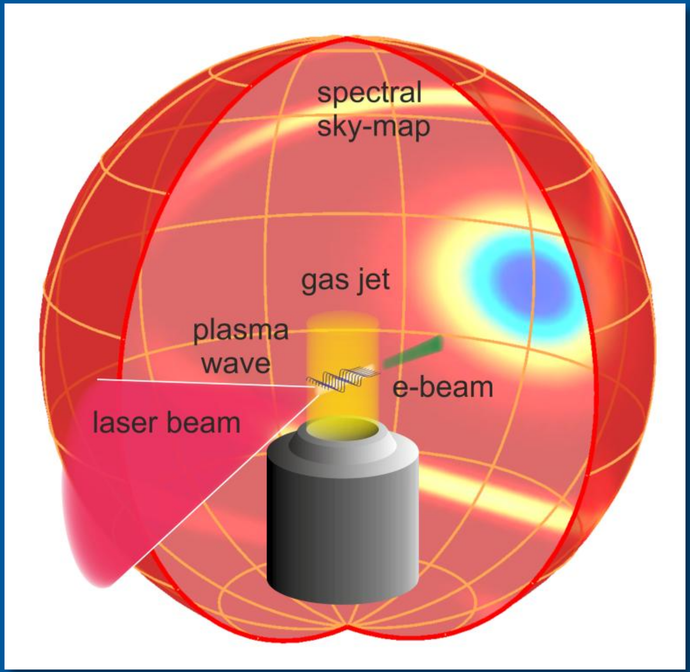
Laser-plasma based electron acceleration



Arie Irman

*Institute of Radiation Physics,
Helmholtz Zentrum Dresden Rossendorf ,
Germany*

First International School on Laser Applications at Accelerators,
14th -19th October 2012, GANIL, CAEN, France

HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

Outline

Part 1

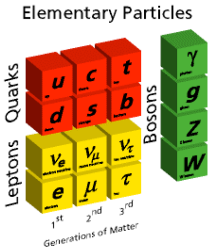
- Introduction
- Characteristics of plasmas
- Plasma wave excitation
- Wave-breaking
- Electron dynamics in plasma wave

Part 2

- Electron injection methods
- Recent progress in laser-plasma electron acceleration experiment
- Conclusion: main challenges
- Overview of LWFA program at HZDR

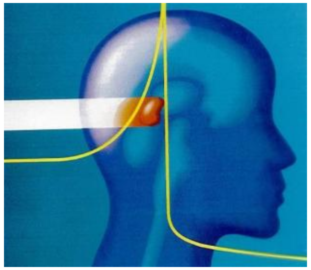
High energy charged particles

- High energy physics:
  fundamental structure of matter and energy

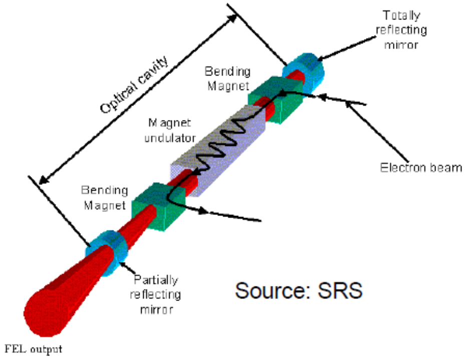


- Material science: semiconductor, phase transition

- Medical physics: cancer therapy



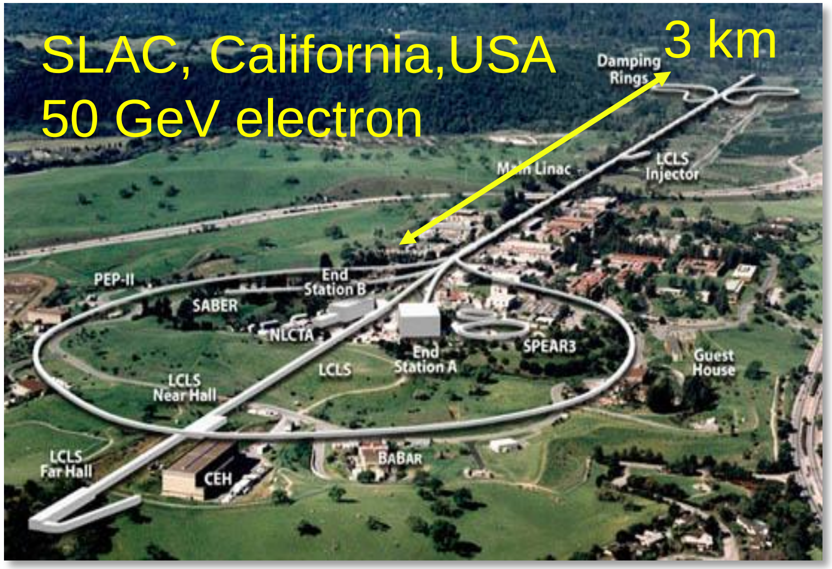
- Coherent radiation and X-ray sources:
  synchrotron and FEL

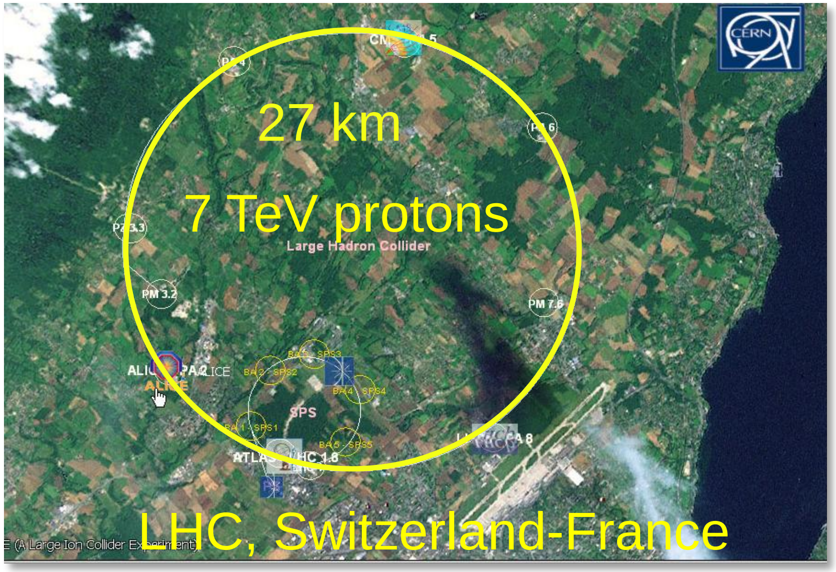


Source: SRS

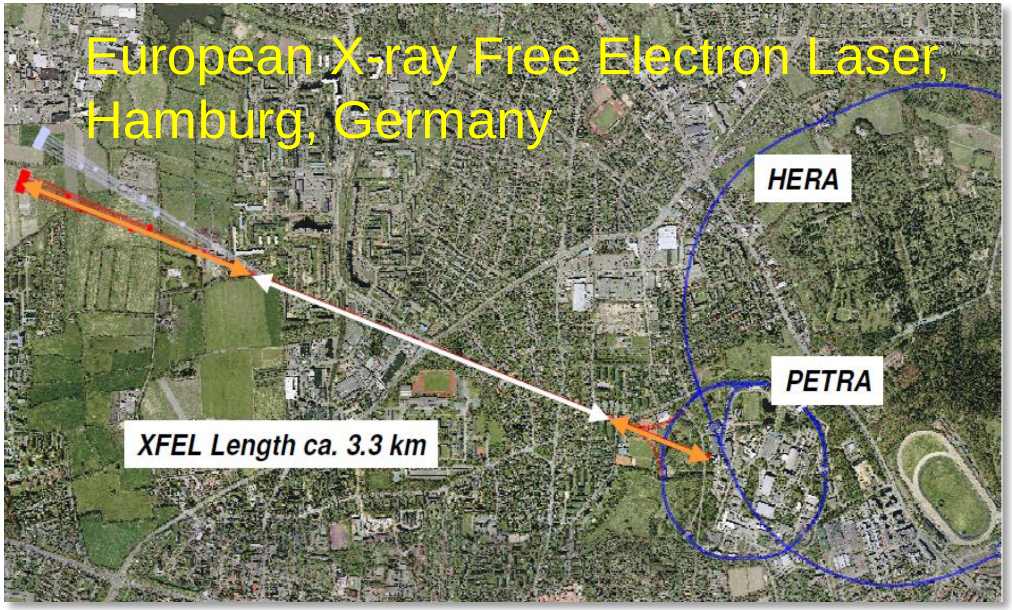
- State of the art technology: vacuum technology, detector

Particle accelerators





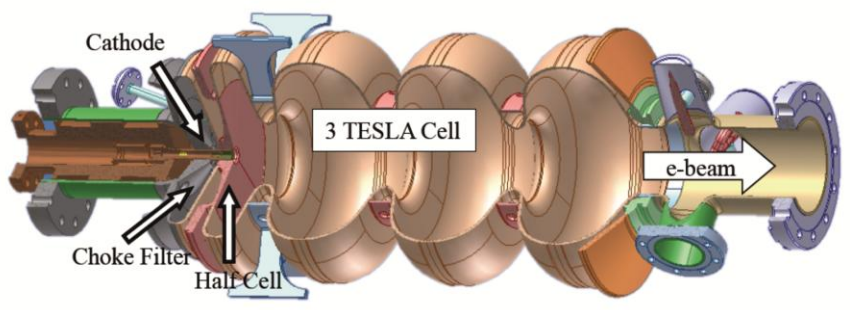
Drawbacks:
- large infrastructure: extremely expensive
- limitted access



RF cavities versus plasmas

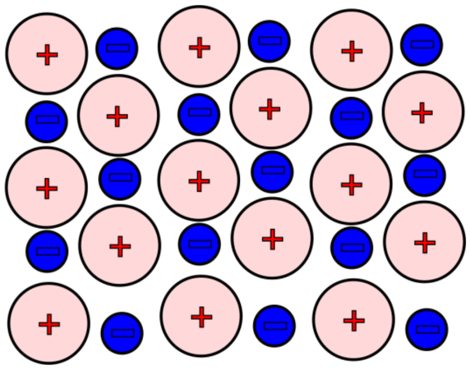
- Linac → high power RF technology
  → accelerating field ~ a few tens of MV/m (vacuum breakdown)

3-1/2 cells Superconduting RF photoinjector ELBE-HZDR



design value:
E_{peak} = 50 MV/m
(TESLA cavities at DESY)
obtained:
E_{peak} ≈ 20 MV/m

- Plasmas → neutral particles, hot ions and electrons
  → space-charge electric fields >> RF electric fields in linac



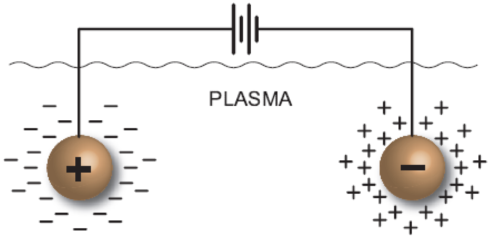
$$E \propto \sqrt{n_p}$$

Characteristics of plasmas

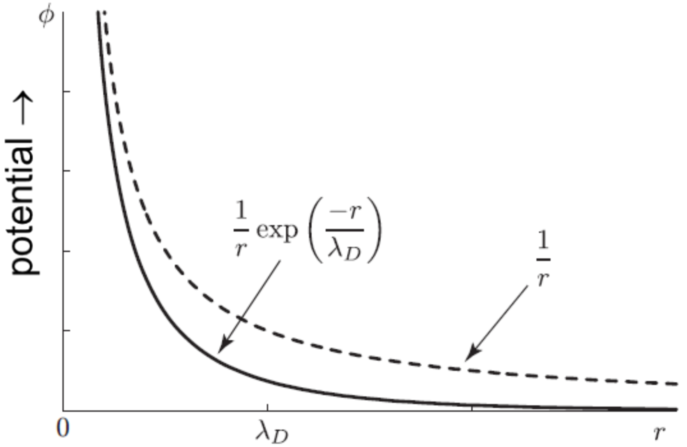
- Collective behavior
  - $\rightarrow$ not depend on local conditions
  - $\rightarrow$ long-range interaction (collision is negligible)

- Debye shielding, Debye length, Debye sphere

$$\lambda_D = \sqrt{\frac{k_B T_e}{4\pi n_0 e^2}} \qquad \lambda_D << L \qquad N_D = \frac{4\pi n_0 \lambda_D^3}{3} \qquad N_D >> 1$$



$\rightarrow$ response time $\geq \dfrac{1}{\omega_p}$

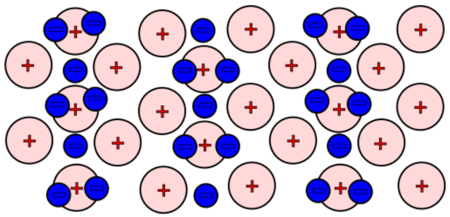


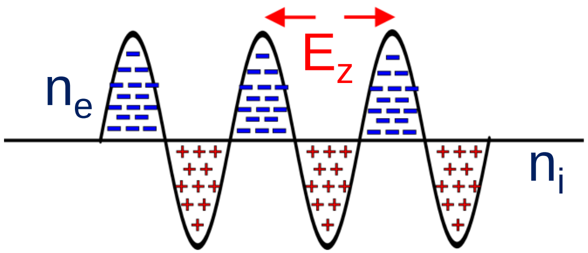
$$n_0 = 10^{18} cm^{-3} \qquad T_e = 10\ eV (\approx 1.16 \times 10^5 K)$$
$$\lambda_D \approx 23\ nm \qquad N_D \approx 54\ electrons$$

