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Application of AdS/CFT potential on the meson mass using two-body Dirac equations

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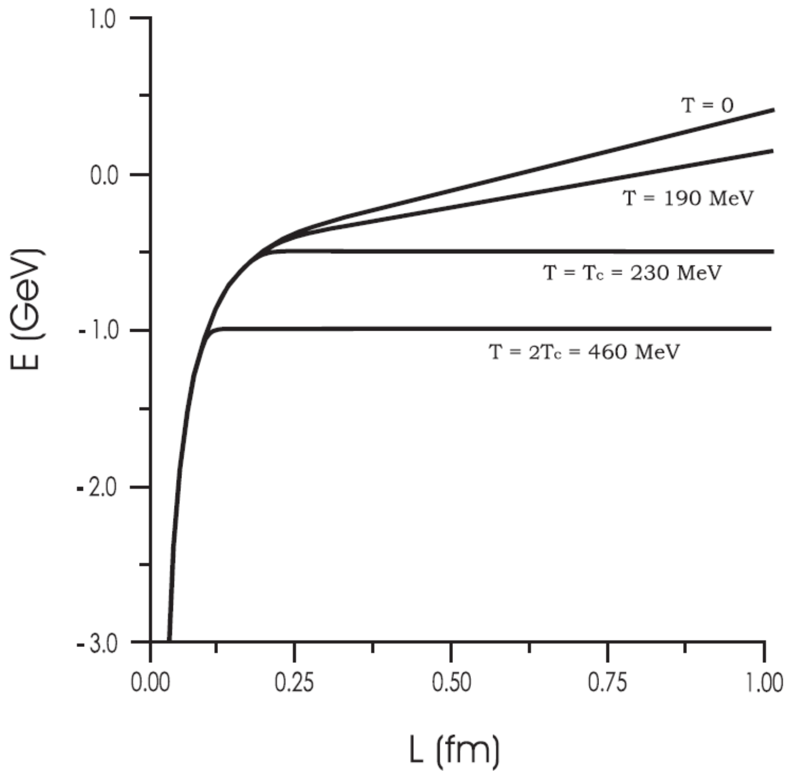
Introduction

- In heavy ion collisions
  - Quark-Gluon-Plasma
  - Dissociation temperature of quarkonia
- For Anti-de Sitter space-time
  - The potential in a function of the temperature
  - [Boschi-Filho, Braga & Ferreira, PRD 74, 086001 (2006)]



Introduction

- The final target
  - the thermal properties of QGP by using AdS/CFT correspondence

- In this stage
  - Check validity of the AdS/CFT potential for quarkonia systems
  - Implied 2-body Dirac-eq. to find mass spectrum of $Q\bar{Q}$ systems

Two-body Constraint Dynamics

- 2 free spinless particles with the mass-shell constraint
  - $H_i = {p_i}^2 + {m_i}^2 \approx 0$
  - m depends only on the space separation perpendicular to the total 4-momentum
- 2 free spin-1/2 particles with the generalized mass-shell constraint
  - $H_i = {p_i}^2 + {\widetilde{m}_i}^2 = {p_i}^2 + {m_i}^2 + \Phi_i(x, p_1, p_2) \approx 0$
  - potential depends on the space like separation only & the components of p perpendicular to the total 4-momentum

[P. van Alstine & H. Crater, J.Math.Phys.23(9), 1982]

Two-body Constraint Dynamics

- **Pauli reduction**[Salpeter, PR 87, 328 (1952)]

  + scale transformation

  [Schwinger, “Particles, Sources, and Fields”,Vol2,pp.348]
- **16-comp. Dirac Eq.**

  $\rightarrow$ **4-comp. rel. Schrödinger-type Eq.**
- $\boldsymbol{H = \frac{\varepsilon_1 H_1 + \varepsilon_2 H_2}{\omega} = p^2 + \Phi_\omega(\sigma_1, \sigma_2, p_\perp, A(r), S(r))}$

