Low energy charmonium-hadron scattering in Lattice QCD

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Introduction

- Recently many charmonium($c\bar{c}$) and bottomonium($b\bar{b}$) like resonance XYZ and $Y_b Z_b$ are observed in several high energy experiments in the world.

- Among them, some resonances such as X(3872), Z(4430), and Z_b have very narrow widths and are observed near the hadron-hadron thresholds.

- These experimental observations draw attention to the properties and the structures of such resonances.

- It is important to understand these interesting resonances from Lattice QCD.

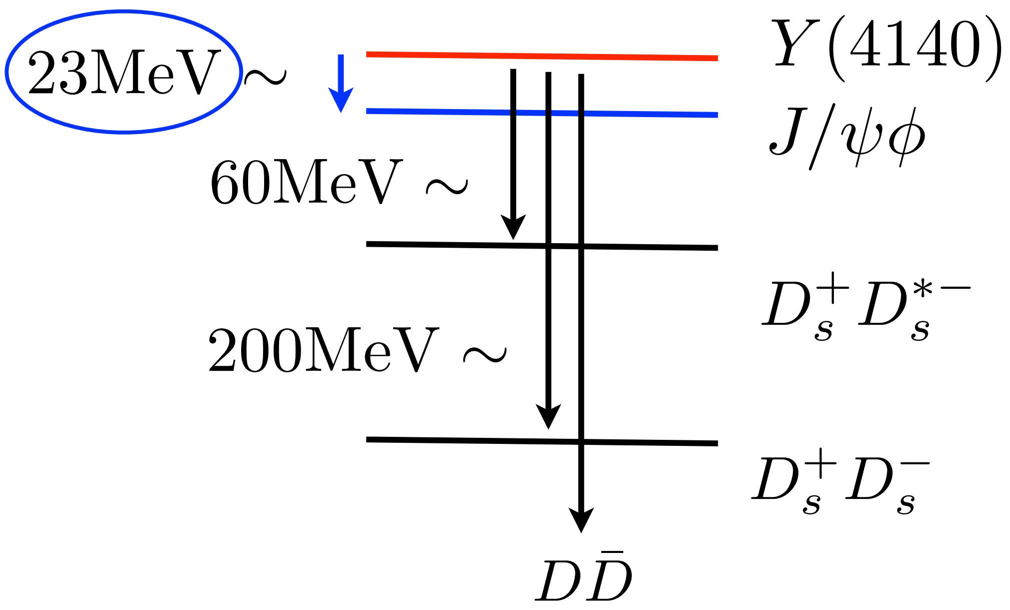
In 2009, a new hadron resonance Y(4140) are reported from B-meson decay by CDF collaboration. T. Aaltonen et al, PRL 102, 242002 (2009)

$$B \to J/\psi \phi K$$

$Y(4140)$

$$M_Y = 4143.0 \pm 2.9 \pm 1.2\mathrm{MeV}$$

$$\Gamma_Y = 11.6^{+8.3}_{-5.0} \pm 3.7\mathrm{MeV}$$ quite narrow width



It seems that Y(4140) states do not couple to open charm channels.

⟶ Is there a specific selection rule?

These features should be related to the structure of Y states, and charmonium(J/ψ)-hadron interactions.

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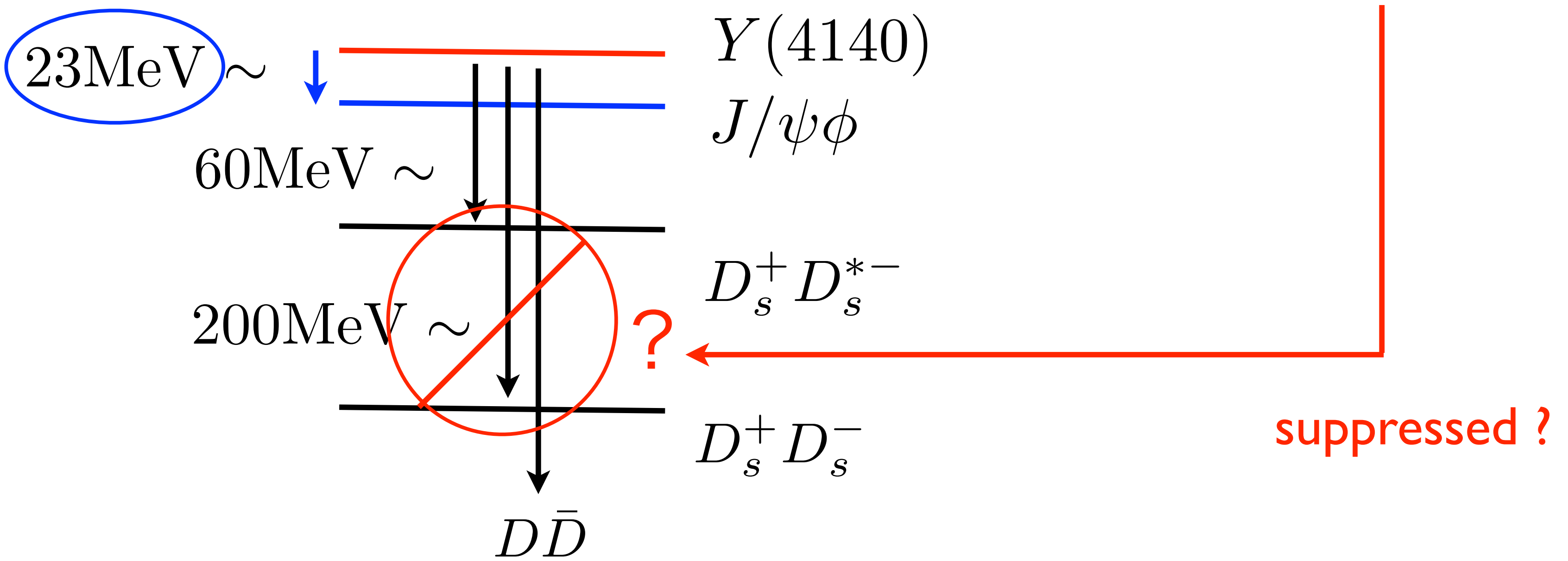

