Matrix elements + PYTHIA8

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Outline

- CKKW-L matrix element merging in parton showers.
- The NL^3 prescription[1]: NLO merging in PYTHIA8.
- Results for $\mathrm{W}+0$ and $\mathrm{W}+1$ at NLO.
- Ideas beyond NL^3.

[1] For Nils Lavesson and Leif Lönnblad.

The MEPS merging problem

Problem: We want to describe soft/collinear and hard jets together, because we don't know the boundary between "soft" and "hard".

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- But just adding both gives massive double counting.

$\longrightarrow$ Use ME above a cut t_{MS}, and PS below t_{MS}.

- This introduces another problem: Cut dependence.

$\longrightarrow$ Apply the same weights above and below the cut. E.g. if the state would come with a different α_{s} below the cut, weight state above the cuts with α_{s} ratio.

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- ☐ Construct all shower histories $(S_{+0}, \rho_0), \ldots, (S_{+n}, \rho_n)$, pick one probabilistically.
- ☐ Generate the Sudakov factor by trial showering (i.e. use Pythia).

  Reweight with α_{s} factors and PDF factors.
- ☐ Start shower from last reconstructed scale, then
  - ◇ If n is the highest multiplicity, continue;
  - ◇ If n is not the highest multiplicity, veto events with shower splittings above t_{MS}.

$$
\begin{aligned}
w_{\mathrm{CKKWL}} \quad = \quad & \frac{f_n^+(x_n^+,\rho_n)}{f_n^+(x_n^+,\mu_f)} \frac{f_n^-(x_n^-,\rho_n)}{f_n^-(x_n^-,\mu_f)} \times \prod_{i=1}^{n} \left[\frac{\alpha_{\mathrm{s}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu_r)} \frac{f_{i-1}^+(x_{i-1}^+,\rho_{i-1})}{f_{i-1}^+(x_{i-1}^+,\rho_i)} \frac{f_{i-1}^-(x_{i-1}^-,\rho_{i-1})}{f_{i-1}^-(x_{i-1}^-,\rho_i)} \right. \\
& \left. \Pi_{S_{+i-1}}(x_{i-1},\rho_{i-1},\rho_i) \right] \Pi_{S_{+n}}(x_n,\rho_n,\rho_{n+1},>t_{\mathrm{MS}}) \ .
\end{aligned}
$$

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- ☐ Start shower from last reconstructed scale, then
  - ◇ If n is the highest multiplicity, continue;
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- Combine histograms for all ME multiplicities to get distributions for ME+PS merging.

Results of CKKW-L merging at the LHC

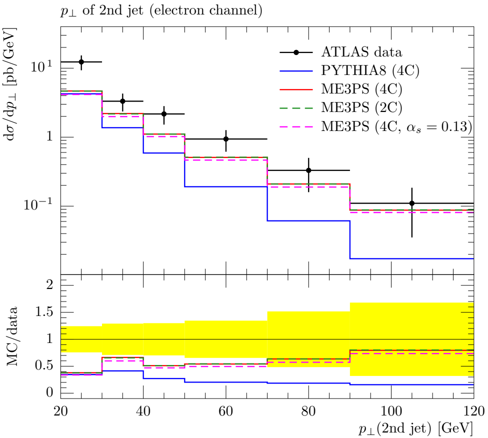

Figure: p_T of 2nd hardest jet in W+jets, for three jet merging, using slightly different tunes PYTHIA8 tunes, compared to ATLAS_2010_S8919674.

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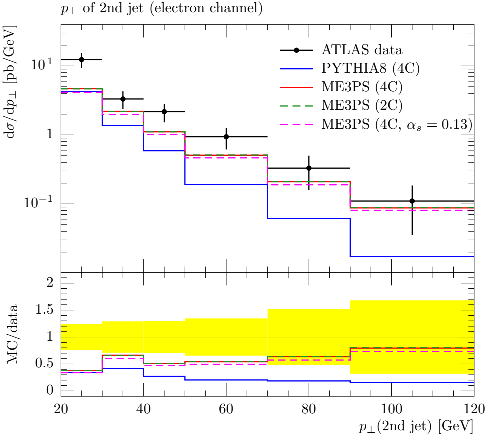

Figure: p_T of 2nd hardest jet in W+jets, for three jet merging, using slightly different tunes PYTHIA8 tunes, compared to ATLAS_2010_S8919674.

⇒ For more exclusive observables, CKKW-L does better than the

Is CKKW-L enough?

Pro: Systematic, possible for any jet multiplicity and process.

Pro: PS resummation not changed.
$\rightarrow$ Little retuning anticipated.

Pro: Gives prediction at accuracy of the shower $+$ a bit.

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Con: Does not preserve inclusive cross section.
$\rightarrow$ Difficult to assess what is bug, and what feature.

Con: From $\mathcal{O}\left(\alpha_{\mathsf{s}}^2\right)$ on, different above / below merging scale.

Con: Gives prediction at accuracy of the shower + a bit.
$\rightarrow$ Fuzzy statement. Is hard MPI less important than e.g. a $\mathcal{O}\left(\alpha_{\mathsf{s}}^2\right)$-correct $\mathrm{W}+1$ relatively soft jet?

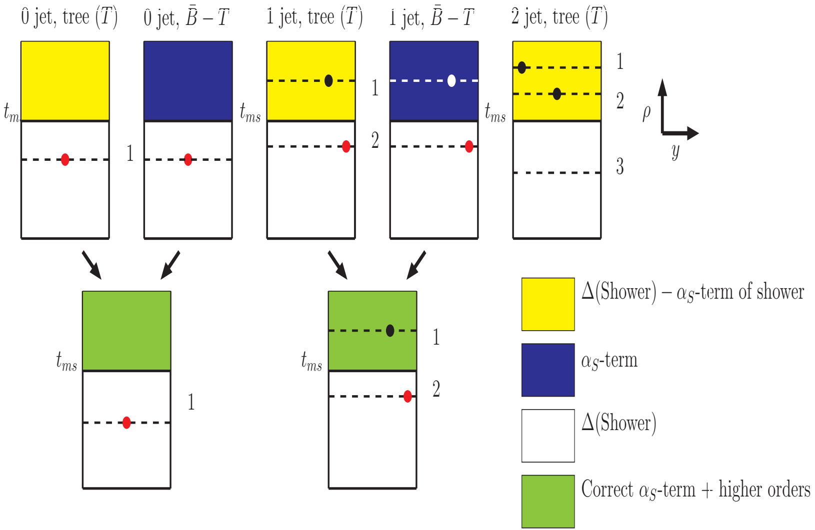
Multi-jet NLO + PS

Let us for the moment focus on getting hard jets right.
$\Rightarrow$ We want NLO accuracy for hard jets.