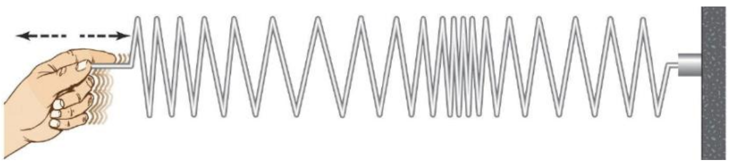
Plasmas as accelerators

- Longitudinal wave with phase velocity ~ speed of light

plasma oscillations







$$\lambda_p = \frac{2\pi}{\omega_p} \qquad \omega_p = \sqrt{\frac{4\pi\ n_e\ e^2}{m_e}}$$

$$n_p = 10^{18} cm^{-3}$$
$$\lambda_p \approx 33\ \mu m$$

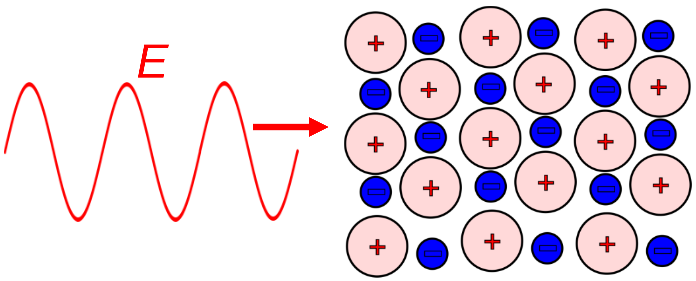
- Maximum accelerating field strength:

$$E_{z,\max}[V/cm] \approx\ 0.96\ \sqrt{n_p[cm^{-3}]}\ \sqrt{2(\gamma_g - 1)}$$

$$n_p = 10^{18} cm^{-3}$$
$$E_{z,\max} \approx\ 800\ \ GV/m$$

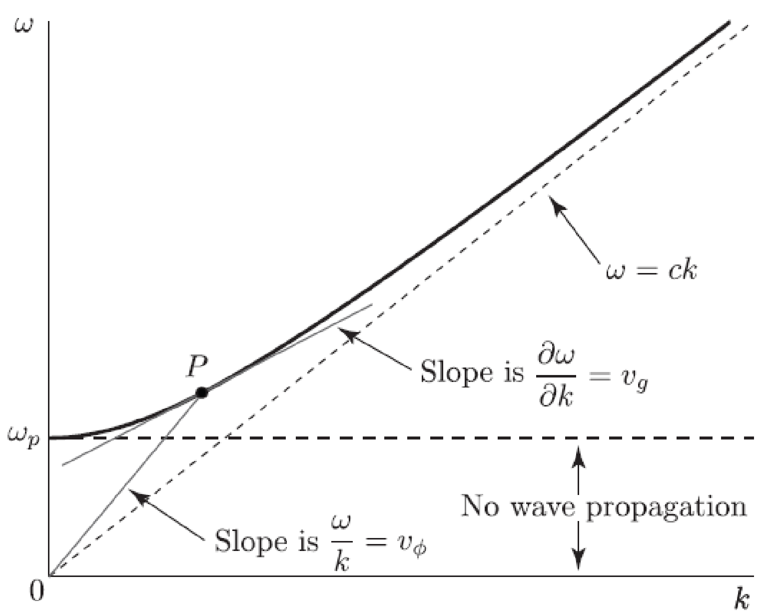
- Injection of particles into the wave $\rightarrow$ the central problem !!

Propagation of light in plasmas



- Electrons wiggle in the E-field (via Lorentz force)
  - ions remain immobile in their positions
  - fast time ~ laser period
  - the net charge separation is zero

$$\omega^2 = \omega_p^2 + k^2 c^2$$ → dispersion relation



- Phase velocity

$$v_\phi = \frac{\omega}{k} = \sqrt{c^2 + \frac{\omega_p^2}{k^2}} = \frac{c}{\eta}$$

- Group velocity

$$v_g = \frac{\partial \omega}{\partial k} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

Propagation of light in plasmas

- Plasma refractive index $\eta = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$

$\omega_p > \omega \rightarrow \eta$ Plasma refractive index becomes imaginary
$\rightarrow$ Plasma becomes not transparent
$\rightarrow$ Light will reflected by the plasma (like a mirror)

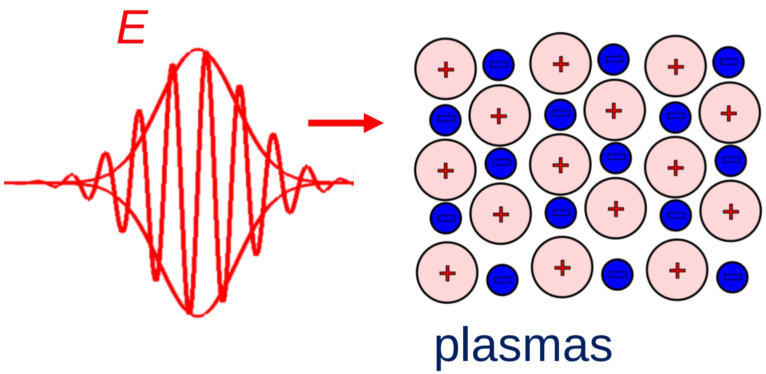
- Critical density $n_c[cm^{-3}] = \frac{m_e \omega^2}{4\pi e^2} \approx \frac{1.1 \times 10^{21}}{\lambda[\mu m]^2}$

$\lambda = 0.8\ \mu m \quad n_c = 1.7 \cdot 10^{21}\ cm^{-3}$ $\rightarrow$ ~ 35 bar Helium gas

$\omega_p > \omega,\ n_p > n_c$ Overdense plasma

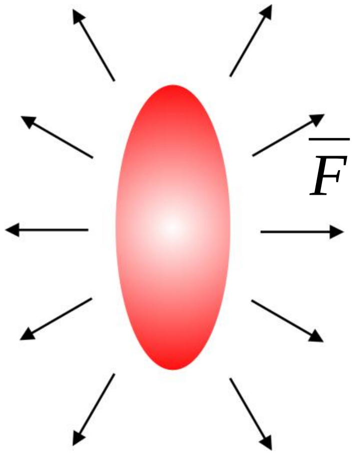
$\omega_p < \omega,\ n_p < n_c$ Underdense plasma
$\rightarrow$ Regime for laser electron acceleration

The Ponderomotive force



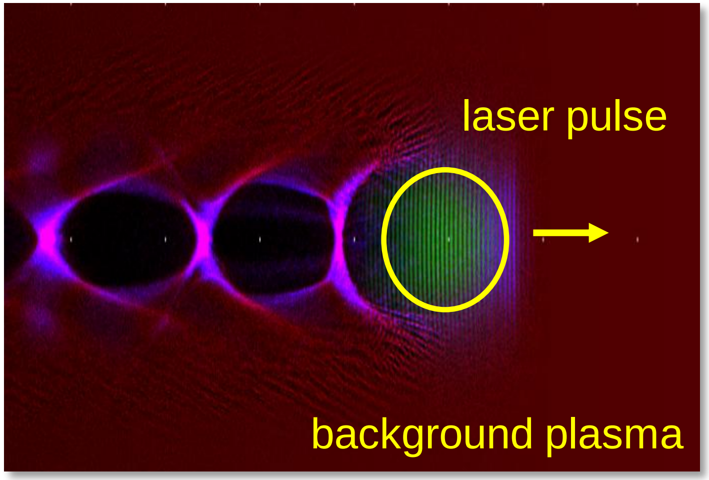
- Electrons wiggle in the E-field (via Lorentz force)
  - time-averaged force is not zero
  - fast time ~ laser period
  - slow time ~ laser envelope
  - the net charge separation is not zero

- The ponderomotive force ("light pressure") expels electrons from high intensity region



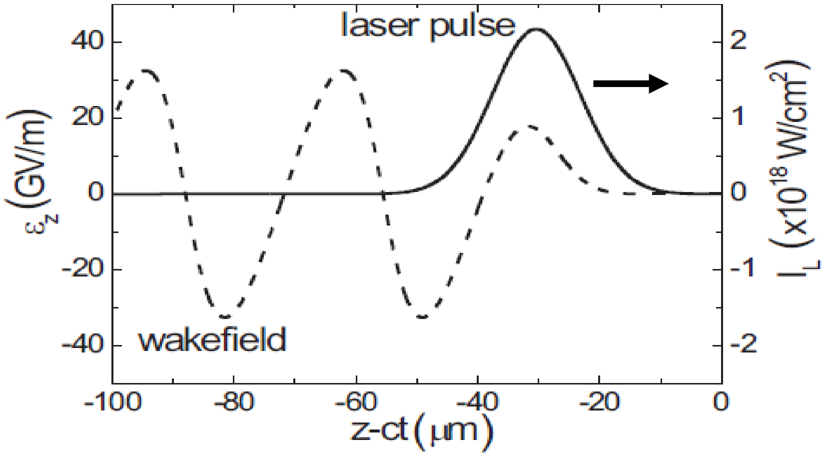
$$\overline{F}_{pond} = \left\langle m_e \frac{d\overline{v}}{dt} \right\rangle = -\frac{e^2}{4\, m_e \omega^2} \overline{\nabla} E^2(r)$$

Plasma wave excitation



- A high intensity laser pulse can excite plasma waves > 10^{17} W/cm^2
- Driver pulse length $\ll \lambda_p$
- Ions remains immobile
- $v_\phi^{plasma} = v_g^{laser} \sim c$

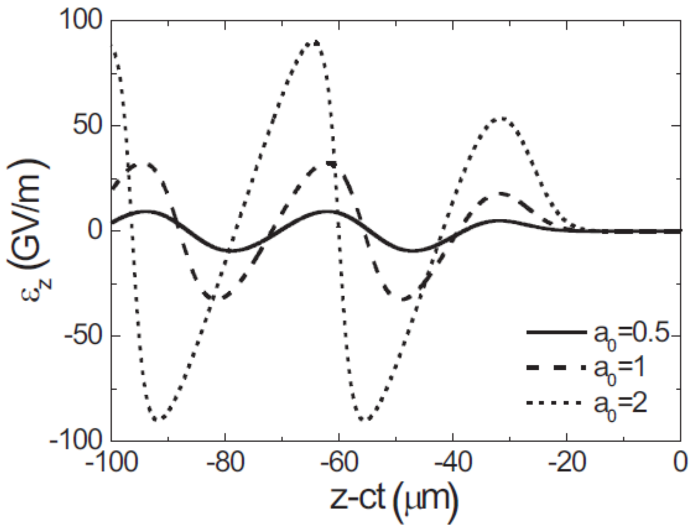
T.Tajima and J.Dawson, PRL 43, 267(1979)

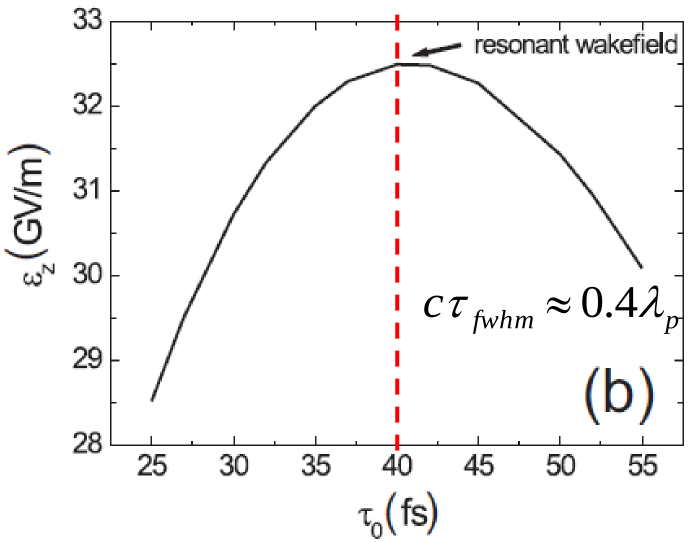


water wave behind a boat

from DWPicture.com.au

Characteristics of laser wakefield: 1D

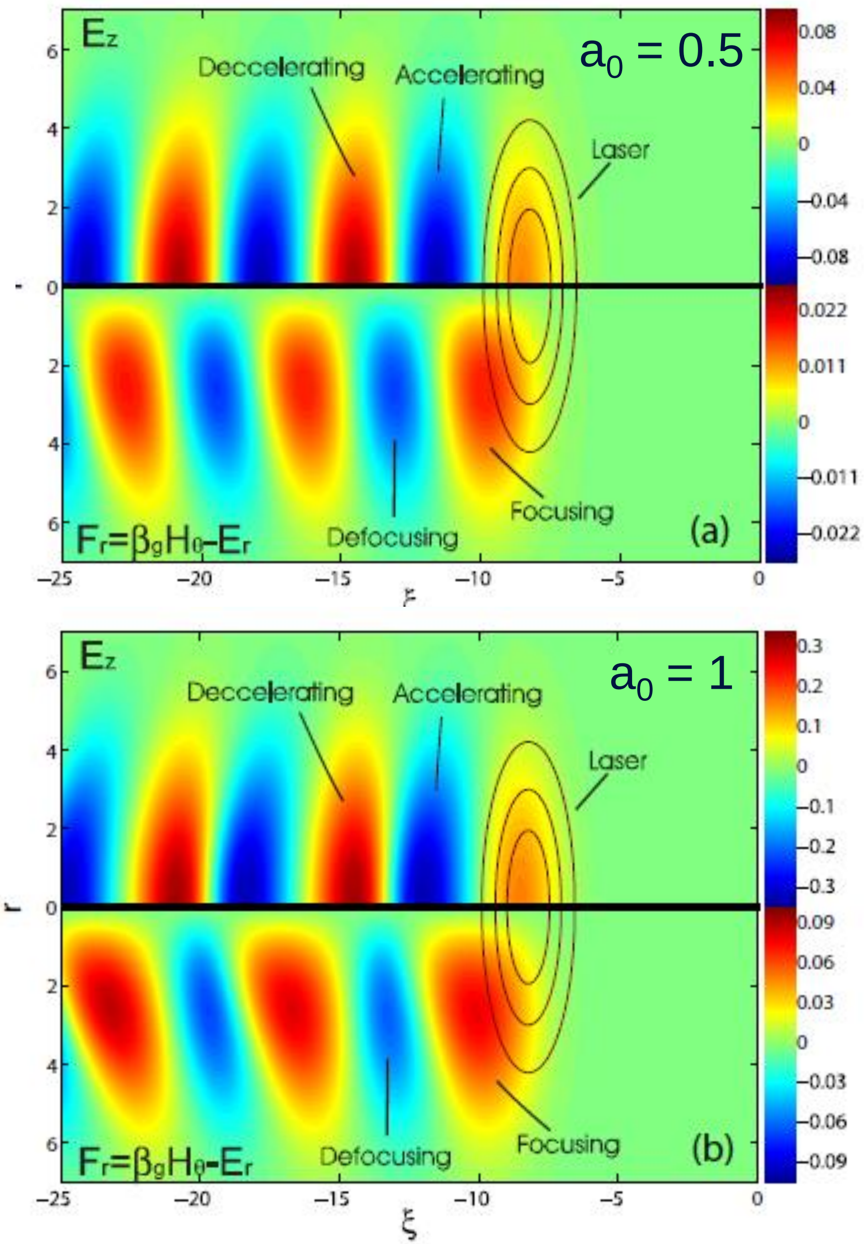




- The amplitude of wakefield increases with increasing the laser intensity
- Linear regime: the wakefield has a sinusoidal shape
- Non-linear regime: the wakefield becomes steeper
- Plasma wavelength increases with increasing the laser intensity
- The resonance occurs when the driver laser length is ~ 0.4 λ_p

