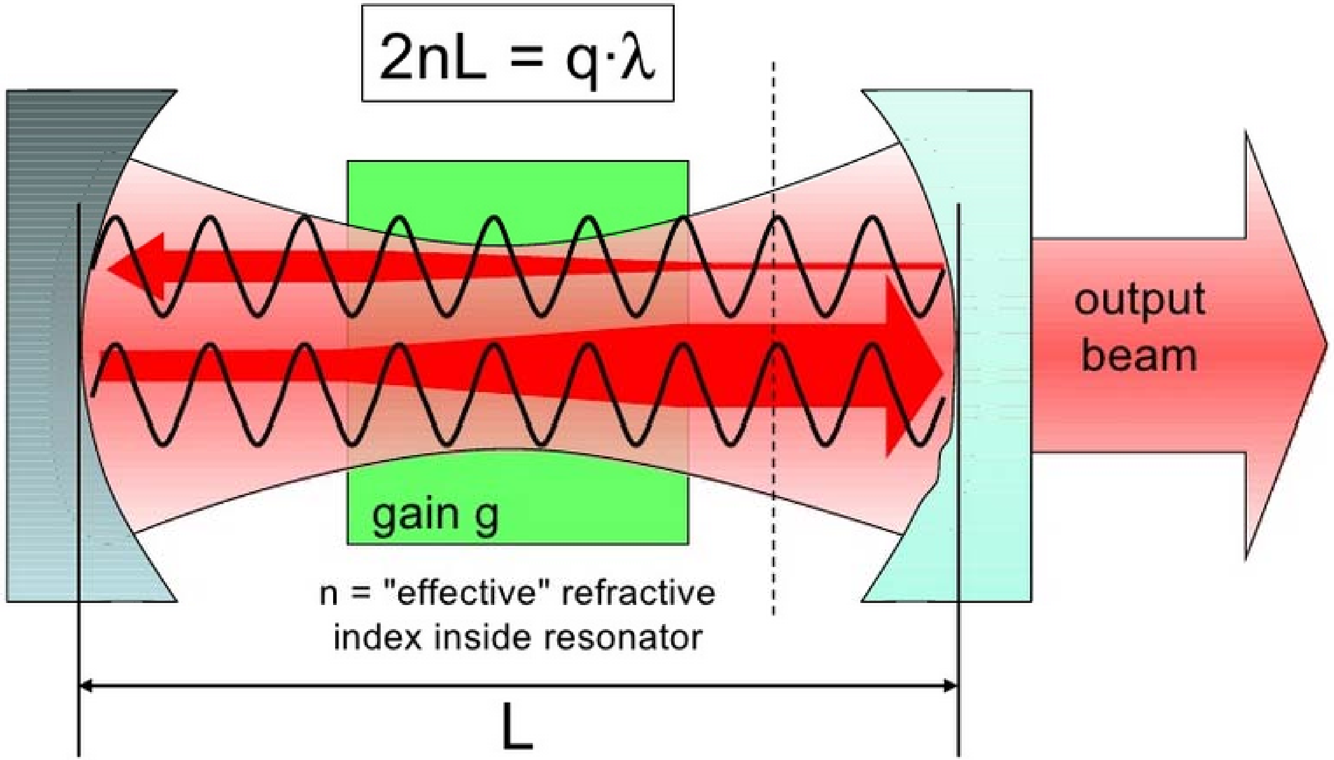


Analogy: RF/optical resonator

a resonator has resonance frequencies !

What is a Laser ?

Resonator + Gain Medium



Problem: Missing Link

Powerful ion sources

High Current
Small Emittance

200 mA p
ε=1mm mrad

Classic transport
50% losses !

Rfq
< 10% losses !

Powerful accelerators

High energies
good efficiency
bunches

Here: Multiple use of same rf field.

History

- 1970 Kapchinskij and Teplyakov propose the Rfq idea
- 1974 experimental test at USSR Institute for High Energy Physics in Protvino. A 148.5 MHz RFQ accelerated 100-KeV protons to 620 KeV with an efficiency of 50%.
- 1977 RFQ concept is published in the western world. Strong interest in Los Alamos National Laboratory (USA). Decision to test the RFQ principle. Developments of computer codes for RFQ design.
- 1979 Start of P.O.P. (Proof-of-principle) experiment at Los Alamos . 425 MHz RFQ accelerates a 100-keV proton beam to 640 keV with an efficiency of 90%, as predicted by the codes.
- Nowadays hundreds of RFQ accelerator are operating in the world.

Three goals

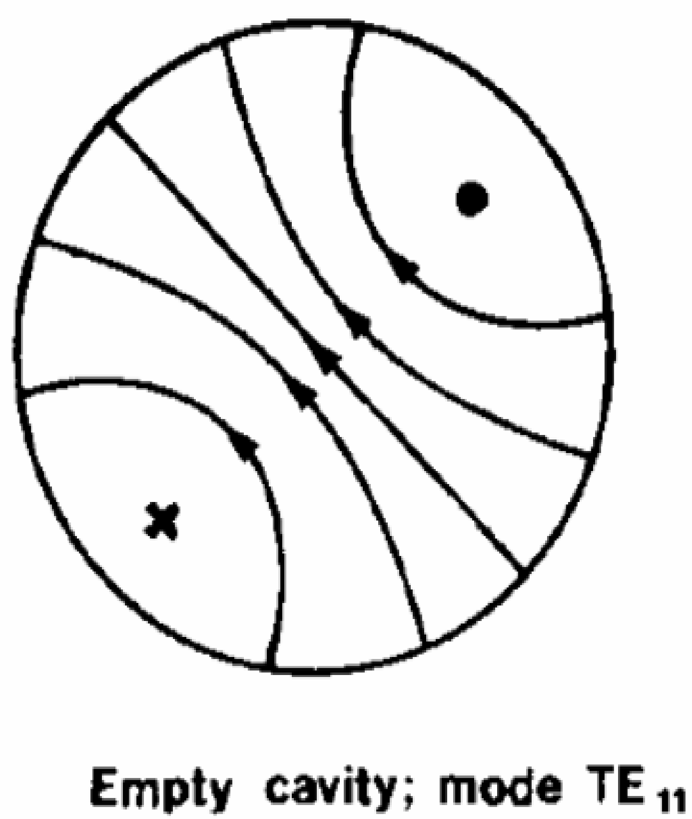
- Focus
- Bunch
- Accelerate

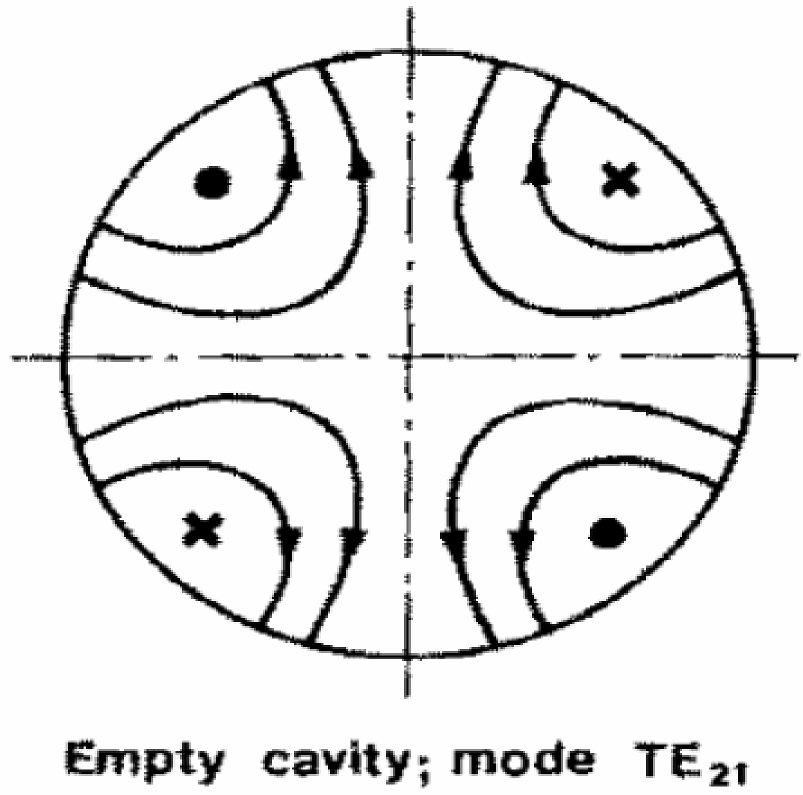
a dc ion beam with high efficiency.