What we have to do:

- Avoid double counting states.
  $\Rightarrow$ Define NLO exclusive cross section.
- Avoid double counting orders of α_{s}.
  $\Rightarrow$ Expand CKKW cross section in α_{s},
  remove things we want to NLO accuracy,
  and add back true NLO.
- Implement, and check.

Multi-jet NLO + PS: General idea



NL^3 prerequisites: "Exclusive" NLO cross sections

Tree level configurations for n partons can contain $n-1$ resolved jets.

NLO configurations for n partons can contain $n-1$ and $n+1$ resolved jets.

To avoid counting states with $n+1$ resolved jets twice, we need define an NLO n-jet cross section that contains exactly n resolved jets.

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$\Rightarrow$ POWHEG's $\bar{\mathrm{B}}$ (with all real emissions integrated), with a jet cut, works fine. Or, instead of cutting, we can subtract phase space points with $n+1$ resolved jets by hand:

$$
\begin{aligned}
d\sigma_n^{NLO} &= d\phi_n \left\{ \mathcal{T}_n + \mathcal{V}_n + \mathcal{I}_n + \int d\Phi_{\mathrm{rad}} \left(\mathcal{R}_{n+1} - \mathcal{D}_n \right) \right\} \\
&- d\phi_n \int_{\rho_{MS}} d\Phi_{\mathrm{rad}} \mathcal{R}_{n+1}
\end{aligned}
$$

Merging conditions

We can now set up conditions to ensure NLO accuracy, while taking all higher orders from the shower:

$$\begin{aligned} \alpha_s^n t_n w_T + \alpha_s^{n+1} v_n w_V + \alpha_s^{n+1} r_n w_R &= \alpha_s^n t_n + \alpha_s^{n+1}(v_n + r_n) \\ &+ \alpha_s^n t_n \sum_{i=2}^{\infty} \alpha_s^i w_{PS,i} \end{aligned} \tag{1}$$

and

$$\alpha_s^{n+1} t_{n+1} w_H = \alpha_s^{n+1} t_{n+1} \sum_{i=0}^{\infty} \alpha_s^i w_{PS,i} \ , \tag{2}$$

For (2), we can immediately put

$$w_H = w_{\text{CKKW-L}}$$

For (1), we can use

$$\begin{aligned} w_T &= w_{\text{CKKW-L}} - w_{\text{CKKW-L}}\big|_{\alpha_s} \\ w_V &= 1 = w_R \end{aligned}$$

The NL^3 prescription

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For tree-level sample:

1. Generate events according to tree-level matrix element.
2. For each event, generate the CKKW-L weight $w_{\text{CKKW-L}}$ and subtract the $\mathcal{O}\left(\alpha_s^1\right)$ term $w_{\text{CKKW-L}}\big|_{\alpha_s}$.
3. Start shower from the last reconstructed scale ρ_n. Veto emissions above ρ_{MS}.

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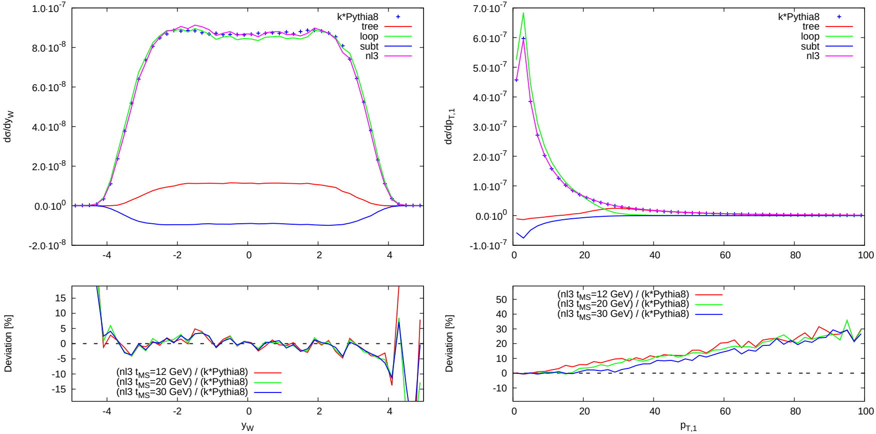
For $\mathcal{O}\left(\alpha_s^1\right)$ (virtual + insertion + regularised unresolved real) sample:

1. Generate events according to NLO exclusive n-jet cross section. If the $\mathcal{O}\left(\alpha_s^1\right)$ sample was not generated with a cut on resolved real emissions, remove the $+1$-resolved jet phase space points.
2. For each event, start shower at ρ_{MS}.

Results: NL_0^3



Nicely boring, since the POWHEG-BOX phase space mapping keeps the W-rapidity fixed, and $p_{\perp,1}$ is LO only.

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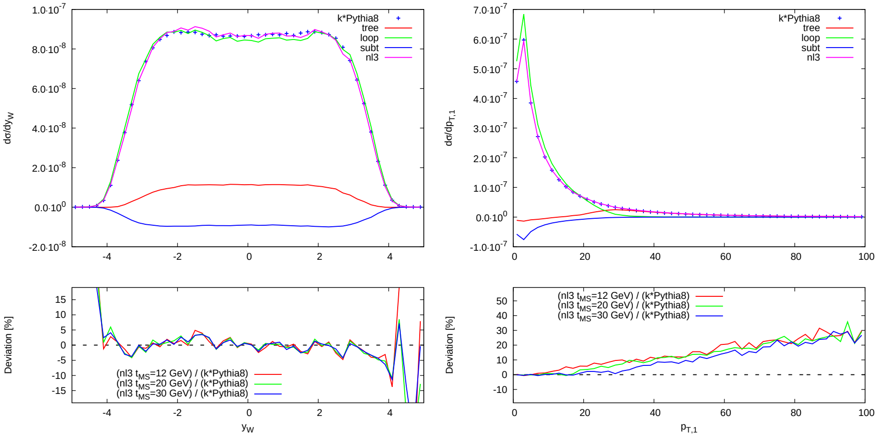
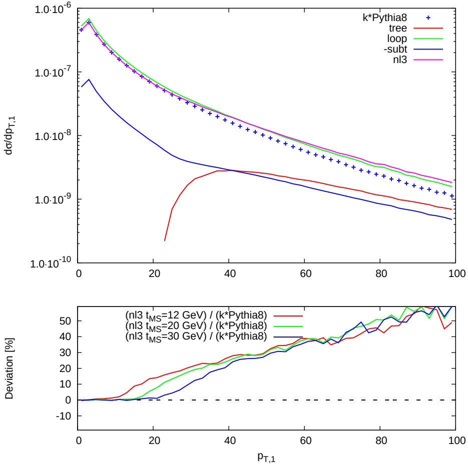


Nicely boring, since the POWHEG-BOX phase space mapping keeps the W-rapidity fixed, and $p_{\perp,1}$ is LO only.