Characteristics of laser wakefield: 3D

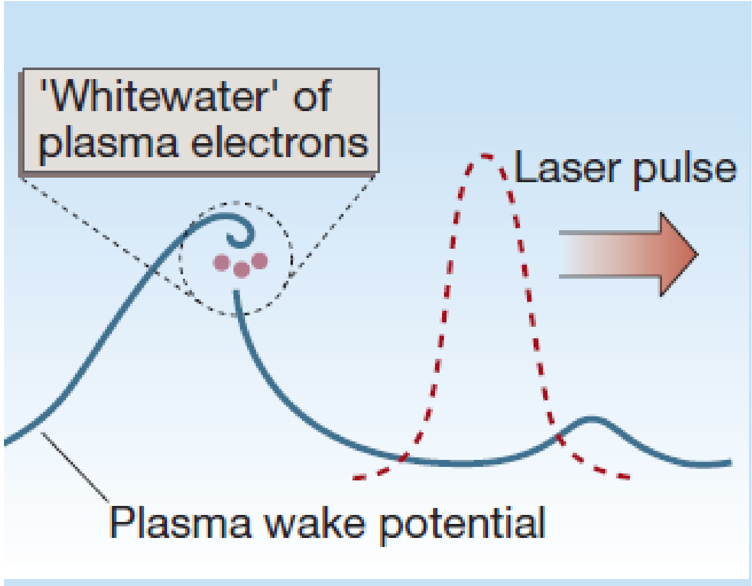
HZDR



- Existence of transverse force fields
- Focusing fields bring electrons to the axis.
- Defocusing fields scatter electrons out of the axis.
- The optimum accelerating region is the overlap region between the accelerating and the focusing region.
- The overlap region becomes larger in the non-linear wakefield.

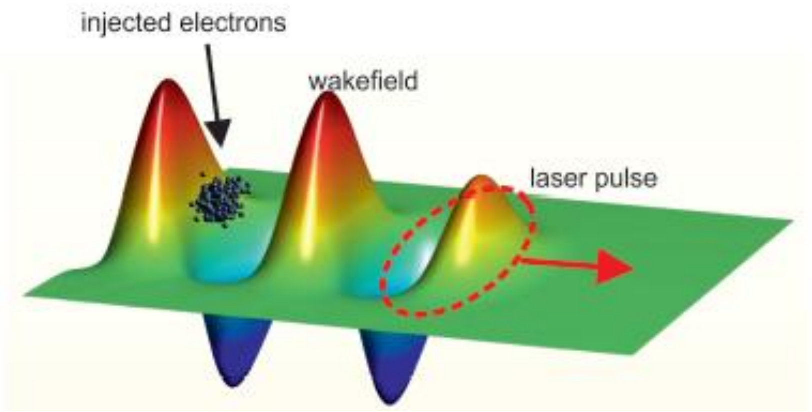
Wave-breaking

- Wave-breaking limits the maximum accelerating field strength.
- Electron longitudinal velocity > plasma wave phase velocity



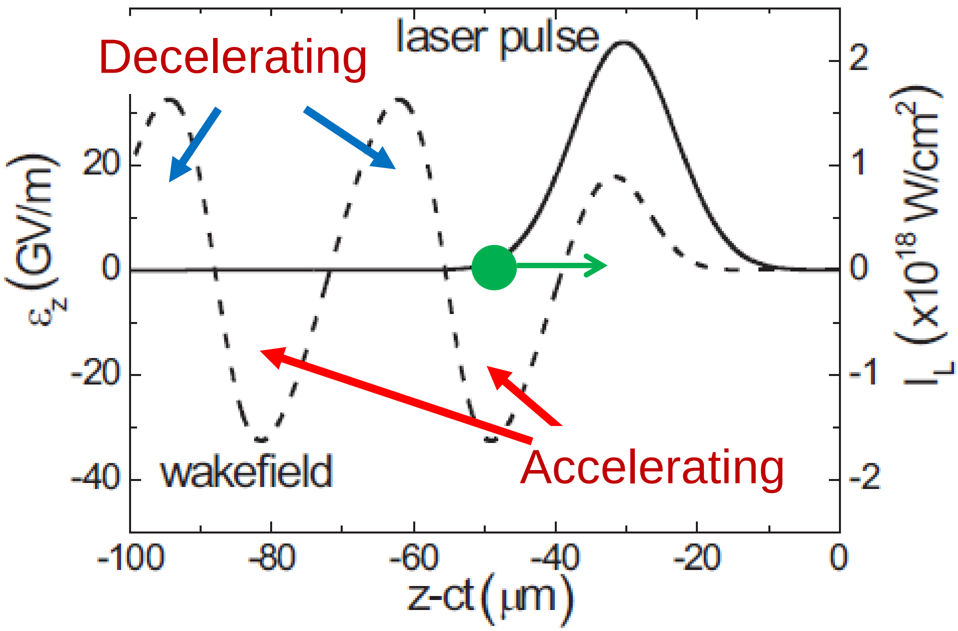
T.Katsouleas, Nature 431, 515(2004)

LWFA: a compact electron accelerator



Electron orbit in phase space:

- electron trapping mechanism
- acceleration
- dephasing



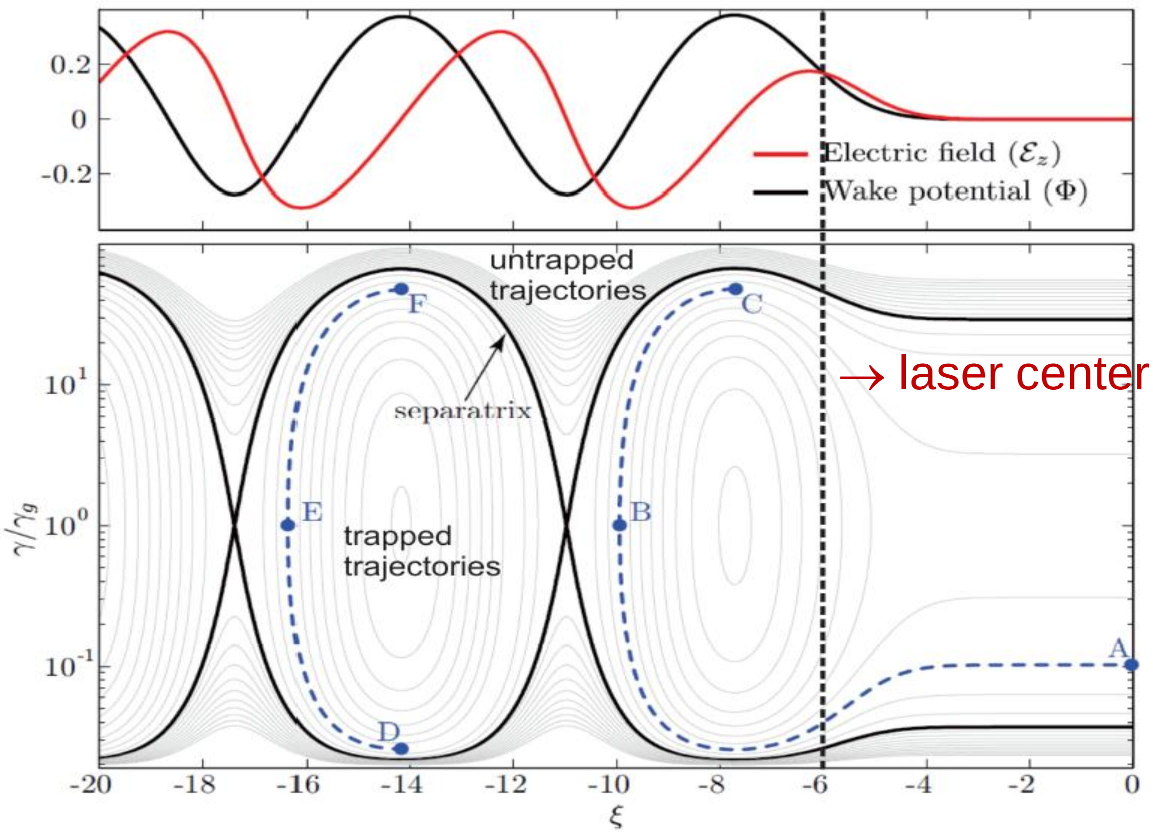
Equation of motion of a test electron in wakefields with amplitude E_z

$$H(\gamma_e, \xi) = \gamma_e - \gamma_e \beta_e \beta_g - \phi(\xi)$$

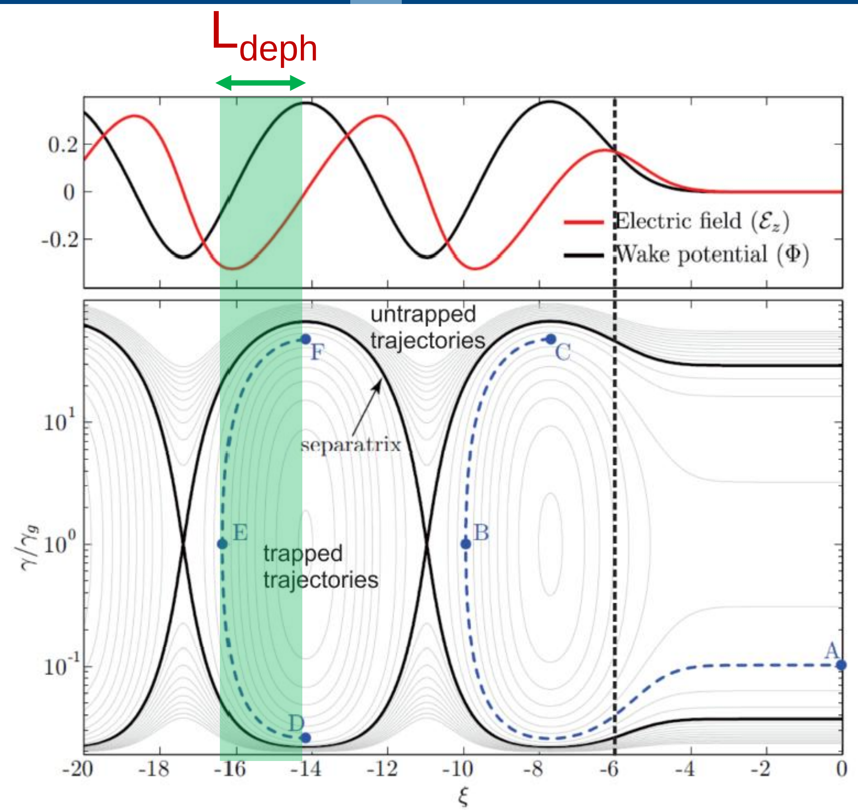
$$\frac{dH(\gamma_e, \xi)}{d\tau} = 0$$

Hamiltonian constant along a given electron orbit

The separatrix:
orbit that separates the region of trapped and untrapped electrons in the longitudinal phase space



Maximum energy gain



- Dephasing length: distance for electron to gain energy before entering the decelerating phase

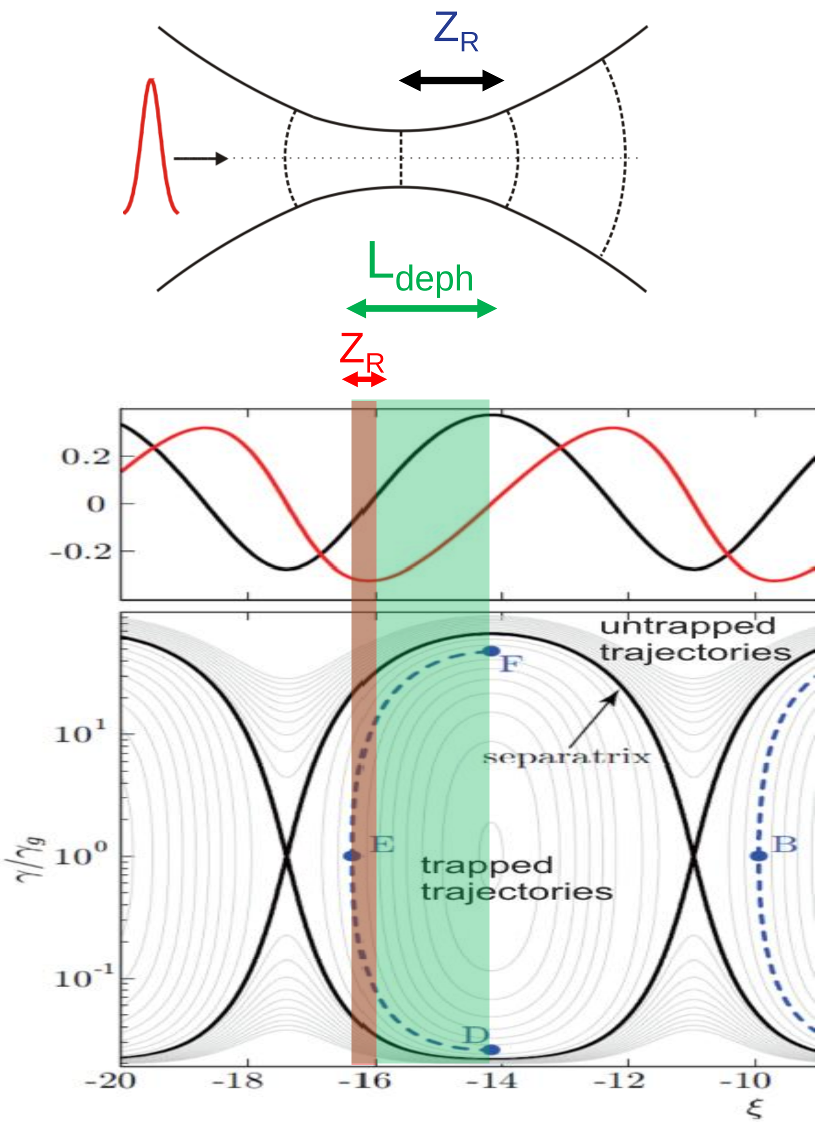
$$L_{deph} \approx \gamma_g^2 \lambda_p \qquad L_{deph} \sim n_p^{-3/2}$$