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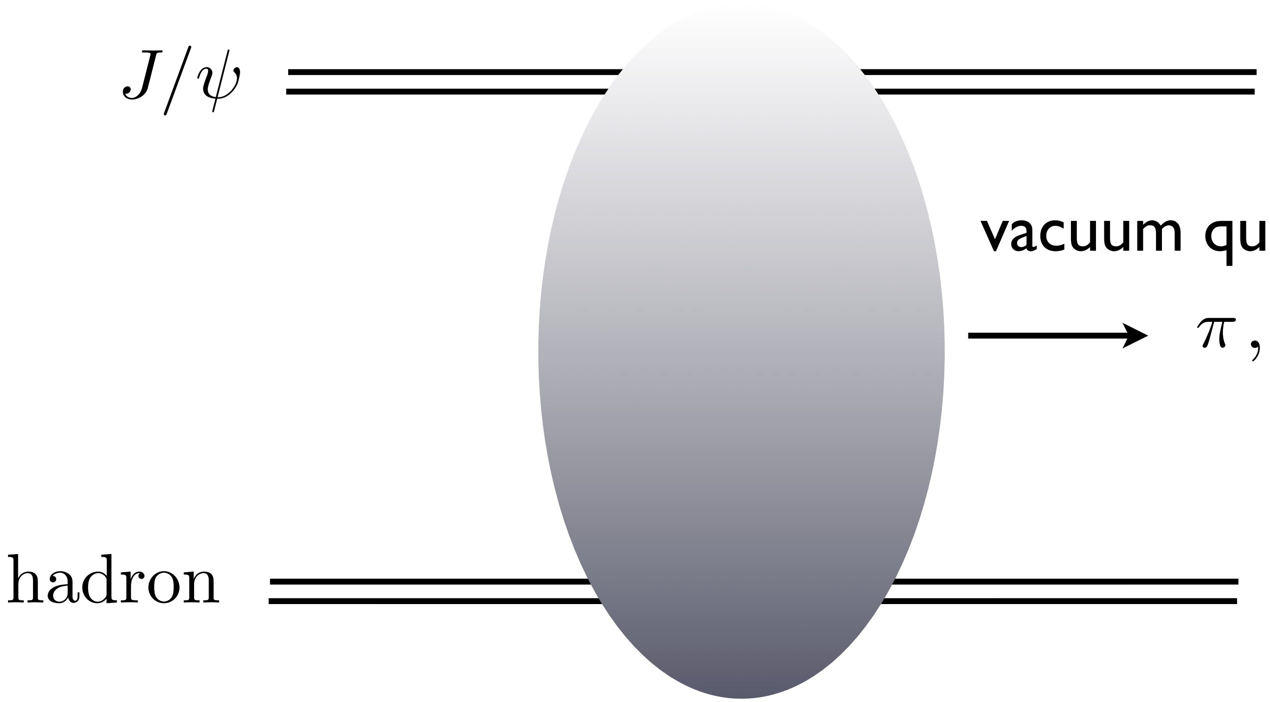
These features should be related to the structure of Y states, and charmonium(J/ψ)-hadron interactions.