$\Rightarrow$ Yet another POWHEG W-production interface, but now with MENLOPS for free.

Results: p_T of the hardest jet, NL_{01}^3

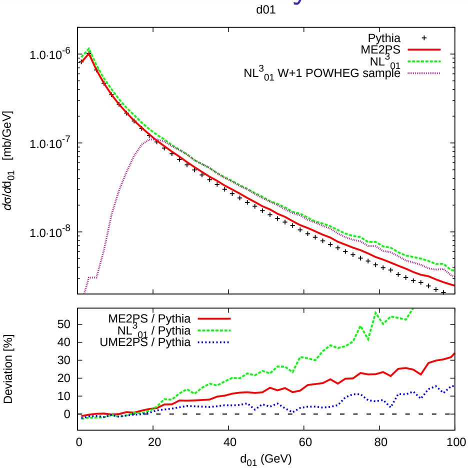


New: Combined W and W + 1 jet at NLO.

The increase in the tail partly from the W+2 jet tree-level ME (25% at 100 GeV), and partly the effect of the p_T-dependence of the W+1 NLO cross section.

Figure: W+0@NLO and W+1@NLO from POWHEG-BOX.

Be careful what you wish for!



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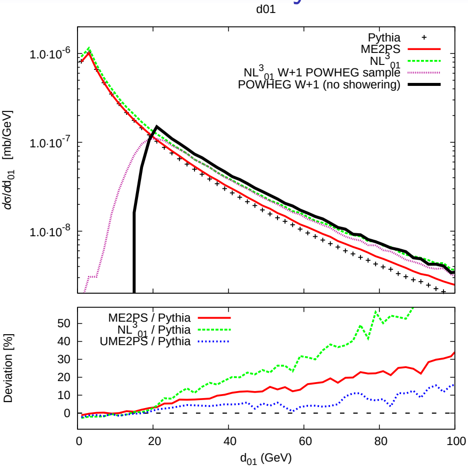

Figure: W+0@NLO and W+1@NLO.

Well, it is NLO, but. . .

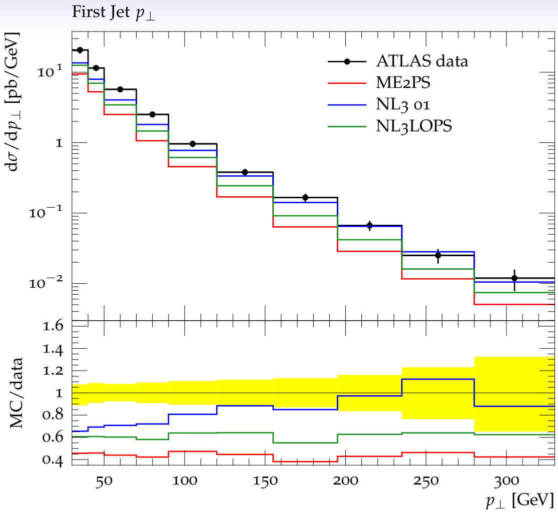

Figure: W+0@NLO and W+1@NLO, compared to ATLAS_2012_I1083318.


CORN
FLAKES
MILK

A cautionary tale

What we wished for:

- Take well-defined fixed-order calculations.
- Merge them with the shower, ensure that we remain (NLO) accurate.
- Beyond the accuracy of the fixed-order calculation, keep exactly the PS higher orders, i.e. CKKW-L terms (we implemented CKKW-L in the first place so that we could continue to NL3!)

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Are we allowed to change the higher orders?

Yes, if we remove problems in CKKW-L.

Unitarity and MEPS

$$
\begin{aligned}
&f_0(x_0,\mu_f)B_0\Bigg[1-\int_{\rho_{ms}}^{\rho_{max}}\frac{d\rho}{\rho}\int\frac{dz}{z}\frac{\alpha_s(\rho)}{2\pi}\frac{f_1(\frac{x_0}{z},\rho)}{f_0(x_0,\rho)}P_1\\
&\quad+\frac{1}{2}\left(\int_{\rho_{ms}}^{\rho_{max}}\frac{d\rho}{\rho}\int\frac{dz}{z}\frac{\alpha_s(\rho)}{2\pi}\frac{f_1(\frac{x_0}{z},\rho)}{f_0(x_0,\rho)}P_1\right)^2+\ldots\Bigg]\\
&+f_0(x_0,\mu_f)B_1\alpha_{\mathrm{s}}(\rho_1)\frac{f_1(x_1,\rho_1)}{f_0(x_0,\rho_1)}\Bigg[1-\int_{\rho_1}^{\rho_{max}}\frac{d\rho}{\rho}\int\frac{dz}{z}\frac{\alpha_s(\rho)}{2\pi}\frac{f_1(\frac{x_0}{z},\rho)}{f_0(x_0,\rho)}P_1\\
&\quad-\int_{\rho_{ms}}^{\rho_1}\frac{d\rho}{\rho}\int\frac{dz}{z}\frac{\alpha_s(\rho)}{2\pi}\frac{f_2(\frac{x_1}{z},\rho)}{f_1(x_1,\rho)}P_2+\ldots\Bigg]\\
&+f_0(x_0,\mu_f)B_2\alpha_{\mathrm{s}}(\rho_1)\alpha_{\mathrm{s}}(\rho_2)\frac{f_1(x_1,\rho_1)}{f_0(x_0,\rho_1)}\frac{f_2(x_2,\rho_2)}{f_1(x_1,\rho_2)}
\end{aligned}
$$

The blue and green terms cancel.
In standard parton shower, the red terms cancel.
In CKKW, they do not cancel! $\Rightarrow$ Replace (e) by integrated (f) $\Rightarrow$ Unitary to $\mathcal{O}\left(\alpha_s^2\right)$.