- Maximum electron energy: $W_{\max} \sim E_z L_{deph}$ $\qquad E_z \sim n_p^{1/2}$

$$W_{\max} \sim n_p^{-1/2}$$

End of Part 1

1. Long dephasing length ≈ acceleration distance

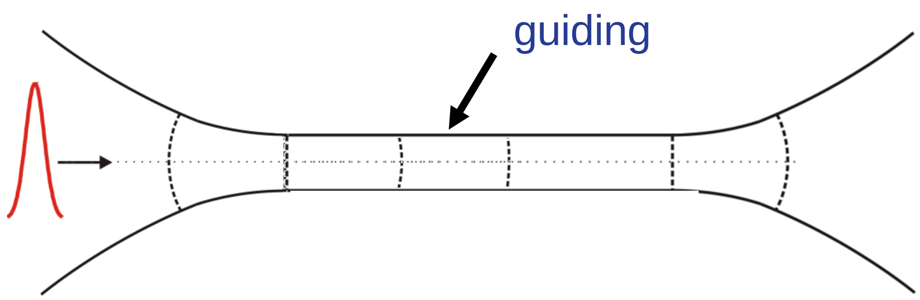


- High intensity laser over long distance
- Optical diffraction limits the acceleration length to the Rayleigh length
- Example:

$$\lambda = 0.8\mu m \quad w_0 = 10\mu m \quad Z_R \approx 400\ \mu m$$

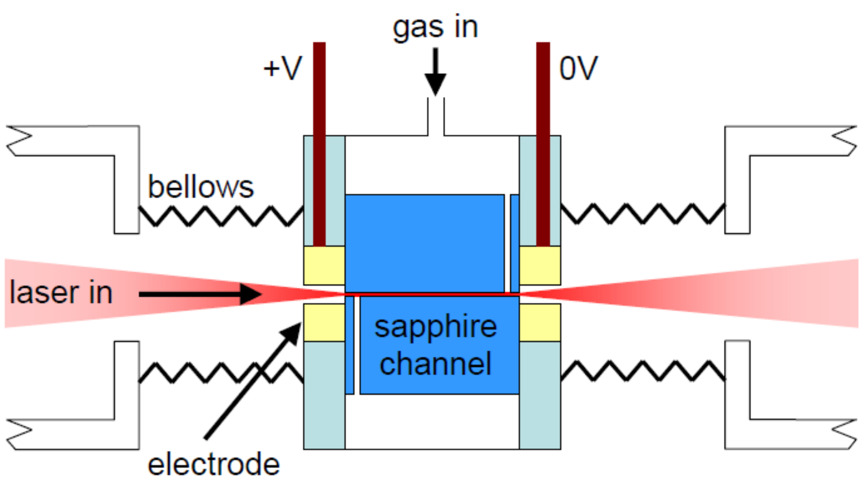
$$n_p \approx 10^{18}\ cm^{-3} \quad L_{deph} \approx 6\ cm$$

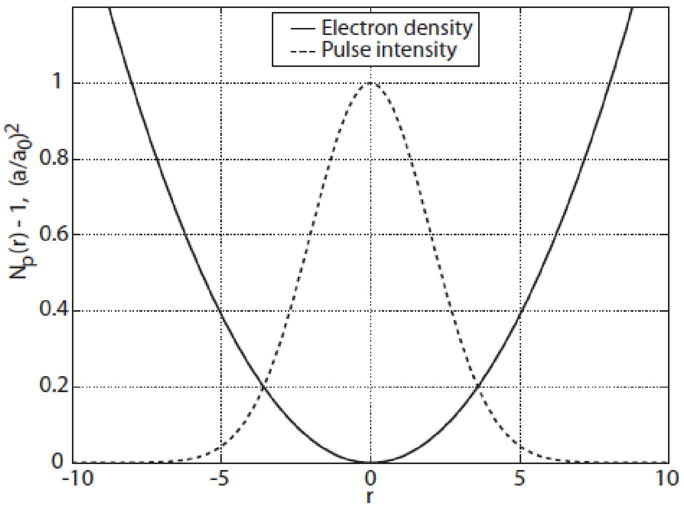
1. Long dephasing length ≈ acceleration distance



- Relativistic self-focusing
- Preformed plasma channel

Hydrogen filled 3.3 cm long plasma waveguide
dia ~ 190-310 μm, n_p ~ 1-5 x 10^{18} cm^{-3}

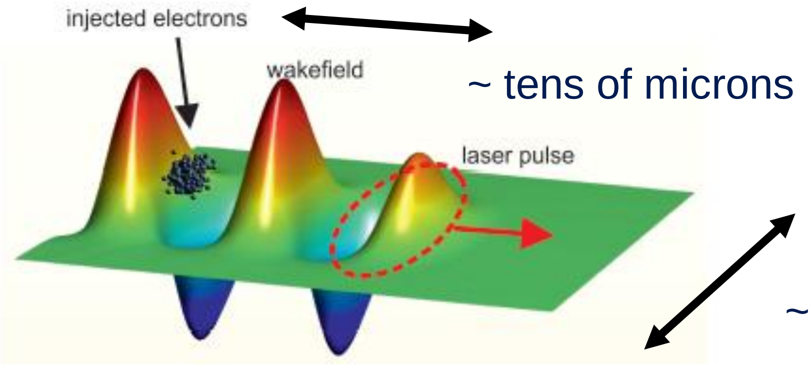




A.Gosalves, PhD Thesis 2006, Oxford University

2. fs bunch and fs time for injection → the central problem !!

- size < laser beam size
- length < plasma wavelength
- synchronization < laser pulse duration
- initial energy > trapping threshold



~ tens of microns

~ tens of microns

Self-injection in the bubble regime

Monoenergetic beams of relativistic electrons from intense laser–plasma interactions

S. P. D. Mangles[1], C. D. Murphy[1,2], Z. Najmudin[1], A. G. R. Thomas[1], J. L. Collier[2], A. E. Dangor[1], E. J. Divall[2], P. S. Foster[2], J. G. Gallacher[3], C. J. Hooker[2], D. A. Jaroszynski[3], A. J. Langley[2], W. B. Mori[4], P. A. Norreys[2], F. S. Tsung[4], R. Viskup[3], B. R. Walton[1] & K. Krushelnick[1]

[1]*The Blackett Laboratory, Imperial College London, London SW7 2AZ, UK*
[2]*Central Laser Facility, Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, UK*
[3]*Department of Physics, University of Strathclyde, Glasgow G4 0NG, UK*
[4]*Department of Physics and Astronomy, UCLA, Los Angeles, California 90095, USA*

High-quality electron beams from a laser wakefield accelerator using plasma-channel guiding

C. G. R. Geddes[1,2], Cs. Toth[1], J. van Tilborg[1,3], E. Esarey[1], C. B. Schroeder[1], D. Bruhwiler[4], C. Nieter[4], J. Cary[4,5] & W. P. Leemans[1]

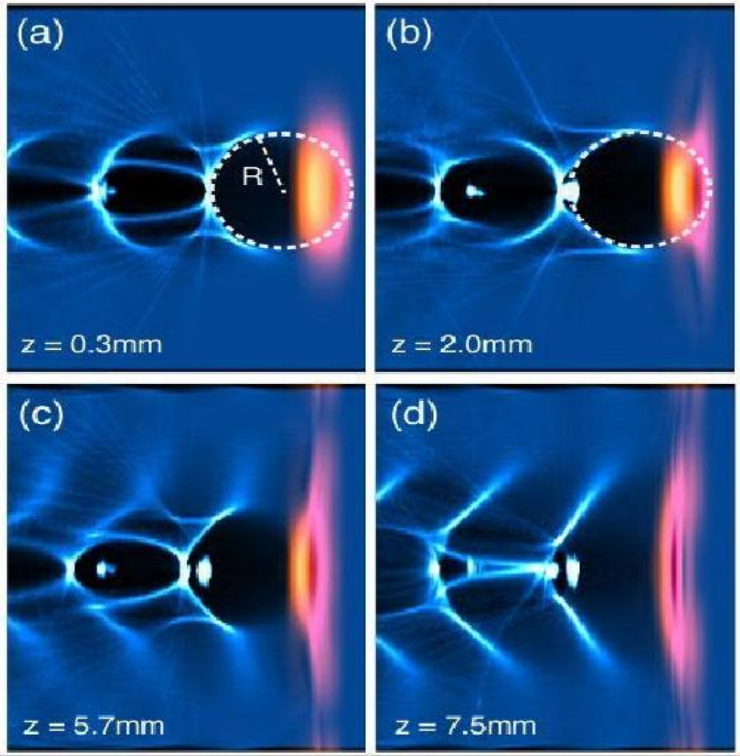
[1]*Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 94720, USA*
[2]*University of California, Berkeley, California 94720, USA*
[3]*Technische Universiteit Eindhoven, Postbus 513, 5600 MB Eindhoven, the Netherlands*
[4]*Tech-X Corporation, 5621 Arapahoe Ave. Suite A, Boulder, Colorado 80303, USA*
[5]*University of Colorado, Boulder, Colorado 80309, USA*

A laser–plasma accelerator producing monoenergetic electron beams

J. Faure[1], Y. Glinec[1], A. Pukhov[2], S. Kiselev[2], S. Gordienko[2], E. Lefebvre[3], J.-P. Rousseau[1], F. Burgy[1] & V. Malka[1]

[1]*Laboratoire d'Optique Appliquée, Ecole Polytechnique, ENSTA, CNRS, UMR 7639, 91761 Palaiseau, France*
[2]*Institut fur Theoretische Physik, 1, Heinrich-Heine-Universitat Duesseldorf, 40225 Duesseldorf, Germany*
[3]*Département de Physique Théorique et Appliquée, CEA/DAM Ile-de-France, 91680 Bruyères-le-Châtel, France*

Self-injection in the bubble regime[1,2,3,many more]



Lu, W., *et al*, PRSTAB 10,061301(2007)

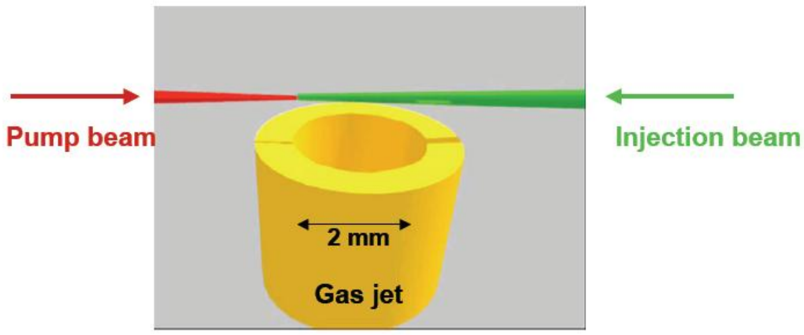
- A highly nonlinear mechanism: self focusing + self steepening
- Electrons completely blown out by the F_p
- No spatial and temporal problems with injection
- Inherent problem: shot-to-shot stability is very sensitive to the fluctuation of laser's and plasma's parameters

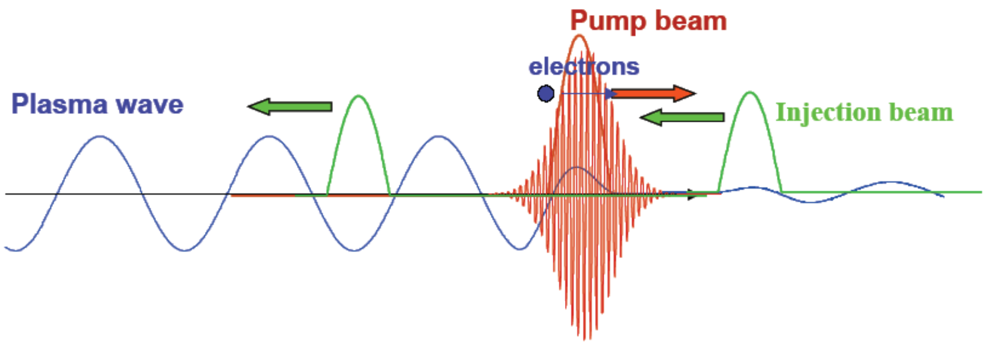
[1]Pukhov, A., *et al*, *Appl.Phys.B.*,74,355(2002), [2]Leemans, W.P., *et al*, *Nature physics2,696(2006),*[3]Mangles, S.P.D., *et al*, *Nature 431,535(2004),*[4] Geddes, C.G.R, *et al*, *Nature 431,538(2004),* [5]Faure, J., *et al*, *Nature 431,541(2004)*, many more

Simulation bubble regime PIConGPU

A. Hübl, R. Widera, A. Debus, M. Bussmann
Junior research group, Computer assisted radiation physics (Michael Bussmann, et al.)
PIConGPU.hzdr.de

Injection by colliding pulse scheme[1,2,3]