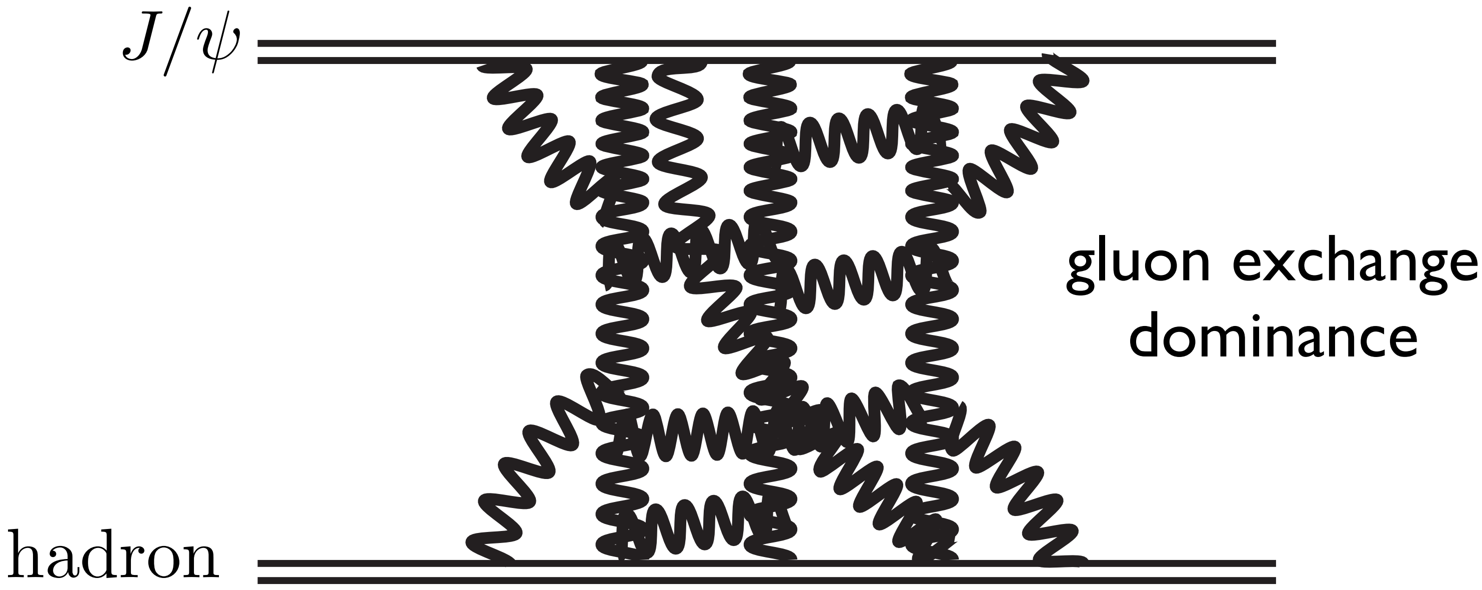
Charmonium-hadron interactions



vacuum quantum number

→ π , ρ exchanges are forbidden!

Charmonium-hadron interactions



So if there exist a Y resonance in charmonium-hadron system, gluons would play very interesting role!

Non-perturbative method such as lattice QCD is really needed to study this system.

In last year, CDF again reported the same peak with higher statistics.

T. Aaltonen et al, arXiv:1101.6058

On the other hand, two photon scattering experiment by Belle and B-meson decay in LHCb experiment did not observe Y(4140) yet.

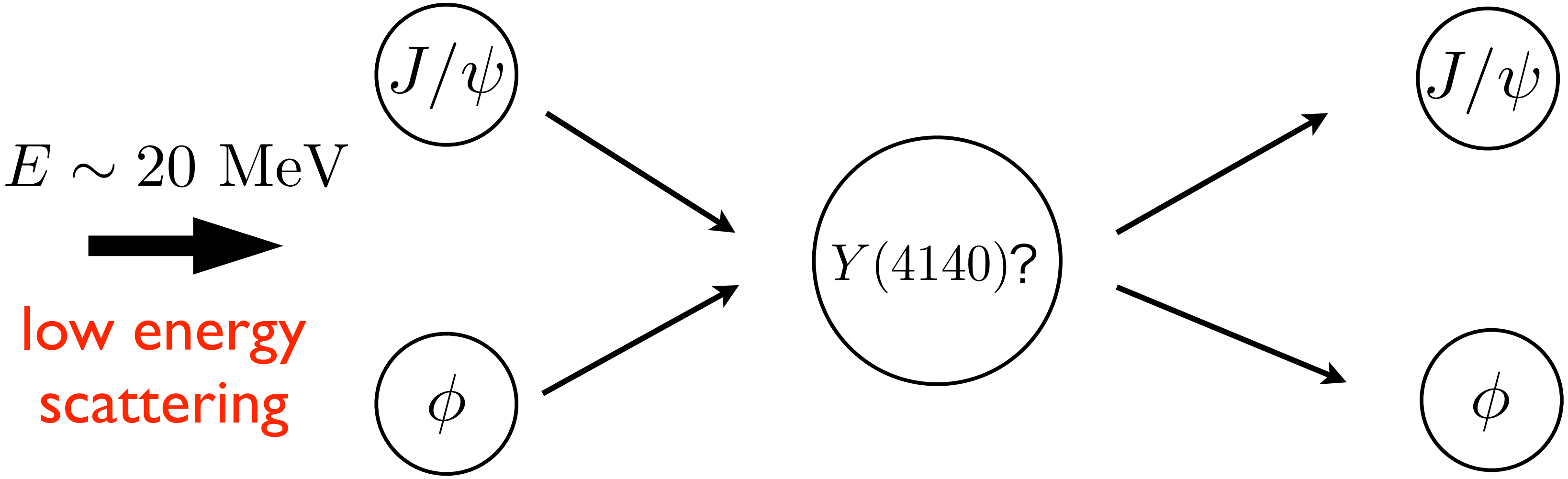
C. P. Shen et al PRL104, 112004 (2010)
R. Aaij et al, PRD85, 091103 (2012)

If Y(4140) exits, it has interesting features, but its existence is still controversial experimentally.

