Birefringent photon spectrum from a nonlinear interaction with strong magnetic field

Koichi Hattori (Yonsei Univ.)
in collaboration with Kazunori Itakura (KEK)

4th ATHIC, 15th Nov., 2012

KH and K. Itakura, [hep-ph/1209.2663].
"Vacuum birefringence in strong magnetic fields:
(I) Photon polarization tensor with all the Landau levels"

KH and K. Itakura, in preparation.
"Vacuum birefringence in strong magnetic fields:
(II) Complex refractive index in the lowest Landau level approximation"

What is “Birefringence” ?

Two polarization modes of a propagating photon
have different refractive indices.

<u>*How is in the vacuum with external magnetic field ?*</u>

+ ~~Lorentz~~ & Gauge symmetries → n ≠ 1 in general

+ Oriented response of the Dirac sea → Vacuum birefringence

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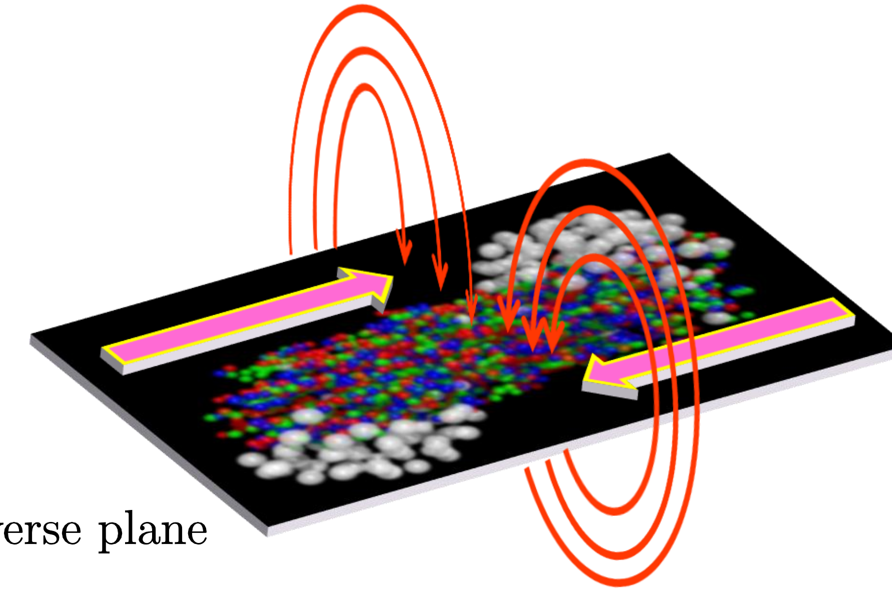
Extremely strong magnetic fields in peripheral collisions induced by strongly accelerated heavy nuclei

$v_N > 0.9999\ c$ $\quad$ $Z = 79$ (Au), 82 (Pb)

Geometry in peripheral collisions



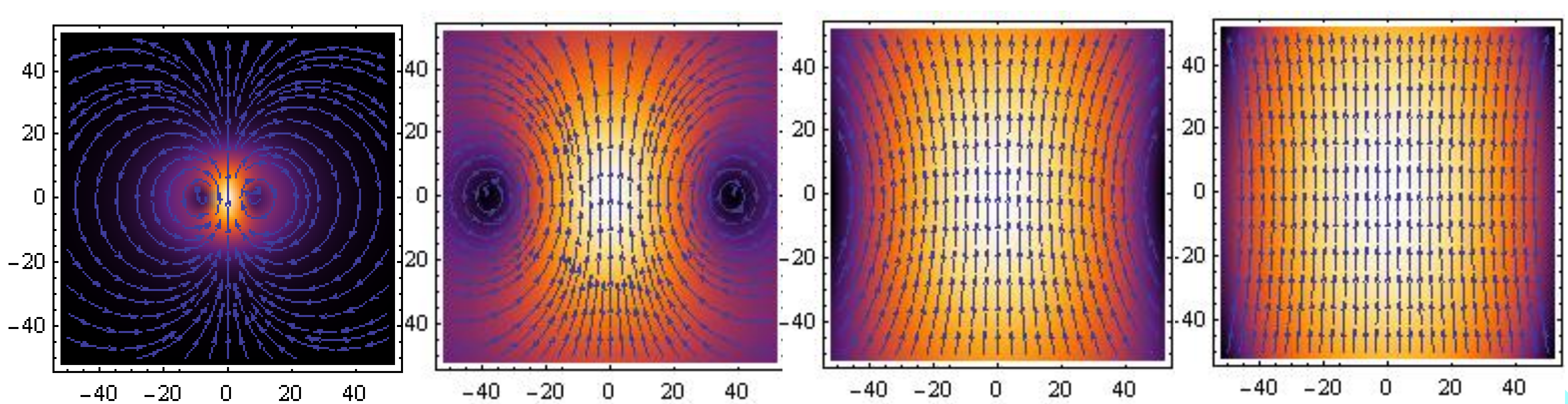
Lienard-Wiechert potential

$$e\boldsymbol{B} = \frac{\alpha_{\rm EM} Z \sinh(Y)\ \ (\boldsymbol{x}_\perp \times \boldsymbol{e}_z)}{[\ |\boldsymbol{x}_\perp|^2 + \tau^2 \sinh^2(Y-\eta)\ ]^{3/2}}$$