Unitary MEPS

Recipe: Subtract what you add. $\Rightarrow$ Replace the probability of unresolved 2-jet states in the 1-jet sample by integrated 2-jet sample. $\Rightarrow$ Unitary to $\mathcal{O}\left(\alpha_s^2\right)$.

$$
\begin{aligned}
& f_0(x_0,\mu_f)B_0\Bigg[1-\int_{\rho_{ms}}^{\rho_{max}}\frac{d\rho}{\rho}\int\frac{dz}{z}\frac{\alpha_s(\rho)}{2\pi}\frac{f_1(\frac{x_0}{z},\rho)}{f_0(x_0,\rho)}P_1 \\
& \quad +\frac{1}{2}\left(\int_{\rho_{ms}}^{\rho_{max}}\frac{d\rho}{\rho}\int\frac{dz}{z}\frac{\alpha_s(\rho)}{2\pi}\frac{f_1(\frac{x_0}{z},\rho)}{f_0(x_0,\rho)}P_1\right)^2+\ldots\Bigg] \\
& +f_0(x_0,\mu_f)B_1\alpha_s(\rho_1)\frac{f_1(x_1,\rho_1)}{f_0(x_0,\rho_1)}\Bigg[1-\int_{\rho_1}^{\rho_{max}}\frac{d\rho}{\rho}\int\frac{dz}{z}\frac{\alpha_s(\rho)}{2\pi}\frac{f_1(\frac{x_0}{z},\rho)}{f_0(x_0,\rho)}P_1 \\
& -\int_{\rho_{ms}}^{\rho_1}\frac{d\rho}{\rho}\int\frac{dz}{z}\frac{\alpha_s(\rho)}{2\pi}\frac{f_2(\frac{x_1}{z},\rho)}{f_1(x_1,\rho)}P_2+\cdots+\int_{\rho_{ms}}^{\rho_1}\frac{d\rho}{\rho}\int\frac{dz}{z}\frac{\alpha_s(\rho)}{2\pi}\frac{f_2(\frac{x_1}{z},\rho)}{f_1(x_1,\rho)}P_2\Bigg] \\
& -\int_{\rho_{ms}}^{\rho_1}\frac{d\rho}{\rho}\int\frac{dz}{z}f_0(x_0,\mu_f)B_2\alpha_s(\rho_1)\alpha_s(\rho_2)\frac{f_1(x_1,\rho_1)}{f_0(x_0,\rho_1)}\frac{f_2(x_2,\rho_2)}{f_1(x_1,\rho_2)} \\
& +f_0(x_0,\mu_f)B_2\alpha_s(\rho_1)\alpha_s(\rho_2)\frac{f_1(x_1,\rho_1)}{f_0(x_0,\rho_1)}\frac{f_2(x_2,\rho_2)}{f_1(x_1,\rho_2)}
\end{aligned}
$$

UMEPS: Preliminary results

For the nightmare case of restricting the phase space of the second emission explicitly, we find:

$\sigma_{inclusive}$	$\sigma_{ME2PS}(t_{\mathrm{MS}} = 20 GeV)$	$\sigma_{ME2PS}(t_{\mathrm{MS}} = 30 GeV)$
5.126e-06	5.246e-06	5.195e-06

$\Rightarrow$ About 2.3% change from $\sigma_{inclusive}$.

$\sigma_{inclusive}$	$\sigma_{UME2PS}(t_{\mathrm{MS}} = 20 GeV)$	$\sigma_{UME2PS}(t_{\mathrm{MS}} = 30 GeV)$
5.126e-06	5.121e-06	5.122e-06

$\Rightarrow$ About 0.9‰ change from $\sigma_{inclusive}$.

UMEPS: Preliminary results

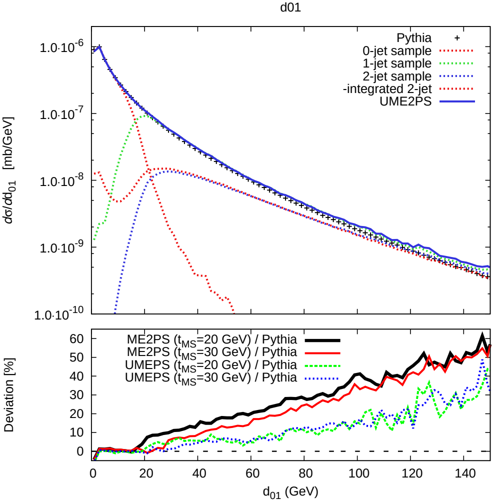

Figure: CKKW-L vs. UMEPS: d_{01} between last $k_\perp$ cluster and beam, in W+jets events at LHC7, for two different merging scale values.

Back on track: Unitary NLOPS

Proceed as in unitary MEPS, but shift inclusive cross section to NLO result. Add-subtract NLO samples.

Pro: Preserves inclusive cross section.

Pro: Resolves the NNLL problems of NL^3.

Pro: Step towards improving the PS accuracy of low-multiplicity states.

Con: Trickier than CKKW-L or NL^3 for higher multiplicities.

Outlook

- CKKW-L merging is implemented in PYTHIA8 (public since version 8.157).
- We have implemented a modified NL^3 scheme in PYTHIA8.
- For W + jets, we get what we ask for, but that seems not enough.
- Our next hope is UMEPS and UNLOPS.

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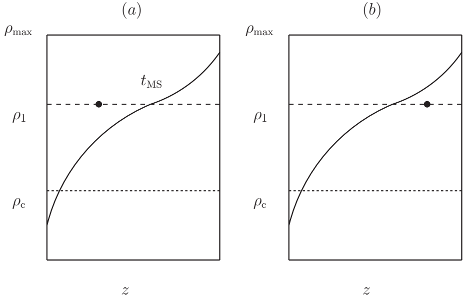
Back up

Example:
One jet above ρ_c

(ρ: evolution p_T

z: Auxiliary variables

t_{ms}: Merging scale)



Take (a) from $+1$ jet matrix element $\mathcal{T}_1$. Reweight with the PS weight, i.e. pick this state with weight

$$\left[\ f_1(\mu_1)\alpha_{\mathrm{s}}(\mu_R)\mathcal{T}_1\ \right] d\Phi_1^{\mathrm{ME}} \times w_{Path} \times \frac{f_0(\rho_0)}{f_1(\mu_1)}$$
$$\times \quad \frac{\alpha_{\mathrm{s}}(\rho_1)}{\alpha_{\mathrm{s}}(\mu_R)} \frac{f_1(\rho_1)}{f_0(\rho_1)} \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho_c)$$

Take (b) from $+0$ jet matrix element $\mathcal{T}_0$, with one shower splitting, i.e. with weight

$$\left[\ f_0(\mu_0)\mathcal{T}_0\ \right] d\Phi_0^{\mathrm{ME}} \times \frac{f_0(\rho_0)}{f_0(\mu_0)}$$
$$\times \quad \alpha_{\mathrm{s}}(\rho_1) \frac{f_1(\rho_1)}{f_0(\rho_1)} P(z)\, d\rho_1 dz_1 \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho_c)$$

