Initial state of heavy ion collisions

Sangyong Jeon

Department of Physics
McGill University
Montréal, QC, CANADA

Initial state fluctuation of heavy ion collisions

- Initial density fluctuations in the nucleus
- Vaccum fluctuations
- This is a QCD problem

Initial state density fluctuations

- Origin: Distribution of nucleons in a nucleus – It's not continuous.
- Probability to make a inelastic collision
- How is the released energy distributed?
- Quantum fluctuations “frozen” by fast evolution?

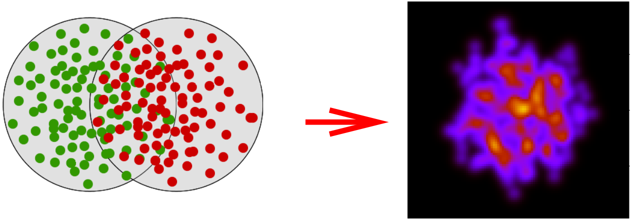
Transverse Fluctuations

Glauber model – Phenomenological

- Sample initial positions according to the WS thickness function:

$$T(\mathbf{x}_\perp) = \int dz\, \rho_{WS}(\mathbf{x})$$

- At each collisions site, assign a gaussian energy density profile – ϵ_0 and σ_0 are parameters



QCD based MV model

- MV Model: Nucleus moving with (almost) the speed of light is dominated by *classical* gluons
- Gluon dynamics governed by the classical EoM

$$[\mathcal{D}_\mu, \mathcal{G}^{\mu\nu}] = J^\nu$$

- The classical source for a nucleus with $v_z = c$:

$$J^\nu = \rho(\mathbf{x}_\perp)\delta(x^-)\delta^{\nu +}$$

where $x^- = (t - z)/\sqrt{2}$.

- Classical source distribution:

$$W[\rho] = \exp\left(- \int d^2 x_\perp \frac{\rho(\mathbf{x}_\perp)\rho(\mathbf{x}_\perp)}{2\mu^2(\mathbf{x}_\perp)} \right)$$

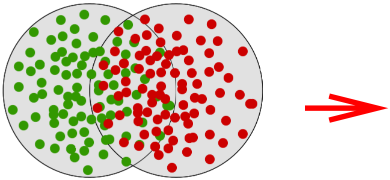
with $\mu^2 \sim xG(x, \mu^2)/\pi R^2$

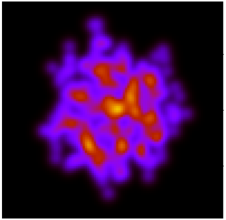
E-by-E implementation of the fluctuations

- Per event: Sampling $W[\rho]$ at each discretized $\mathbf{x}_\perp$ volume introduces e-by-e fluctuations
- Good approximation in the $A \to \infty$ limit.

E-by-E implementation of the fluctuations

- Per event: Sampling $W[\rho]$ at each discretized $\mathbf{x}_\perp$ volume introduces e-by-e fluctuations
- Good approximation in the $A \to \infty$ limit.
- What to do when A is finite?
  - Need to consider *nucleon positions*.
  - Take A to be superposition of nucleons
  - Need to know μ^2 for nucleons (e.g. IP-Sat)





E-by-E implementation of the fluctuations

- Per event: Sampling $W[\rho]$ at each discretized $\mathbf{x}_\perp$ volume introduces e-by-e fluctuations
- Good approximation in the $A \to \infty$ limit.
- What to do when A is finite?
  - Need to consider *nucleon positions*.
  - Take A to be superposition of nucleons
  - Need to know μ^2 for nucleons (e.g. IP-Sat)
- Transverse evolution is determinisitic when $\rho_{\mathrm{e-by-e}}$ is given

