

Enhancement of v_2 in presence of magnetic field in relativistic heavy ion collisions

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Similarities in the nature of density fluctuations in early universe and the plasma in relativistic heavy ion collisions:

Superhorizon fluctuations

Mishra, Mohapatra, Saumia, Srivastava,
PRC 77, 064902 (2008)

In universe: Inflation

In RHICE?

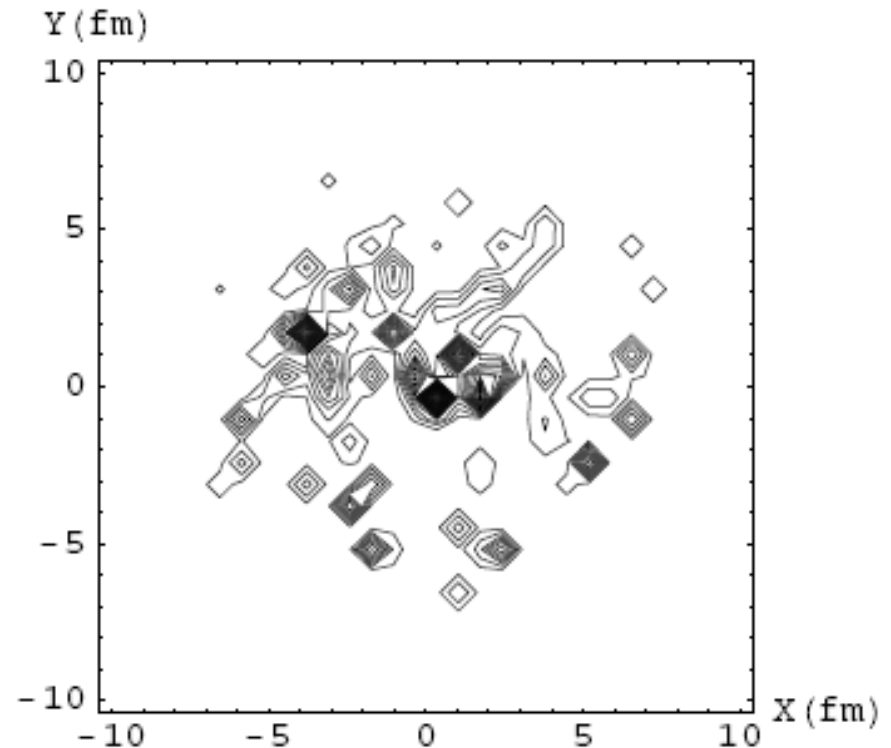
Consider central collision

Initial transverse energy
density fluctuations at 1fm

From HIJING

Nucleon size ~ 1.6 fm

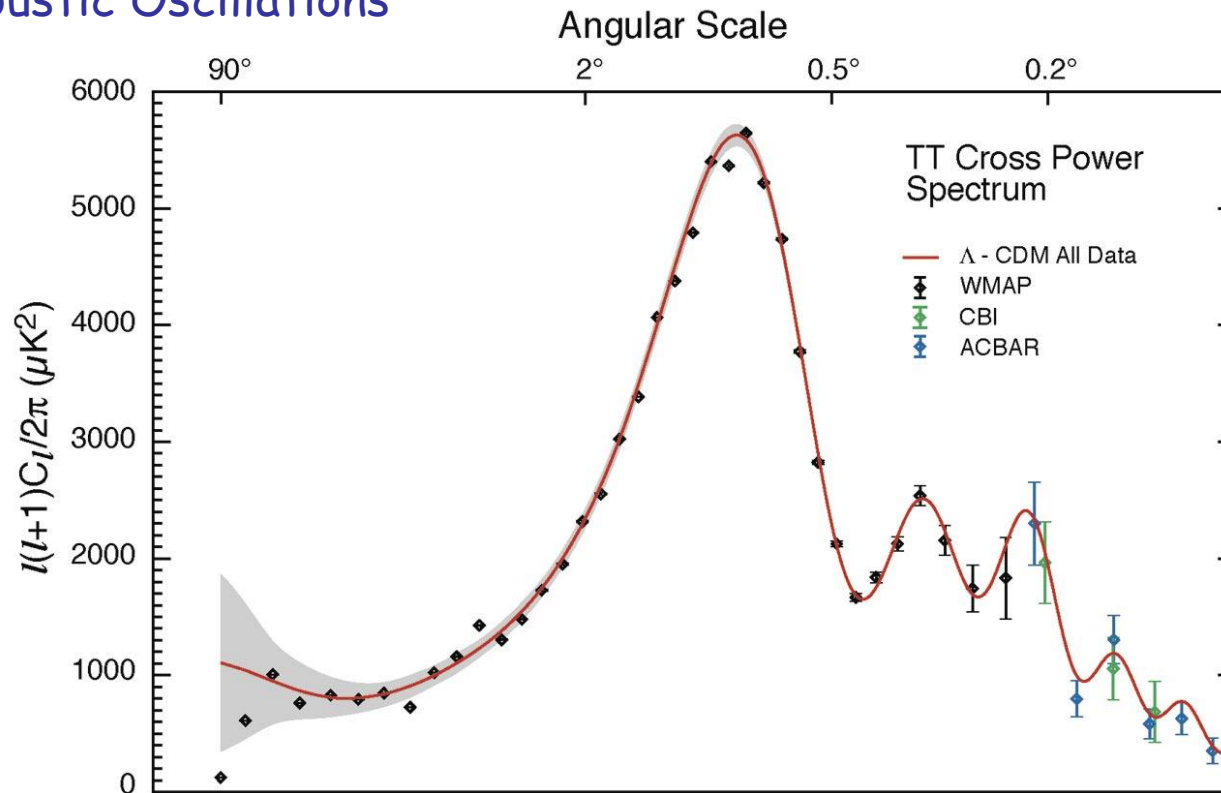
horizon at thermalization ~ 1 fm



Central Au - Au Collision

C M Energy 200 GeV

CMR Acoustic Oscillations