One jet above ρ_c

Combining this, the merged approximation to the inclusive zero-jet cross section is

$$
\begin{aligned}
d\sigma^{CKKW} \quad = \quad & f_0(\rho_0)\Big\{ \\
& \mathcal{T}_1 d\Phi_1^{\mathrm{ME}} w_{Path} \alpha_{\mathrm{s}}(\rho_1) \\
& \Theta(t(S_{+1,me}) - t_{\mathrm{MS}}) \\
& \tfrac{f_1(\rho_1)}{f_0(\rho_1)} \, \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho_c) \\
+ \quad & \mathcal{T}_0 d\Phi_0^{\mathrm{ME}} P\,(z)\, d\rho_1 dz_1 \alpha_{\mathrm{s}}(\rho_1) \\
& \Theta(t_{\mathrm{MS}} - t(S_{+1,ps})) \\
& \tfrac{f_1(\rho_1)}{f_0(\rho_1)} \Pi_{S_{+0}}(\rho_0, \rho_1)\, \Pi_{S_{+1}}(\rho_1, \rho_c) \;\Big\}
\end{aligned}
$$

The merging scale dependence vanishes if the red probabilities are equal, and if the no-emission probabilities Π_{S_+} are identical (because then the blue Θ-functions add to one).

NL^3 prerequisites: Exclusive NLO cross sections ($\bar{\mathrm{B}}$), notation

$\mathcal{J}_n(\phi_n)$	: Jet observable, measured at phase space point ϕ_n, giving an n-jet prediction.
$\mathcal{T}_{n,f_{b1}}$	: Tree level ME with n partons, flavour f_{b1}.
$\mathcal{V}_{n,f_{b1}}$	: Virtual correction with n partons, flavour f_{b1}.
$\mathcal{I}_{n+1,\alpha_r}$	: Integrated subtraction for n partons and flavour f_{b1} (derived from approximate $n+1$ with flavour α_r).
$\mathcal{R}_{n+1,\alpha_r}$	: Real emission ME with $n+1$ partons and flavour α_r.
$\mathcal{D}_{n+1,\alpha_r}$	: Subtraction terms for $n+1$ partons with flavour α_r.

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$$
\begin{aligned}
d\sigma^{NLO}_{n,ex,f_{b1}} \quad = \quad & d\phi_n \mathcal{J}_n(\phi_n)\mathcal{T}_{n,f_{b1}} + d\phi_n \mathcal{J}_n(\phi_n)\left[\mathcal{V}_{n,f_{b1}} + \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} \mathcal{I}_{n+1,\alpha_r}\right] \\
& + d\bar{\phi}_n \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} \int^{\rho_{\mathrm{MS}}} d\Phi^{(0)}_{\mathrm{rad}} \mathcal{J}_n(\bar{\phi}_n)\left(\mathcal{R}_{n+1,\alpha_r} - \mathcal{D}_{n+1,\alpha_r}\right) \\
& + d\bar{\phi}_n \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} \int_{\rho_{\mathrm{MS}}} d\Phi^{(1)}_{\mathrm{rad}} \Big\{ \mathcal{J}_{n+1}(\phi_{n+1})\left(\mathcal{R}_{n+1,\alpha_r} \qquad \right) \\
& \qquad\qquad - \mathcal{J}_n(\bar{\phi}_n)\mathcal{D}_{n+1,\alpha_r} \Big\}
\end{aligned}
$$

The red part contains resolved $n+1$ parton states, and should be zero (by vetoing). $n+1$-parton phase space points will be included in the next higher multiplicity.

$\Rightarrow$ NLO cross section "exclusive" in the same way that tree-level is.

The blue part is what is collected in $\bar{\mathrm{B}}$ in POWHEG.

NL^3 prerequisites: Exclusive NLO cross sections ($\bar{\mathrm{B}}$)

$$
\begin{aligned}
d\sigma^{NLO}_{n,ex,f_{b1}} \quad = \quad & d\phi_n \mathcal{J}_n(\phi_n)\mathcal{T}_{n,f_{b1}} + d\phi_n \mathcal{J}_n(\phi_n)\left[\mathcal{V}_{n,f_{b1}} + \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} \mathcal{I}_{n+1,\alpha_r}\right] \\
& + d\bar{\phi}_n \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} \int^{\rho_{\mathrm{MS}}} d\Phi^{(0)}_{\mathrm{rad}} \mathcal{J}_n(\bar{\phi}_n)\left(\mathcal{R}_{n+1,\alpha_r} - \mathcal{D}_{n+1,\alpha_r}\right) \\
& + d\bar{\phi}_n \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} \int_{\rho_{\mathrm{MS}}} d\Phi^{(1)}_{\mathrm{rad}} \Bigg\{ \mathcal{J}_{n+1}(\phi_{n+1})\left(\mathcal{R}_{n+1,\alpha_r} - \mathcal{C}_{n+1,\alpha_r}\right) \\
& \qquad - \mathcal{J}_n(\bar{\phi}_n)\mathcal{D}_{n+1,\alpha_r} \Bigg\}
\end{aligned}
$$

The red part contains resolved $n+1$ parton states, and should be zero (by vetoing or phase space subtraction). $n+1$-parton phase space points will be included in the next higher multiplicity.

$\Rightarrow$ NLO cross section "exclusive" in the same way that tree-level is.

The blue part is what is collected in $\bar{\mathrm{B}}$ in POWHEG.

Rescaled CKKW-L cross sections, K-factor in POWHEG

POWHEG rescales the "seed" cross section B by a phase space dependent K-factor $\bar{\mathrm{B}}/\mathrm{B}$). Schematically:

$$
\begin{aligned}
d\sigma^{\mathrm{PH}} &= d\phi_0 \bar{\mathrm{B}} \times \left[\Delta(\rho_{max}, \rho_c) O(\phi_0) + \int_{\rho_c} d\Phi_{\mathrm{rad}} \, \frac{\mathrm{R}}{\mathrm{B}} \, \Delta(\rho_{max}, \rho(\Phi_{\mathrm{rad}})) O(\phi_1) \right] \\
&= d\phi_0 \mathrm{B} \frac{\bar{\mathrm{B}}}{\mathrm{B}} \times \left[\Delta(\rho_{max}, \rho_c) O(\phi_0) + \int_{\rho_c} d\Phi_{\mathrm{rad}} \frac{\mathrm{R}}{\mathrm{B}} \Delta(\rho_{max}, \rho(\Phi_{\mathrm{rad}})) O(\phi_1) \right] \\
&= d\phi_0 \mathrm{B} \frac{\bar{\mathrm{B}}}{\mathrm{B}} \times \left[\left(1 - \int_{\rho_c} d\Phi_{\mathrm{rad}} \frac{\mathrm{R}}{\mathrm{B}} + \frac{1}{2} \left(- \int_{\rho_c} d\Phi_{\mathrm{rad}} \frac{\mathrm{R}}{\mathrm{B}} \right)^2 \right) O(\phi_0) \right. \\
&\qquad \left. + \int_{\rho_c} d\Phi_{\mathrm{rad}} \frac{\mathrm{R}}{\mathrm{B}} \left(1 - \int_{\rho_c} d\Phi_{\mathrm{rad}}{}' \frac{\mathrm{R}}{\mathrm{B}} \right) O(\phi_1) \; + \; \mathcal{O}\left(\alpha_{\mathrm{s}}^3\right) \right]
\end{aligned}
$$