IP-Sat

- This is for a nucleon
- Dipole cross-section

$$\frac{d\sigma}{d^2b_\perp}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = 2[1 - \exp(-F(r_\perp, x, \mathbf{b}_\perp)]$$

with $\exp(-F(r_\perp, x, \mathbf{b}_\perp) = \dfrac{\pi^2}{2N_c} r_\perp^2 \alpha_S(\tilde{\mu}^2) xg(x, \tilde{\mu}^2) T_b(\mathbf{b}_\perp)$

and $\tilde{\mu}^2 = \dfrac{4}{r_\perp^2} + \tilde{\mu}_0^2 = 2Q_{s,p}^2 + \tilde{\mu}_0^2$

- $Q_{s,p}^2 = 2/r_\perp^2$ determined by $F(r_S, x, \mathbf{b}_\perp) = 1/2$.
  Density profile $T_p(\mathbf{b}_\perp) = \dfrac{1}{2\pi B_G} \exp(-b_\perp^2 / \left\langle b^2 \right\rangle)$ with $\langle b \rangle \sim 0.5\,\mathrm{fm}$
  and $x = \langle p_\perp \rangle / \sqrt{s}$

IP-Glasma

[Gale, Jeon, Schenke, Tribedy and Venugopalan]

- Sample A nucleon position ($\mathbf{x}_\perp^i$) from the thickness function $T(\mathbf{x}_\perp) = \int dz\, \rho_{WS}(\mathbf{x})$
- For each, we have $\tilde{\mu}_p^2(x, \mathbf{b}_\perp)$.
- At $\mathbf{x}_\perp$,

$$\tilde{\mu}_A^2(x, \mathbf{x}_\perp) = \sum_{i=1}^{A} \tilde{\mu}_p^2(x, \mathbf{x}_\perp - \mathbf{x}_\perp^i)$$

- From $\mu_A^2(x, \mathbf{x}_\perp)$ and $\mu_B^2(x, \mathbf{x}_\perp)$ calculate classical gluon field just below the light cone
- Use continuity conditions to get the classical gluon field just above the light cone $\Longrightarrow$ Evolve up to τ_0 $\Longrightarrow$ Get $T^{\mu\nu}$
- Get ϵ and u^μ to use as the initial condition for MUSIC

MUSIC

MUScl for Ion Collisions

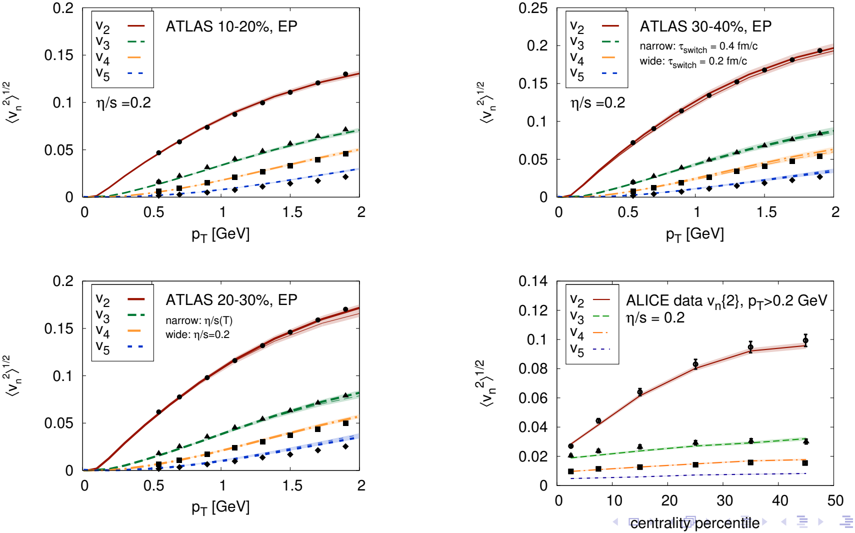
MUSCL: Monotone Upstream-centered Schemes for Conservation Laws