The purpose of this study is to develop an approach to investigate low energy hadron-hadron scatterings and to search for a narrow resonance.

Then, we apply our approach to J/psi-phi system and try to gain a new insight into Y(4140) from Lattice QCD.

Low energy J/ψ-Φ scattering and narrow resonance Y(4140) in C.M. system (zero total momentum)



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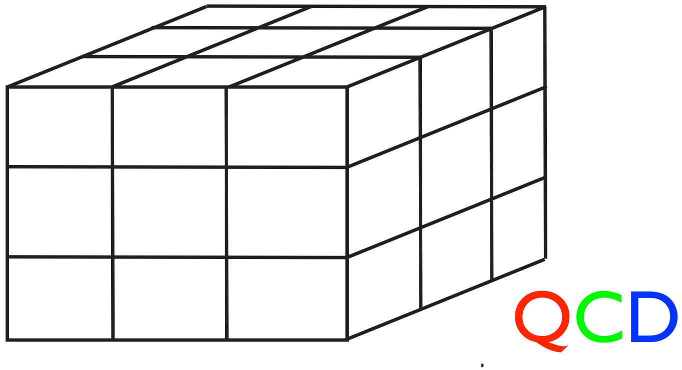
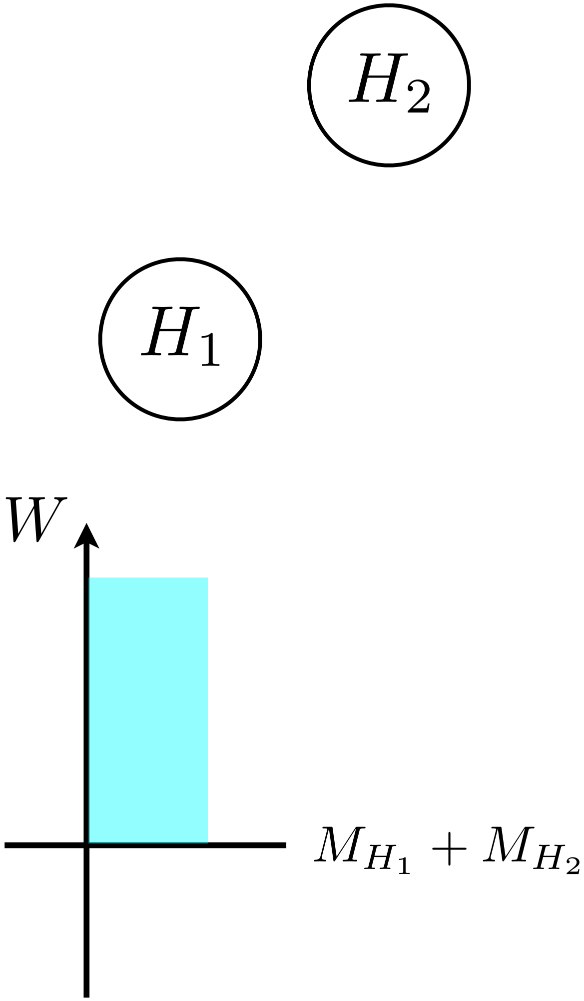
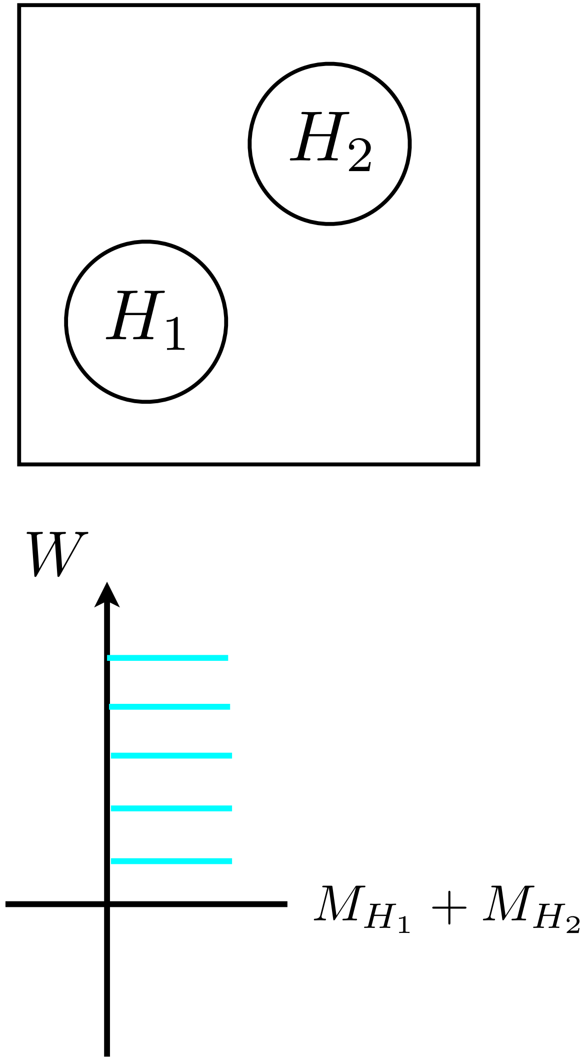