Plot shows variance of various spherical harmonic components Y_{lm} as a function of l .

Similar effects in heavy ion collisions?

CMBR temperature anisotropies analyzed using Spherical Harmonics

$$\frac{\Delta T}{T}(\theta, \phi) = a_{lm} Y_{lm}(\theta, \phi)$$

Average values of these expansions coefficients are zero due to overall isotropy of the universe

$$\langle a_{lm} \rangle = 0$$

Their standard deviations are non-zero and contain important information.

$$C_l = \langle |a_{lm}|^2 \rangle$$

Apply same technique for RHICE also

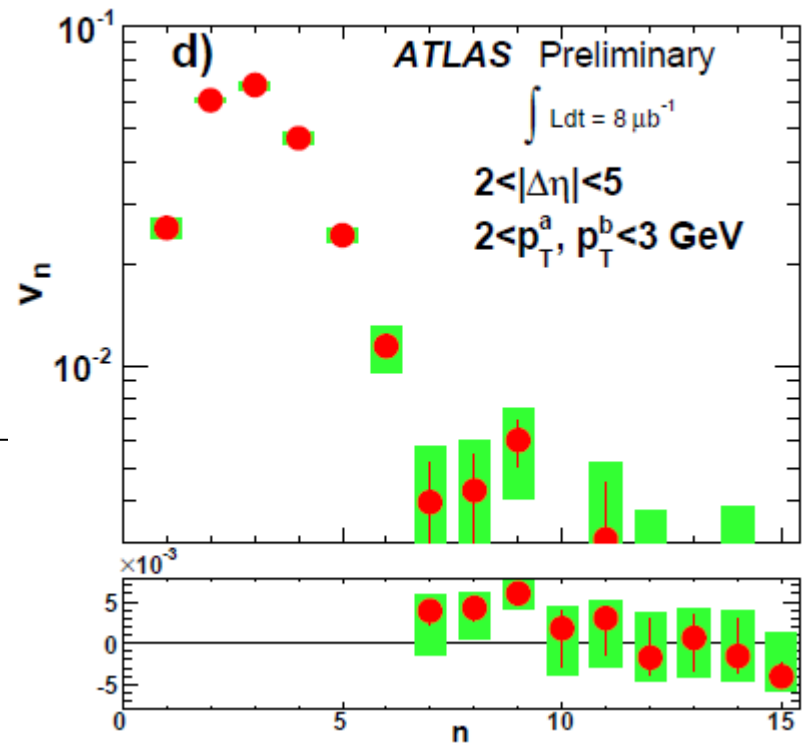
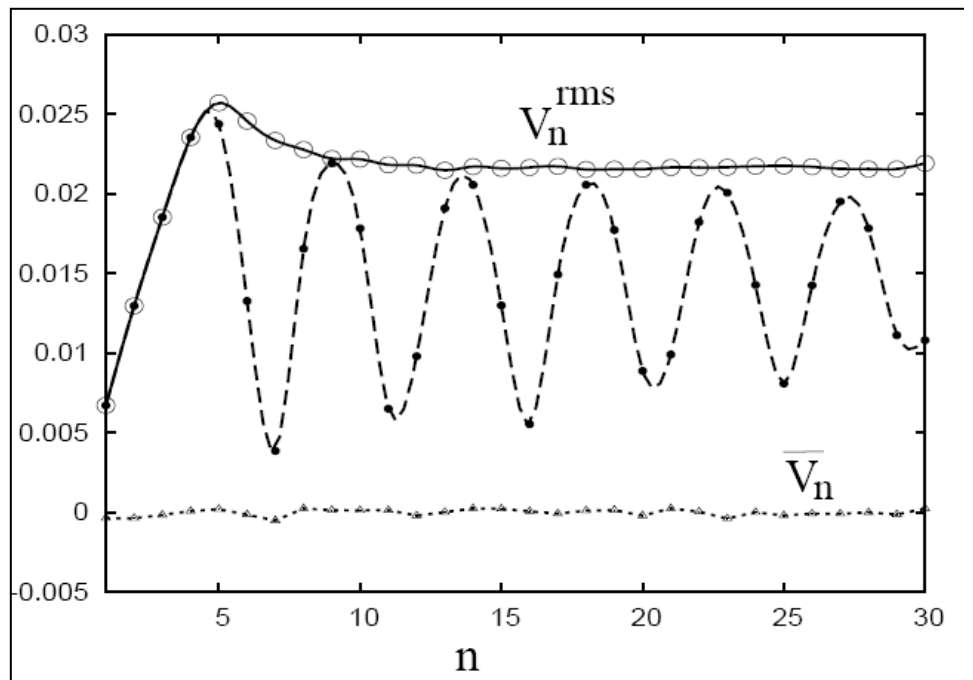
The Analysis:

Calculate the momentum anisotropies in different azimuthal bins in a fixed co-ordinate system.