  $\boldsymbol{= b^2(\omega)}$
  - **where** $b^2(\omega) = \varepsilon_\omega^2 - m_\omega^2$ **with** $\varepsilon_\omega = \frac{\omega^2 - m_1^2 - m_2^2}{2\omega}$ **and** $m_\omega = \frac{m_1 m_2}{\omega}$

    [Crater, Becker, Wong & vanAlstine, PRD 46, 5117 (1992)]

Two-body Constraint Dynamics

- Φ_ω = central potentials + darwin + SO + SS + Tensor + etc.
  - $\Phi_\omega = 2m_\omega S + S^2 + 2\varepsilon_\omega A - A^2 + \Phi_D + \boldsymbol{L}\cdot(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\Phi_{SO} + \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\Phi_{SS} + (3\boldsymbol{\sigma}_1\cdot\hat{r}\boldsymbol{\sigma}_2\cdot\hat{r} - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\Phi_T + \boldsymbol{\sigma}_1\cdot\hat{r}\boldsymbol{\sigma}_2\cdot\hat{r}\boldsymbol{L}\cdot(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\Phi_{SOT} + \boldsymbol{L}\cdot(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\Phi_{SOD} + \mathrm{i}\boldsymbol{L}\cdot\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2\Phi_{SOX}$
- For singlet state with the same mass,
  - no spin-orbit, spin-orbit-tensor contribution
  - Darwin, spin-spin, tensor terms are altogether vanishes.
  - $H = p^2 + 2m_\omega S + S^2 + 2\varepsilon_\omega A - A^2$

[Crater, Yoon & Wong, PRD,79,034011(2009)]

Two-body Constraint Dynamics

- **For S-state,**
  - $(-\frac{d^2}{dr^2} + 2m_\omega S + S^2 + 2\varepsilon_\omega A - A^2 + \Phi_D - 3\Phi_{SS})v_0 = b^2 v_0$
- **For mixture of S & D-states,**
  - $(-\frac{d^2}{dr^2} + 2m_\omega S + S^2 + 2\varepsilon_\omega A - A^2 + \Phi_D + \Phi_{SS})u_+ + \frac{2\sqrt{2}}{3}(3\Phi_T - 6\Phi_{SOT})u_- = b^2 u_+$
  - $(-\frac{d^2}{dr^2} + \frac{6}{r^2} + 2m_\omega S + S^2 + 2\varepsilon_\omega A - A^2 + \Phi_D - 6\Phi_{SO} + \Phi_{SS} - 2\Phi_T + 2\Phi_{SOT})\, u_- + \frac{2\sqrt{2}}{3}(3\Phi_T)u_+ = b^2 u_-$

Advantage of 2-body Dirac constraints

- Already tested in e^+e^- binding system(QED)
  [Todorov, PRD 3, 2351 (1971)]
- Already tested in quark system(QCD)
  [Crater, Yoon & Wong, PRD 79, 034011 (2009)]
- 16-comp. dynamics →
  4-comp. dynamics
- Leads to simple Schrödinger-type equation.
- Particles interacts through scalar and vector interactions.
- Spin-dependence is determined naturally.

Comparing Two Types of Potentials

- Simple QCD based model

  [Crater, Yoon & Wong, PRD79, 034011 (2009)]

  - $V_{CYW}(r) = \frac{8\pi\Lambda^2 r}{27} - \frac{16\pi}{27r\ \ln(Ke^2 + B/(\Lambda r)^2}$