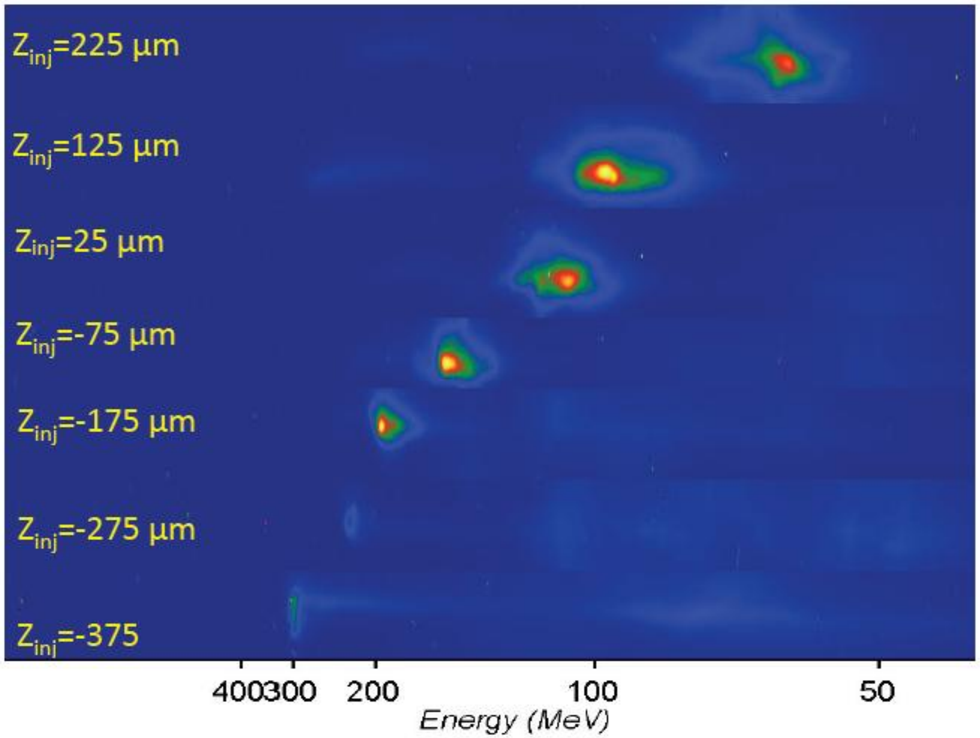


- The pump beam to drive the wakefield, the injection beam to heat up the back-ground electrons
- Injection is local and in first bucket
- Better shot to shot reproducibility
- Better control over electron parameters, energy, total charge,

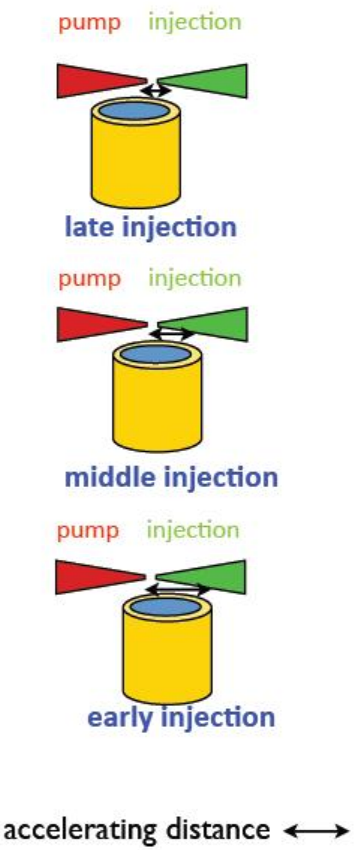
[1]Faure, J., *et al*, *Nature 444,737 (2006)*, [2]Esarey, E., *et.al*., *PRL79.2682(1997)*, [3]Kotaki, H., *et.al*., *PoP 11 (2004)*

Injection by colliding pulse scheme

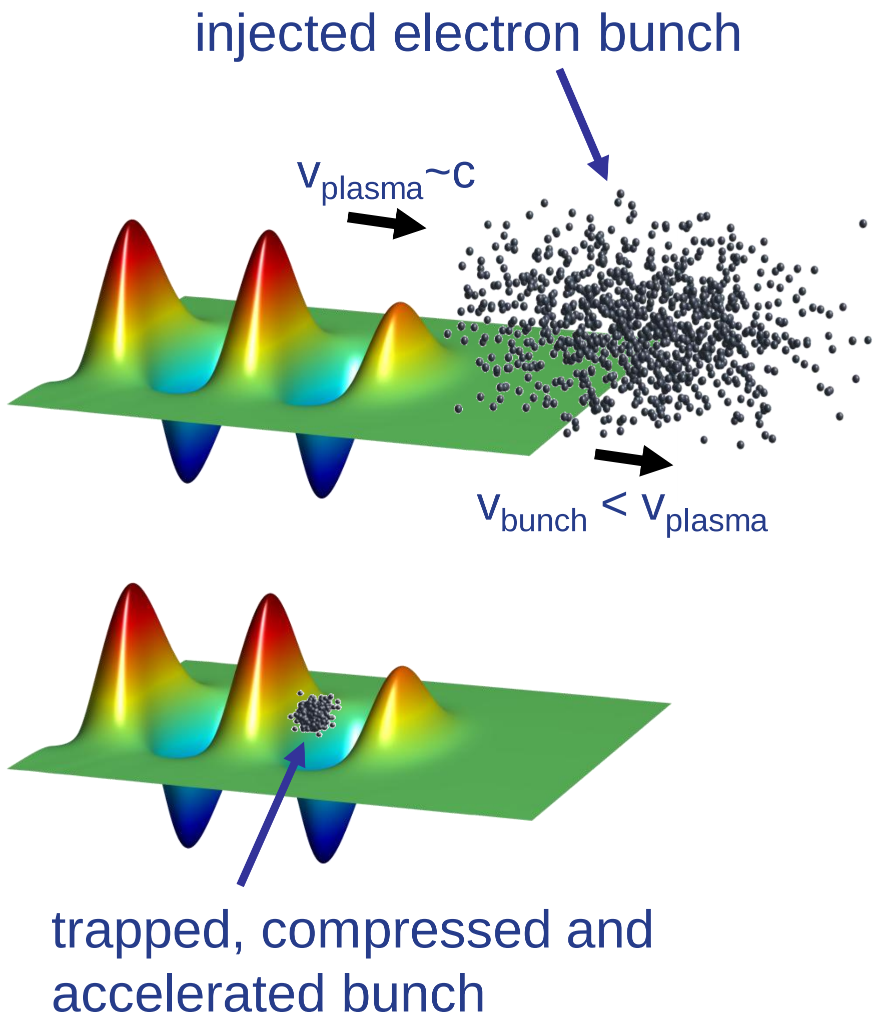
- Tuneable electron energy



Faure, J., *et al*, *Nature 444,737 (2006)*



External electron injection



- Linear to weakly nonlinear wakefield
- No need ultra-short injected electron bunch
- No need fs synchronization
- No need precise transverse positioning
- Easy control over the injection time
- Scaleable to higher energies
- Promising candidate for a controlled acceleration

[1]A. G. Khachatryan *et. al Nucl.Instrum.Methods Phys. Res.A*, *566, 244 (2006),* A. G. Khachatryan *et. al., Phys. Rev. ST Accel. Beams 7, 121301 (2004),* A. G. Khachatryan, *Phys. Rev. E 65, 046504(2002)*, A. G. Khachatryan, *JETP Lett. 74, 371 (2001).*

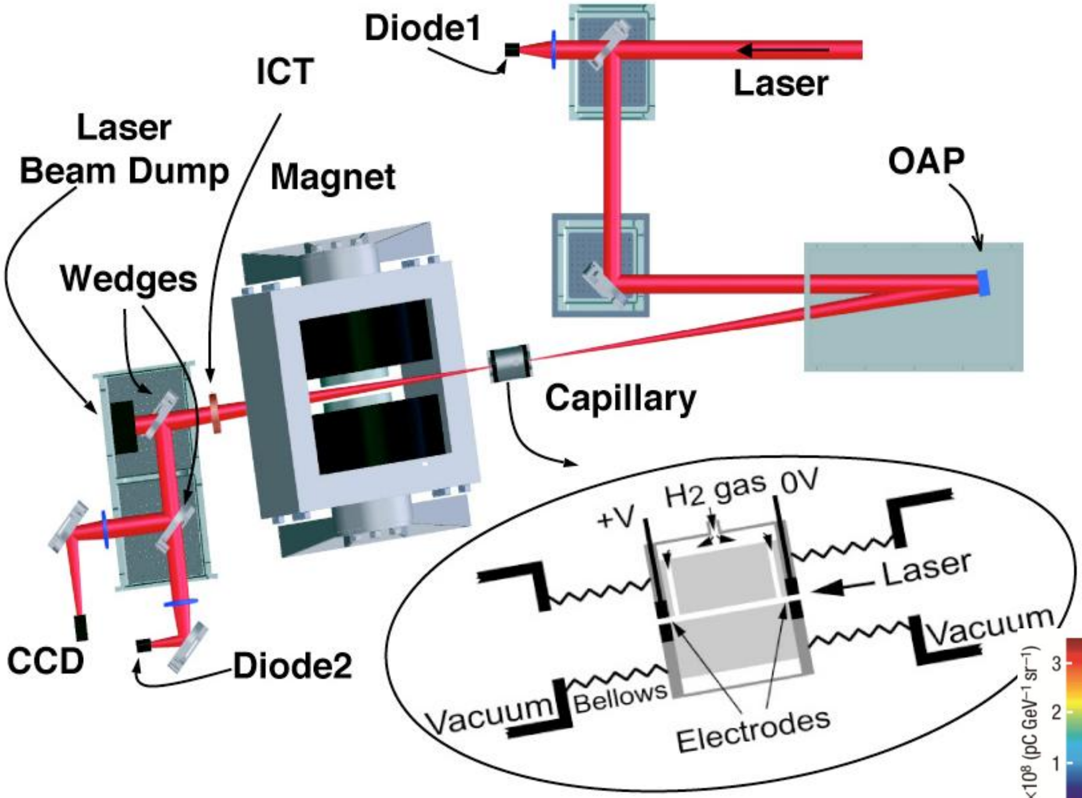
Recent progress in LWFA experiment

LETTERS

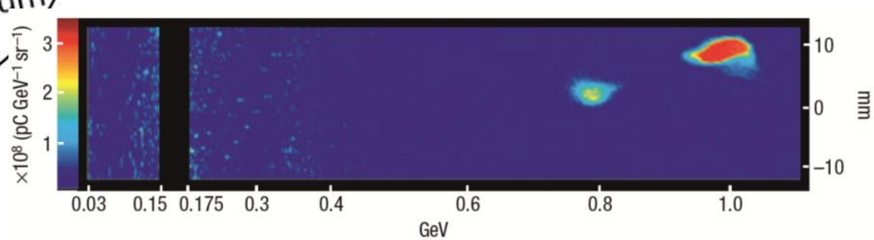
Nature Physics 2, 696(2006)

GeV electron beams from a centimetre-scale accelerator

W. P. LEEMANS[1*†], B. NAGLER[1], A. J. GONSALVES[2], Cs. TÓTH[1], K. NAKAMURA[1,3], C. G. R. GEDDES[1], E. ESAREY[1*], C. B. SCHROEDER[1] AND S. M. HOOKER[2]

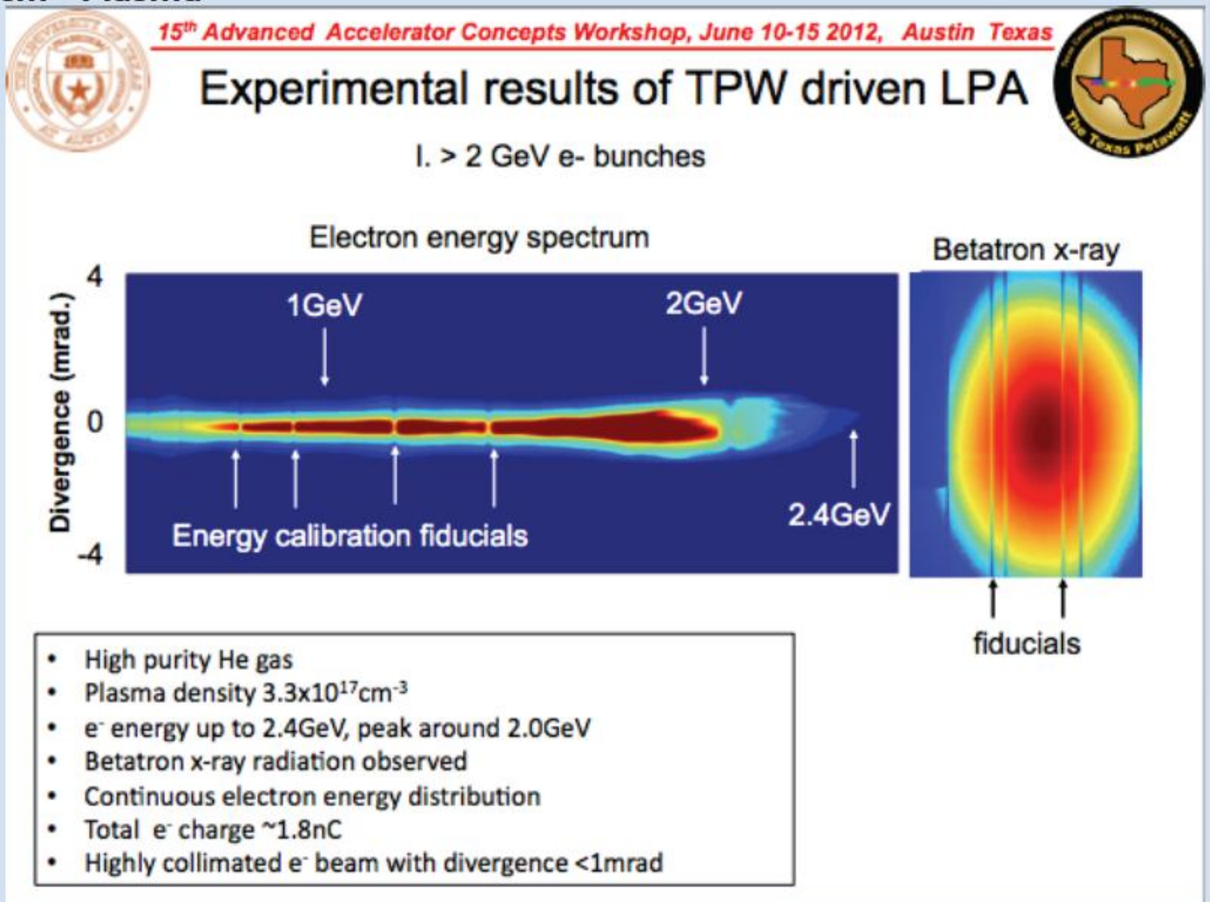


- 40 TW laser pulse, plasma density $n_p = 4.3 \times 10^{18}$ cm^{-3}, 30 pC at 1 GeV with 2.5 % rms energy spread
- Acceleration distance < 3 cm, $E_z > 33$ GV/m