!!! Preserve the emittance !!!

Both the focusing as well as the bunching and acceleration are performed by the RF field.

TE Modes

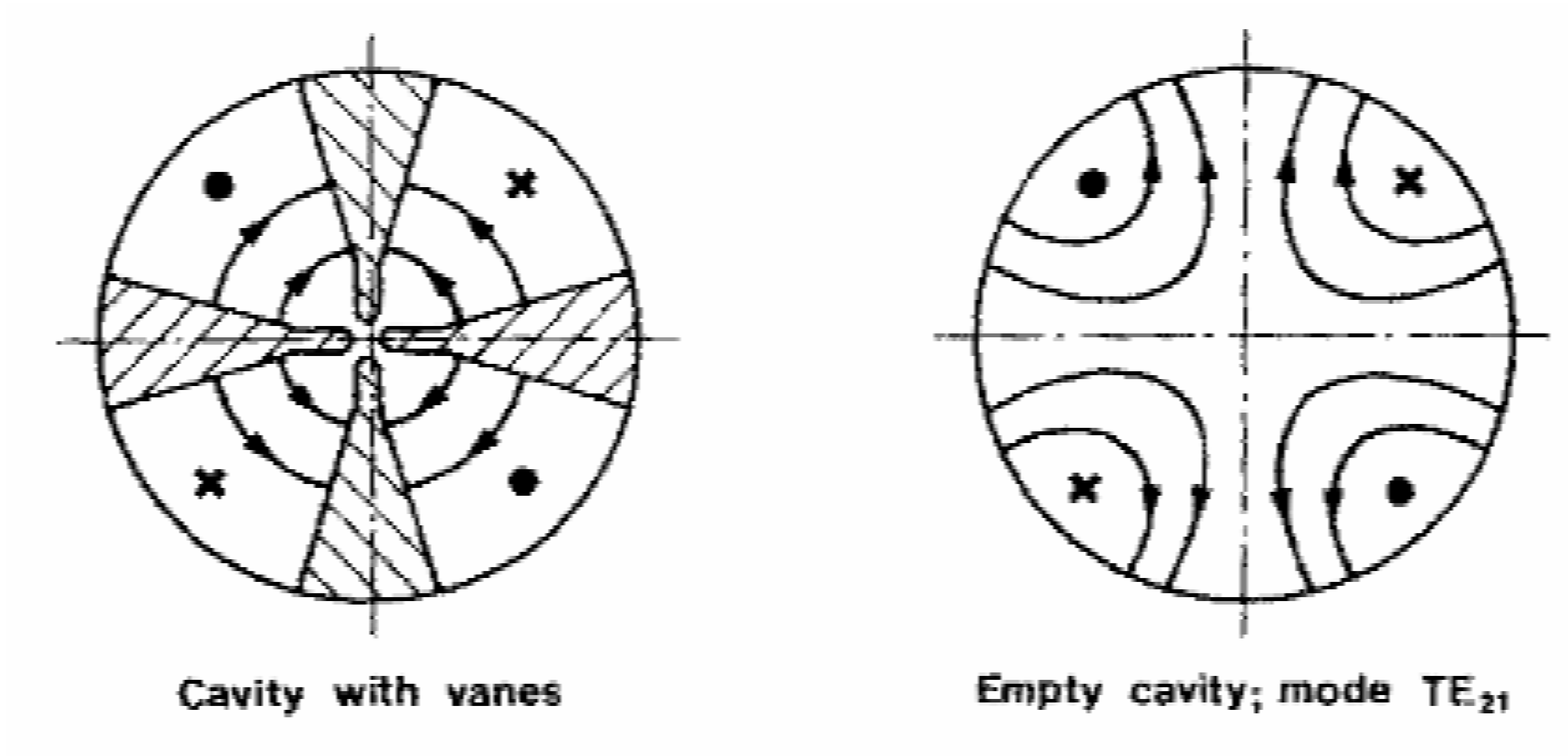




Dipole Mode

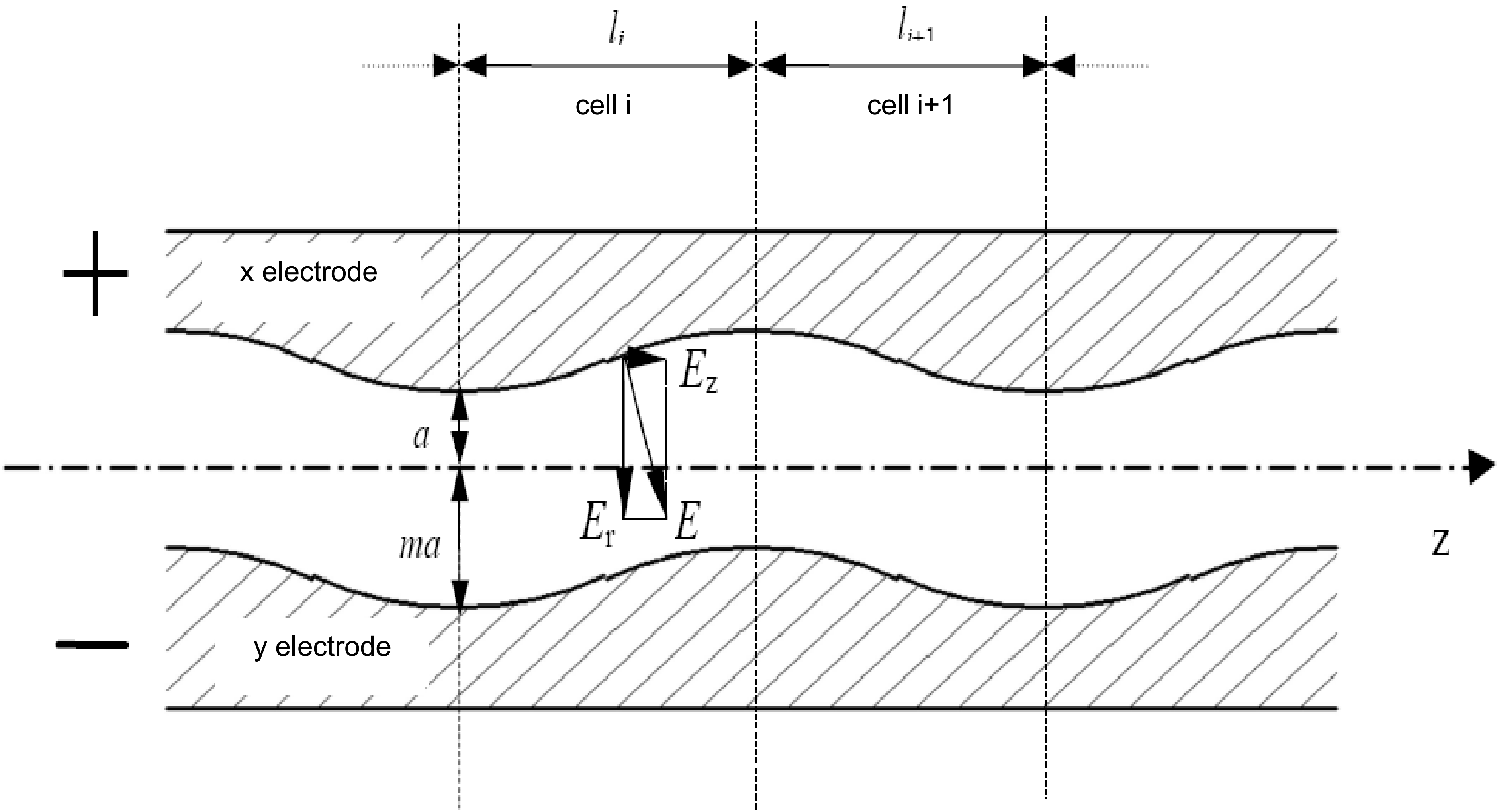
Quadrupole Mode: Rfq.