  - From Richardson Potential which satisfies asymptotic freedom

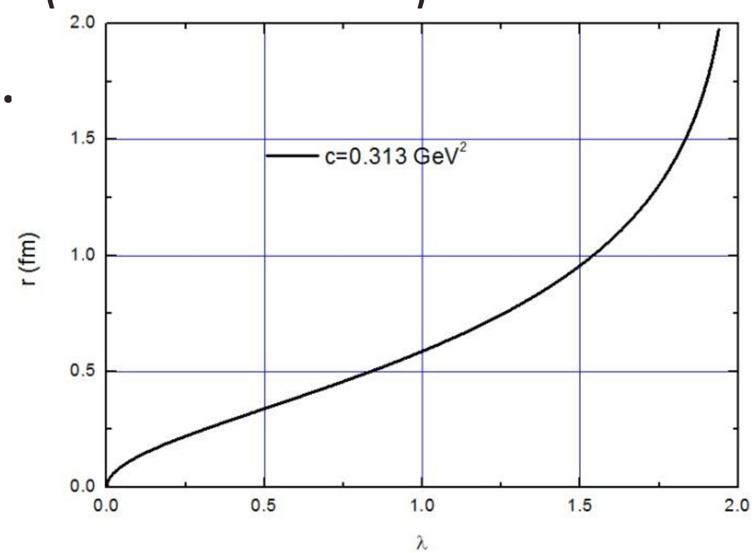
  - Λ, K, B are parameters.

Comparing Two Types of Potentials

- **AdS/CFT Potential**

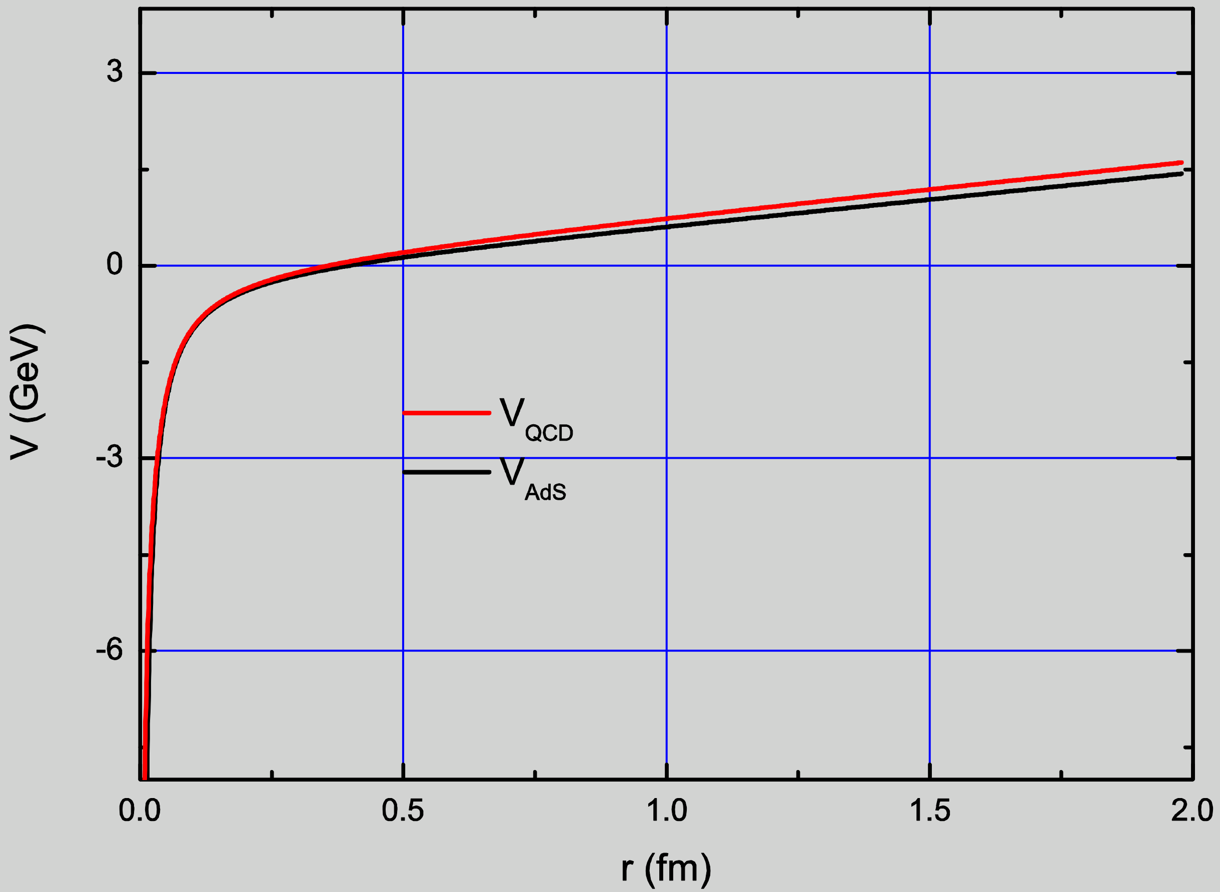
  [Andreev & Zakharov, PRD74, 025023(2006)]