Energy scaling with laser power

Xiaoming Wang (Texas) - Electron Acceleration to >2GeV by Petawatt Laser-driven Wakefield in 10^{17} cm^{-3} Plasma



- Texas Petawatt: 150 J focused into 7cm He gas cell
- Highly-collimated <1 mrad, high charge ~1.8nC
- Betatron x-ray radiation observed
- Observed quasi-monoenergetic beams: ~1.1GeV, ΔE~0.2GeV , 64 pC

Visualization of laser wakefields

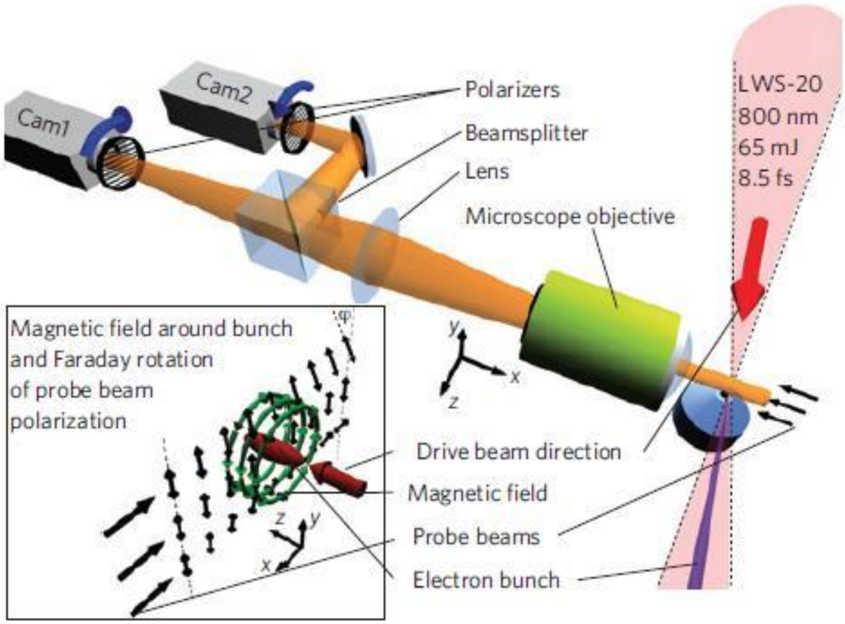
nature physics

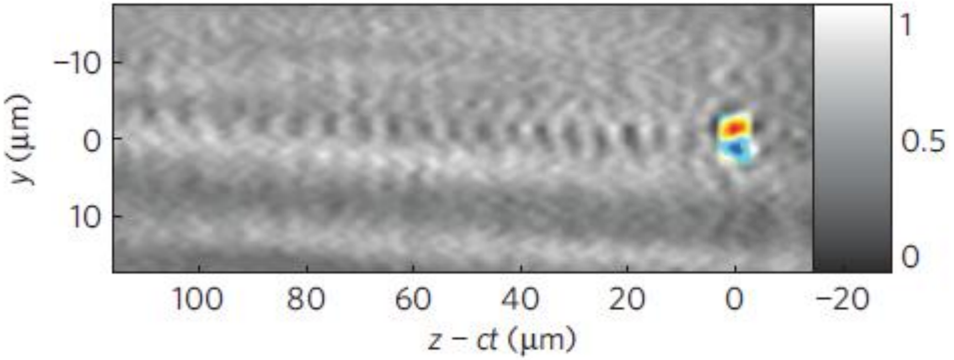
LETTERS

PUBLISHED ONLINE: 13 MARCH 2011 | DOI: 10.1038/NPHYS1942

Real-time observation of laser-driven electron acceleration

Alexander Buck[1,2]★, Maria Nicolai[3], Karl Schmid[1,2], Chris M. S. Sears[1], Alexander Sävert[3], Julia M. Mikhailova[1], Ferenc Krausz[1,2], Malte C. Kaluza[3,4] and Laszlo Veisz[1]★

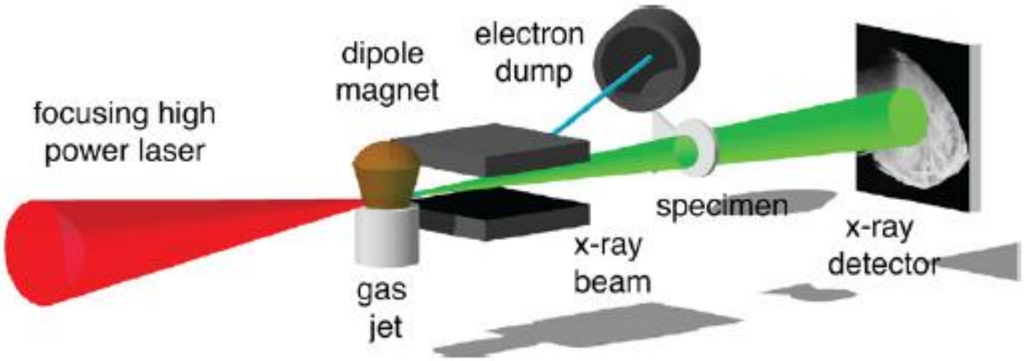


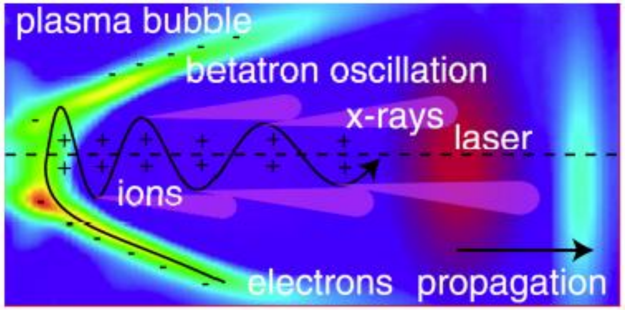


APPLIED PHYSICS LETTERS **99**, 093701 (2011)

X-ray phase contrast imaging of biological specimens with femtosecond pulses of betatron radiation from a compact laser plasma wakefield accelerator

S. Kneip,[1,2,a)] C. McGuffey,[2] F. Dollar,[2] M. S. Bloom,[1] V. Chvykov,[2] G. Kalintchenko,[2] K. Krushelnick,[2] A. Maksimchuk,[2] S. P. D. Mangles,[2] T. Matsuoka,[2] Z. Najmudin,[1] C. A. J. Palmer,[1] J. Schreiber,[1] W. Schumaker,[2] A. G. R. Thomas,[2] and V. Yanovsky[2]
[1]*Blackett Laboratory, Imperial College London, London SW7 2AZ, United Kingdom*
[2]*Center for Ultrafast Optical Science, University of Michigan, Ann Arbor 48109, USA*





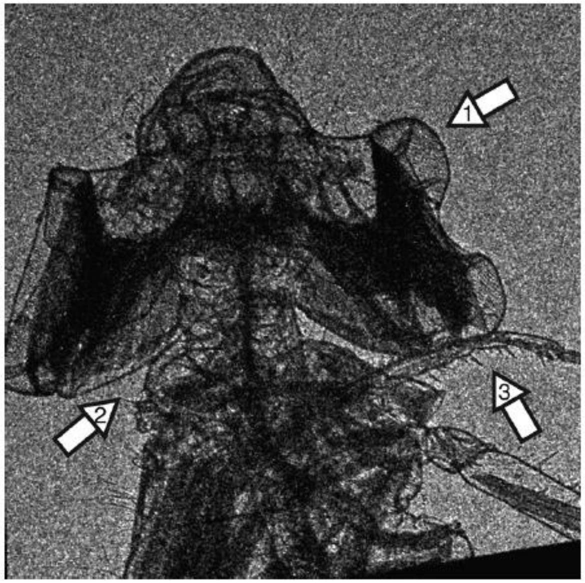


FIG. 3. Single shot 30 fs exposure x-ray phase contrast image of the head of a damselfly. Notice details of the compound eye (1), exoskeleton (2), and leg with hairs (3).

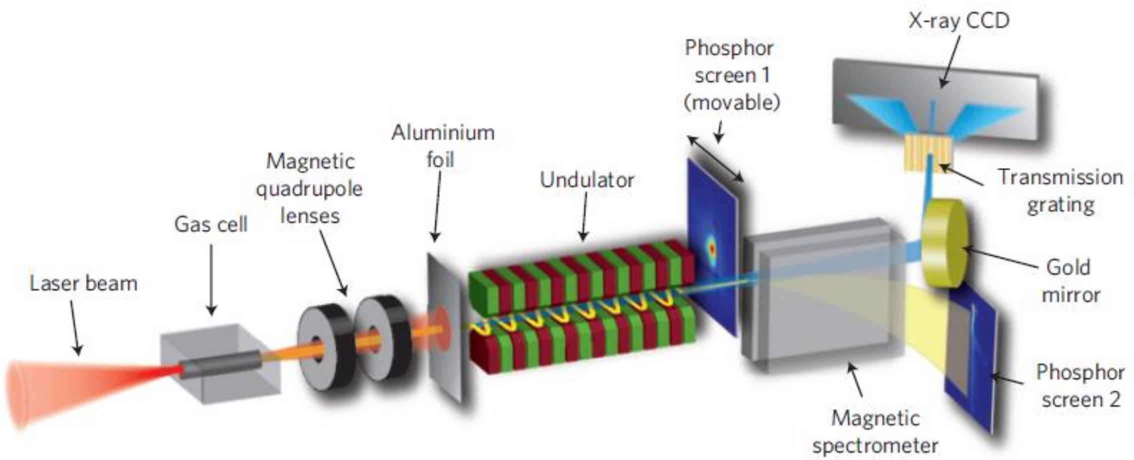
nature physics

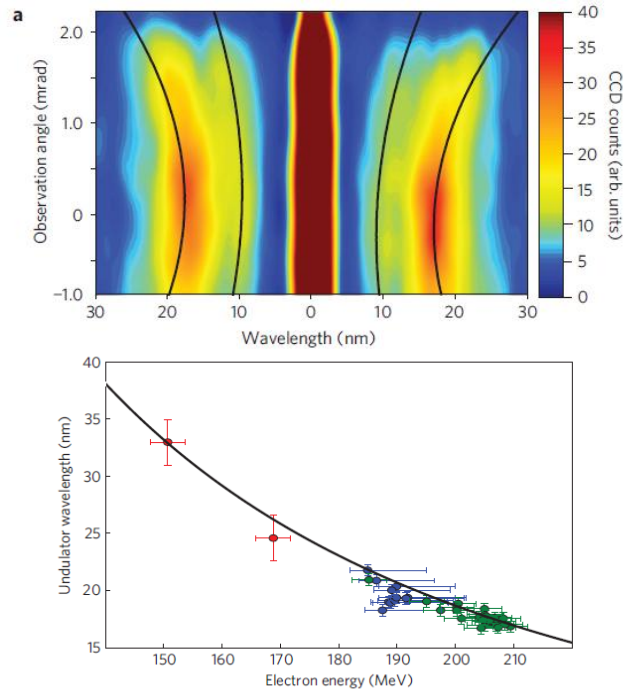
LETTERS

PUBLISHED ONLINE: 27 SEPTEMBER 2009 | DOI: 10.1038/NPHYS1404

Laser-driven soft-X-ray undulator source

Matthias Fuchs[1,2], Raphael Weingartner[1,2], Antonia Popp[1], Zsuzsanna Major[1,2], Stefan Becker[2], Jens Osterhoff[1,2], Isabella Cortrie[2], Benno Zeitler[2], Rainer Hörlein[1,2], George D. Tsakiris[1], Ulrich Schramm[3], Tom P. Rowlands-Rees[4], Simon M. Hooker[4], Dietrich Habs[1,2], Ferenc Krausz[1,2], Stefan Karsch[1,2]★ and Florian Grüner[1,2]★

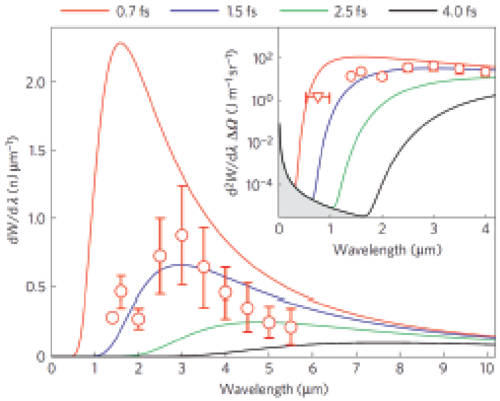




LWFA: unique bunch properties

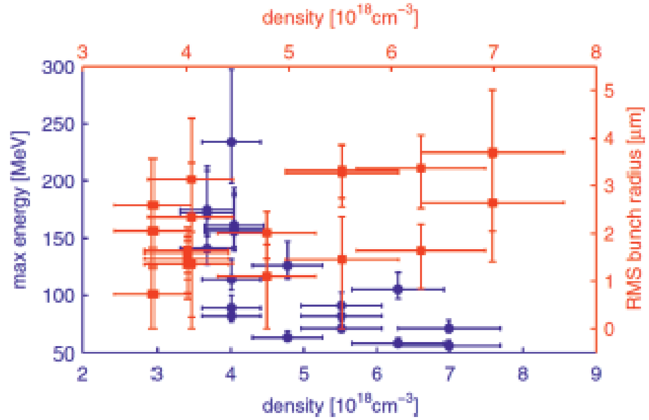
- Ultra-small size ($\sigma_{rms} << \lambda_p$) and ultra-short bunch duration ($\tau_{fwhm} << \lambda_p/c$)

CTR technique



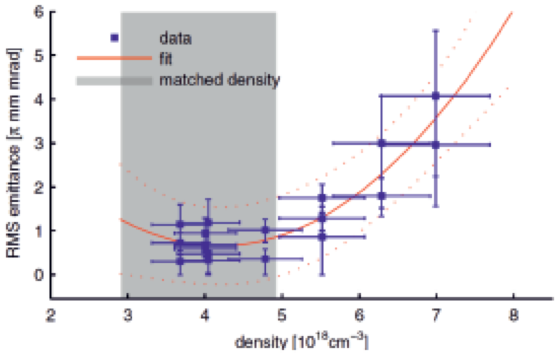
O. Lund, et.al., Nature Physics 7, 219(2011)

Betatron Xray source size



S.Kneip, et.al. Phys. Rev.ST. Accel. Beams., 15, 021302(2012)