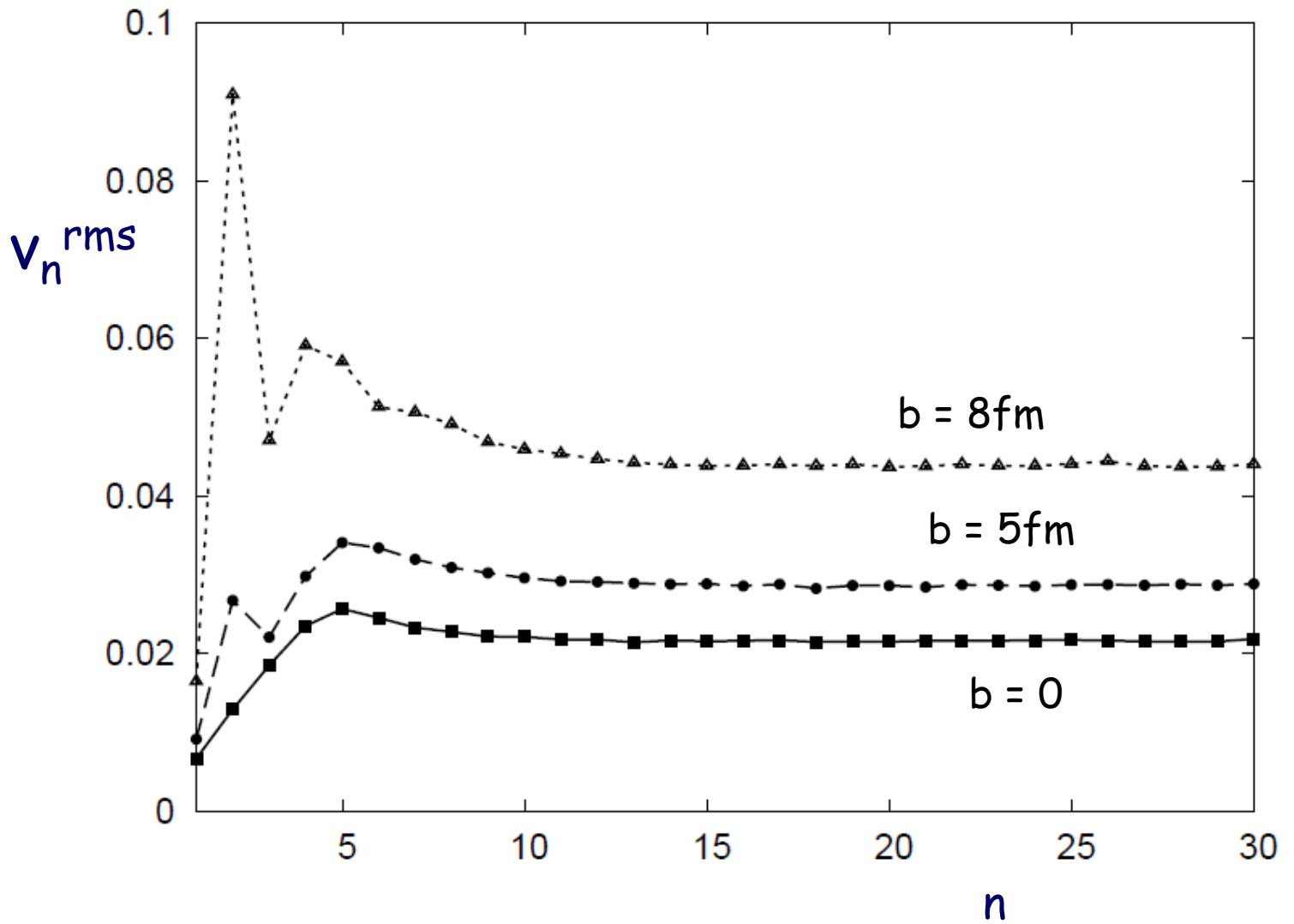
Calculate the Fourier coefficients of the momentum anisotropies.

Find the root mean square values which will give different flow coefficients.

