Renewed look at η' in medium

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contents based on :
Y. Kwon, S.H. Lee, K. Morita, & Gy. Wolf [Phys.Rev. D **86** (2012) 034014]

Goal: Theoretical explanation of η' mass at finite temperature

Motivation

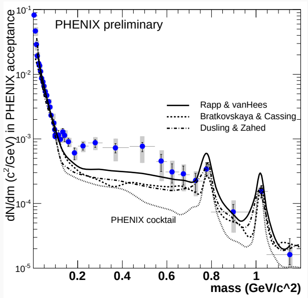
- $U_A(1)$ anomaly relation:

$$\partial_\mu J_5^\mu = \frac{2N_f}{N_c}\frac{g^2}{16\pi^2}G\tilde{G}$$

  - pseudo Goldstone boson ($m_{\eta'} \sim 1\,\mathrm{GeV}$)

- Dilepton data at RHIC: unexplained
  - η' mass reduction required:
    $\Rightarrow$ Effective $U_A(1)$ restoration

    Csörgo, Vertesi, & Sziklai [Phys.Rev.Lett. 105 (2010) 182301]



Strategy

- Chiral symmetry breaking: chiral order parameter
  - Banks-Casher formula
    Banks & Casher [Nucl.Phys. B 169 (1980) 103]
  - in correlation functions
    Cohen [Phys.Rev. D 54 (1996) 1867], Lee & Hatsuda [Phys.Rev. D 54 (1996) 1871]
- In-medium η' mass relation:
  - Witten-Veneziano formula
    Witten [Nucl.Phys. B156 (1979) 269], Veneziano [Nucl.Phys. B 159 (1979) 213]
  - generalization to finite temperature
- Coupling to η': $\langle 0|G\tilde{G}|\eta'\rangle$
- Qualitative estimation of η' mass at finite temperature

Banks-Casher formula

Banks & Casher [Nucl.Phys. B 169 (1980) 103]

- Quark condensate:

$$\langle \bar{q}q \rangle = \frac{-1}{Z} \int \mathcal{D}[A]\, e^{-S_{\rm YM}} \mathrm{Det}\,[\not{D} + m]\, \mathrm{Tr}[S(0,0)] + \cdots$$

$$\mathrm{Tr}[S(0,0)] = \mathrm{Tr}\left[\left\langle 0 \left| \frac{1}{\not{D} + m} \right| 0 \right\rangle\right] = \int \mathrm{d}\lambda\, \rho(\lambda) \frac{2m}{\lambda^2 + m^2} \xrightarrow{m \to 0} \pi \rho(\lambda = 0)$$

- Banks-Casher formula in chiral limit:

$$\langle \bar{q}q \rangle = -\pi \langle \rho(\lambda = 0) \rangle$$

$\sigma - \pi$ correlator

$$\frac{1}{V}\int \mathrm{d}^4x \,[\langle \bar{q}(x)q(x),\ \bar{q}(0)q(0)\rangle - \langle \bar{q}(x)\tau^a i\gamma_5 q(x),\ \bar{q}(0)\tau^a i\gamma_5 q(0)\rangle]$$

$$= \quad -\mathrm{Tr}\,[S(x,0)S(0,x)] \quad + \quad \mathrm{Tr}\,[\tau^a i\gamma_5 S(x,0)\tau^a i\gamma_5 S(0,x)]$$

$$+\,\langle \mathrm{Tr}\,[S(x,x)]\,\mathrm{Tr}\,[S(0,0)]\rangle$$

- Generalized Banks-Casher formula in the chiral limit:

$$= c(\pi\rho(\lambda = 0))^2$$

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$\eta' - \pi$ correlator

$$\frac{1}{V}\int d^4x\,[\langle \bar{q}(x) i\gamma_5 q(x),\, \bar{q}(0) i\gamma_5 q(0)\rangle - \langle \bar{q}(x)\tau^a i\gamma_5 q(x),\, \bar{q}(0)\tau^a i\gamma_5 q(0)\rangle]$$

$$= \quad -\mathrm{Tr}\,[i\gamma_5 S(x,0) i\gamma_5 S(0,x)] \quad + \quad \mathrm{Tr}\,[\tau^a i\gamma_5 S(x,0)\tau^a i\gamma_5 S(0,x)]$$

$$+\langle \mathrm{Tr}\,[i\gamma_5 S(x,x)]\,\mathrm{Tr}\,[i\gamma_5 S(0,0)]\rangle$$

$$+\frac{1}{V}\int d^4x\,\left\langle \bar{u}_0(x) d_0(0)\bar{d}_0(0) u_0(x)\int d^4y\,\bar{s}_0(y) m_s s_0(y) + \text{perm.}\right\rangle$$

$$< \quad (\pi\rho(\lambda = 0))^2 + [\text{const. for SU(2)}] \times \prod_{q>3} m_q \langle \bar{q}q\rangle$$

Cohen [Phys.Rev. D 54 (1996) 1867], Lee & Hatsuda [Phys.Rev. D 54 (1996) 1871]

◎ $U_A(1)$ symmetry will be effectively restored up to quark mass terms in SU(3).