3+1D Event-by-Event Viscous Hydrodynamics

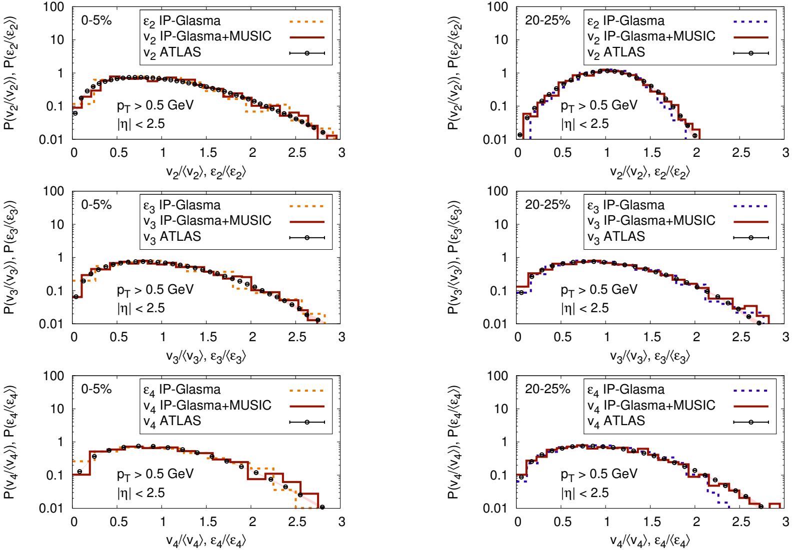
- 3+1D parallel implementation of new *Kurganov-Tadmor Scheme* [Jour. of Comp. Phys. **160**, 241 (2000)] in (τ, η) with an additional baryon current (No need for a Riemann Solver. OK to take $\Delta t \to 0$.)
- Ideal *and* Viscous Hydro
- Event-by-Event fluctuating initial condition
- Sophisticated Freeze-out surface construction
- Full resonance decay (3+1D version of Kolb and Heinz)
- Many different equation of states including the newest from Huovinen and Petreczky
- *New Developement*: UrQMD afterburner

IP-Glasma + MUSIC – v_n

[Gale, Jeon, Schenke, Tribedy and Venugopalan, arXiv:1209.6330]
Best value $\eta/s = 0.2 = 2.5/(4\pi)$.



IP-Glasma + MUSIC – Scaled v_n and ϵ_n distribution



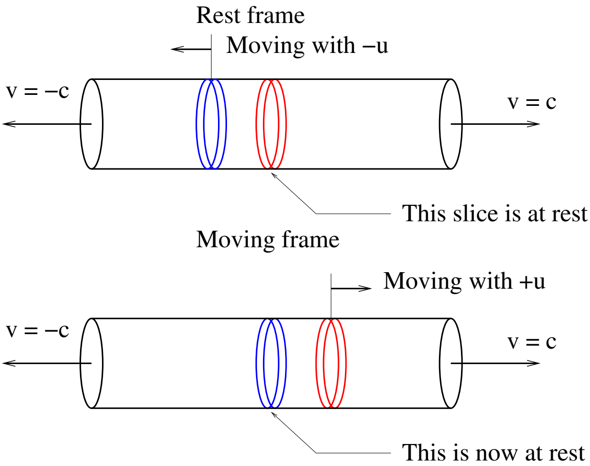
Hydro – Faithful mapping of ϵ_n to v_n

Longitudinal Fluctuations[1]

[1] L.Pang, Q.Wang and X.N.Wang have been working on this.

Simple model of energy distribution in η

- Consider two particles pulling a string.
- Further suppose these particles are moving with $\pm c$.



Simple model of energy distribution in η

- Consider two particles pulling a string.
- Further suppose these particles are moving with $\pm c$.
  - In any inertial frame, the ends are moving with $\pm c$.
    $\Longrightarrow$ Cannot distiguish the frames
  - In rapidity, ends of the string are moving with $y = \pm\infty$
  - The string occupies the whole rapidity space
  - Boost changes

$$t = \tau \cosh(\eta) \to t' = \tau \cosh(\eta + \Delta y)$$
$$z = \tau \sinh(\eta) \to z' = \tau \sinh(\eta + \Delta y)$$