Scattering states in Lattice

Infinite volume system



Finite volume system



$$k = \sqrt{2\mu E}$$

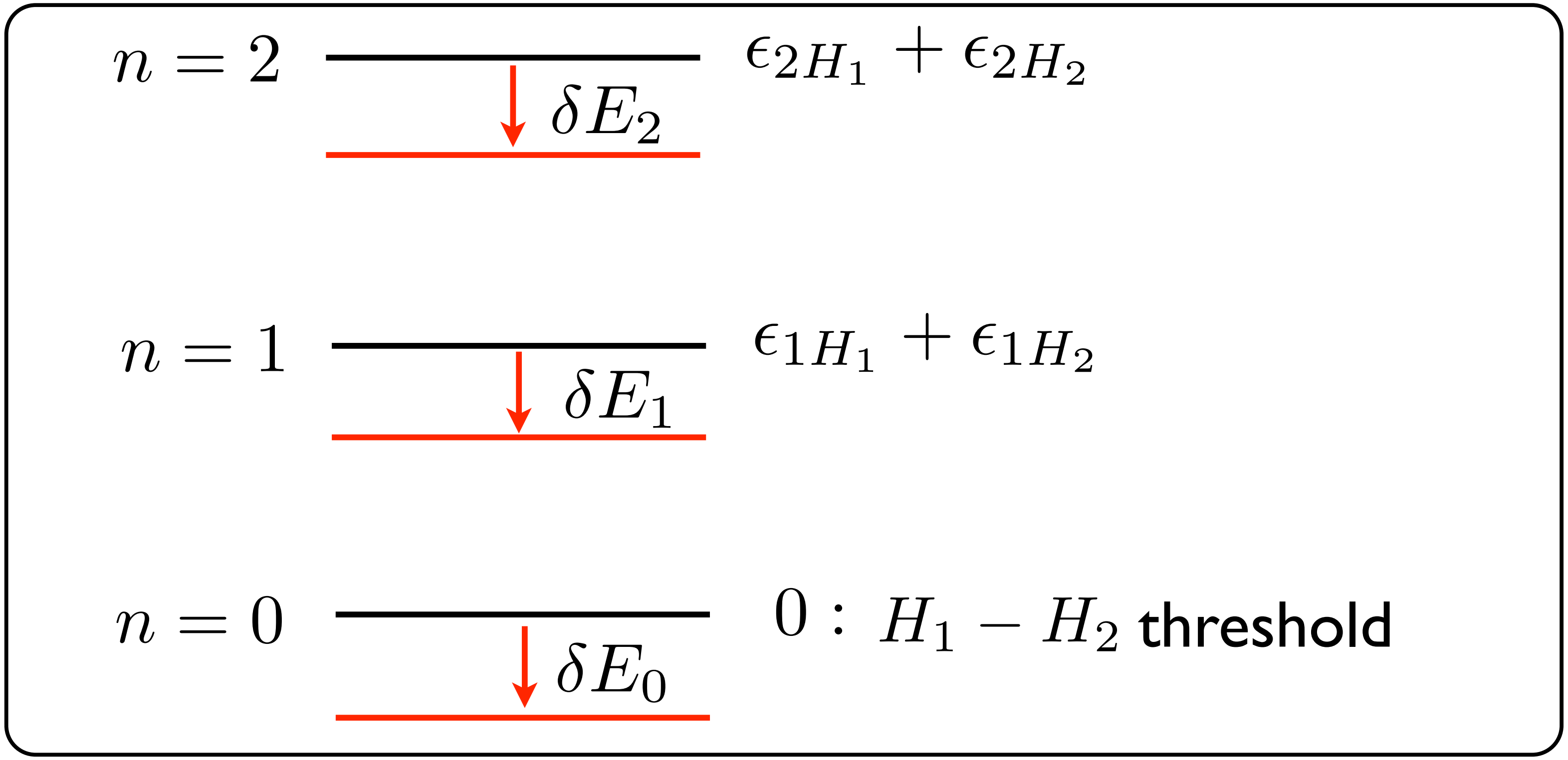
$$E_{nH} = \sqrt{\left(\frac{2\pi}{L}n\right)^2 + M_H^2}$$