$Y = \tanh^{-1} v_{\rm N}$: beam rapidity

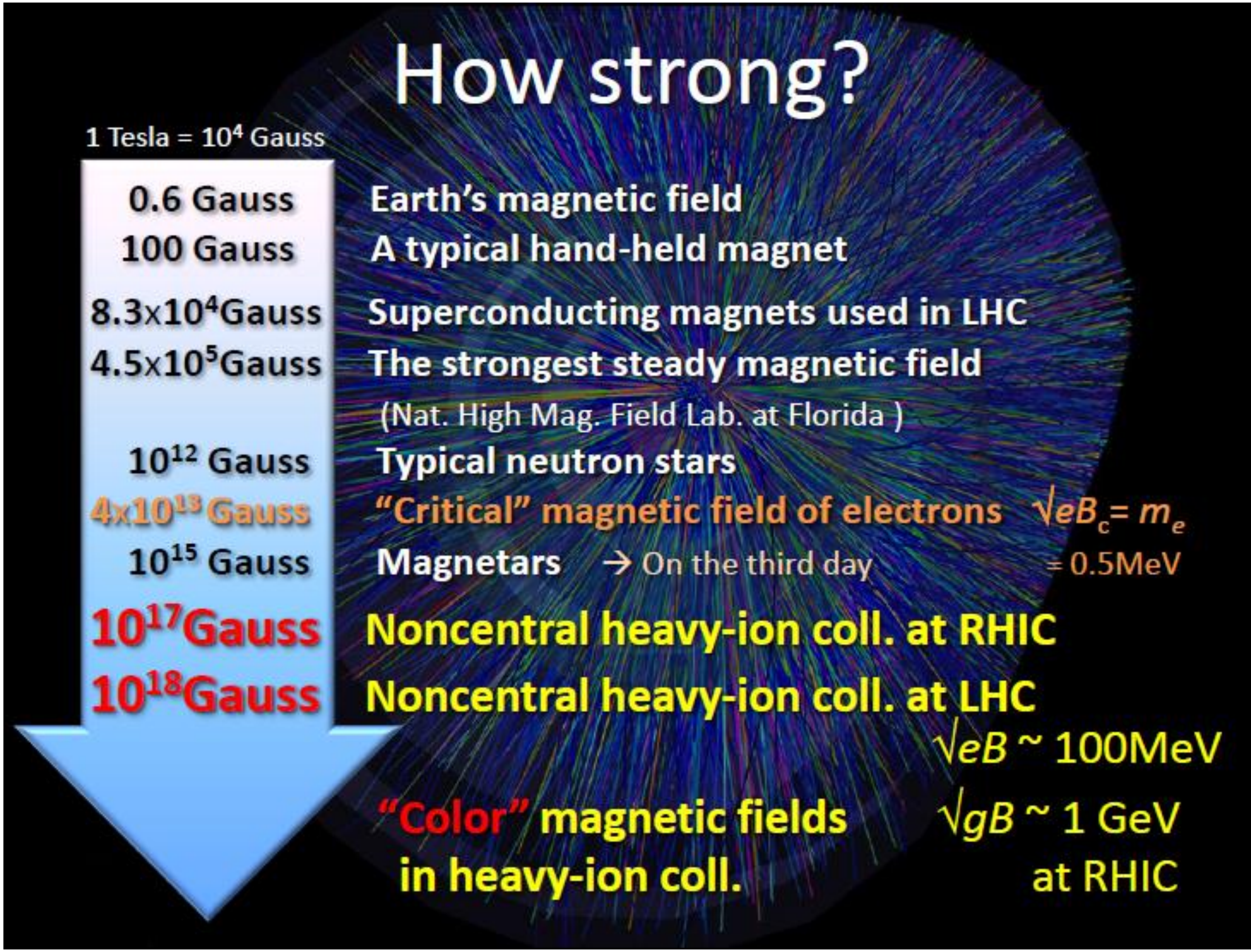
$\boldsymbol{x}_\perp$: distance from the beam in the transverse plane

Superposition of circulating magnetic fields in the transverse plane

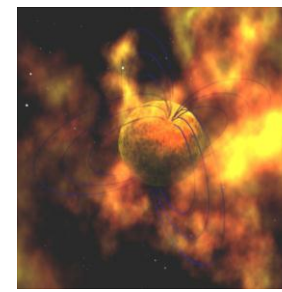


t = 0.1 fm/c $\quad$ 0.5 fm/c $\quad$ 1 fm/c $\quad$ 2 fm/c

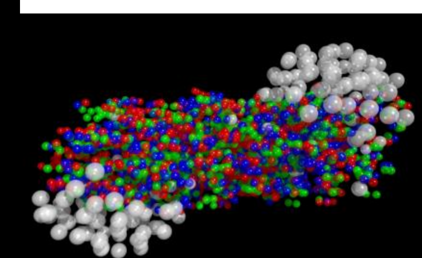
Strong magnetic fields in nature and laboratories



Magnet in Lab.



Magnetar



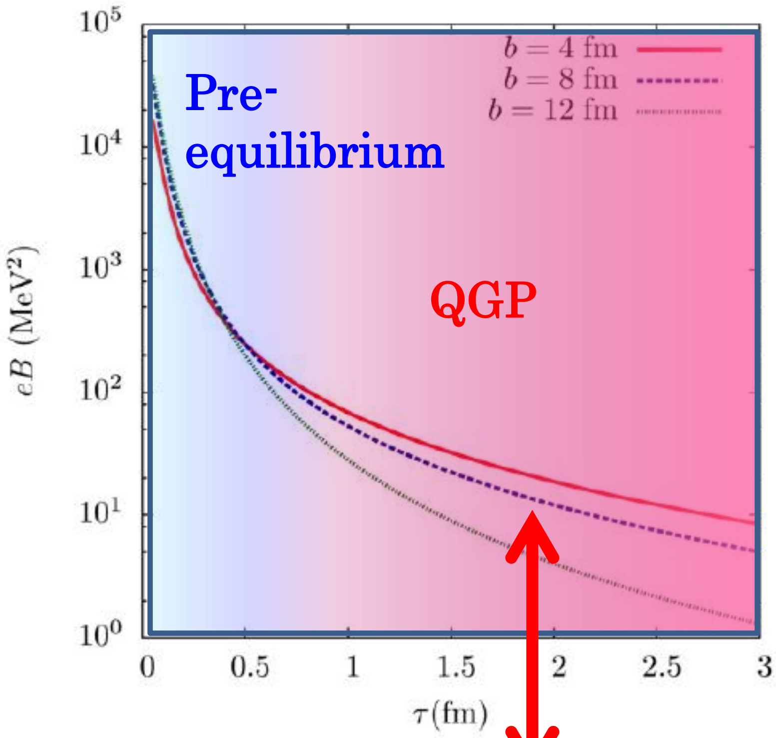
Heavy ion collisions

From Itakura-san's talk in *International conference on physics in intense field 2010 @ KEK*

Time evolution of the magnetic field after collisions

Simple estimates by Lienard-Wiechert potential

Analytical modeling of colliding nuclei,
Kharzeev, McLerran, Warringa, NPA (2008)



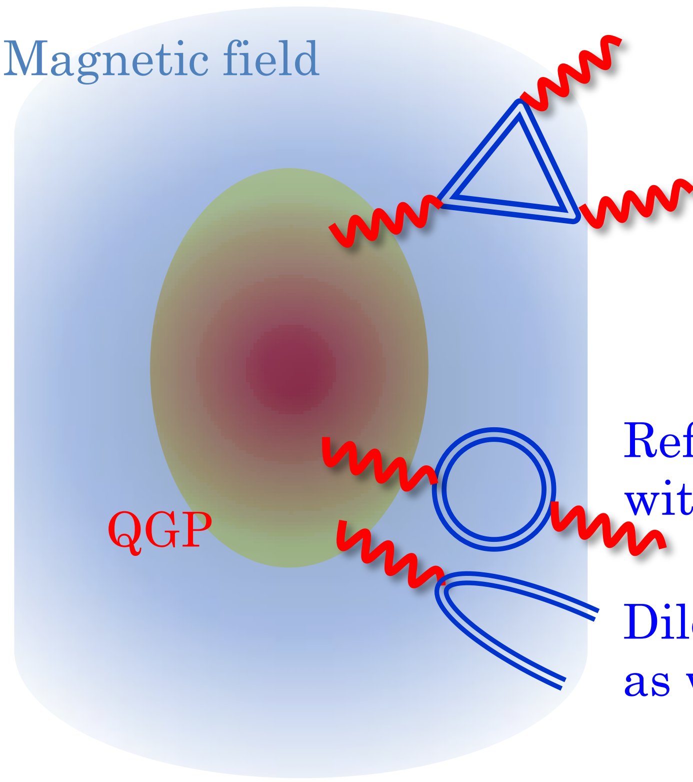
$$\frac{eB}{eB_c} \sim 10^{2\sim3}$$

$$eB_c = m_e^2 \sim 0.25 \quad \text{MeV}^2$$

Still a few orders stronger than the "critical field"

Modifications of photon propagations by nonlinear QED effects

EM probes would carry away info of initial stage.