- Implies that the energy is distributed along the string in a boost-invariant way or

$$\frac{dE}{d\eta} = f(\tau)$$

Simple model of energy distribution in η

- This aregument good for event-averaged $d\langle E\rangle/d\eta$.
- Does not have to be true for an individual event.
- What is required: Fluctuation spectrum is independent of η or

$$\frac{d}{d\eta}\langle E^n\rangle = F_n(\tau)$$

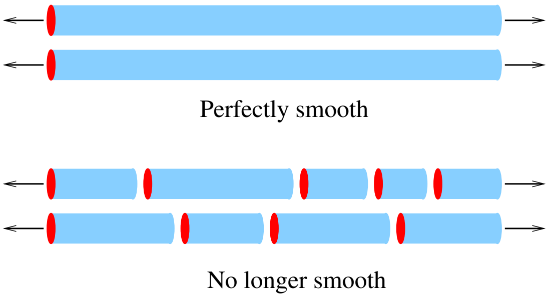
η distribution in 2+1D hydro

- 2+1D hydro assumes that the two nuclei are pulling away with $\pm c$ stretching the system
- Same argument. But $\mathbf{x}_\perp$ dependence also possible.

$$\frac{d}{d\eta}\langle E^n\rangle = F_n(\tau, \mathbf{x}_\perp)$$

The origin of fluctuations in the η direction

- One may argue: If the string pulling is the right picture, and if the string *does not break*, then there can be no fluctuations. – If the string is perfectly smooth, where would it break?



- Quantum fluctuations must introduce inhomogeneity for a string to be able to find a breaking point
- In addition: Jets and minijets also induce fluctuations

Formulation of the problem[2]

[2] Gelis, Venugopalan, Fukushima, Lappi, Dusling and collaborators have worked on similar problems

Degrees of freedom in strong field QCD

- ρ: Color density of the nucleus
- $\mathcal{A}_\mu$: Classical gluon field
- a_μ: Quantum fluctuation on top of $\mathcal{A}_\mu$
- Natural way to describe them?

Schwinger Keldysh formalism

- Scalar theory for simplicity
- Feynman path-integral is for time ordered products of operators

$$\langle\phi_{\rm fin}|T(\hat{O}(t_2)\hat{O}(t_1))|\phi_{\rm init}\rangle = \int_{\phi_{\rm init}}^{\phi_{\rm fin}} \mathcal{D}\phi\, \exp\left(iS[\phi]\right) O(t_1)O(t_2)$$

- Convenient for *transition amplitudes*
- Not so convenient for averages:

$$\left\langle \hat{O}(t_2)\hat{O}(t_1)\right\rangle_{\rm init} = \langle\phi_{\rm init}|\hat{O}(t_1)\hat{O}(t_2)|\phi_{\rm init}\rangle$$

or vacuum averages

$$\left\langle \hat{O}(t_1)\hat{O}(t_2)\right\rangle_{\rho} = {\rm Tr}\hat{\rho}_{\rm v}\hat{O}(t_1)\hat{O}(t_2)$$
$$= \int [d\phi_1^{\rm init}] \int [d\phi_2^{\rm init}]\, \rho_{\rm v}[\phi_1^{\rm init}, \phi_2^{\rm init}] \langle\phi_2^{\rm init}|\hat{O}(t_1)\hat{O}(t_2)|\phi_1^{\rm init}\rangle$$

SK for scalar

- Insert

$$1 = \int [d\phi_{\text{fin}}] |\phi_{\text{fin}}\rangle\langle\phi_{\text{fin}}|$$

to get

$$\begin{aligned} &\left\langle \hat{O}(t_1)\hat{O}(t_2)\right\rangle_\rho \\ &= \int [d\phi_{\text{fin}}] \int [d\phi_1^{\text{init}}] \int [d\phi_2^{\text{init}}] \rho_{\text{v}}[\phi_1^{\text{init}}, \phi_2^{\text{init}}] \\ &\qquad \underbrace{\langle \phi_2^{\text{init}} | \hat{O}(t_1) | \phi_{\text{fin}} \rangle}_{\int \mathcal{D}\phi_2 e^{-iS[\phi_2]} O} \underbrace{\langle \phi_{\text{fin}} | \hat{O}(t_2) | \phi_1^{\text{init}} \rangle}_{\int \mathcal{D}\phi_1 e^{iS[\phi_1]} O} \end{aligned}$$