Load Cavity with 4 Electrodes



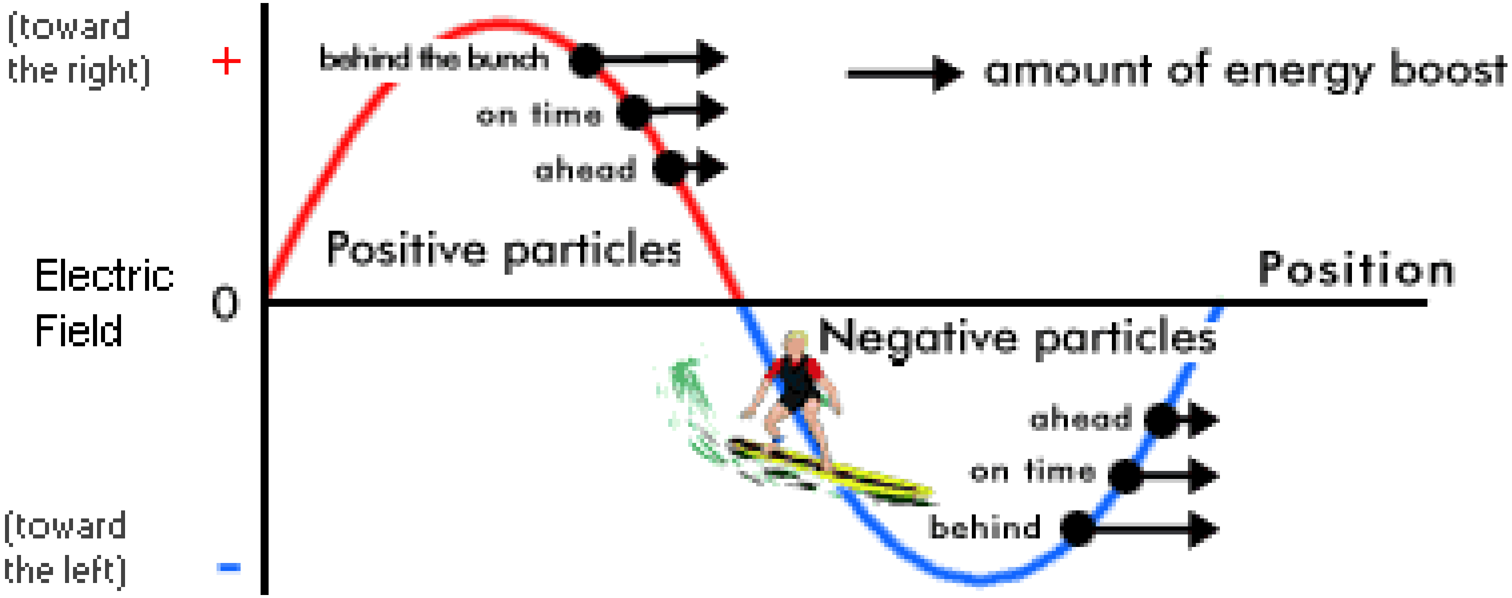
TE_{210} mode.