Photon splitting: γ+B → 2γ,
avoiding Furry's theorem

Refraction of photon,
without Lorentz symmetry

Dilepton emission from real photon decay,
as well as virtual photon $\gamma^* \to e^+e^-$

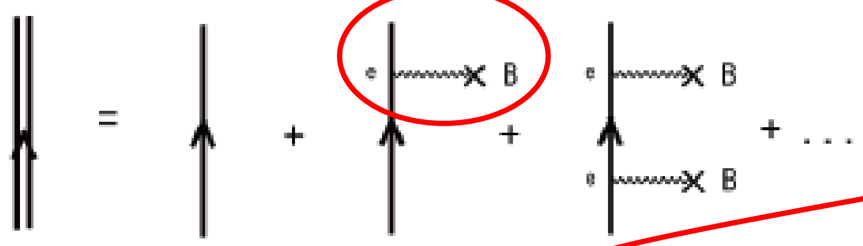
Modified Maxwell eq. : $\left\{ q^2\eta^{\mu\nu} - q^\mu q^\nu - \Pi^{\mu\nu}_{\rm ex}(q^2) \right\} A_\nu(q) = 0$

Photon vacuum polarization tensor with the dressed fermion propagators:

$$i\Pi^{\mu\nu}_{\rm ex}(q) = -(-ie)^2 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\, \gamma^\mu G(p|A) \gamma^\nu G(p+q|A) \,\right]$$

Break-down of naïve perturbation in strong magnetic fields

Dressed fermion propagator



$$1 + O(eB/m_e^2) + O((eB/m_e^2)^2) + \dots$$

Critical field strength
$B_c = m_e^2 / e$

In heavy ion collisions,
$B/B_c \sim O(10^4) >> 1$

Naïve perturbation breaks down when $B > B_c$

→ Need to take into account all-order diagrams

Resummation w.r.t external legs by "proper-time method " Schwinger

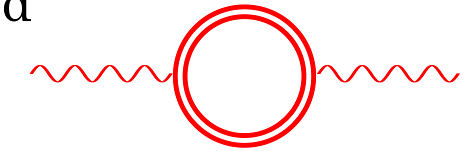
$$G(p|A) = \frac{i\,(\not{p} - e\not{A} + m)}{(\not{p} - e\not{A})^2 - m^2 + i\epsilon}$$

$$= i\,(\not{p} - e\not{A} + m) \times \frac{1}{i}\int_0^\infty d\tau\; e^{i\tau\left\{(\not{p}-e\not{A})^2-(m^2-i\varepsilon)\right\}}$$

τ : proper-time

Nonlinear to the external field

Photon propagation in a constant external magnetic field



Lorentz and gauge symmetries lead to a tensor structure,

$$\Pi^{\mu\nu}_{\rm ex}(q^2) = -\left\{\chi_0 P_0^{\mu\nu} + \chi_1 P_1^{\mu\nu} + \chi_2 P_2^{\mu\nu}\right\}$$

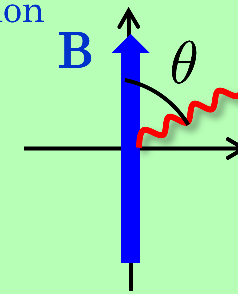
$$q_\mu \Pi^{\mu\nu}_{\rm ex}(q) = 0$$

$$P_0^{\mu\nu} = q^2\eta^{\mu\nu} - q^\mu q^\nu$$

$$P_1^{\mu\nu} = q_\parallel^2\eta_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu$$
$$P_2^{\mu\nu} = q_\perp^2\eta_\perp^{\mu\nu} - q_\perp^\mu q_\perp^\nu$$

θ: angle btw B-field and photon propagation

$$q^\mu = (q^0, q_\perp, 0, q^3)$$
$$q_\parallel^\mu = (q^0, 0, 0, , q^3)$$
$$q_\perp^\mu = (0, q_\perp, 0, 0)$$



Eigen-equations from the modified Maxwell eq.

$$\begin{cases} \{\ (1+\chi_0)\ q^2\ \}\ \pi_0^\mu = 0 \\ \{\ (1+\chi_0)\ q^2 + \chi_2\ q_\perp^2\ \}\ \pi_1^\mu = 0 \\ \{\ (1+\chi_0)\ q^2 + \chi_1\ q_\parallel^2\ \}\ \pi_2^\mu = 0 \\ \xi_g^{-1}\ q^2\ \pi_3^\mu = 0\ . \end{cases}$$

"Vacuum birefringence" Dielectric constant/refractive index : $\epsilon = n^2 = \dfrac{|\boldsymbol{q}|^2}{\omega^2}$

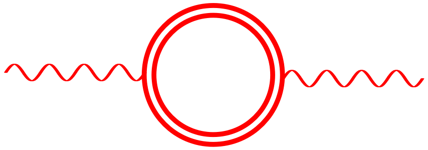
Following from the tensor structure, we obtain distinct eigenmodes!!

$$\begin{cases} \epsilon_\parallel = \dfrac{1+\chi_0+\chi_1}{1+\chi_0+\chi_1\cos^2\theta} \\ \epsilon_\perp = \dfrac{1+\chi_0}{1+\chi_0+\chi_2\sin^2\theta} \end{cases}$$ with eigenvectors, $$\begin{cases} \boldsymbol{E}_\parallel \propto ((1+\chi_0+\chi_1)\cos\theta, 0, -(1+\chi_0)\sin\theta) \\ \boldsymbol{E}_\perp \propto (0,1,0) \end{cases}$$

Melrose and Stoneham

$\epsilon_\parallel = \epsilon_\perp = 1$ when $\theta = 0$, because of a boost invariance along B-field

Scalar coefficient functions χ by the proper-time method



Given by a double integral wrt proper time variables associated with two fermion lines

$$\chi_i(r_\parallel^2, r_\perp^2, B_\mathrm{r}) = \frac{\alpha B_\mathrm{r}}{4\pi} \int_{-1}^{1} d\beta \int_0^\infty d\tau \ \frac{\Gamma_i(\tau,\beta)}{\sin(\tau)} \ e^{-i(\phi_\parallel + \phi_\perp)\tau}$$