$$E = W - (M_{H_1} + M_{H_2})$$

$$= [W - (E_{nH_1} + E_{nH_2})] + [E_{nH_1} - M_{H_1}] + [E_{nH_2} - M_{H_2}]$$

$$E = \delta E_n + \epsilon_{nH_1} + \epsilon_{nH_2}$$

Interaction strength   Energy of free 2 particles



Measurement of δE_n

Two-point function

$$G^{H_1}(t, t_{src}) =< \hat{O}_{H_1}(t)\hat{O}^{\dagger}_{H_1}(t_{src}) >$$

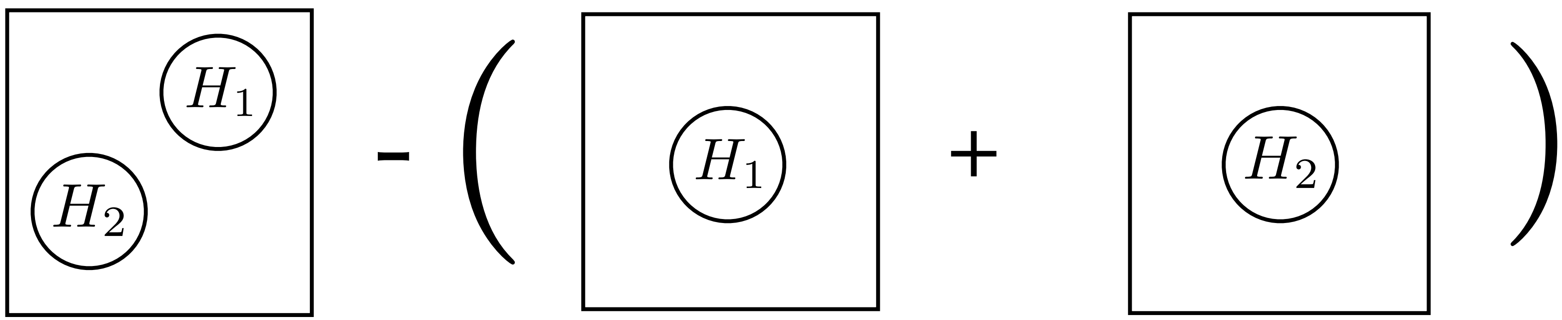
$$G^{H_2}(t, t_{src}) =< \hat{O}_{H_2}(t)\hat{O}^{\dagger}_{H_2}(t_{src}) >$$

Four-point function

$$G^{H_1-H_2}(t, t_{src}) =< \hat{O}_{H_1}(t)\hat{O}_{H_2}(t)[\hat{O}_{H_1}(t_{src})\hat{O}_{H_2}(t_{src})]^{\dagger} >$$

$$\frac{G^{H_1-H_2}(t, t_{src})}{G^{H_1}(t, t_{src})G^{H_2}(t, t_{src})} \sim e^{-\delta E_n t}$$

δE_n =



Luscher and Rummukainen-Gottlieb finite size formula

M. Luscher, NPB354, 531, (1991)

K. Rummukainen and S. Gottlieb, NPB450, 397, (1995)

$$\cot\delta_0 = \frac{1}{\pi^{3/2} q} Z_{00}^{(\vec{d})}(1; q^2) \quad , \qquad q = \frac{Lk}{2\pi}$$

with the generalized zeta-funciton

$$Z_{lm}^{\vec{d}}(1; q^2) = \sum_{\vec{r} \in \Gamma_{\vec{d}}} \frac{\mathcal{Y}_{lm}(\vec{r})}{\vec{r}^2 - q^2} \quad , \qquad \mathcal{Y}_{lm}(\vec{r}) = |\vec{r}|^l Y_{lm}(\Omega_r)$$

$$\Gamma_{\vec{d}} = \{\vec{r} | \vec{r} = \vec{n} + \frac{\vec{d}}{2}, \ \vec{n}, \vec{d} \in Z^3\}$$

Finite size formula is the relation which connects energy eigenvalue in a finite volume with scattering phase shift in infinite volume.

In order to search a "narrow" resonance in "low energy" regions near thresholds, we introduce the twisted boundary condition.

Periodic Boundary Condition

$$\phi(\vec{x}+L\vec{\epsilon}_i)=\phi(\vec{x}) \quad , \quad i=x,y,z$$

$$\longrightarrow \vec{k}=\frac{2\pi}{L}\vec{n}$$

$$E_1=k_1^2/2\mu \sim 100\ \mathrm{MeV} \longrightarrow$$ Bad resolution

Twisted Boundary Condition (TBC)

P.F. Bedaque, PLB593 (2004) 84

$$\phi(\vec{x}+L\vec{\epsilon}_i)=e^{i\theta_i}\phi(\vec{x})$$

$$\longrightarrow \vec{k}=\frac{2\pi}{L}(\vec{n}+\vec{d}) \quad , \quad \vec{d}=(\frac{\theta_x}{2\pi},\frac{\theta_y}{2\pi},\frac{\theta_z}{2\pi})$$

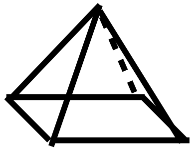
We can investigate low energy scatterings and search for a narrow resonance with a good energy resolution.