Acceleration in an Rfq



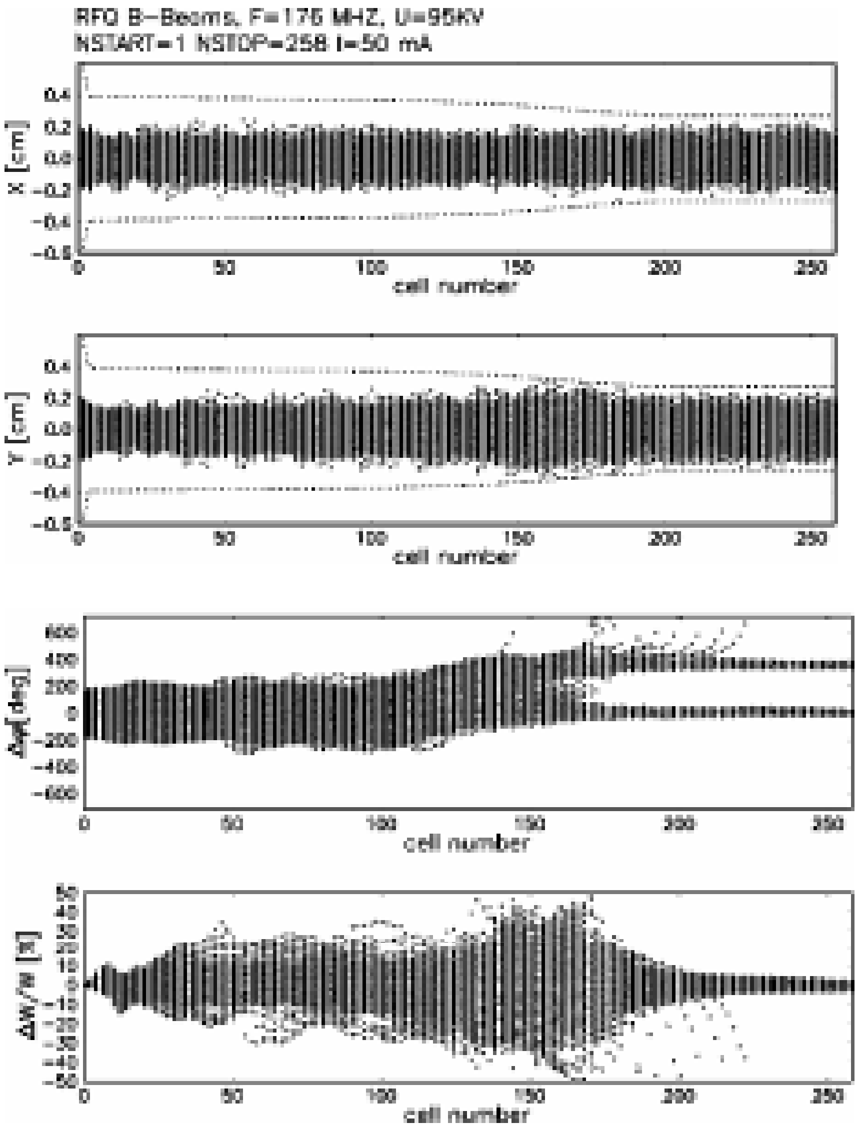
Linac 2: Phase Focusing

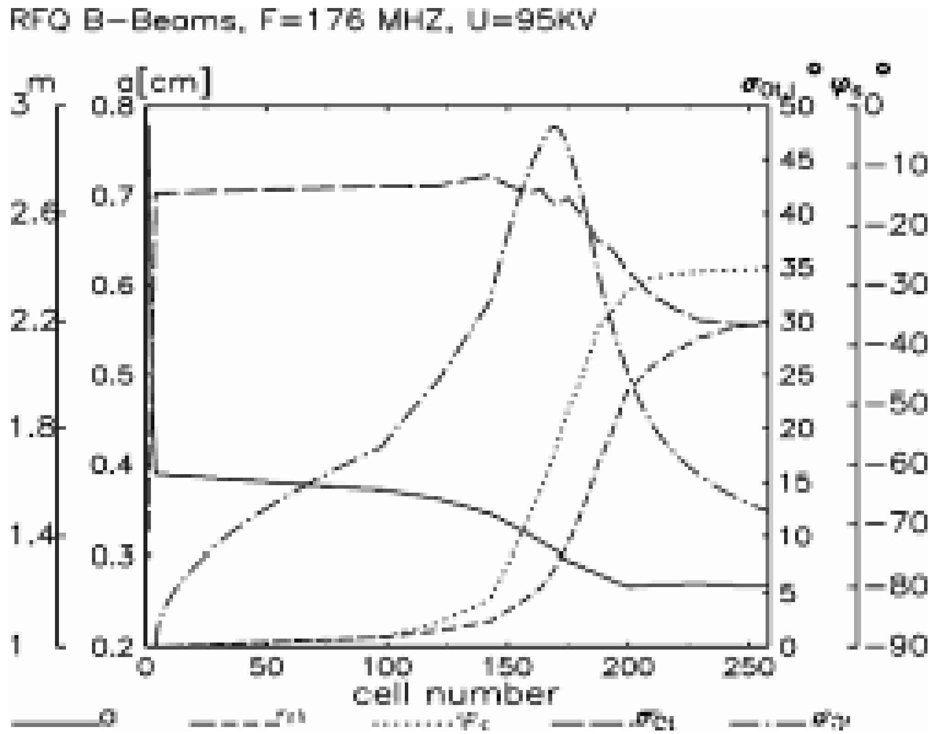
Why do particles stay within bunch ?



- E. M. McMillan – V. Veksler (1945)
- Field is synchronized so that slower particles get more acceleration
- Energy/time focus in laser acceleration ?

Overall Design





Rfq Sections

Radial matching to adapt the beam to a time-varying focusing system		
		aperture smoothly brought to the average value
Shaping to give the beam a longitudinal structure		
Taper phase to –80°,-60°	Start modulation	aperture such that focusing is constant
Bunching to bunch and begin acceleration		
Taper phase to –30°,-20°	Max. modulation	aperture such that focusing is constant
Acceleration to bring the beam to the final energy		
Constant Phase	Constant modulation	Constant aperture
Output matching to adapt the beam to the downstream user's need.		

Ion Optics: Basics

- Standard: Some 10^9 particles/bunch
- Always: ΔE (SR, rest gas, fringe fields, etc.)

How to guide particles ? $\vec{F} = e \cdot \left(\vec{E} + \vec{v} \times \vec{B} \right)$

Examples: *B = 1T* √
E= 10^8 V/m

$$\frac{1}{R(x,z,s)} = \frac{e}{p} B_z(x,z,s)$$

Basics cont'd

- Develop *B* into Taylor series

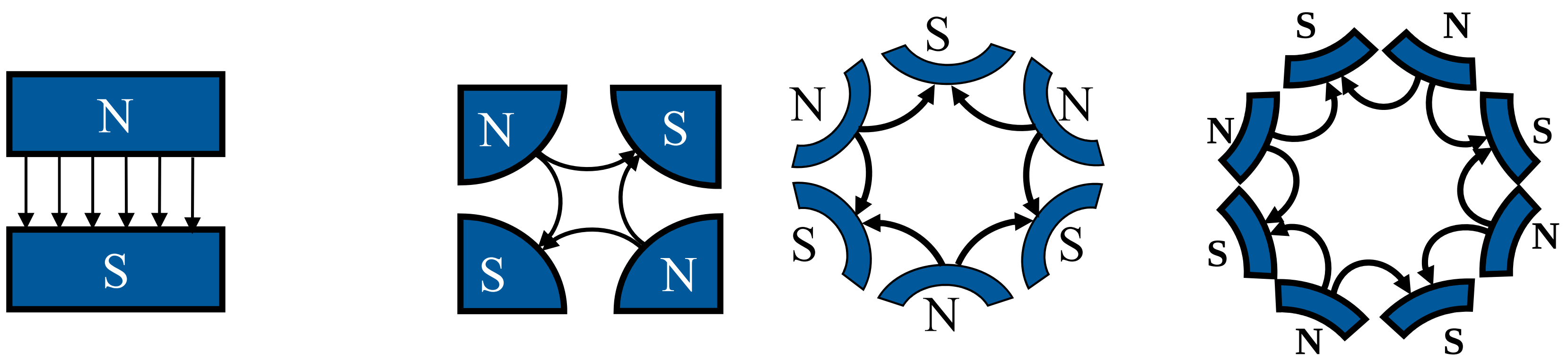
$$\frac{e}{p}B_z(x) = \frac{e}{p}B_{z0} + \frac{e}{p}\frac{dB_z}{dx}x + \frac{1}{2!}\frac{e}{p}\frac{d^2B_z}{dx^2}x^2 + \frac{1}{3!}\frac{e}{p}\frac{d^3B_z}{dx^3}x^3 \ldots$$

$$= \frac{1}{R} + kx + \frac{1}{2!}mx^2 + \frac{1}{3!}ox^3 \ldots$$

Magnet Definitions

- **2n-pole**:

dipole | quadrupole | sextupole | octupole …

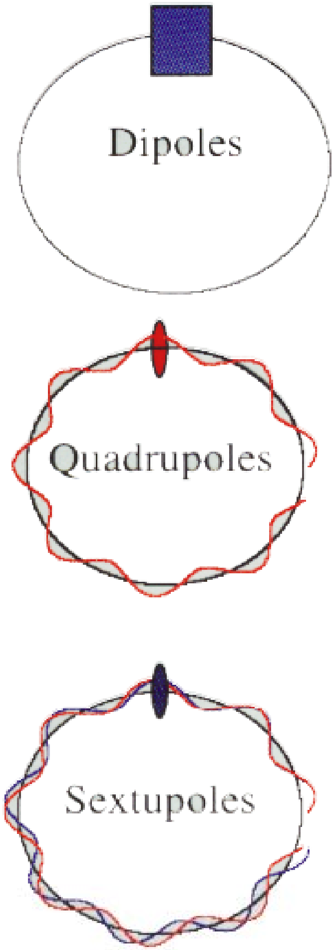


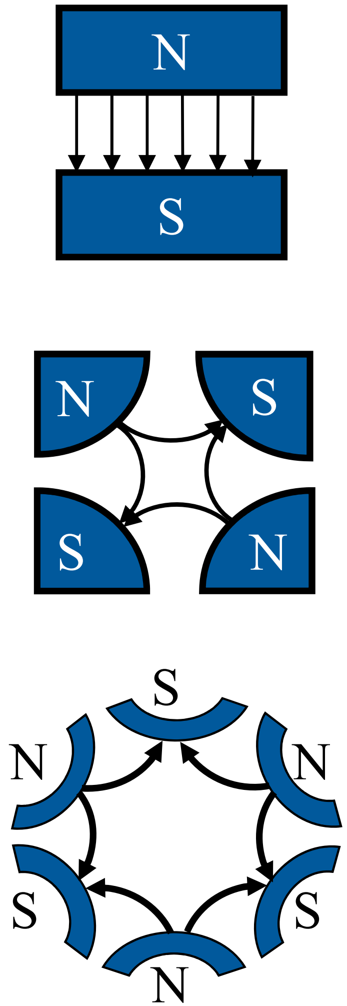
n: 1 2 3 4 …

- Normal: gap appears at the horizontal plane
- Skew: rotate around beam axis by **π/2n** angle
- Symmetry: rotating around beam axis by **π/n** angle, the field is reversed (polarity flipped)

D. Robin, MSU

Functions of Magnetic Elements





- Bending;
- Focusing;
- Correction of chrom. aberrations.

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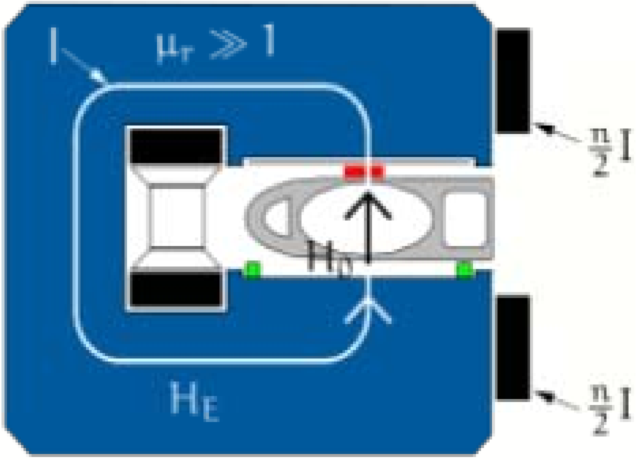
Dipole Magnets

- Used for beam bending



Something to remember:

$$\frac{1}{\rho}\left[m^{-1}\right] = 0.2998\,\frac{B_0[T]}{p[GeV/c]}$$

Field Calculation

$$\oint \vec{H}\,\vec{ds} = hH_0 + lH_E$$

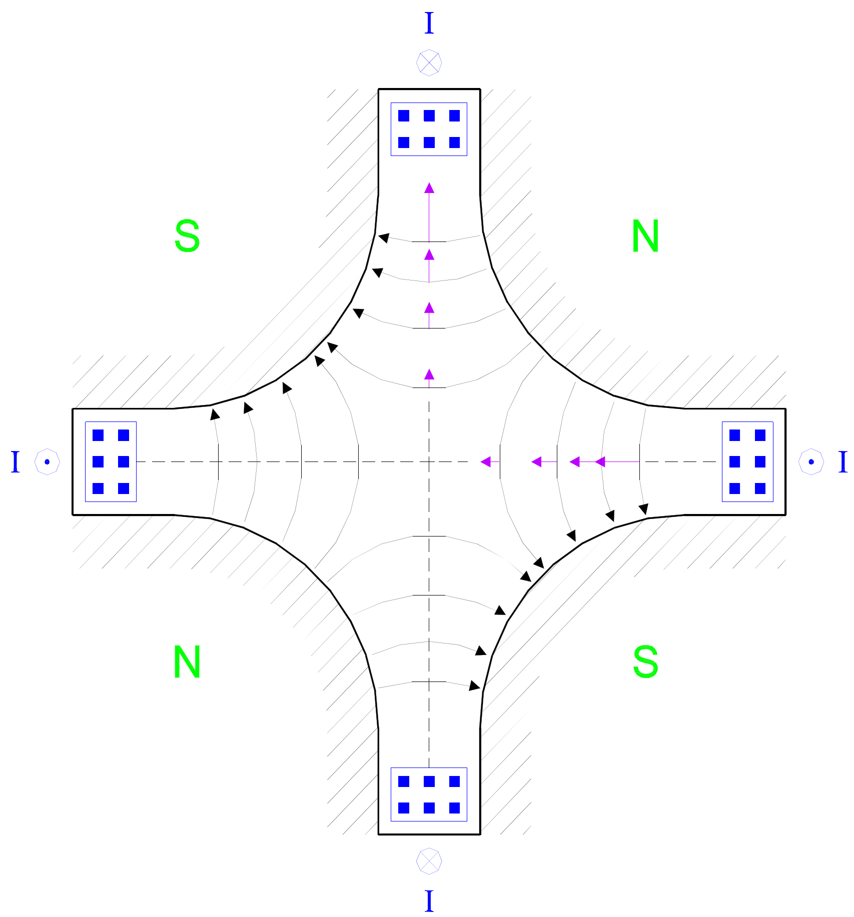
$$H_E = \frac{1}{\mu_r} H_0$$

if μ_r>>1, then

$$B_0 = \frac{\mu_0 n I}{h}$$

with h = gap height

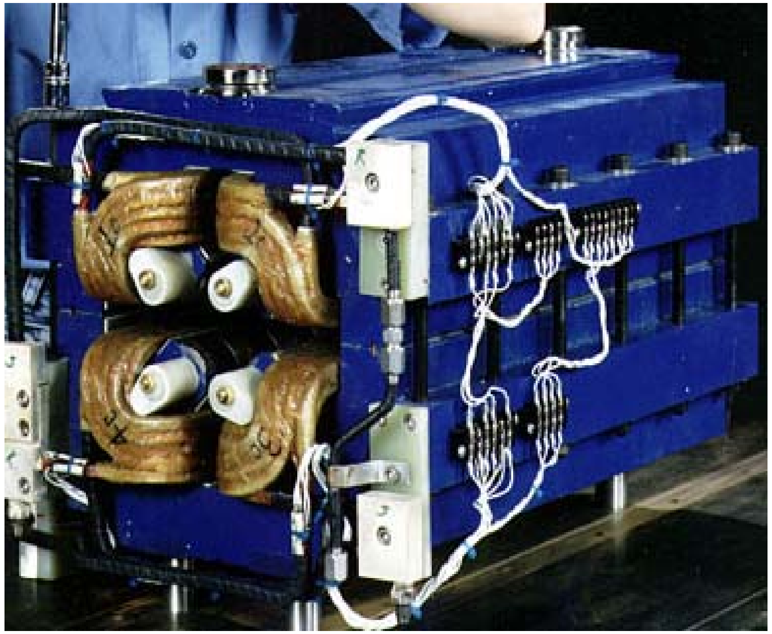
Focusing: Quadrupoles



- Quadrupole produces a constant gradient g=-dB$_z$/dx.
  - Focusing forces increase linear with displacement
  - Important: no coupling

- Optical lenses are either focusing or defocusing.
- Magnetic lenses focus in one plane but are defocusing in the orthogonal plane (from Maxwell's equations)