- Normalized transverse emittance (ϵ_N<0.5 πmm-mrad) ~ linacs

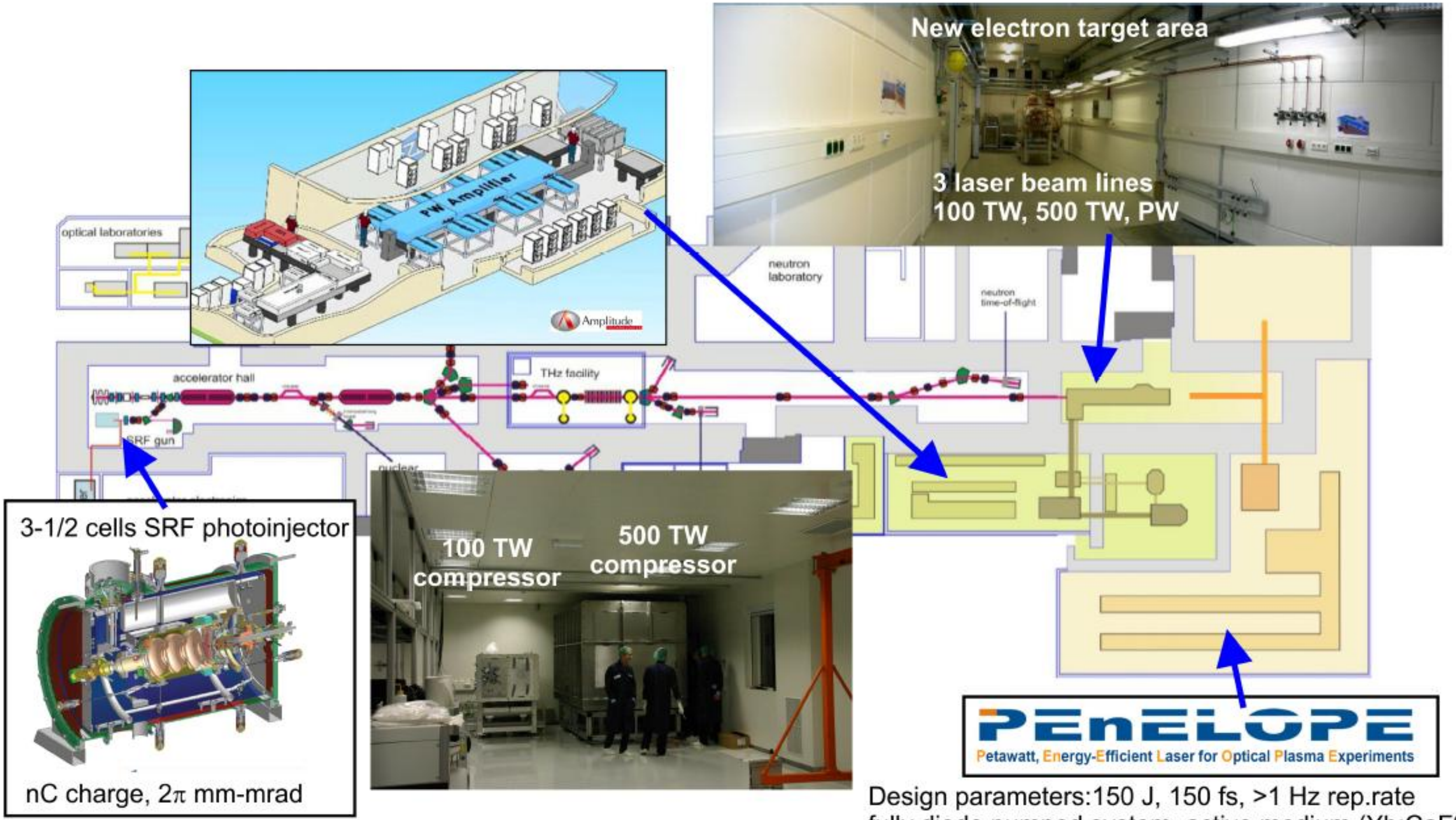


S.Kneip, et.al. Phys. Rev.ST. Accel. Beams., 15, 021302(2012)

Main challenges in LWFA

- Improved control over electron injection into wakefields
  1. Coliding pulse mechanism (J.Faure, et.al.Nature 444,737(2006))
  2. Density down-ramp injection (C.G.R.Geddes,et.al.,Phys.Rev.Lettl,100,215004(2008))
- Improved beam stability: shot-to-shot reproducibility of charge, energy and energy spread, emittance
  1. Reduce the fluctuation of plasma's parameters: gas cell (J.Osterhoff ,et.al., Phys.Rev.Lett.,101,085002(2008))
  2. Full control over crucial laser's parameters:pulse's front tilt (A.Popp, et.al.,Phys.Rev.Lett.,105,215001(2010))
- Scaleability to higher energies
  1. Multi-staging technology (W.P.Leemans and E.Esarey, Phys.Today, March 2009)

Center for High Power Radiation Sources

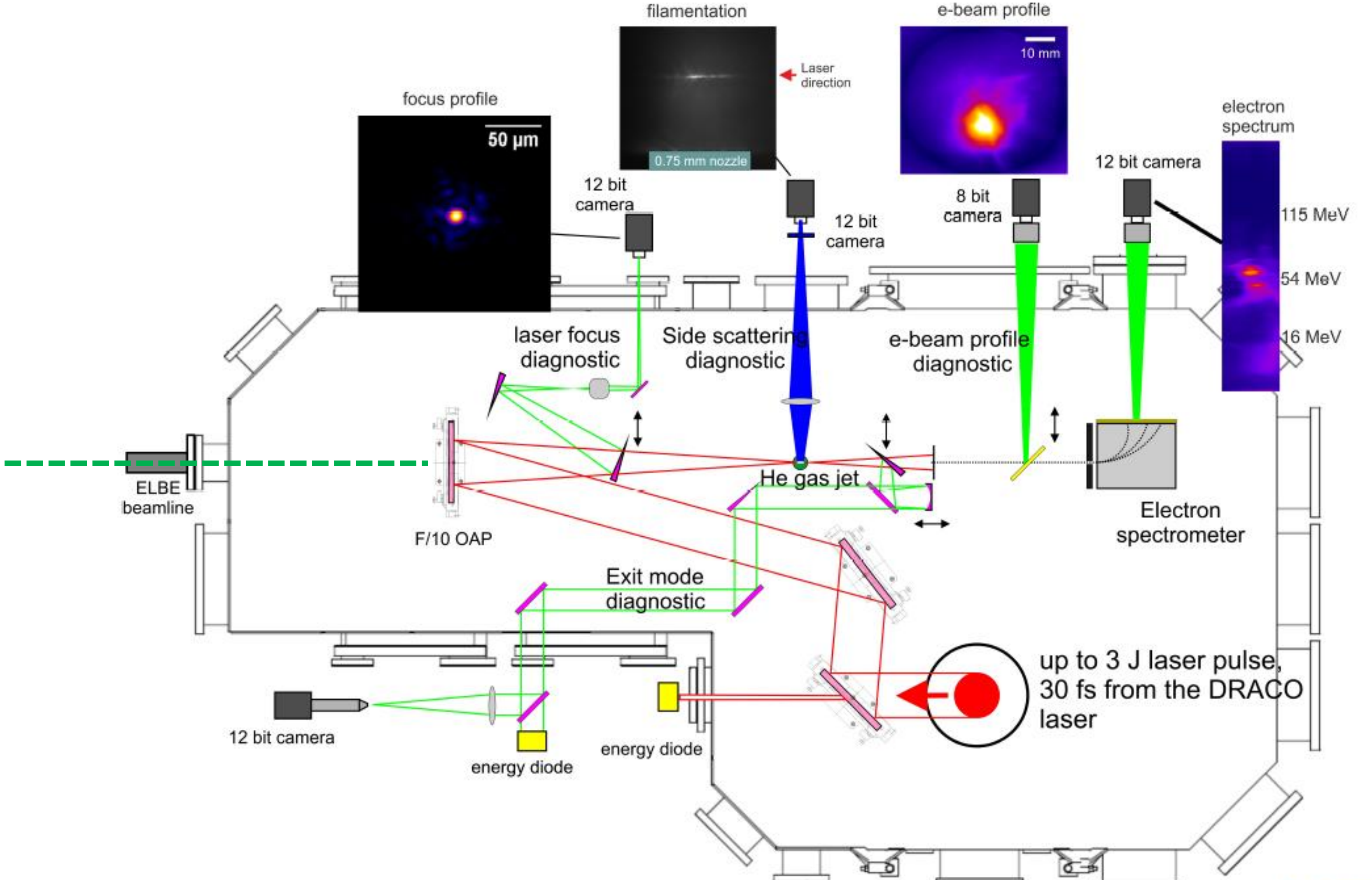


Design parameters:150 J, 150 fs, >1 Hz rep.rate
fully diode pumped system, active medium (Yb:CaF2)

- Electron Linac for beams with high Brilliance and low Emittance (ELBE)
- Dresden lAser aCceleration sOurce (DRACO)

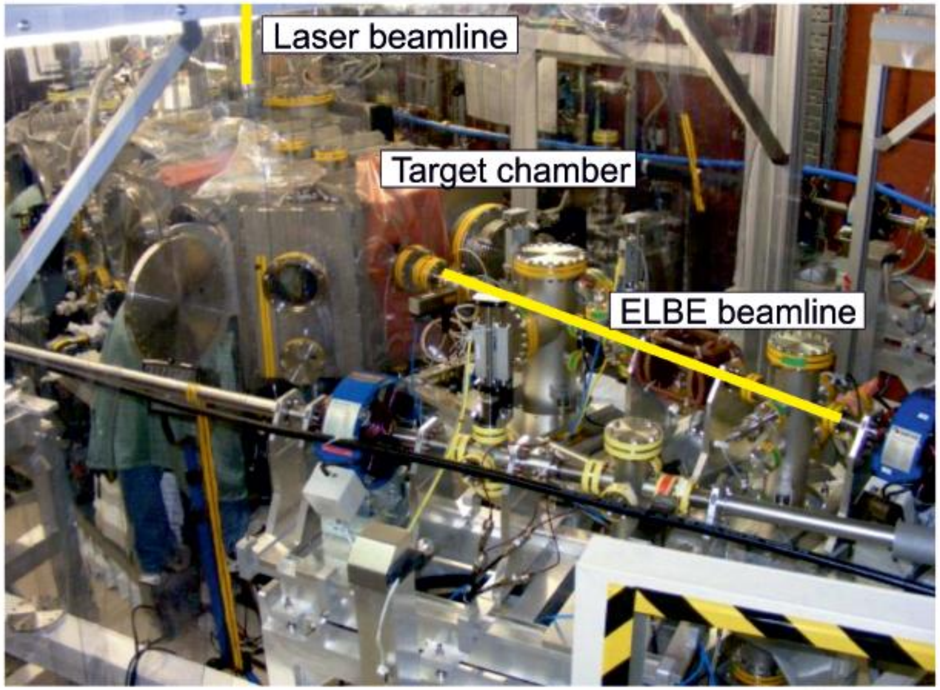
LWFA program at HZDR

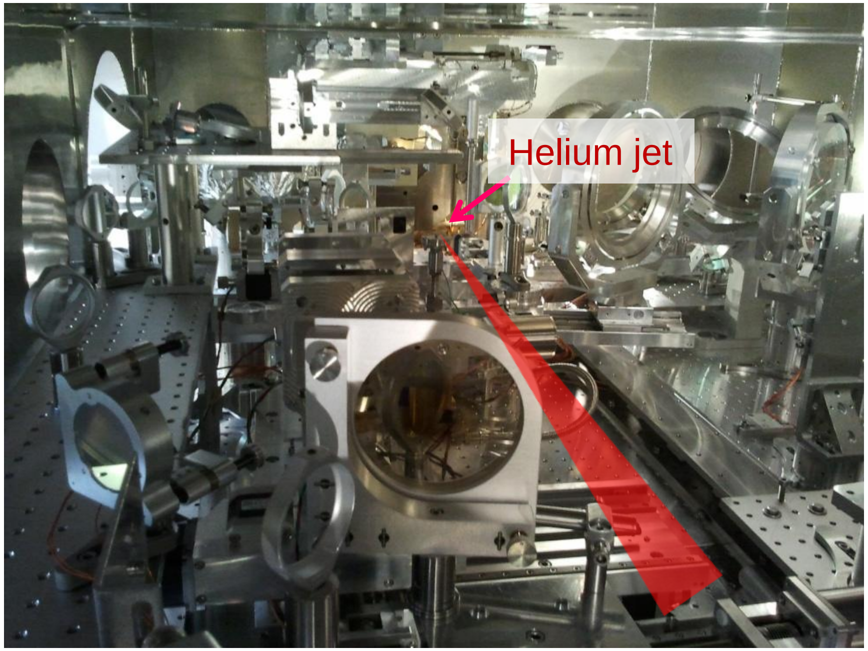
Direct access in the bubble regime and external injection