Symmetries with TBC in C.M. System

In the case of $\vec{\theta} = 0$ $\longrightarrow$ Cubic symmetry: O_h

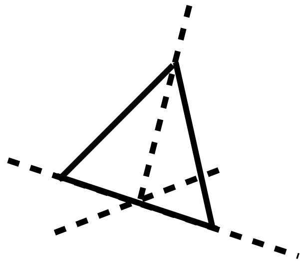
i) $\vec{\theta} = (0, 0, \theta)$

$\longrightarrow$ Tetragonal: C_{4v}



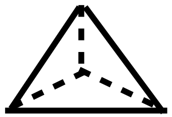
ii) $\vec{\theta} = (\theta, \theta, 0)$

$\longrightarrow$ Rhombic: C_{2v}

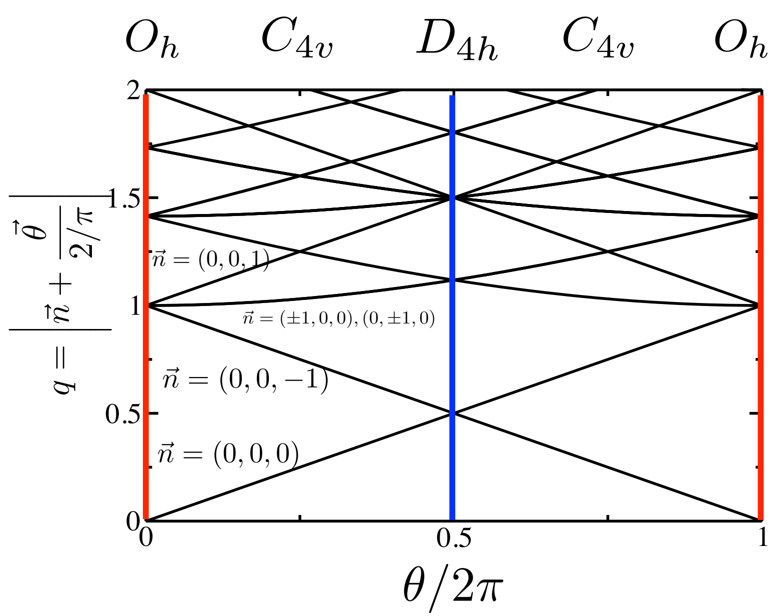


iii) $\vec{\theta} = (\theta, \theta, \theta)$

$\longrightarrow$ Trigonal: C_{3v}



i) $\vec{\theta} = (0, 0, \theta)$



- ▶ C_{nv} DO NOT have parity symmetry.
- ▶ Actually, these symmetries are same to two particle system of different mass and non-zero total momentum.

Z. Fu, PRD85, 014506 (2012)

L. Leskovec and S. Prelovsek, PRD85, 114507(2012)

M. Doring, et al, arXiv:1205. 4838 [hep-lat]

- With good parity symmetry, we can apply Luscher and Rummukainen-Gottlieb finite size formula

$$\cot\delta_0 = \frac{1}{\pi^{3/2} q} Z_{00}^{(\vec{d})}(1; q^2)$$

- In the case of a finite twisted angle

Finite size formula with twisted BC with (0,0, θ)

For A1 sector

$$\cot\delta_0 = m_{00} + \frac{|m_{10}|^2}{\cot\delta_1 - m_{11}}$$

mixing term from p-wave

$$Z_{lm}^{\vec{\theta}}(1; q^2) = \sum_{\vec{r} \in \Gamma_{\vec{\theta}}} \frac{\mathcal{Y}_{lm}(\vec{r})}{\vec{r}^2 - q^2}$$

$$\Gamma_{\vec{\theta}} = \{\vec{r} | \vec{r} = \vec{n} + \frac{\vec{\theta}}{2\pi}, \vec{n} \in Z^3\}$$

where

$$m_{00} = \frac{1}{\pi^{3/2} q} Z_{00}^{\vec{\theta}}(1; q^2)$$

$$m_{11} = \frac{1}{\pi^{3/2} q} Z_{00}^{\vec{\theta}}(1; q^2) + \frac{2}{\sqrt{5}\pi^{3/2} q^3} Z_{20}^{\vec{\theta}}(1; q^2)$$

$$m_{10} = \frac{i}{\pi^{3/2} q^2} Z_{10}^{\vec{\theta}}(1; q^2)$$

Here, we neglect higher wave contributions above l=2 owing to low energy scatterings near the threshold.

Our approach

i) Firstly we calculate δ_0 at $\vec{\theta}$= (0,0,0), (0,0,π),(π,π,0), (π,π,π) data with parity symmetry from Luscher and Rummukainen-Gottlieb formula:

$$\underline{\cot\delta_0} = \frac{1}{\pi^{3/2}q} Z_{00}^{\vec{\theta}}(1;q^2)$$

ii) From this δ_0, we calculate δ_1 with (θ, θ, θ) data from the formula:

$$\underline{\cot\delta_1} = \tilde{m}_{11} + \frac{|\tilde{m}_{10}|^2}{\underline{\cot\delta_0} - \tilde{m}_{00}}$$

$$\tilde{m}_{00} = \frac{1}{\pi^{3/2}q} Z_{00}^{\vec{\theta}}(1;q^2)$$

$$\tilde{m}_{11} = \frac{1}{\pi^{3/2}q} Z_{00}^{\vec{\theta}}(1;q^2) + \frac{2\sqrt{6}}{\pi^{3/2}\sqrt{5}q^3} \mathrm{Im}(Z_{22}^{\vec{\theta}}(1;q^2))$$

$$\tilde{m}_{10} = \frac{\sqrt{3}i}{\pi^{3/2}q^2} Z_{10}^{\vec{\theta}}(1;q^2)$$

iii) Using the δ1, we also calculate low energy δ0 with (0, 0, θ) data from the formula:

$$\cot\delta_0 = m_{00} + \frac{|m_{10}|^2}{\underline{\cot\delta_1} - m_{11}}$$

where

$$m_{00} = \frac{1}{\pi^{3/2} q} Z_{00}^{\vec{\theta}}(1; q^2)$$

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$$m_{10} = \frac{i}{\pi^{3/2} q^2} Z_{10}^{\vec{\theta}}(1; q^2)$$

We obtain information of low energy s-wave and p-wave scattering phase shifts.