Quadrupole Magnets



- Field rises linear with distance

$$B_x(y) = -g \cdot y$$
$$B_y(x) = -g \cdot x$$

If *R* is distance (centre - pole) and current *I* flows through *n* coil turns

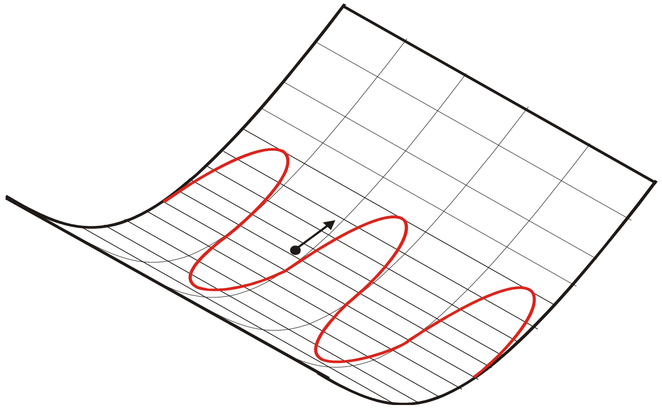
$$g = \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = \frac{2\mu_0 n I}{R^2}$$

Following dipole logic: $k\left[m^{-2}\right] = 0.2998\,\frac{g\left[T/m\right]}{p\left[GeV/c\right]}$

Transverse Particle Motion

Vertical displacement *y* in quad of length *ds* and strength *k*:

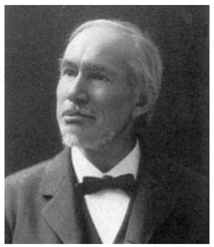
$$dy' = -y \cdot k \cdot ds$$



Leads to Hill's equation:

$$y''(s) \pm k(s)\,y(s) = 0$$

Generalized expression:



George Hill

$$K(s) = \begin{cases} -k(s) + \dfrac{1}{\rho^2(s)} & \textit{horizontal} \\ k(s) & \textit{vertical} \end{cases}$$

Solution of Hill's Equation

- Harmonic oscillator with variable spring constant.

  Generalized coordinate u(s) = x or y.

$$u(s) = a\sqrt{\beta(s)} \cdot e^{\pm i(\Phi(s)+\Phi_0)}$$

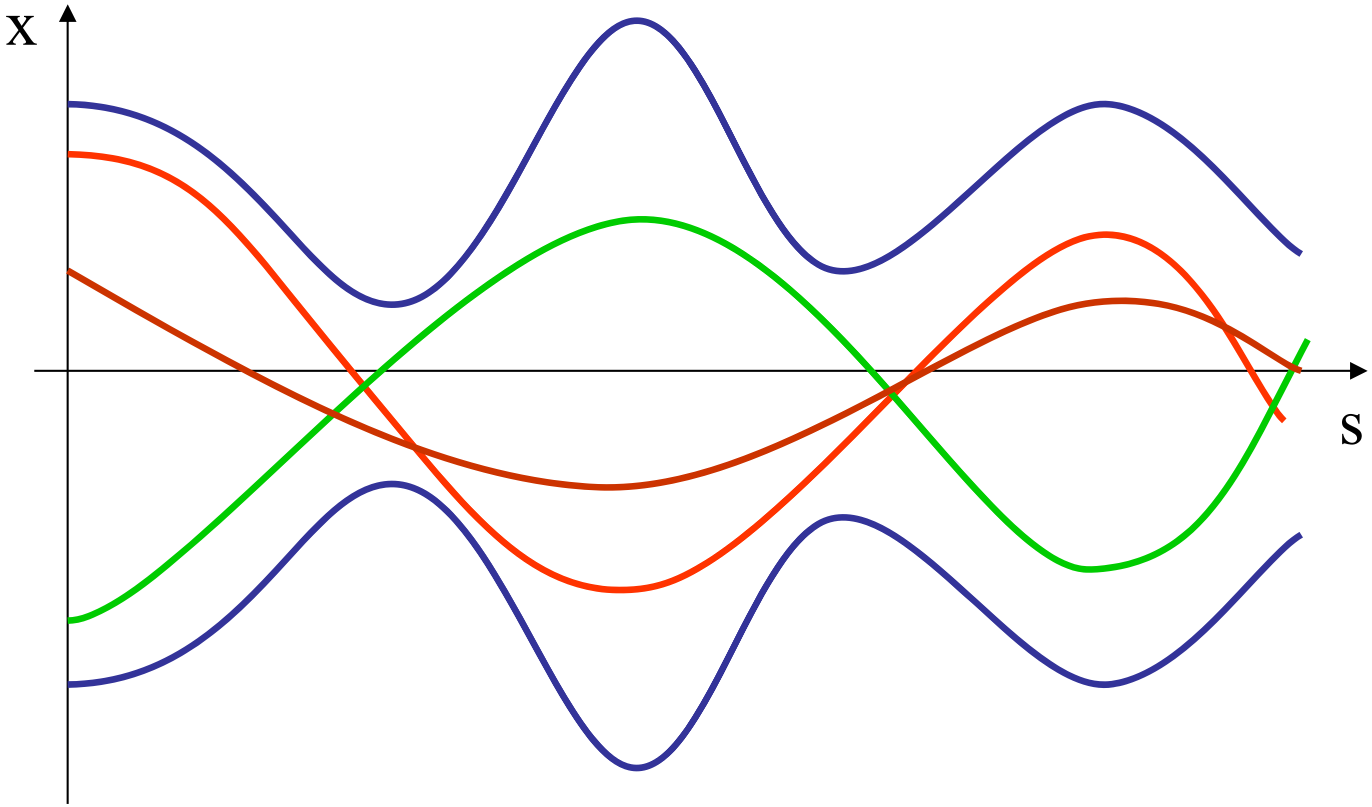
where $\Phi'(s) = \frac{1}{\beta(s)}$ $and\ \alpha = const.$