$i = 0, \ 1, \ 2$

Schwinger, Adler, Shabad, Urrutia, Tsai and Eber, Dittrich and Gies

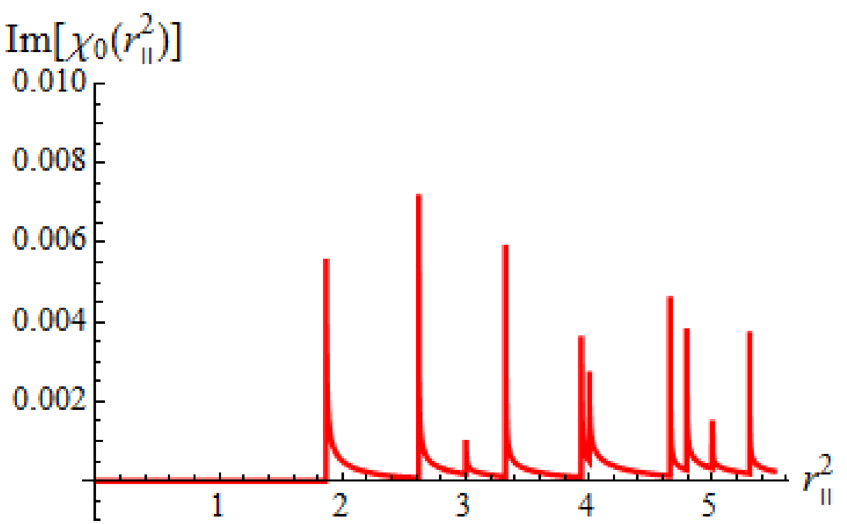
Analytic integration without any approximation

Decomposition into a double infinite sum

$$\chi_i = \frac{\alpha B_\mathrm{r}}{4\pi} e^{-\eta} \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} \Omega_{\ell i}^{n}$$

Polarization tensor has an imaginary part above

$$q_\parallel^2 = \left[\sqrt{m^2 + 2\ell eB} + \sqrt{m^2 + 2(\ell+n)eB} \ \right]^2$$

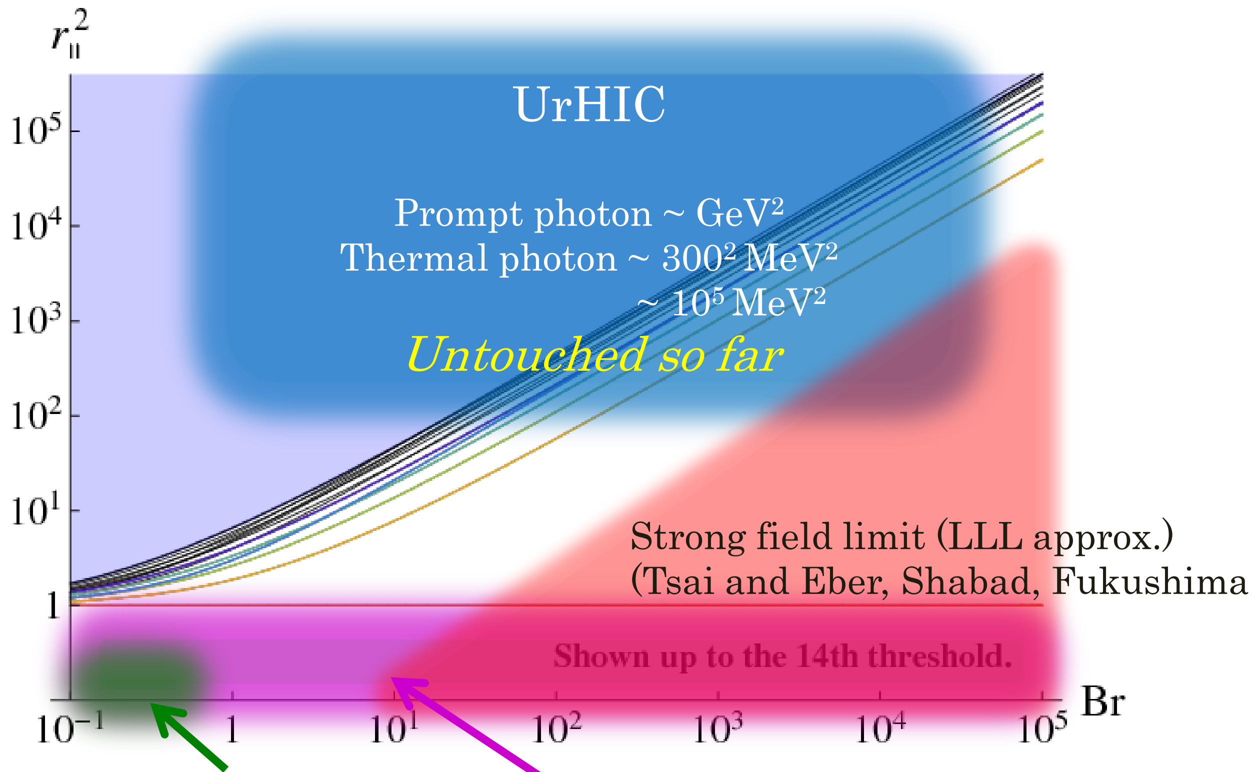


ℓ and n: "Landau levels" of a pair exchitation

Summary of relevant scales and available calculations for χ's

$$r_\parallel^2 = \frac{q_\parallel^2}{4m_e^2}$$

(Photon momentum)



The first threshold
(lowest Landau level):
$q_\parallel^2 = 4m_e^2 = 1$ MeV^2

Soft photon & weak field limit
(Adler)

Numerical integration
(Kohri, Yamada)

$$B_r = B/B_c$$

Complex dielectric constants
from the vacuum polarization tensor

$$\epsilon_{\parallel} = \frac{1+\chi_0+\chi_1}{1+\chi_0+\chi_1\cos^2\theta}$$
$$\epsilon_{\perp} = \frac{1+\chi_0}{1+\chi_0+\chi_2\sin^2\theta}$$

and the complex refractive indeices

$$n_{\parallel}^2 = \epsilon_{\parallel}$$
$$n_{\perp}^2 = \epsilon_{\perp}$$

Dielectric constant at the lowest-Landau-level

The first term $(\ell, n) = (0,0)$ in the double infinite series :

$$\chi_0 = 0$$

$$\chi_1 = \frac{\alpha B_{\mathrm{r}}}{4\pi} \, e^{-\frac{|\boldsymbol{q}_\perp|^2}{2|eB|}} \times \frac{1}{r_\parallel^2} \left\{ I_{0\Delta}^0(r_\parallel^2) - 2 \right\}$$

$$\chi_2 = 0$$

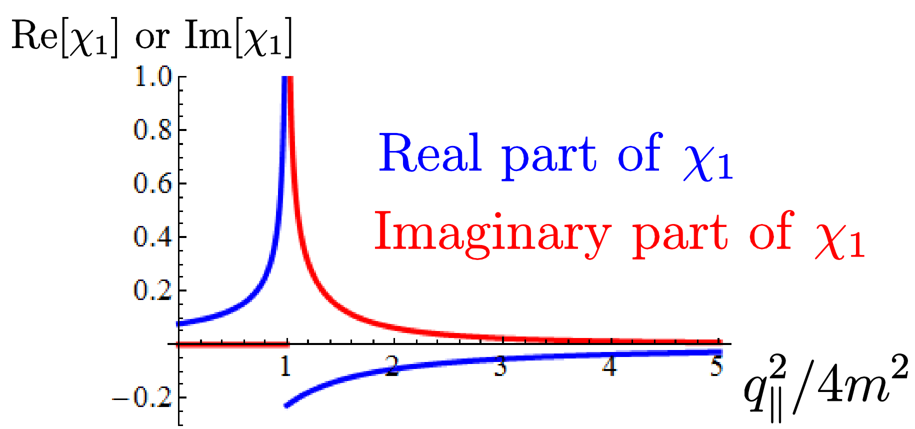
$$I_{0\Delta}^0(r_\parallel^2) = \frac{2}{\sqrt{r_\parallel^2(1-r_\parallel^2)}} \arctan\left(\frac{r_\parallel^2}{\sqrt{r_\parallel^2(1-r_\parallel^2)}} \right)$$