- *Two* path integrals are needed

SK Lagrangian and classical-quantum separation

- SK Lagrangian

$$\mathcal{L} = L(\phi_1) - L(\phi_2) + J_1\phi_1 - J_2\phi_2$$

- Degrees of freedom doubles. ϕ_1 only interacts with ϕ_1 and ϕ_2 only interacts with ϕ_2.

The Keldysh rotation

- Define

$$\phi_c = (\phi_1 + \phi_2)/2$$
$$\phi_q = (\phi_1 - \phi_2)$$

or

$$\phi_1 = \phi_c + \phi_q$$
$$\phi_2 = \phi_c - \phi_q$$

- Taylor expanding $L(\phi_1) - L(\phi_2)$ yields

$$\mathcal{L}_{SK} = \phi_q \frac{\delta \mathcal{L}(\phi_c)}{\delta \phi_c} + \frac{1}{3!}\phi_q\phi_q\phi_q \frac{\delta^3 \mathcal{L}(\phi_c)}{\delta \phi_c^3} + J_c\phi_q + J_q\phi_c$$

- Key points:
  - Without ϕ_q^3 term this is completely *classical*
  - EoM $\delta\mathcal{L}/\delta\phi + J_c = 0$ emerges naturally

The Keldysh rotation

- Define

$$\phi_c = (\phi_1 + \phi_2)/2$$
$$\phi_q = (\phi_1 - \phi_2)$$

or

$$\phi_1 = \phi_c + \phi_q$$
$$\phi_2 = \phi_c - \phi_q$$

- Propagators

$$\langle \phi_c(x)\phi_q(y)\rangle = G_R(x, y)$$
$$\langle \phi_q(x)\phi_c(y)\rangle = G_A(x, y)$$
$$\langle \phi_c(x)\phi_c(y)\rangle = G_S(x, y)$$

The fact that $G_S \neq 0$ reflects we have a quantum vacuum. For instance, in the Minkowski vacuum $G_S(p) = (1/2)(2\pi)\delta(p^2 - m^2)$

Strong source

- In the presence of a physical source, $J_c = (J_1 + J_2)/2 = J_{\rm phys}$.
- If $J_c = O(1/g)$ with $g \ll 1$, $\phi_c = O(1/g)$ $\Longrightarrow$ Classical d.o.f. dominates $\Longrightarrow$ Can ignore ϕ_q^3 term
- Observables

$$\left\langle \hat{\mathcal{O}} \right\rangle_J \approx \int [d\phi_r^{\rm init}][d\dot{\phi}_r^{\rm init}]\, \rho_W[\phi_r^{\rm init}, \dot{\phi}_r^{\rm init}]\mathcal{O}(\varphi_{\rm cl}[J_{\rm phys}, \phi_r^{\rm init}, \dot{\phi}_r^{\rm init}])$$

with the classical solution satisfying $\mathcal{L}'(\varphi_{\rm cl}) + J_{\rm phy} = 0$ and the boundary conditions given by ρ_W.

Complete formulation of fluctuations

- All fluctuations included:

$$\int [dJ_{\text{phys}}]\, W[J_{\text{phys}}] \left\langle \hat{\mathcal{O}} \right\rangle_J$$
$$\approx \int [dJ_{\text{phys}}]\, W[J_{\text{phys}}]$$
$$\int [d\phi_r^{\text{init}}][d\dot{\phi}_r^{\text{init}}]\, \rho_W[\phi_r^{\text{init}}, \dot{\phi}_r^{\text{init}}]\mathcal{O}(\varphi_{\text{cl}}[J_{\text{phys}}, \phi_r^{\text{init}}, \dot{\phi}_r^{\text{init}}])$$