In the last lines, POWHEG rescales the $\mathcal{O}\left(\alpha_{\mathrm{s}}^2\right)$ terms. Since the subsequent showering is unitary, this "rescaled seed cross section" multiplies all approximate higher orders introduced by the shower.

Merging conditions, notation

$$
\begin{aligned}
\alpha_{\mathrm{s}}^{n} t_n &= \sum_{f_{b1}} d\phi_n \mathcal{J}_n(\phi_n) \mathcal{T}_{n,f_{b1}} \\
\alpha_{\mathrm{s}}^{n+1} v_n &= \sum_{f_{b1}} d\phi_n \mathcal{J}_n(\phi_n) \left[\mathcal{V}_{n,f_{b1}} + \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} \mathcal{I}_{n+1,\alpha_r} \right] \\
\alpha_{\mathrm{s}}^{n+1} r_n &= \sum_{f_{b1}} \sum_{\alpha_r \in \{\alpha_r | f_{b1}\}} d\bar{\phi}_n \left\{ \int^{\rho_{\mathrm{MS}}} d\Phi_{\mathrm{rad}} \mathcal{J}_n(\bar{\phi}_n) \left[\mathcal{R}_{n+1,\alpha_r} - \mathcal{D}_{n+1,\alpha_r} \right] \right. \\
&\qquad \left. - \int_{\rho_{\mathrm{MS}}} d\Phi_{\mathrm{rad}} \mathcal{J}_n(\bar{\phi}_n) \mathcal{D}_{n+1,\alpha_r} \right\}
\end{aligned}
$$

NLO merging weights

These conditions can be checked order by order. Then, from the ansatz

$$w_{T,V,R} \quad = \quad a_{T,V,R,0} + \sum_{i=1}^{\infty} b_{T,V,R,i}\alpha_{\mathrm{s}}^{i} + \sum_{i=1}^{\infty} c_{T,V,R,i}\left(\frac{1}{\alpha_{\mathrm{s}}}\right)^{i}$$

we find the solutions

$$w_T = w_{\text{CKKW-L}} - w_{\text{CKKW-L}}\big|_{\alpha_{\mathrm{s}}} + \sum_{i=1}^{\infty} c_{T,i}\left(\frac{1}{\alpha_{\mathrm{s}}}\right)^{i}$$

$$w_V = 1 + \sum_{i=2}^{\infty} c_{V,i}\left(\frac{1}{\alpha_{\mathrm{s}}}\right)^{i} \quad , \quad w_R = 1 + \sum_{i=2}^{\infty} c_{R,i}\left(\frac{1}{\alpha_{\mathrm{s}}}\right)^{i}$$

of which

$$\begin{aligned} w_T &= w_{\text{CKKW-L}} - w_{\text{CKKW-L}}\big|_{\alpha_{\mathrm{s}}} \\ w_V &= 1 = w_R \end{aligned}$$

is a special case, as expected. The PS resummation is still encoded in the merging weight $w_{\text{CKKW-L}}$. The weights $w_{\text{CKKW-L}}\big|_{\alpha_{\mathrm{s}}}$ are the coloured terms we have found in the expansion of the merging weight.

NLO merging weights

$$w_T = w_{\text{CKKW-L}} - w_{\text{CKKW-L}}\big|_{\alpha_s} + \sum_{i=1}^{\infty} c_{T,i} \left(\frac{1}{\alpha_s}\right)^i$$

$$w_V = 1 + \sum_{i=2}^{\infty} c_{V,i} \left(\frac{1}{\alpha_s}\right)^i \;, \quad w_R = 1 + \sum_{i=2}^{\infty} c_{R,i} \left(\frac{1}{\alpha_s}\right)^i$$

We do not want to include spurious $\mathcal{O}\left(\alpha_s^{n-i}\right)$ terms, and do not want to disturb the intricate cancellations between v_n and r_n:

$$c_{T,1} = 0 \qquad c_{T,i} + c_{V,i} + c_{R,i} = 0 \qquad c_{V,i} = c_{R,i}$$

For example, we can also allow

$$\begin{aligned} w_T &= w_{\text{CKKW-L}} - w_{\text{CKKW-L}}\big|_{\alpha_s} - \sum_{i=2}^{\infty} 2c_{V,i} \left(\frac{1}{\alpha_s}\right)^i \\ w_V &= w_R = 1 + \sum_{i=2}^{\infty} c_{V,i} \left(\frac{1}{\alpha_s}\right)^i \end{aligned}$$

We choose the simplest form since we could not think of a useful, shower-producible factor that has an expansion in negative powers of α_s.

Consequences of NL^3 weights

- Algorithm works on exclusive NLO n-jet cross section.
  $\Rightarrow$ Define a cut in the NLO calculation.
  If not possible (e.g. in the POWHEG-BOX), the calculation can be made exclusive by subtracting phase space points not passing the cut.[2]

[2] The phase space subtraction can be constructed by reclustering the next higher multiplicity tree-level events.

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- Algorithm works on exclusive NLO *n*-jet cross section.
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  If not possible (e.g. in the POWHEG-BOX), the calculation can be made exclusive by subtracting phase space points not passing the cut.[2]
- The condition $w_V = 1$ also means that the merging scale has to be defined by the jet algorithm (ρ) used for shower evolution.
- We want to keep the NLO 0-jet inclusive cross section fixed. Merging multiple NLO calculations introduces terms $\propto \alpha_s^2 \ln^2 \frac{1}{\rho_{\mathrm{MS}}}$, which are beyond the control of the PS.
  $\Rightarrow$ Need to assess if these terms will prove a problem for reasonable ρ_{MS} values.