ArcTan : source of an imaginary part above the lowest threshold

Dielectric constant at the LLL

$$\begin{cases} \epsilon_\parallel = \frac{1+\chi_1}{1+\chi_1 \cos^2\theta} \\ \epsilon_\perp = 1 \end{cases}$$

Polarization excites only along the magnetic field



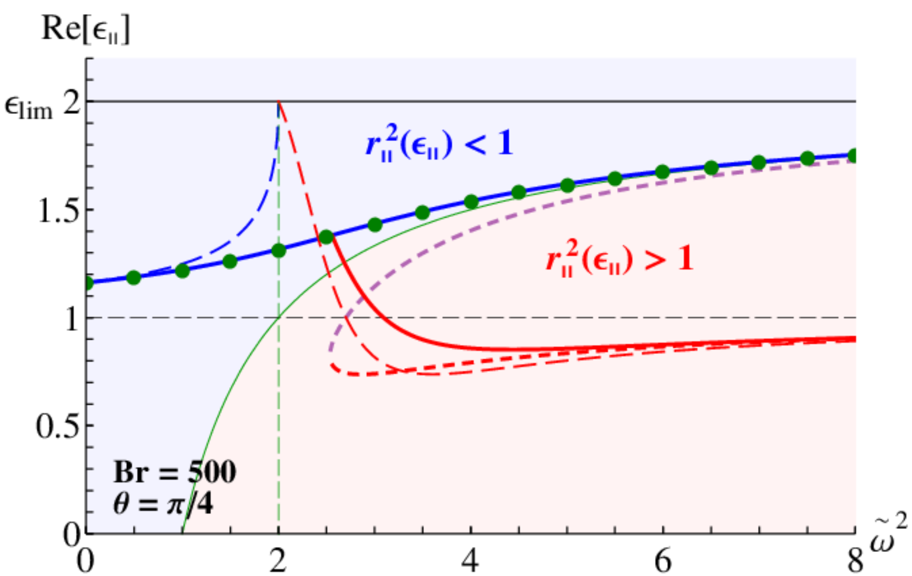
Consistent solution wrt the real and imag. parts

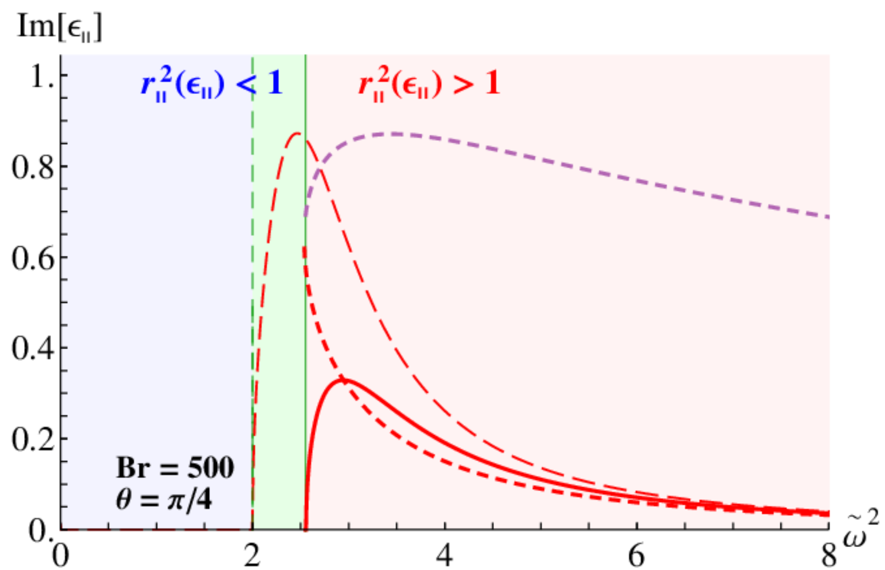
$$\epsilon = \frac{1+\chi_1}{1+\chi_1 \cos^2\theta}$$

$\chi_1(q_\parallel^2, q_\perp^2; B_\mathrm{r})$

$$q_\parallel^2 = \omega^2 - q_z^2 = \omega^2(1-\epsilon\cos^2\theta)$$
$$q_\perp^2 = -|\boldsymbol{q}_\perp|^2 = -\epsilon\,\omega^2\sin^2\theta$$

Damping of the incident photon field due to decay into a fermion pair





Complex refractive index

$$\begin{aligned} \epsilon &= \epsilon_r + i\epsilon_i \\ n &= n_r + i n_i \end{aligned}$$

$$n^2 = \epsilon \quad \Rightarrow \quad \begin{cases} n_r = \frac{1}{\sqrt{2}}\sqrt{|\epsilon| + \epsilon_r} \\ n_i = \frac{1}{\sqrt{2}}\sqrt{|\epsilon| - \epsilon_r} \end{cases} .$$

Some plots of refractive index were shown here.

For details, please contact the speaker (Koichi Hattori)
by email: khattori@yonsei.ac.kr

Phenomenological aspects

Based on the plots of the refractive index, we discussed application to the photon spectrum in HIC here.

Effects on photon HBT interferometry

Modified refraction index induces a distorted HBT image

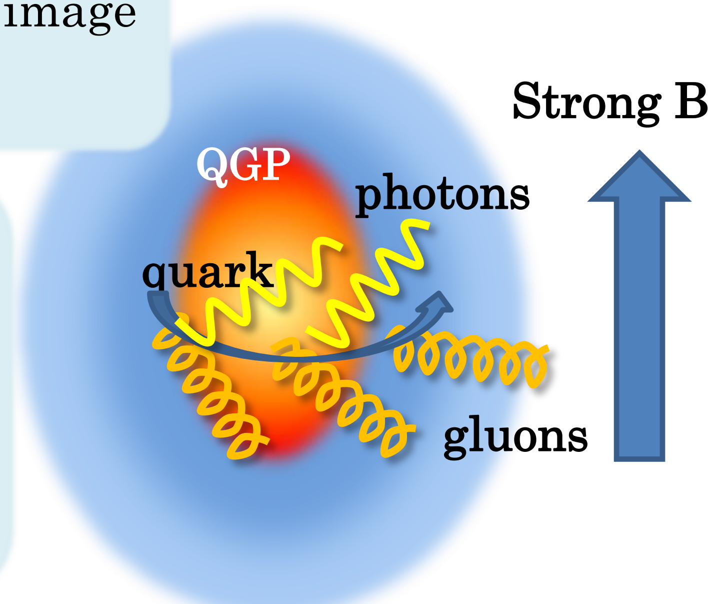
K.Itakura and KH, (2011)

Magnetic field induced photon/gluon emissions

Synchrotron radiation of photon/gluon from quark

K.Tuchin, PRC (2010), [hep-ph/1209.0799] (2012)

See K.Itakura's talk in "Parallel 3C"



Summary

- We analytically evaluated the photon vacuum polarization tensor in external magnetic fields.