Witten-Veneziano formula

Witten [Nucl.Phys. B 156 (1979) 269], Veneziano [Nucl.Phys. B 159 (1979) 213]

$$P(k) = -i \int \mathrm{d}x\, e^{ikx} \left\langle G\tilde{G}(x)\, G\tilde{G}(0) \right\rangle$$

- Contirbutions of pure glue: $P_0(k=0) \neq 0$ from low energy theorem.
- Adding massless quarks,

$$P(k) = -i \int \mathrm{d}x\, e^{ikx} \left\langle \partial^\mu j^5_\mu(x),\, \partial^\nu j^5_\nu(0) \right\rangle \propto k^\mu k^\nu P_{\mu\nu} \xrightarrow{k=0} 0$$

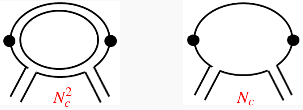
- Large N_c argument:

$$P(k) = \sum_{glueballs} \frac{\left\langle 0 \,|\, G\tilde{G} \,|\, glueball \right\rangle^2}{k^2 - m^2} + \sum_{mesons} \frac{\left\langle 0 \,|\, G\tilde{G} \,|\, meson \right\rangle^2}{k^2 - m^2}$$

- Need η'-meson with $m^2_{\eta'} \equiv \mathcal{O}\left(\frac{1}{N_c}\right)$: $P(k=0) = P_0(0) - \frac{\left\langle 0 \,|\, G\tilde{G} \,|\, \eta' \right\rangle^2}{m^2_{\eta'}} = 0$

Witten-Veneziano formula in medium

- Thermal gluon or quark interactions:



According to N_c counting, the same holds as in the vacuum.

$$P_0(0) = \frac{\left\langle 0 \,|\, G\tilde{G} \,|\, \eta' \right\rangle^2}{m_{\eta'}^{*2}}$$

Low energy theorem

- Low energy theorem

(Novikov, Shifman, Vainstein, Zharkarov), (Ellis, Kapusta, Tang)

$$\frac{d}{d\left(-1/4g_0^2\right)}\langle Op\rangle = -i\int dx\, e^{ikx}\left\langle Op(x), g_0^2 GG(0)\right\rangle$$

$$\langle Op\rangle_T = const\left[M_0 \exp\left(-\frac{8\pi^2}{bg_0^2}\right)\right]^d + c'T^d$$

$$\frac{d}{d\left(-1/4g_0^2\right)}\langle Op\rangle_T = \frac{32\pi^2}{b}\left(d - T\frac{\partial}{\partial T}\right)\langle Op\rangle_T$$

- Low energy theorem at finite temperature:

Lee & Zahed [Phys.Rev. C 63 (2001) 045204]

$$P_0(k=0) = \left(\frac{4\pi}{3\alpha}\right)^2 \frac{2}{11}\left(4 - T\frac{\partial}{\partial T}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle$$

$$\frac{\left\langle 0\,|\,G\tilde{G}\,|\,\eta'\right\rangle^2}{m_{\eta'}^{*2}} = \left(\frac{4\pi}{3\alpha}\right)^2 \frac{2}{11}\left(4 - T\frac{\partial}{\partial T}\right)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle$$

Coupling to η'

$$P(k) = \int d^4x\, e^{ikx} \left\langle G\tilde{G}(x)\, G\tilde{G}(0) \right\rangle$$

$$= \frac{k^4 \left\langle 0 \,|\, G\tilde{G} \,|\, \eta' \right\rangle^2}{k^2 - m_{\eta'}^2} + \cdots$$

$$= k^\mu k^\nu \int d^4x\, e^{ikx} \left(\frac{2\pi N}{\alpha N_f}\right)^2 \left[\langle \bar{q}(x) i\gamma_\mu\gamma_5 q(x),\, \bar{q}(0) i\gamma_\nu\gamma_5 q(0)\rangle - \langle \bar{q}(x) i\gamma_\mu q(x),\, \bar{q}(0) i\gamma_\nu q(0)\rangle\right]$$

$$\xrightarrow{\text{chiral symmetry restored phase}} 0$$

- Therefore, $\left\langle 0 \,|\, G\tilde{G} \,|\, \eta' \right\rangle \to 0$

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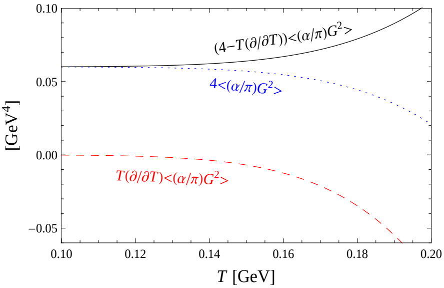
η' mass at finite temperature

$$\frac{\left\langle 0 \,|\, G\tilde{G} \,|\, \eta' \right\rangle^2}{m_{\eta'}^{*2}} = \left(\frac{4\pi}{3\alpha}\right)^2 \frac{2}{11} \left(4 - T\frac{\partial}{\partial T}\right) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle$$

◎ T-dependence of gluon condensate:



Summary

- Witten-Veneziano formula, together with low energy theorem, makes a relation of $m_{\eta'}$ with the gluon condensate.

- η' mass is related to quark condensate and thus should reduce in medium.
  - could serve as signature of chiral symmetry restoration
  - dilepton in heavy ion collision
  - measurements from nuclear targets

Thank you for your attention!