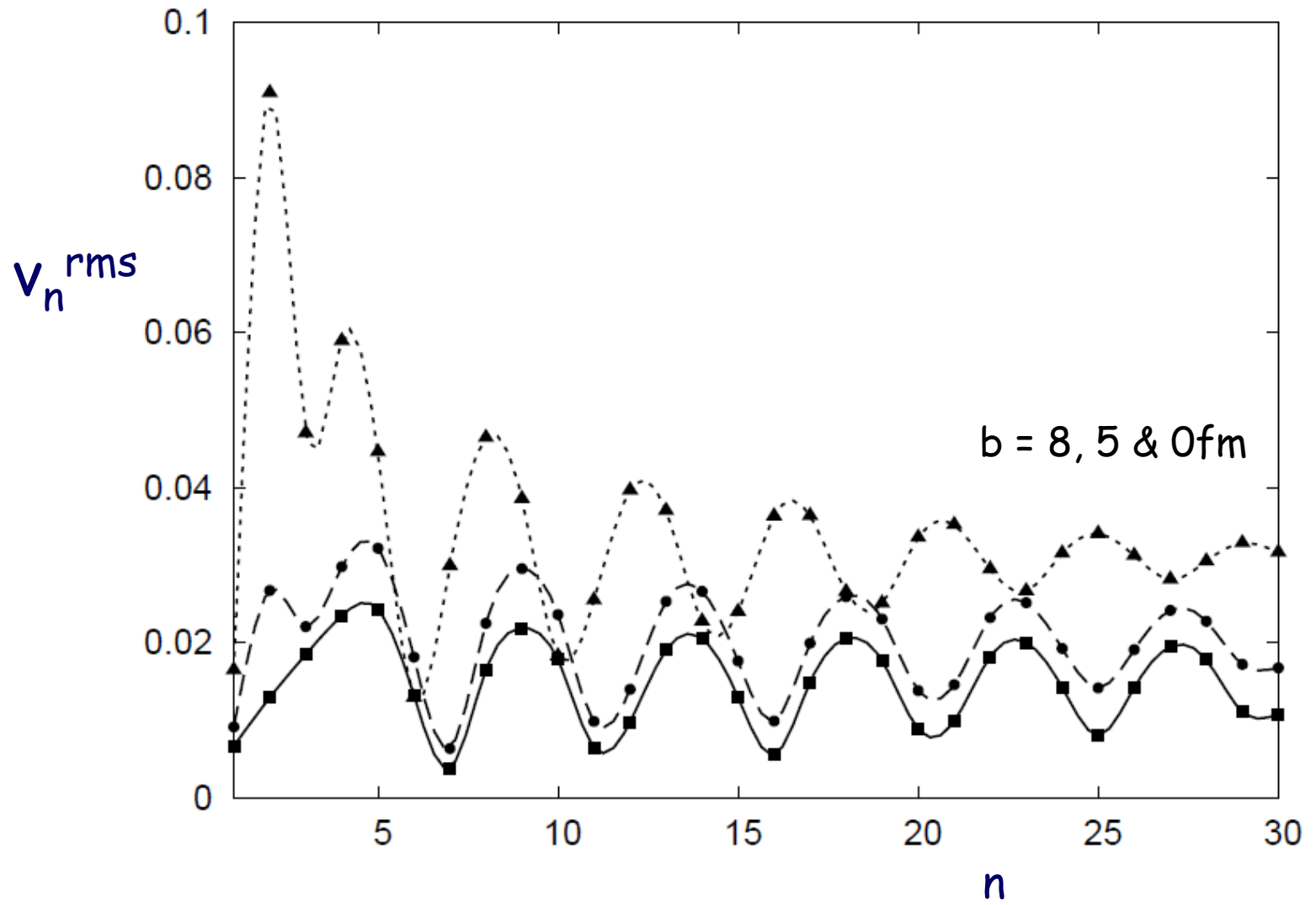
$$V_n^{rms} = \sqrt{\langle V_n^2 \rangle}$$



Plot taken from T. Hirano's talk.



Mishra, Mohapatra, Saumia, Srivastava, PRC
81,034903 (2010)



Effect of Magnetic Field

On acoustic peaks of CMBR:

Primordial magnetic fields are present in the universe.

Plasma will evolve according to the equations of magnetohydrodynamics.

In presence of magnetic field, there are three types of waves in the plasma in place of ordinary sound waves.