- We inspected the complex dielectric constant/refraction index around the lowest-Landau-level threshold with a self-consistent treatment.

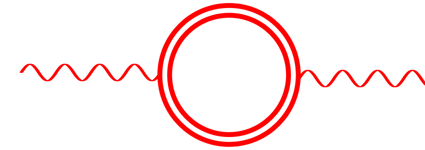
Prospects

- Application to photon spectra in heavy-ion collisions. In progress

- Quarkonia in strong fields
  In progress, in collaboration with S. H. Lee

Let's see a direct approach to the initial stage in HIC events by probing phenomena that uniquely occur in strong fields

Scalar coefficient functions χ in the proper-time method



Given by a double integral wrt proper time variables associated with two fermion lines

$$\chi_i(r_\parallel^2, r_\perp^2, B_\mathrm{r}) = \frac{\alpha B_\mathrm{r}}{4\pi} \int_{-1}^{1} d\beta \int_0^\infty d\tau \; \frac{\Gamma_i(\tau,\beta)}{\sin(\tau)} \; e^{-i(\phi_\parallel + \phi_\perp)\tau}$$

$$i = 0,\ 1,\ 2$$

Dimesionless variables

$$B_\mathrm{r} = \frac{B}{B_c}\ , \quad r_\parallel^2 = \frac{q_\parallel^2}{4m^2}\ , \quad r_\perp^2 = \frac{q_\perp^2}{4m^2}$$

$$\begin{aligned} q^\mu &= (q^0, q_\perp, 0, q^3) \\ q_\parallel^\mu &= (q^0, 0, 0, , q^3) \\ q_\perp^\mu &= (0, q_\perp, 0, 0) \end{aligned}$$

Integrands having strong oscillation

$$\begin{cases} \Gamma_0(\tau,\beta) = \cos(\beta\tau) - \beta\sin(\beta\tau)\cot(\tau) \\ \Gamma_1(\tau,\beta) = (1-\beta^2)\cos(\tau) - \Gamma_0(\tau,\beta) \\ \Gamma_2(\tau,\beta) = 2\frac{\cos(\beta\tau)-\cos(\tau)}{\sin^2(\tau)} - \Gamma_0(\tau,\beta) \quad . \end{cases}$$

Schwinger, Adler, Shabad, Urrutia, Tsai and Eber, Dittrich and Gies

$$\phi_\parallel(r_\parallel^2, B_\mathrm{r}) = \frac{1}{B_\mathrm{r}} \left\{1 - (1-\beta^2)\ r_\parallel^2\right\}$$

$$\phi_\perp(r_\parallel^2, B_\mathrm{r}) = -\frac{2r_\perp^2}{B_\mathrm{r}} \cdot \frac{\cos(\beta\tau) - \cos(\tau)}{\sin(\tau)}$$

Exponentiated trig-functions generate strongly oscillating behavior with arbitrarily high frequency.

Analytic calculation of the double integral

$$\chi_i(r_\parallel^2, r_\perp^2, B_\mathrm{r}) = \frac{\alpha B_\mathrm{r}}{4\pi}\int_{-1}^{1} d\beta \int_0^\infty d\tau \; \frac{\Gamma_i(\tau,\beta)}{\sin(\tau)} \; e^{-i(\phi_\parallel+\phi_\perp)\tau}$$

$$\phi_\perp(r_\parallel^2, B_\mathrm{r}) = \eta \cdot \frac{\cos(\beta\tau) - \cos(\tau)}{\sin(\tau)}$$

$$\eta = -2\frac{r_\parallel^2}{B_\mathrm{r}} = \frac{|\boldsymbol{q}_\perp|^2}{2|eB|}$$

$$u = \eta / \sin\tau$$

Two important relations

★ $$e^{-iu\cos(\beta\tau)} = \sum_{n=0}^{\infty}(2-\delta_{n0}) I_n(-iu) \; e^{in\beta\tau}$$

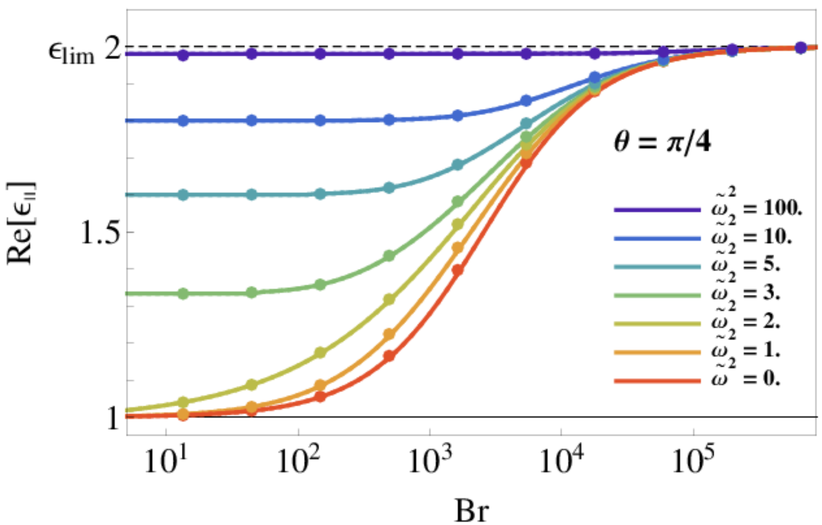
★ $$e^{i\eta\cot\tau} \; I_n(-iu) = \; \eta^n \; e^{-\frac{|\boldsymbol{q}_\perp|^2}{2|eB|}} \sum_{\ell=0}^{\infty} \frac{\ell!}{(\ell+n)!} \; [L_\ell^n(\eta)]^2 \; \left(1 - e^{-2i\tau}\right) e^{-(2\ell+n)i\tau}$$

Associated Laguerre polynomial

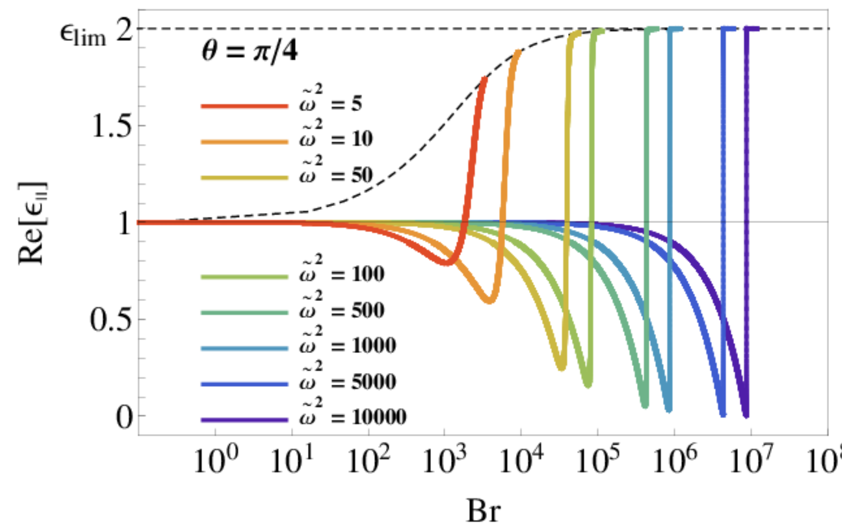
Any term reduces to either of elementary integrals.