Phase advance per period *l* is

$$\mu = \mu(s,l) = \int_s^{s+l} \frac{1}{\beta(t)} dt$$

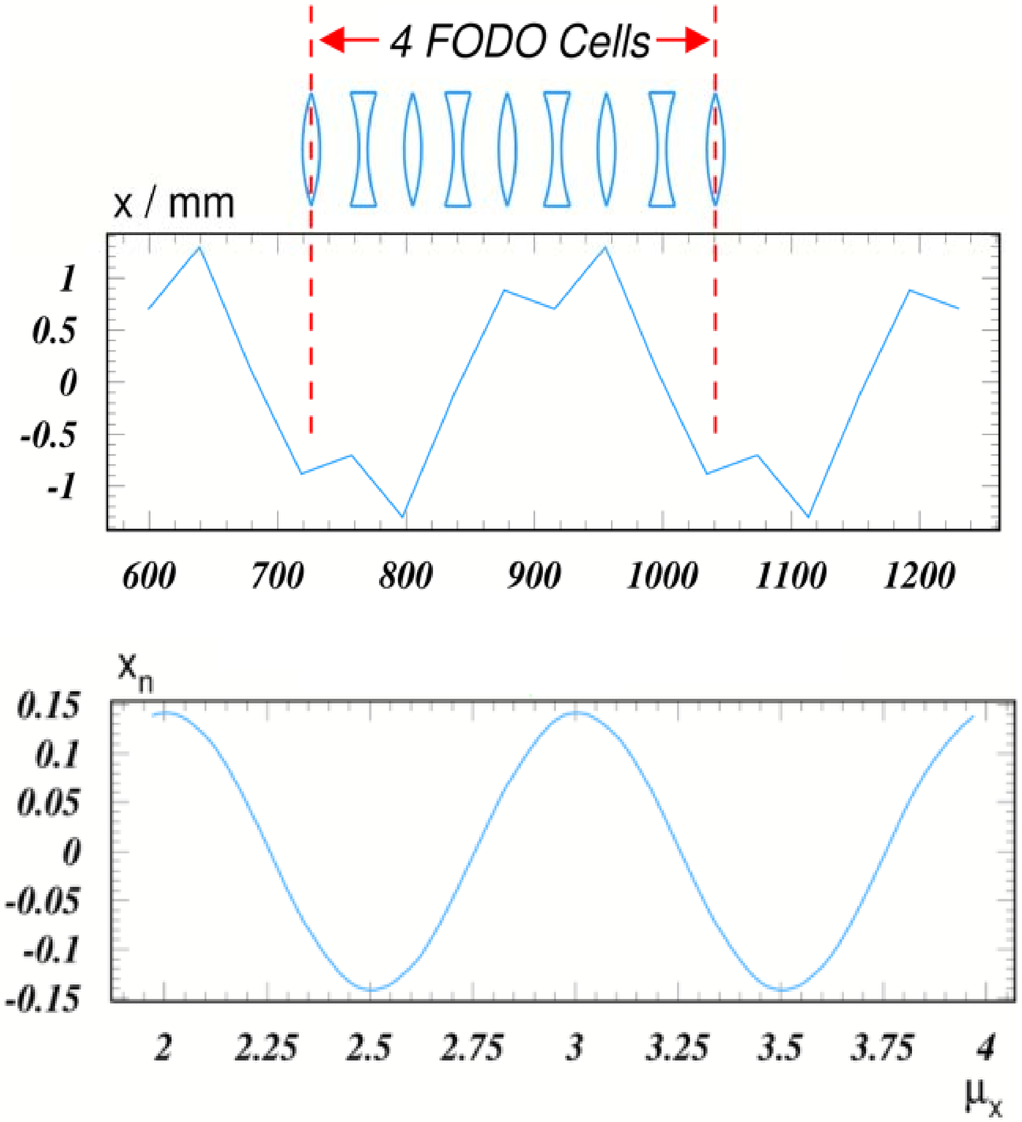
Only single particles !

Normally not of major interest.



➡ Find „Envelope"…

Why use β ?



- Coordinate transformation
- Make use of periodicity
- Normalized representation

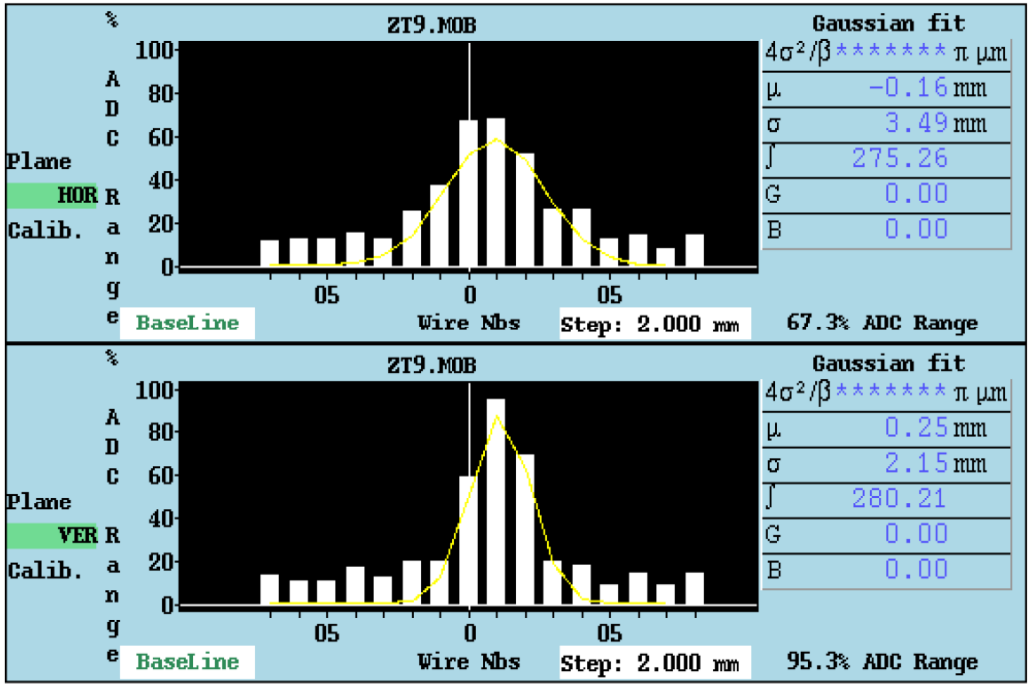
$$X(s) = x(s)/\beta(s)$$

Envelope

- Appr. Gaussian shape. Def.: $\sigma(s)=\sqrt{\varepsilon\cdot\beta(s)}$

Therefore:

$$\varepsilon=\frac{\sigma^2(s)}{\beta(s)}$$



Max. possible emittance (mech. aperture) defines the acceptance.

Dispersion

Until now ideal motion with $\Delta p/p=0$

Introduce dispersion:

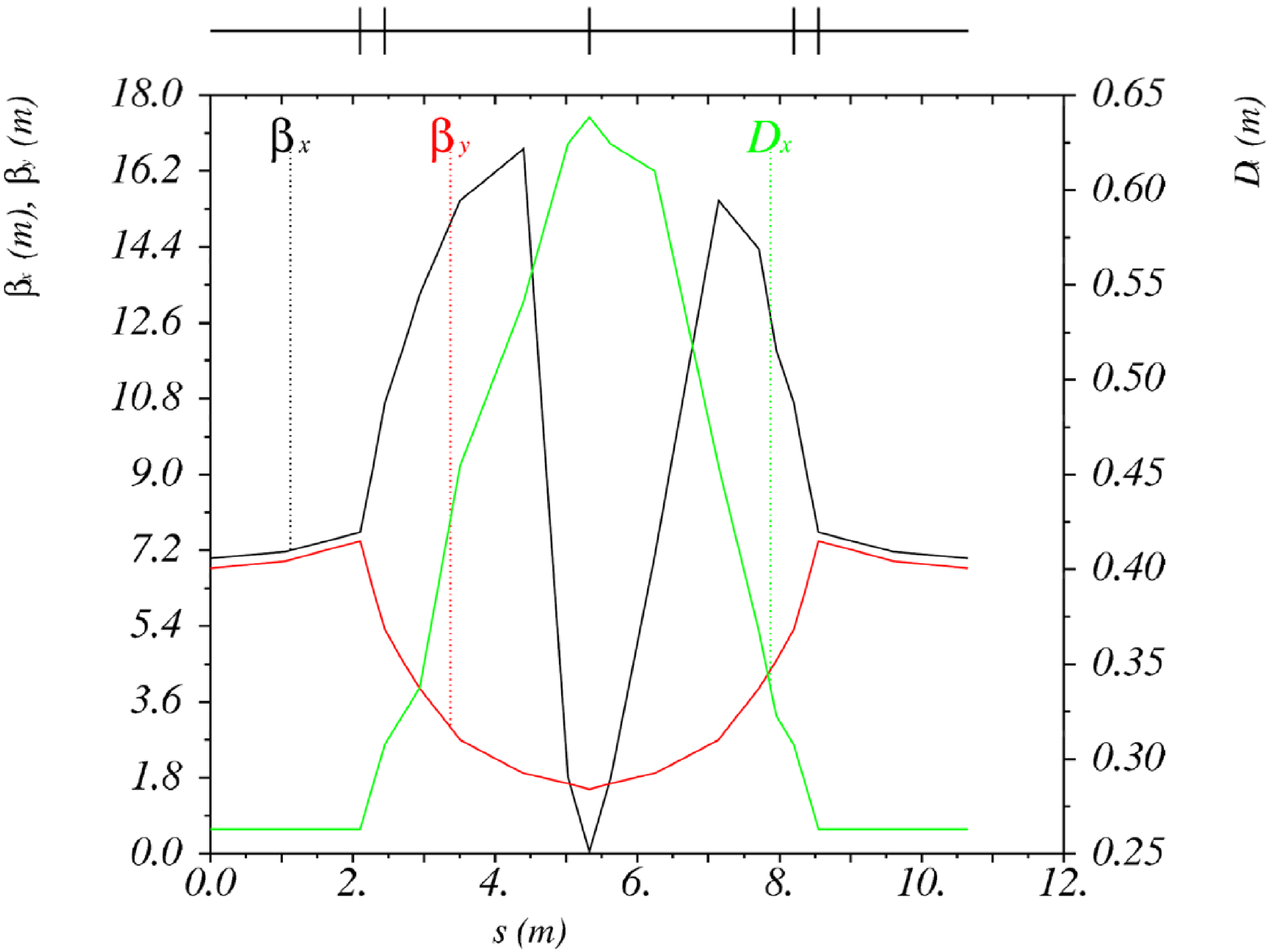
$$x''(s)+\left(\frac{1}{R^2(s)}-k(s)\right)x(s)=\frac{1}{R(s)}\frac{\Delta p}{p}$$