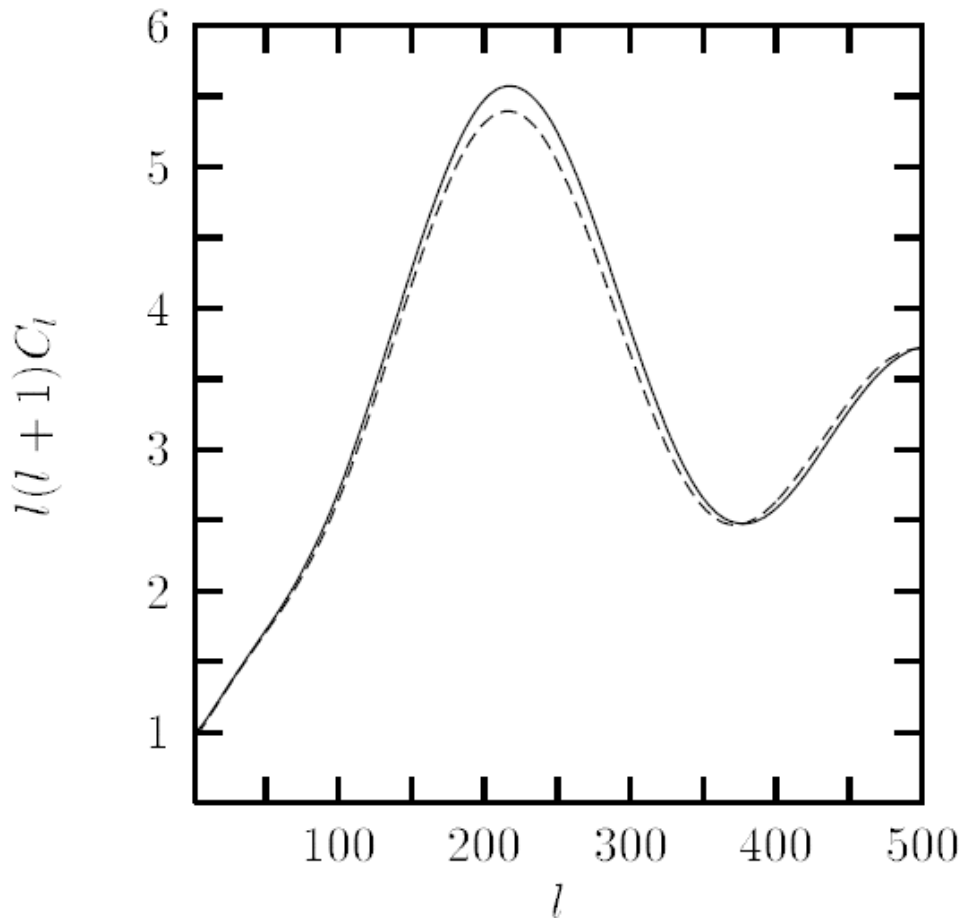
Fast magnetosonic waves: Generalised sound waves with significant contributions from the magnetic pressure.

Their velocity is given by $c^2_+ \sim c^2_s + v^2_A \sin^2\theta$ where θ is the angle between the magnetic field B_0 and the wave vector and the Alfvén velocity $v_A = B_0 / \sqrt{4\pi\rho}$

Slow magnetoacoustic waves: Sound waves with strong magnetic guidance. $c^2_- = v^2_A \cos^2\theta$

Alfvén waves: Propagation of magnetic field perturbations.

The magnetic field effects distort the CMBR acoustic peaks. The distortion can be seen as an effect due to the modified sound velocity (fast magnetosonic waves) with some modulation from slow magnetosonic waves.



Jenni Adams et al.
(1996)

Strong magnetic field $B \sim 10^{15}$ Tesla is expected to be present in the early stages of non-central collisions.

How long this magnetic field remains strong?

The magnetic field arises from the valence charges of the initial nuclei.

It peaks to the maximum value just at the passing time of the nuclei ~ 0.2 fm for RHIC energies and decays as τ^{-2} .

The effect of magnetic field on flow will be negligible in this case, since flow develops in several fm.

At LHC, the magnetic field formed is expected to be more than an order of magnitude larger than that at RHIC.

But still, if it decays rapidly that flow may not be affected significantly.

A full MHD simulation is required to study this in detail.

We know that the equilibrated matter is formed within 0.6 fm when the magnetic field is near its maximum.

That means the magnetic field is rapidly decaying **in the plasma**.

It will induce circular currents in the plasma which produce an induced magnetic field in the direction of the external field.

This slows down the relaxation of the external field.

For QGP, from lattice simulations, $\sigma \cong 0.3 T$.

For $T = 200 \text{ MeV}$, $\sigma = 0.3 \text{ fm}^{-1}$

The relaxation time is given by $\tau \cong R^2 \sigma / 4$.

R is the size of the plasma. **For $R = 5\text{-}6 \text{ fm}$, $\tau = 2\text{-}3 \text{ fm}$.**

K.Tuchin, Phys. Rev. **C 82**, 034904 (2010)

In quasi-static limit (field variation time scale large for the System length/time scales), one can neglect the Maxwell's Displacement current. Then (Tuchin, PRC 82, 034904 (2010)) :

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \vec{\nabla} \cdot \vec{B} = 0, \vec{\nabla} \times \vec{H} = \vec{J}, \vec{J} = \sigma \vec{E}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = -\sigma \frac{\partial \vec{B}}{\partial t} \Rightarrow \nabla^2 \vec{B} = \mu\sigma \frac{\partial \vec{B}}{\partial t} \quad \text{Diffusion equation}$$

Green's function for the diffusion eqn. (with $\mu = 1$):

$$G(\vec{r}, t) = \frac{1}{(4\pi t / \sigma)^{3/2}} \exp\left[-\frac{\vec{r}^2}{4t / \sigma}\right]$$

Using this, we can calculate the time evolution for B for a given initial profile of B. Initial profile of B at $t = t_0$ is taken as:

$$B(\vec{r}, t) = \vec{B}_0 \exp\left[-\frac{\rho^2}{R^2}\right]$$

ρ is the transverse coordinate, and $R \sim 5-6$ fm the system size

The solution for the magnetic field is found to be:

$$B(\vec{r}, t) = \vec{B}_0 \frac{R^2}{R^2 + 4(t - t_0) / \sigma} \exp\left[-\frac{\rho^2}{R^2 + 4(t - t_0) / \sigma}\right]$$

Note: for time up to about $\tau = \frac{R^2 \sigma}{4}$, the magnetic field is essentially constant.