$$\begin{aligned}
F_\ell^n(r_\parallel^2, B_\mathrm{r}) &= \frac{i}{B_\mathrm{r}} \int_{-1}^{1} d\beta \int_0^\infty d\tau \; e^{-i\left(\phi_\parallel + 2\ell - n\beta + n\right)\tau} \\
G_\ell^n(r_\parallel^2, B_\mathrm{r}) &= \frac{i}{B_\mathrm{r}} \int_{-1}^{1} d\beta \int_0^\infty d\tau \; \beta \, e^{-i\left(\phi_\parallel + 2\ell - n\beta + n\right)\tau} \\
H_\ell^n(r_\parallel^2, B_\mathrm{r}) &= \frac{i}{B_\mathrm{r}} \int_{-1}^{1} d\beta \int_0^\infty d\tau \; \beta^2 \, e^{-i\left(\phi_\parallel + 2\ell - n\beta + n\right)\tau}
\end{aligned}$$

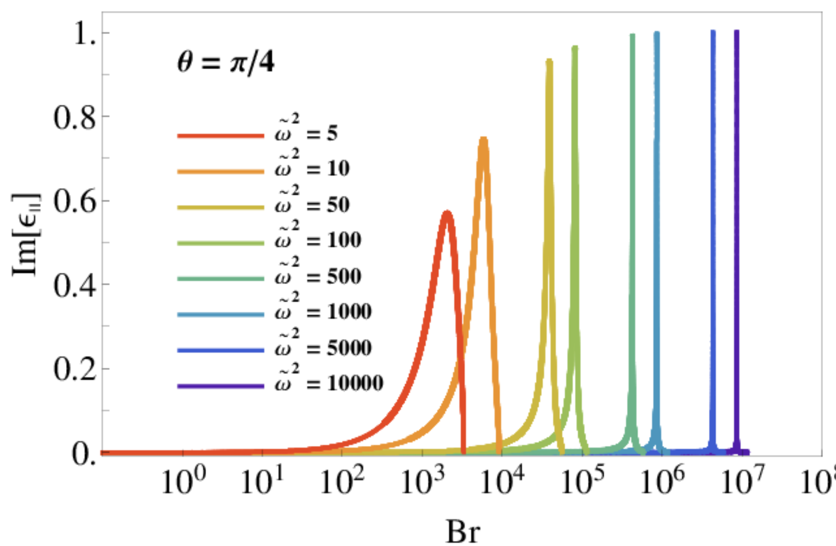
Real part of ε on stable branch



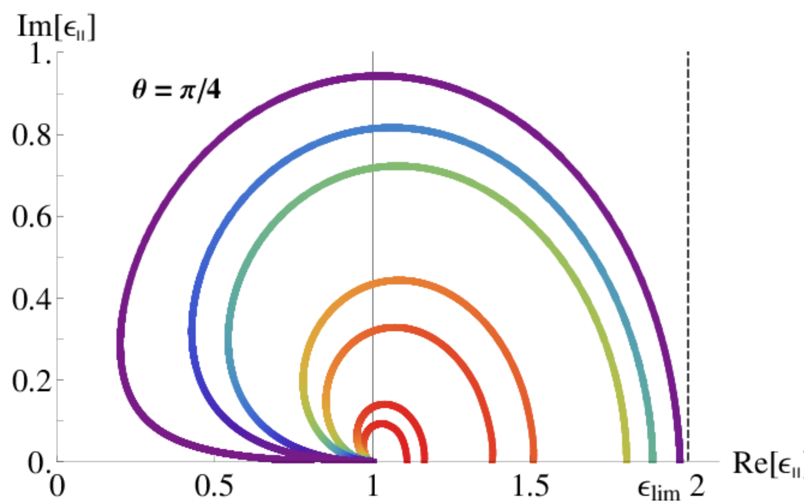
Real part of ε on unstable branch



Imaginary part of ε on unstable branch



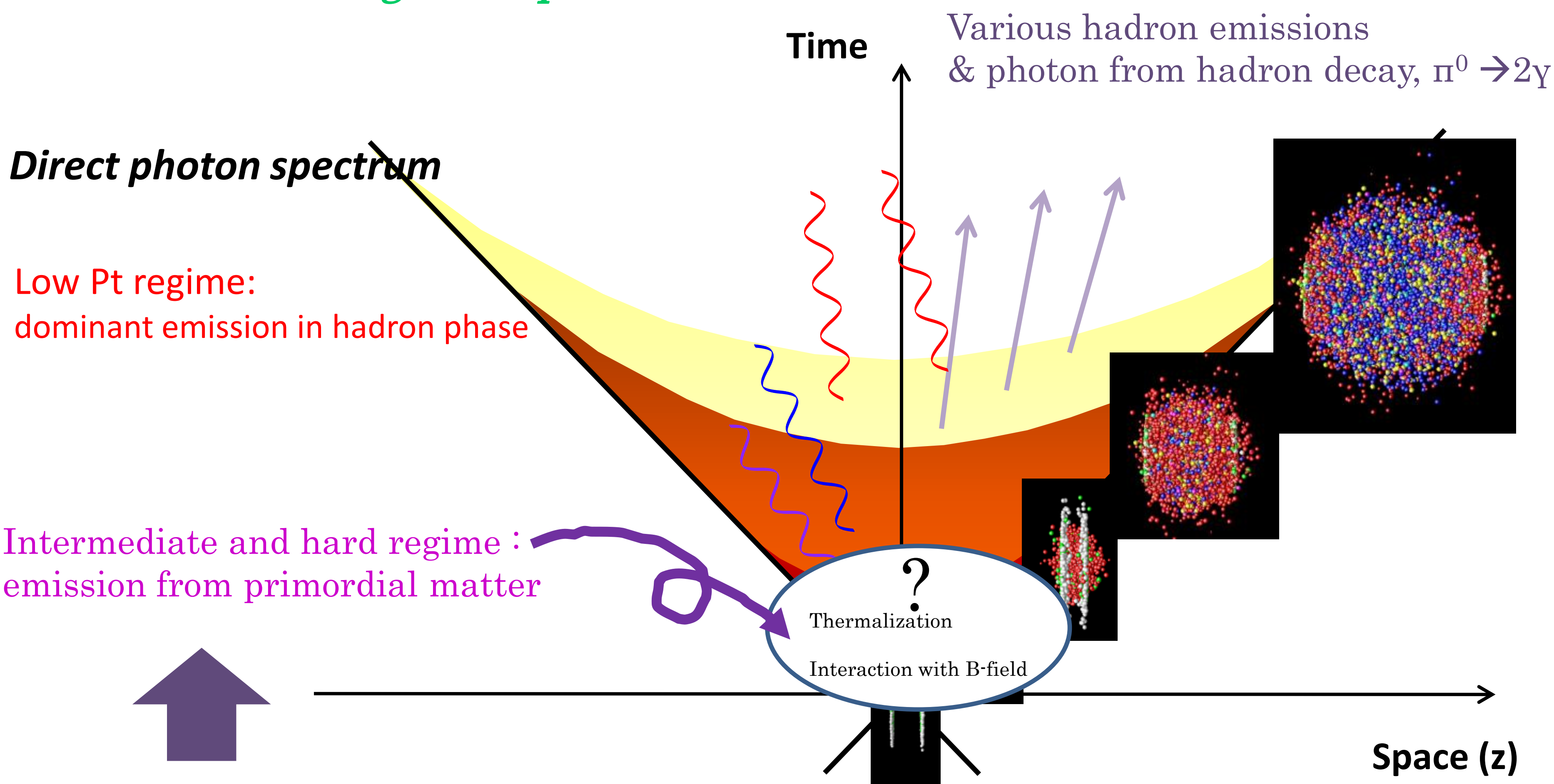
Relation btw real and imaginary parts on unstable branch