[2] The phase space subtraction can be constructed by reclustering the next higher multiplicity tree-level events.

Expansion of the CKKW-L weight to $\mathcal{O}\left(\alpha_s^1\right)$

$$
\begin{aligned}
d\sigma_0^{CKKW} &= f_0(\rho_0)\left|\mathcal{M}_{S_{+0}}\right|^2 d\Phi_0^{\mathrm{ME}}\Bigg[1 + K\big|_{\alpha_s} \\
&\quad - \frac{\alpha_s}{2\pi}\int_{\rho_{\mathrm{MS}}}\frac{f_1(\mu_f)}{f_0(\mu_f)}P_1(z)d\rho dz + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2\left(\int_{\rho_{\mathrm{MS}}}\frac{f_1(\mu_f)}{f_0(\mu_f)}P_1(z)d\rho dz\right)^2\Bigg]
\end{aligned}
$$

Replace blue terms by correct 0-jet NLO expression.

Expansion of the CKKW-L weight to $\mathcal{O}\left(\alpha_s^1\right)$

$$
\begin{aligned}
d\sigma_0^{CKKW} &= f_0(\rho_0)\left|\mathcal{M}_{S_{+0}}\right|^2 d\Phi_0^{\mathrm{ME}}\left[1 + K\right|_{\alpha_s} \\
&- \frac{\alpha_s}{2\pi}\int_{\rho_{MS}}\frac{f_1(\mu_f)}{f_0(\mu_f)}P_1(z)d\rho dz + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2\left(\int_{\rho_{MS}}\frac{f_1(\mu_f)}{f_0(\mu_f)}P_1(z)d\rho dz\right)^2\Bigg] \\
d\sigma_1^{CKKW} &= f_1(\rho_0)\alpha_s\left|\mathcal{M}_{S_{+1}}\right|^2 d\Phi_1^{\mathrm{ME}}\left[1 + K\right|_{\alpha_s} + \frac{\alpha_s}{4\pi}\beta_0\ln\left(\frac{\rho_1}{\mu_r}\right) \\
&+ \left.\frac{f_1(\rho_1)}{f_1(\rho_0)}\right|_{\alpha_s} + \left.\frac{f_0(\rho_0)}{f_0(\rho_1)}\right|_{\alpha_s} \\
&- \frac{\alpha_s}{2\pi}\int_{\rho_1}^{\rho_0}\frac{f_1(\mu_f)}{f_0(\mu_f)}P_1(z)d\rho dz - \frac{\alpha_s}{2\pi}\int_{\rho_{MS}}^{\rho_1}\frac{f_2(\mu_f)}{f_1(\mu_f)}P_2(z)d\rho dz\Bigg]
\end{aligned}
$$

Replace red terms by correct 1-jet NLO expression.

Expansion of the CKKW-L weight to $\mathcal{O}\left(\alpha_s^1\right)$

$$
\begin{aligned}
d\sigma_0^{CKKW} &= f_0(\rho_0)\left|\mathcal{M}_{S_{+0}}\right|^2 d\Phi_0^{\mathrm{ME}}\Bigg[1 + K\big|_{\alpha_s} \\
&- \frac{\alpha_s}{2\pi}\int_{\rho_{MS}} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z)\, d\rho\, dz + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2 \left(\int_{\rho_{MS}} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z)\, d\rho\, dz\right)^2\Bigg] \\
d\sigma_1^{CKKW} &= f_1(\rho_0)\alpha_s\left|\mathcal{M}_{S_{+1}}\right|^2 d\Phi_1^{\mathrm{ME}}\Bigg[1 + K\big|_{\alpha_s} + \frac{\alpha_s}{4\pi}\beta_0 \ln\left(\frac{\rho_1}{\mu_r}\right) \\
&+ \left.\frac{f_1(\rho_1)}{f_1(\rho_0)}\right|_{\alpha_s} + \left.\frac{f_0(\rho_0)}{f_0(\rho_1)}\right|_{\alpha_s} \\
&- \frac{\alpha_s}{2\pi}\int_{\rho_1}^{\rho_0} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z)\, d\rho\, dz - \frac{\alpha_s}{2\pi}\int_{\rho_{MS}}^{\rho_1} \frac{f_2(\mu_f)}{f_1(\mu_f)} P_2(z)\, d\rho\, dz\Bigg] \\
d\sigma_2^{CKKW} &= f_2(\rho_0)\alpha_s^2\left|\mathcal{M}_{S_{+2}}\right|^2 d\Phi_2^{\mathrm{ME}}
\end{aligned}
$$

Keep higher multiplicity ME if no NLO calculation available.

Weight of tree-level samples

$$
\begin{aligned}
d\sigma_0^{NL3} \quad = \quad & f_0(\rho_0)\left|\mathcal{M}_{S_{+0}}\right|^2 d\Phi_0^{\mathrm{ME}}\left\{K\Pi_{S_{+n}}(\rho_0,\rho_{\mathrm{MS}})\right. \\
& \left. - 1 - K\big|_{\alpha_{\mathrm{s}}} + \frac{\alpha_{\mathrm{s}}}{2\pi}\int_{\rho_{\mathrm{MS}}}\frac{f_1(\mu_f)}{f_0(\mu_f)}P_1(z)\,d\rho\, dz\right\}
\end{aligned}
$$

Weight of tree-level samples

$$
\begin{aligned}
d\sigma_0^{NL3} &= f_0(\rho_0)\left|\mathcal{M}_{S_{+0}}\right|^2 d\Phi_0^{\mathrm{ME}}\left\{K\Pi_{S_{+n}}(\rho_0,\rho_{\mathrm{MS}})\right. \\
&\qquad \left. - 1 - K\big|_{\alpha_s} + \frac{\alpha_s}{2\pi}\int_{\rho_{\mathrm{MS}}}\frac{f_1(\mu_f)}{f_0(\mu_f)}P_1(z)d\rho dz\right\} \\
d\sigma_1^{NL3} &= f_1(\rho_0)\alpha_s\left|\mathcal{M}_{S_{+1}}\right|^2 d\Phi_1^{\mathrm{ME}}\left\{K\frac{\alpha_s(\rho_1)}{\alpha_s}\frac{f_1(\rho_1)}{f_1(\rho_0)}\frac{f_0(\rho_0)}{f_0(\rho_1)}\right. \\
&\quad \Pi_{S_{+0}}(\rho_0,\rho_1)\Pi_{S_{+1}}(\rho_1,\rho_{\mathrm{MS}}) \\
&\qquad - 1 - K\big|_{\alpha_s} - \frac{\alpha_s}{4\pi}\beta_0\ln\left(\frac{\rho_1}{\mu_r}\right) - \frac{f_1(\rho_1)}{f_1(\rho_0)}\bigg|_{\alpha_s} - \frac{f_0(\rho_0)}{f_0(\rho_1)}\bigg|_{\alpha_s} \\
&\qquad \left. + \frac{\alpha_s}{2\pi}\int_{\rho_1}^{\rho_0}\frac{f_1(\mu_f)}{f_0(\mu_f)}P_1(z)d\rho dz + \frac{\alpha_s}{2\pi}\int_{\rho_{\mathrm{MS}}}^{\rho_1}\frac{f_2(\mu_f)}{f_1(\mu_f)}P_2(z)d\rho dz\right\}
\end{aligned}
$$