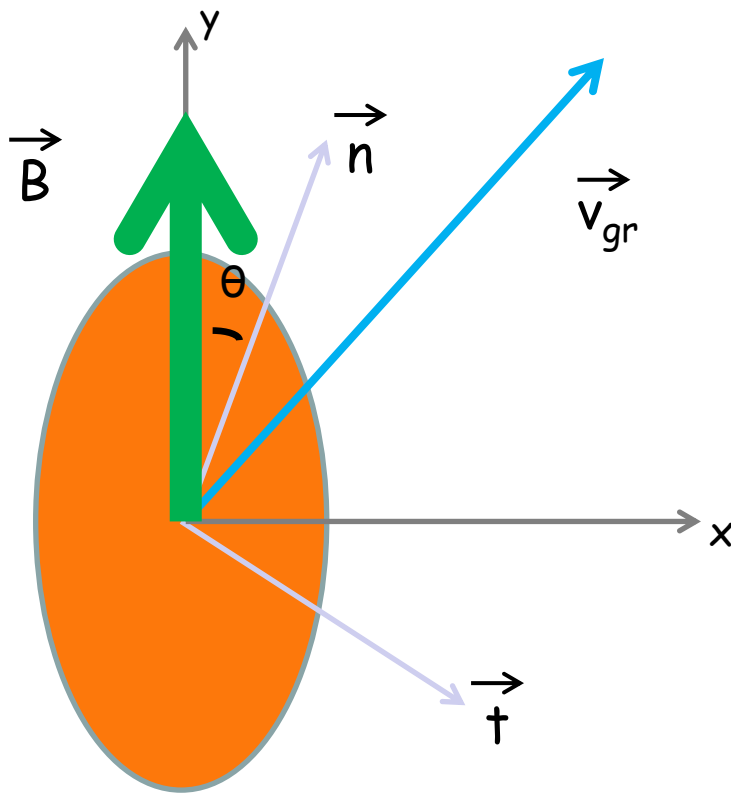
Lattice data for $\sigma = 0.3$ /fm for $T \sim 200$ MeV, (S. Gupta, PLB 597, 57 (2004), H.-T. Ding et al. arXiv:1012.4963)

With $R \sim 5 - 6$ fm, we get $t \sim 2 - 3$ fm

This is the same time scale during which substantial flow develops.

Thus, for this time scale, reasonable to take constant B with peak magnitude of about 10^{15} Tesla.

This may lead to strong variations in the velocities of different types of waves in the plasma.



The velocity varies with the angle between the wave vector and the direction of the magnetic field.

This may affect the development of anisotropic flow.

Mohapatra, Saumia, Srivastava, Mod. Phys. Lett. A26 2477 (2011)

The expression for group velocity in relativistic MHD

$$\mathbf{v}_{gr} = v_{ph} \left[\mathbf{n} + \mathbf{t} \frac{\sigma \pm 2\delta(a \mp (1 + \delta \cos^2 \theta)) \sin \theta \cos \theta}{2(1 + \delta \cos^2 \theta \pm a)a} \right]$$

$$v_{ph} = \mathbf{n} \frac{\left[\left(\frac{4\rho}{3\omega} \right) c_s^2 + v_A^2 \right]^{1/2}}{2} (1 + \delta \cos^2 \theta \pm a)^{1/2}$$

$$\mathbf{n} = \mathbf{k}/k, \quad \mathbf{t} = \left[\left(\frac{\mathbf{B}}{B} \right) \times \mathbf{n} \right] \times \mathbf{n}$$

$$a^2 = (1 + \delta \cos^2 \theta) - \sigma \cos^2 \theta,$$

$$\delta = \frac{c_s^2 v_A^2}{\left(\frac{4\rho}{3\omega} \right) c_s^2 + v_A^2}, \quad \sigma = \frac{4 c_s^2 v_A^2}{\left[\left(\frac{4\rho}{3\omega} \right) c_s^2 + v_A^2 \right]^2}, \quad \omega = \left(\frac{4\rho}{3} \right) + B^2$$

Note that the direction of group velocity depends on the coefficient of \mathbf{t} above, which is a function of the local energy density, pressure etc. *i.e.*, the direction of group velocity will vary in the radial direction also and may result in complex flow patterns.