has to be non-zero

Most of the time dispersion orbit is defined as

$$\Delta p / p = 1$$

Machine Design: Orbit and D(s)

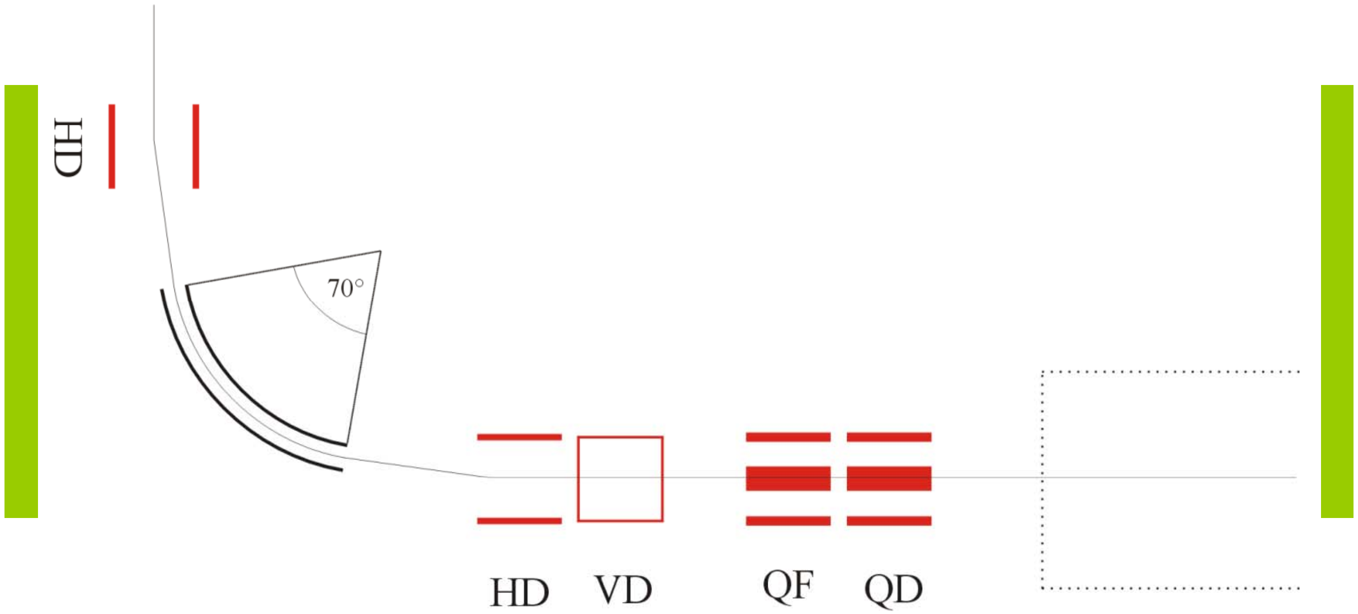


Diagnostics needs to measure/control this !

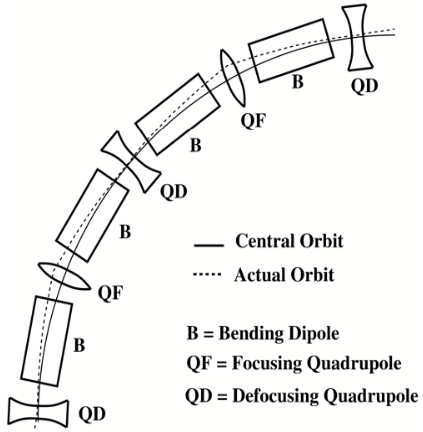
Complex Structures

Use individual matrices

$$\left(M_B\right)\left(M_{QF}\right)\left(M_{QD}\right)\left(M_D\right)\left(M_B\right)\left(M_D\right)\cdots$$



Complex Structures



- Drift

$$M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

- Thin Quadrupole where 1/f=kl

$$M_{Q,short} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -kL & 1 \end{pmatrix}$$

- Large Quadrupole

$$M_{QF} = \begin{pmatrix} \cos\left(L\sqrt{|k|}\right) & \frac{1}{\sqrt{|k|}}\sin\left(L\sqrt{|k|}\right) \\ -\sqrt{|k|}\sin\left(L\sqrt{|k|}\right) & \cos\left(L\sqrt{|k|}\right) \end{pmatrix}$$

$$M_{QD} = \begin{pmatrix} \cosh\left(L\sqrt{|k|}\right) & \frac{1}{\sqrt{|k|}}\sinh\left(L\sqrt{|k|}\right) \\ -\sqrt{|k|}\sinh\left(L\sqrt{|k|}\right) & \cosh\left(L\sqrt{|k|}\right) \end{pmatrix}$$

4x4 Matrices

- Combine the matrices for each plane

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) \\ C'_x(s) & S'_x(s) \end{pmatrix} \begin{pmatrix} x_0(s) \\ x'_0(s) \end{pmatrix}$$

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_y(s) & S_y(s) \\ C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} y_0(s) \\ y'_0(s) \end{pmatrix}$$

- ...to get a total 4x4 matrix

$$\begin{pmatrix} x(s) \\ x'(s) \\ y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} C_x(s) & S_x(s) & 0 & 0 \\ C'_x(s) & S'_x(s) & 0 & 0 \\ 0 & 0 & C_y(s) & S_y(s) \\ 0 & 0 & C'_y(s) & S'_y(s) \end{pmatrix} \begin{pmatrix} x_0(s) \\ x'_0(s) \\ y_0(s) \\ y'_0(s) \end{pmatrix}$$

Summary

- (very brief) overview of accelerator history;
- Linacs at the heart of most facilities – important: Rfq;
- From Maxwell's equations to cavity modes;
- Similiarities between rf cavities and laser resonators.
- (simplified) beam optics and application to accelerator layout;

All of these will turn up many times during this week !

Thanks for your attention.