Br = (50,100,500,1000,5000,10000, 50000)

Direct photon from initial stage in UrHIC

Initial and background photons



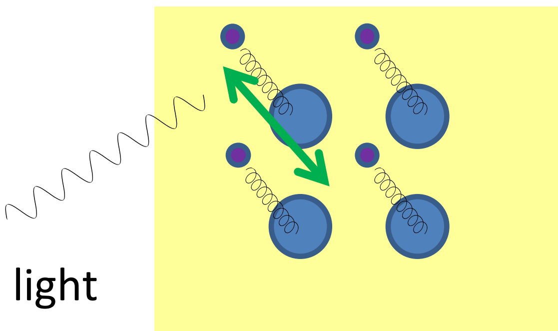
Directly accessible to the initial stage and QGP

Detecting photon from the initial stage

⇔ Detecting effects of B-field

Schematic picture of the birefringence

Polarization in dielectric medium :
a classical argument



Incident light

$$\boldsymbol{P} = qN\boldsymbol{x} \quad \begin{cases} q \text{ : charge} \\ N \text{ : density of dipoles} \\ \boldsymbol{x} \text{ : displacement} \end{cases}$$

$$m\ddot{\boldsymbol{x}} + \gamma\dot{\boldsymbol{x}} + U\boldsymbol{x} = q\boldsymbol{E}$$

Dissipation

Linear bound force

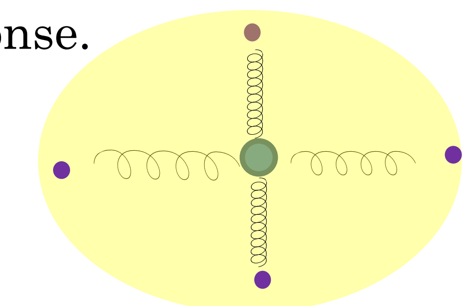
Incident light field

Lorentz-type dispersion :

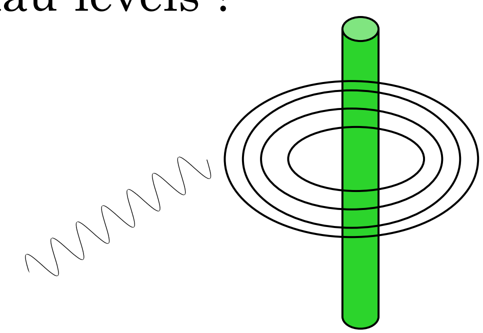
$$\chi \propto \frac{1}{\omega^2 - \omega_0^2 + i\gamma\omega}$$

with characteristic frequency : $\omega_0^2 = U/m$

Anisotropic constants result
in an anisotropic response.



What happens with the anisotropic (discretized)
spectrum by the Landau-levels ?



Close look at the integrals

What dynamics is encoded in the scalar functions ?

$$\begin{aligned} F_\ell^n(r_\parallel^2, B_\mathrm{r}) &= I_{\ell\Delta}^n(r_\parallel^2) \\ G_\ell^n(r_\parallel^2, B_\mathrm{r}) &= \mathcal{G}_\ell^n[\ I_{\ell\Delta}^n(r_\parallel^2)\ ;\ r_\parallel^2, B_\mathrm{r}\ ] \\ H_\ell^n(r_\parallel^2, B_\mathrm{r}) &= \mathcal{H}_\ell^n[\ I_{\ell\Delta}^n(r_\parallel^2)\ ;\ r_\parallel^2, B_\mathrm{r}\ ] \end{aligned}$$

$$I_{\ell\Delta}^n(r_\parallel^2) = \frac{2}{\sqrt{4ac-b^2}}\left[\ \arctan\left(\frac{b+2a}{\sqrt{4ac-b^2}}\right) - \arctan\left(\frac{b-2a}{\sqrt{4ac-b^2}}\right)\ \right]$$

$$a = r_\parallel^2\ ,\quad b = -nB_\mathrm{r}\ ,\quad c = (1-r_\parallel^2) + (2\ell+n)B_\mathrm{r}$$

An imaginary part representing a real photon decay

$$b^2 - 4ac = 0 \quad \Leftrightarrow \quad (-nB_\mathrm{r})^2 - 4r_\parallel^2\left[(1-r_\parallel^2) + (2\ell+n)B_\mathrm{r}\right] = 0$$

$$\Leftrightarrow \quad q_\parallel^2 = \left[\ \sqrt{m^2 + 2\ell eB} + \sqrt{m^2 + 2(\ell+n)eB}\ \right]^2$$

Invariant mass of a fermion-pair in the Landau levels

Analytic results!

Applicable to any momentum regime and field strength !
Applicable to both on-shell and off-shell photon!

$$\chi_i = \frac{\alpha B_{\mathrm{r}}}{4\pi} e^{-\eta} \underbrace{\sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty}}_{\text{Sum wrt Landau levels}} (2-\delta_{n0}) \underbrace{\Omega^n_{\ell\, i} [\, F^n_\ell,\ G^n_\ell,\ H^n_\ell\,]}_{\text{Combination of known functions}}$$

Photon decay channel opens at every Landau level

$$I^n_{\ell\Delta}(r_\parallel^2) = \begin{cases} \frac{1}{2\sqrt{(r_\parallel^2 - s_-)(r_\parallel^2 - s_+)}} \log\left|\frac{a-c-\sqrt{b^2-4ac}}{a-c+\sqrt{b^2-4ac}}\right| & (r_\parallel^2 < s_-) \\ \frac{1}{\sqrt{|(r_\parallel^2 - s_-)(r_\parallel^2 - s_+)|}} \left[\arctan\left(\frac{b+2a}{\sqrt{4ac-b^2}}\right) - \arctan\left(\frac{b-2a}{\sqrt{4ac-b^2}}\right) \right] & (s_- < r_\parallel^2 < s_+) \\ \frac{1}{2\sqrt{(r_\parallel^2 - s_-)(r_\parallel^2 - s_+)}} \left[\log\left|\frac{a-c-\sqrt{b^2-4ac}}{a-c+\sqrt{b^2-4ac}}\right| + 2\pi i \right] & (s_+ < r_\parallel^2) \end{cases}$$