  - $V_{AdS}(r) = \frac{4}{ca^2}\sqrt{\frac{c}{\lambda}}\left(-1 + \int_0^1 dv\, v^{-2} \times \left[e^{(1/2)\lambda v^2}\left(1 - v^4 e^{\lambda(1-v^2)}\right)^{-(1/2)} - 1\right]\right)$
  - where $r = 2\sqrt{\frac{\lambda}{c}}\int_0^1 dv\, v^2 e^{(1/2)\lambda(1-v^2)}\left(1 - v^4 e^{\lambda(1-v^2)}\right)^{-(1/2)}$
  - $a = 2.46\ \mathrm{GeV}^{-1}$ is string tension.
  - c is parameter



Comparing Two Types of Potentials

CYW Potential	AdS/CFT Potential

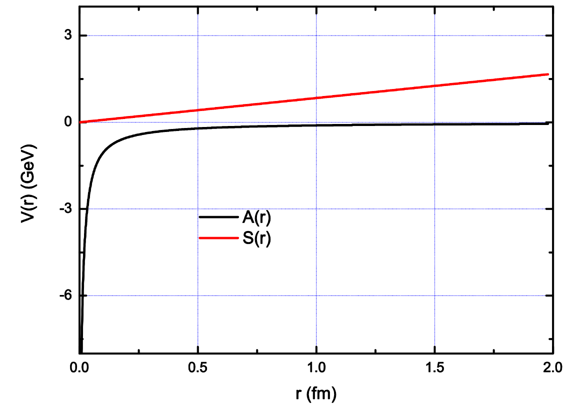


Comparing Two Type of Potentials

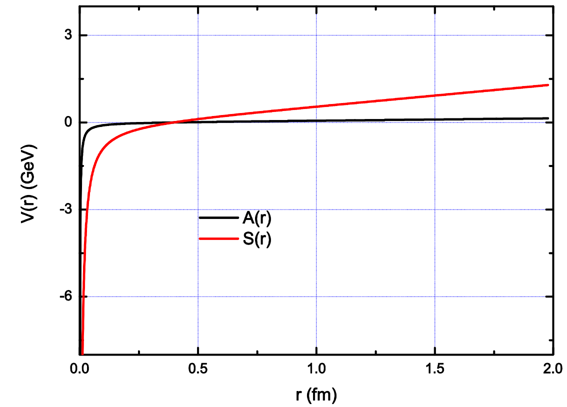
- $V_{CYW}(r) = \frac{8\pi\Lambda^2 r}{27} - \frac{16\pi}{27r\ln(Ke^2+B/(\Lambda r)^2}$

Scalar potential

Vector potential



- $V_{AdS}(r) =$
$$\frac{4}{ca^2}\sqrt{\frac{c}{\lambda}}\left(-1+\int_0^1 dv\, v^{-2}\times\left[e^{(1/2)\lambda v^2}\left(1-v^4e^{\lambda(1-v^2)}\right)^{-(1/2)}-1\right]\right)$$
  - **No distinction between S and A**