- $\rho_W[\phi_r^{\text{init}}, \dot{\phi}_r^{\text{init}}]$: Wigner transform of $W[\phi_1, \phi_2]$. Integration by part to get $\phi_q L'(\phi_c)$ naturally leads to this.
- This now includes
    - Fluctuations in the source: $W[J_{\text{phys}}]$.
    - Quantum vacuum fluctuations: $\rho_W[\phi_r^{\text{init}}, \dot{\phi}_r^{\text{init}}]$

Fluctuation spectrum

- Vacuum wave functional obtained by solving the functional Schrödinger equation in $\tau - \eta$

$$
i\partial_\tau \Psi_0[\phi] = \int d\eta d^2x_\perp \left(-\frac{1}{2\tau} \frac{\delta^2}{\delta\phi(\eta, x_\perp)^2} + \frac{\tau}{2}\phi(\eta, x_\perp) \left(-\frac{\partial_\eta^2}{\tau^2} - \nabla_\perp^2 + m^2 \right) \phi(\eta, x_\perp) \right) \Psi_0[\phi]
$$

- In the $\tau \to 0$ limit,

$$
\lim_{\tau \to 0} \Psi_0(\tau) = \mathcal{N}(0) \exp\left(-\frac{1}{2} \int d^3\tilde{k}\, \phi(\tilde{k})\phi(-\tilde{k}) k_\eta \theta(k_\eta) \right)
$$

- Even if $J_{\rm phy}$ alone yields a boost invariant solution, due to the vacuum fluctuation, $\varphi_{\rm cl}$ is *not* boost invariant.
- Dynamics is still classical. Fluctuations are in the initial states.

Strong source QCD

- Much more complicated due to gauge symmetry, but doable.
- Again,

$$\langle \mathcal{O} \rangle = \int [d\rho] W[\rho] \int [da_r^{\rm init}][d\dot{a}_r^{\rm init}] \rho_W[a_r^{\rm init}, \dot{a}_r^{\rm init}] \mathcal{O}(\mathcal{A}_{\rm cl} + a)$$

- Strategy from here: Put it on lattice.
- So far only ρ part implemented (With Schenke, Trabedy, Venugopalan, Gale)
- Quantum fluctuations need to be better implemented including the longitudinal fluctuations: Under development

Summary and prospective

- Hydrodynamics: An efficient and faithful translator of the spatial anisotropy to the momentum anisotropy
- Transverse fluctuations under control with a QCD inspired IP-Glasma model
- Longitudinal fluctuations – Model based study is ongoing (thanks to X.-N.)
- QCD based formulation of the longitudinal fluctuation underway
- New Development: UrQMD afterburner (Come to Sangwook Ryu's talk this afternoon)

Backup Slides

Fluctuations in η direction

- What is a string? $\Longrightarrow$ QCD **E** & **B** field confined in a tube.
- No real theory exists $\Longrightarrow$ Take the cue from the experiment
- Assume: soft particles in *pp* come from strings.
  $\Longrightarrow$ Knowing average $d\left\langle E\right\rangle_{pp}/d\eta$ and the width $d\left\langle(\Delta E)^2\right\rangle_{pp}/d\eta$, we can model the *AA* initial condition.
- One *AA* collision $\approx N_{\rm bin}$ *pp* collisions
- In a pinch, one may substitute at mid-rapidity

$$\frac{d}{d\eta}\left\langle E^n\right\rangle_{pp} = \left\langle m_T\right\rangle^n \frac{d}{d\eta}\left\langle N^n\right\rangle$$

- Not enough. There is a strong correlation in η: We need to know

$$G(\eta,\eta') = \left\langle \frac{dE}{d\eta}\frac{dE}{d\eta'}\right\rangle \approx g_1((\eta+\eta')/2)g_2(\eta-\eta')\left\langle m_T\right\rangle^2\left\langle \frac{dN}{d\eta}\frac{dN}{d\eta'}\right\rangle$$