We simulate the effect of magnetic field on v_2 as follows:

Generate initial density fluctuations from HIJING

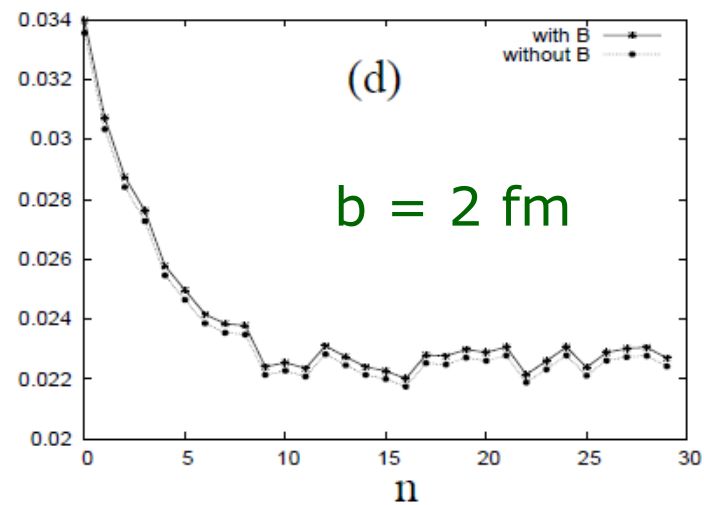
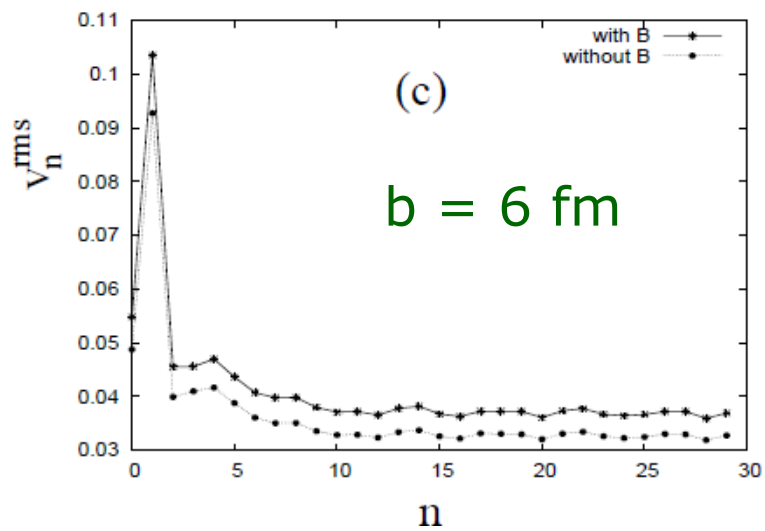
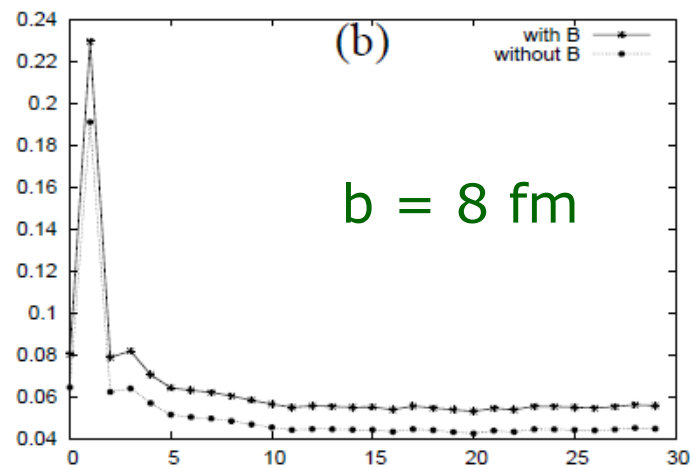
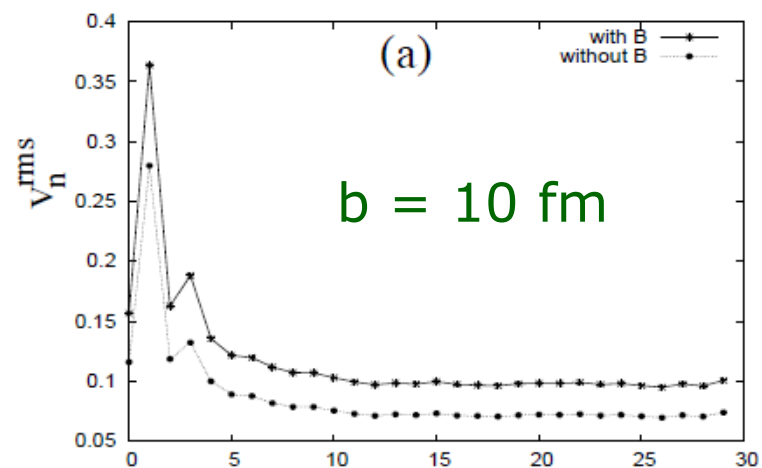
Calculate F_n (the Fourier coefficients of spatial anisotropy)