Results

CYW potential parameters	Commonly used masses	AdS/CFT potential parameters
$\Lambda = 0.4218\ \text{GeV}$		$c = 0.323$
$B = 0.05081$		$A:S \approx 1:9$
$K = 4.198$		
$\boldsymbol{m_c = 1.476\ \text{GeV}}$	$\boldsymbol{m_c = 1.18\sim1.34\ \text{GeV}}$	$\boldsymbol{m_c = 1.346\ \text{GeV}}$
$\boldsymbol{m_b = 4.844\ \text{GeV}}$	$\boldsymbol{m_b = 4.13\sim4.37\ \text{GeV}}$	$\boldsymbol{m_b = 4.747\ \text{GeV}}$

Results

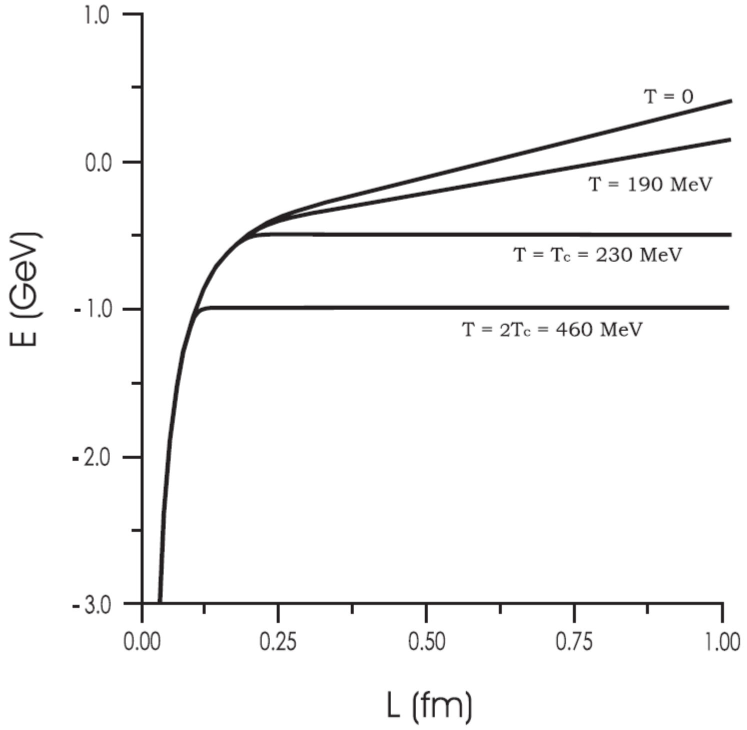
Meson	Exp. (GeV)	CYW (GeV)	Exp.-QCD (GeV)	AdS/CFT (GeV)	Exp.-AdS (GeV)
$\eta_c : c\bar{c}1^1S_0$	2.980	2.978	0.002	2.981	-0.001
$J/\psi: c\bar{c}1^3S_1$	3.097	3.140	-0.043	2.992	0.105
$B_c{}^-: b\bar{c}1^1S_0$	6.276	6.276	0.000	6.254	0.022
$\eta_b: b\bar{b}1^1S_0$	9.389	9.345	0.044	9.390	-0.001
$\Upsilon: b\bar{b}1^3S_1$	9.460	9.484	-0.024	9.396	0.064

Summary

- Using Dirac's relativistic constraints, Two-Body Dirac Equations successfully leads to the **relativistic Schrödinger-type Equation.**
- AdS/CFT potential is used for $Q\bar{Q}$-systems
- Applying the weighting factor to the scalar & vector potential
- Similar result on the mass spectrum of meson
- quark masses are close to the commonly used

Work on process & Future Work

- $1^1S_0, 1^3S_1 \rightarrow n^{2S+1}L_J$
- **Introduce quark-antiquark potential at finite temperature**
- **By using the AdS/CFT potential, the properties of heavy meson systems will be investigated.**



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