Weight of tree-level samples

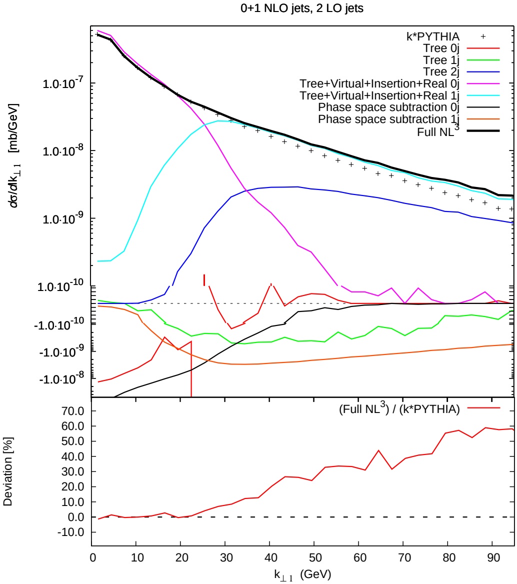
$$
\begin{aligned}
d\sigma_0^{NL3} &= f_0(\rho_0)\left|\mathcal{M}_{S_{+0}}\right|^2 d\Phi_0^{\mathrm{ME}} \Bigg\{ K \Pi_{S_{+n}}(\rho_0, \rho_{\mathrm{MS}}) \\
&\qquad - 1 - K\big|_{\alpha_s} + \frac{\alpha_s}{2\pi}\int_{\rho_{\mathrm{MS}}} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho dz \Bigg\} \\
d\sigma_1^{NL3} &= f_1(\rho_0)\alpha_s \left|\mathcal{M}_{S_{+1}}\right|^2 d\Phi_1^{\mathrm{ME}} \Bigg\{ K \frac{\alpha_s(\rho_1)}{\alpha_s} \frac{f_1(\rho_1)}{f_1(\rho_0)} \frac{f_0(\rho_0)}{f_0(\rho_1)} \\
&\quad \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho_{\mathrm{MS}}) \\
&\qquad - 1 - K\big|_{\alpha_s} - \frac{\alpha_s}{4\pi}\beta_0 \ln\left(\frac{\rho_1}{\mu_r}\right) - \frac{f_1(\rho_1)}{f_1(\rho_0)}\bigg|_{\alpha_s} - \frac{f_0(\rho_0)}{f_0(\rho_1)}\bigg|_{\alpha_s} \\
&\qquad + \frac{\alpha_s}{2\pi}\int_{\rho_1}^{\rho_0} \frac{f_1(\mu_f)}{f_0(\mu_f)} P_1(z) d\rho dz + \frac{\alpha_s}{2\pi}\int_{\rho_{\mathrm{MS}}}^{\rho_1} \frac{f_2(\mu_f)}{f_1(\mu_f)} P_2(z) d\rho dz \Bigg\} \\
d\sigma_2^{NL3} &= f_2(\rho_0)\alpha_s^2 \left|\mathcal{M}_{S_{+2}}\right|^2 d\Phi_2^{\mathrm{ME}} K \frac{\alpha_s(\rho_1)}{\alpha_s} \frac{\alpha_s(\rho_2)}{\alpha_s} \frac{f_0(\rho_0)}{f_0(\rho_1)} \frac{f_1(\rho_1)}{f_1(\rho_2)} \frac{f_2(\rho_2)}{f_2(\rho_0)} \\
&\quad \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho_2) \Pi_{S_{+2}}(\rho_2, \rho_{\mathrm{MS}})
\end{aligned}
$$

Implementation

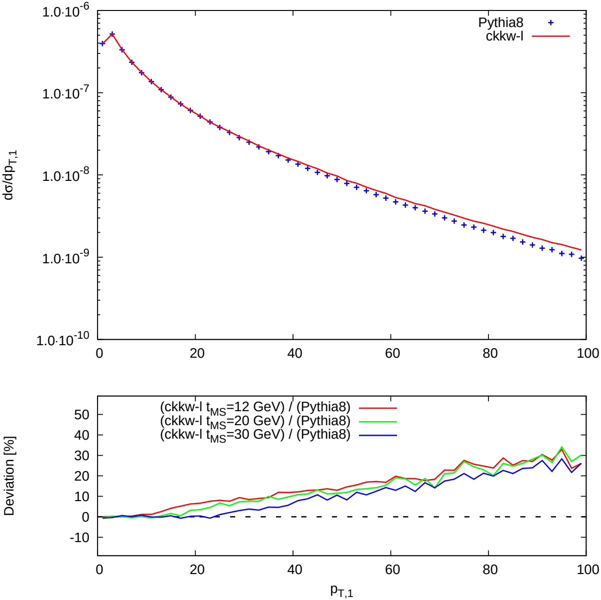
To implement this scheme, we need to know how to generate the weights needed for the tree-level samples:

K	: Calculate the (fixed) K-factor beforehand by dividing the cross sections.
$\frac{\alpha_s}{4\pi}\beta_0 \ln(\frac{\rho_1}{\mu_r})$	: Easily calculated by evaluating the logarithm.
$\frac{f_i(\rho_j)}{f_i(\rho_k)}\Big\vert_{\alpha_s}$	: Evolve $f_i(\rho_j)$ to ρ_k according to (integrated) DGLAP equation, then use numerical integration to calculate integral.
$\frac{\alpha_s}{2\pi}\int_{\rho_i}^{\rho_{i-1}} \frac{f_1(\mu_f)}{f_0(\mu_f)} P(z) d\rho dz$	: Generated by counting the PS emissions between ρ_{i-1} and ρ_i, generated with fixed α_s and fixed PDF scales μ_f.

. . . and that's what it looks like:



Merging scale variation for 2-jet CKKW-L merging.



$k_\perp$ dependence $\bar{\mathrm{B}}$ for $W+$ jet.