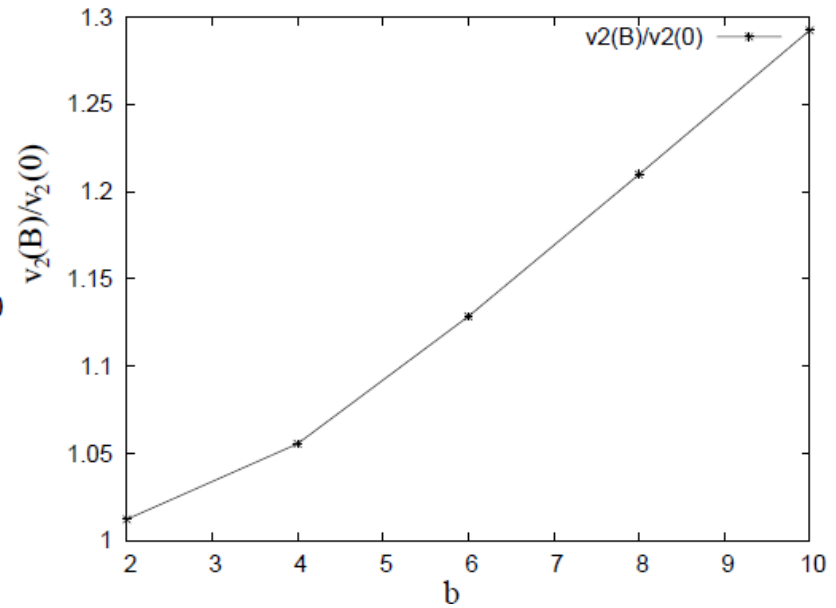
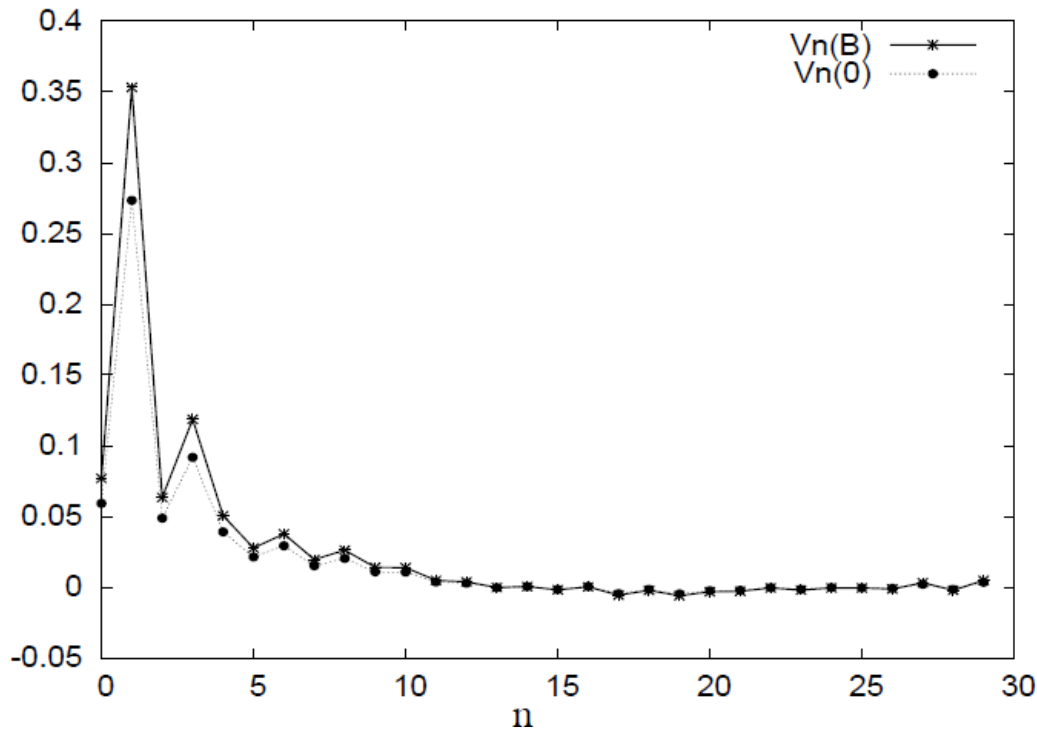
We know, for elliptic flow, $\frac{v_2}{\varepsilon} \propto CS$

v_2 is calculated from F_2 but now the proportionality factor is proportional to **group velocity** ($0.346 \times v_{gr}$ from Bhalerao et. al.) which **is a function of the azimuthal angle**.

Note: We are assuming a constant magnetic field here.

Results





We see that v_2 is larger with magnetic field.

If we include the effect of magnetic field in flow, a larger η/s value can be accommodated.

An important message:

The inflationary density fluctuations in the universe and the initial fluctuations in relativistic heavy-ion collisions are similar.

The evolution of inflationary density fluctuations and their signatures in *CMB* are very well studied.

One can adapt the physics and techniques used in *CMB* physics appropriately to study the fluctuations in *RHICE*.

Eg: Recently we have proposed that a shape analysis of the *CMB* temperature fluctuations (superhorizon) could be used to probe anisotropic expansion history of the universe. We have also shown that the same technique could probe anisotropic expansion of QGP in non-central collisions, and could be used as an alternative technique to determine the reaction plane.

Summary and Conclusions:

- Due to induced currents in the plasma, the initial magnetic field can survive up to times as large as 2-3 fm in RHICE.
- This means that relativistic hydrodynamics is incomplete in understanding flow; one should use relativistic magnetohydrodynamics instead.
- This will affect the flow anisotropies; elliptic flow will be enhanced.
- This implies that the viscosity of the plasma extracted from experimental data using hydro might be underestimated.

Thank You