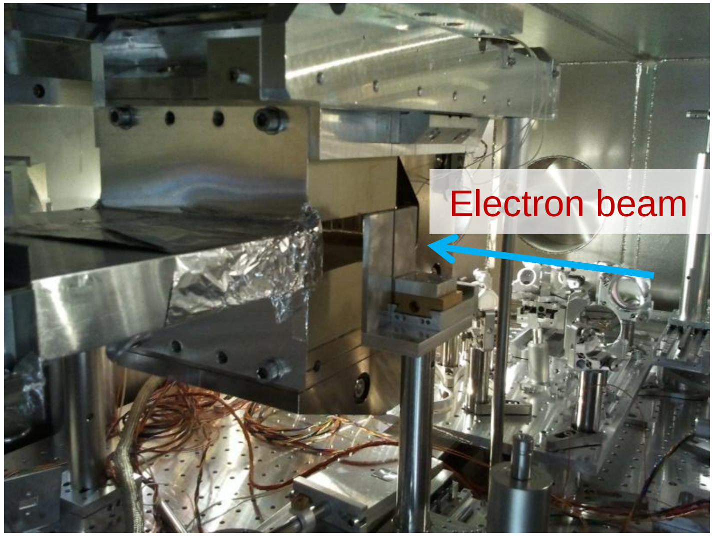


See J. Couperus' poster on Thursday

LWFA program at HZDR

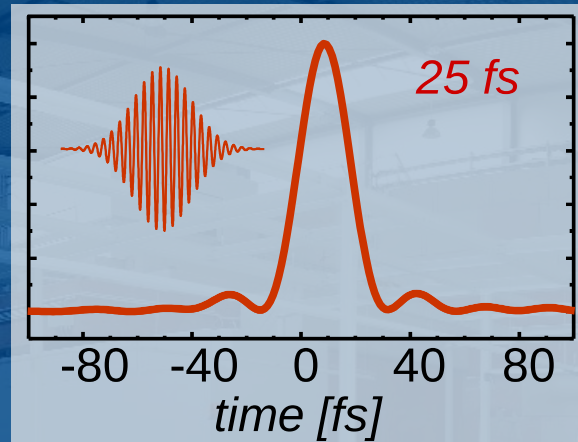


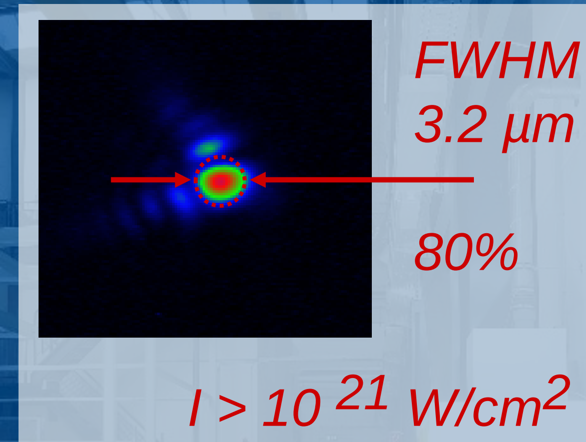


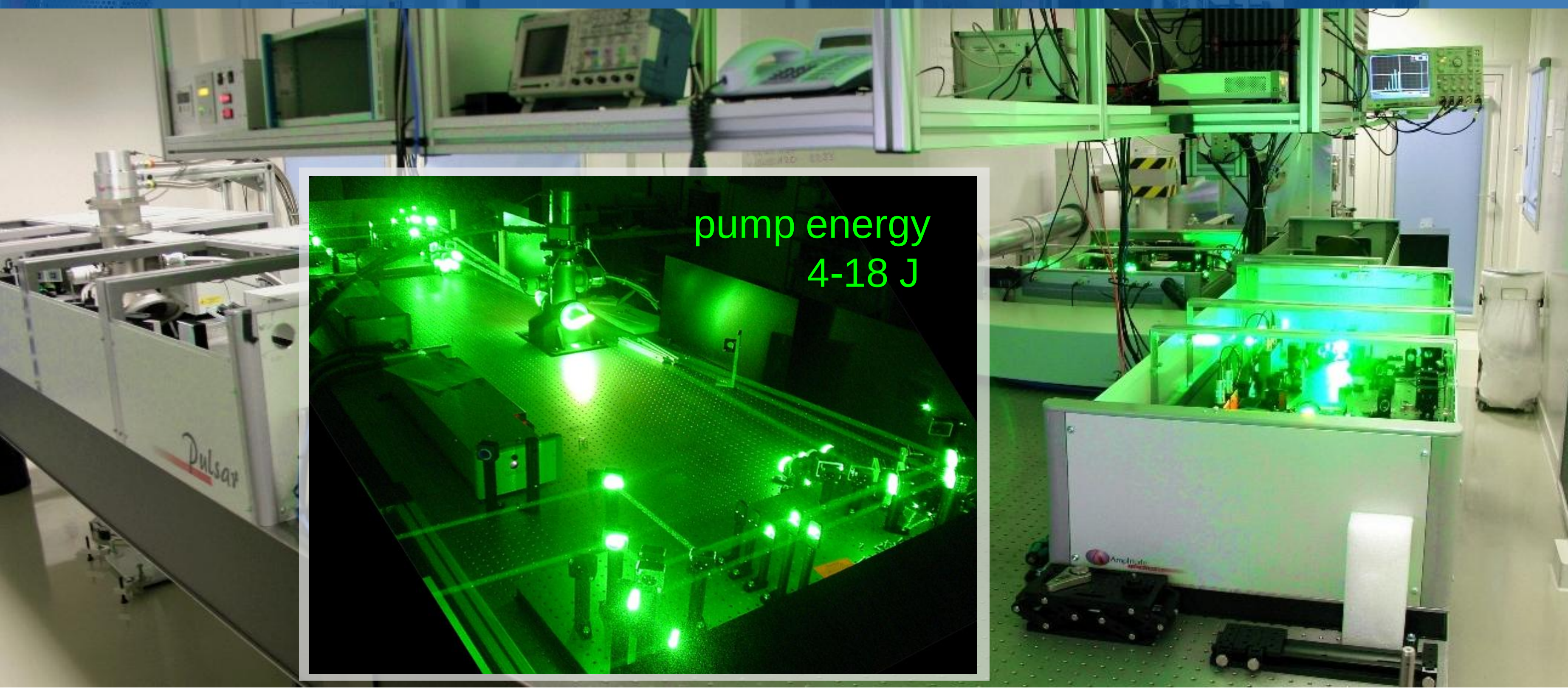


DRACO laser

Ti:Sapphire
CPA laser
>4 J (on target)
10 Hz







*Dresden laser acceleration source

Thank you

Self-consistent fluid model

The huge number of particles in plasmas

$\rightarrow$ impossible to solve Newton's equation for each particle

$\rightarrow$ hydrodynamic approach: study the motion of fluid elements

Main assumptions:

- ○ plasma is fully ionized and initially at thermal equilibrium
- ○ plasma is underdense: $\omega_0 >> \omega_{plasma}$
- ○ ions are immobile
- ○ plasma is cold: plasma electron thermal velocity << plasma wave phase velocity : $v_{th} << v_{ph}$

Towards master equations: starting point

Maxwell's equations
(EM fields)

$$\overline{\nabla} \cdot \overline{B} = 0 \quad \rightarrow \text{Closed loop magnetic fields}$$

$$\overline{\nabla} \times \overline{E} = -\frac{1}{c}\frac{\partial \overline{B}}{\partial t} \quad \rightarrow \text{Faraday induction's law}$$

$$\overline{\nabla} \cdot \overline{E} = 4\pi\rho \quad \rightarrow \text{Poisson's equation}$$

$$\overline{\nabla} \times \overline{B} = \frac{1}{c}\left(4\pi \overline{j} + \frac{\partial \overline{E}}{\partial t}\right) \quad \rightarrow \text{Ampere's law}$$

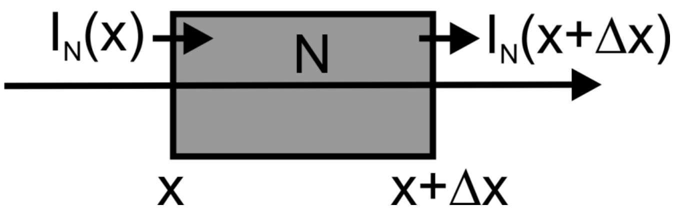
Sources

$$\rho = -e(n_p - n_0) \quad \rightarrow \text{Charge density}$$

$$\overline{j} = -e n_p \overline{v_e} \quad \rightarrow \text{Current density}$$

Continuity equation
(conservation of the number of particles)

$$\frac{\partial n_e}{\partial t} + \overline{\nabla} \cdot (n_e \overline{v}) = 0$$



Lorentz equation
(motion of particles in an EM field)

$$\overline{F} = -e\left[\overline{E} + \left(\frac{\overline{v}}{c} \times \overline{B}\right)\right]$$

HZDR

Fields can be described in term of potentials:

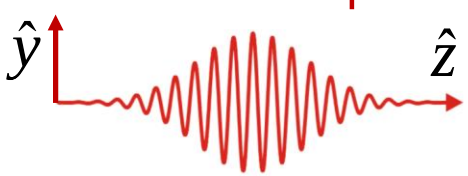
$$\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t},$$
$$\mathbf{B} = \nabla \times \mathbf{A}.$$

where satisfies: $\nabla \cdot \mathbf{A} = 0$

In normalized parameters: $\phi = e\varphi / m_e c^2 \quad a = eA / m_e c^2$

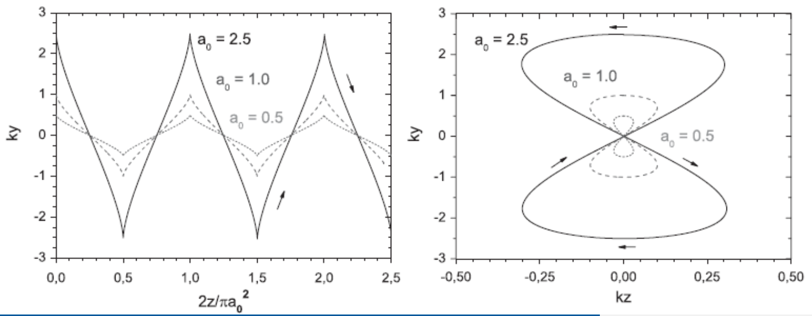
1-Dimensional case

Driver laser pulse



$$\mathbf{a}(z,t) = a_0 \exp\{-((z-ct)/\sigma_z)^2\}\,\hat{y}\exp\{-i(k_0 z - \omega_0 t)\},$$

$$a_0 = \frac{p_{y0}}{m_e c} = \sqrt{\frac{2e^2\lambda_0^2}{m_e^2 c^5 \pi} I_0},$$ $\rightarrow$ the strength of laser-plasma interaction



$a_0 \leq 1$ Non-relativistic regime

$a_0 > 1$ Relativistic regime(**v** x **B**)

λ_0=0.8 μm, a_0=1, I_0= 2.1x10^{18} W/cm^2

Towards master equations

Laboratory frame → laser frame

$$z \qquad \xi = k_p\left(z - v_g t\right)$$

$$t \qquad \tau = \omega_p t$$

$$\frac{\partial}{\partial t} = \omega_p \frac{\partial}{\partial \tau} - k_p v_g \frac{\partial}{\partial \xi} \qquad \frac{\partial^2}{\partial t^2} = \left(k_p v_g \frac{\partial}{\partial \xi} - \omega_p \frac{\partial}{\partial \tau}\right)^2$$

$$\frac{\partial}{\partial z} = k_p \frac{\partial}{\partial \xi} \qquad \frac{\partial^2}{\partial z^2} = k_p^2 \frac{\partial^2}{\partial \xi^2}$$

Master equations

$$k_p \frac{\partial}{\partial \xi}\left[\gamma(1-\beta_g\beta_z)-\phi\right] = -\frac{\omega_p}{c}\frac{\partial}{\partial \tau}\gamma\beta_z,$$ momentum eq.

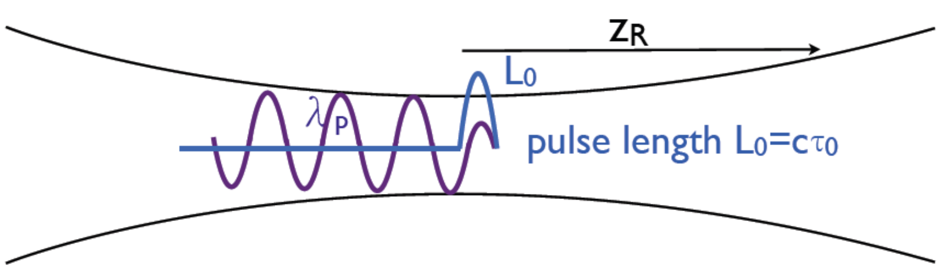
$$k_p \frac{\partial}{\partial \xi}\left[n(\beta_g-\beta_z)\right] = \frac{\omega_p}{c}\frac{\partial}{\partial \tau}n,$$ continuity eq.

$$\left[k_p^2(1-\beta_g^2)\frac{\partial^2}{\partial \xi^2}+2k_p\omega_p\frac{\beta_g}{c}\frac{\partial^2}{\partial \xi \partial \tau}-\frac{\omega_p^2}{c^2}\frac{\partial^2}{\partial \tau^2}\right]a = \beta_g^2 k_p^2 \frac{n}{n_e}\frac{a}{\gamma},$$ laser propagation in plasma eq.

$$\frac{\partial^2}{\partial \xi^2}\phi = \beta_g^2\left[\frac{n}{n_e}-1\right].$$ Poisson eq.

Quasi-static approximation:

- evolution time of the laser envelop >> the plasma response time
- this requires $\tau_{fwhm} << Z_R / c$



Master equations

With quasi-static approximation:

$$k_p \frac{\partial}{\partial \xi}\left[\gamma(1-\beta_g\beta_z)-\phi\right] = -\frac{\omega_p}{c}\frac{\partial}{\partial \tau}\gamma\beta_z \stackrel{}{=} 0, \quad \text{momentum eq.}$$

$$k_p \frac{\partial}{\partial \xi}\left[n(\beta_g-\beta_z)\right] = \frac{\omega_p}{c}\frac{\partial}{\partial \tau}n = 0, \quad \text{continuity eq.}$$

$$\left[k_p^2(1-\beta_g^2)\frac{\partial^2}{\partial \xi^2}+2k_p\omega_p\frac{\beta_g}{c}\frac{\partial^2}{\partial \xi\partial \tau}-\frac{\omega_p^2}{c^2}\frac{\partial^2}{\partial \tau^2}\right]a = \beta_g^2 k_p^2\frac{n}{n_e}\frac{a}{\gamma}, \quad \text{laser propagation in plasma eq.}$$

$$\frac{\partial^2}{\partial \xi^2}\phi = \beta_g^2\left[\frac{n}{n_e}-1\right], \quad \text{Poisson eq.}$$

1-D Laser wakefield equation

$$\frac{d^2\Phi}{d\xi^2} = \beta_g^2\gamma_g^2\left(\beta_g\frac{1}{\sqrt{1-\frac{1}{\gamma_g^2}\frac{1+a^2/2}{\Phi^2}}}-1\right), \qquad E_z = -\frac{1}{\beta_g^2}\frac{d\Phi}{d\xi}.$$

Try with Runge-Kutta

$$\gamma_g^2 = 1/(1-\beta_g^2) \qquad \Phi = 1+\phi$$

Normalized to $E_0 = m_e v_g \omega_p / e$