Lattice set up

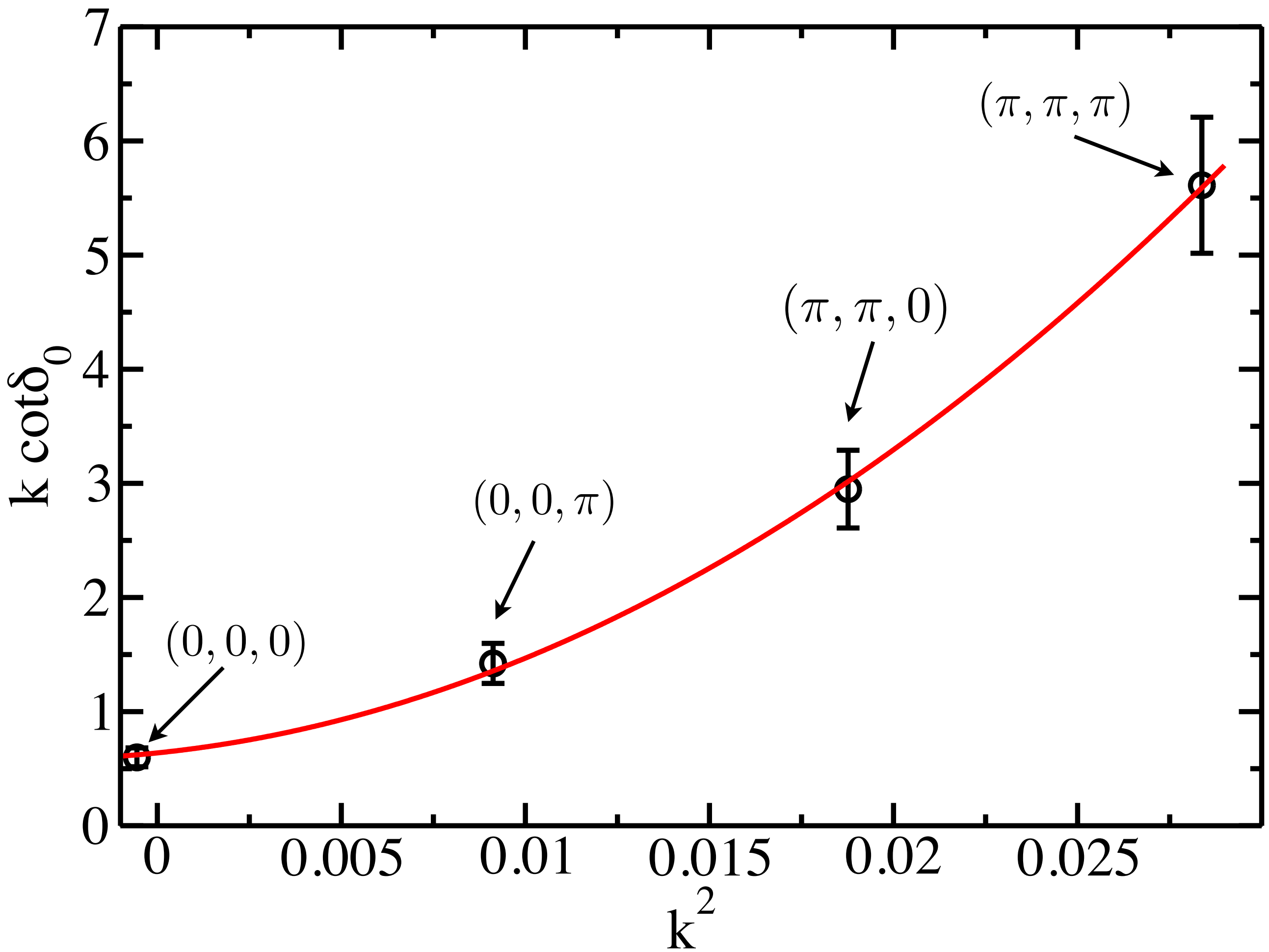
- PACS-CS 2+1 flavor dynamical gauge configurations at $m_\pi = 156\ \mathrm{MeV}$ S.Aoki et al, PRD79, 034503, 2009
  - Iwasaki gauge action + Clover fermion action
  - 32^3 x 64 lattice
  - a = 0.0907(13) fm
  - La ~ 2.9 fm
  - 198 configs
  - Wall source
- Relativistic Heavy Quark (RHQ) action for charm

  Y. Namekawa et al, PRD84:074505, 2011
  - Tsukuba type RHQ action (5 parameters)

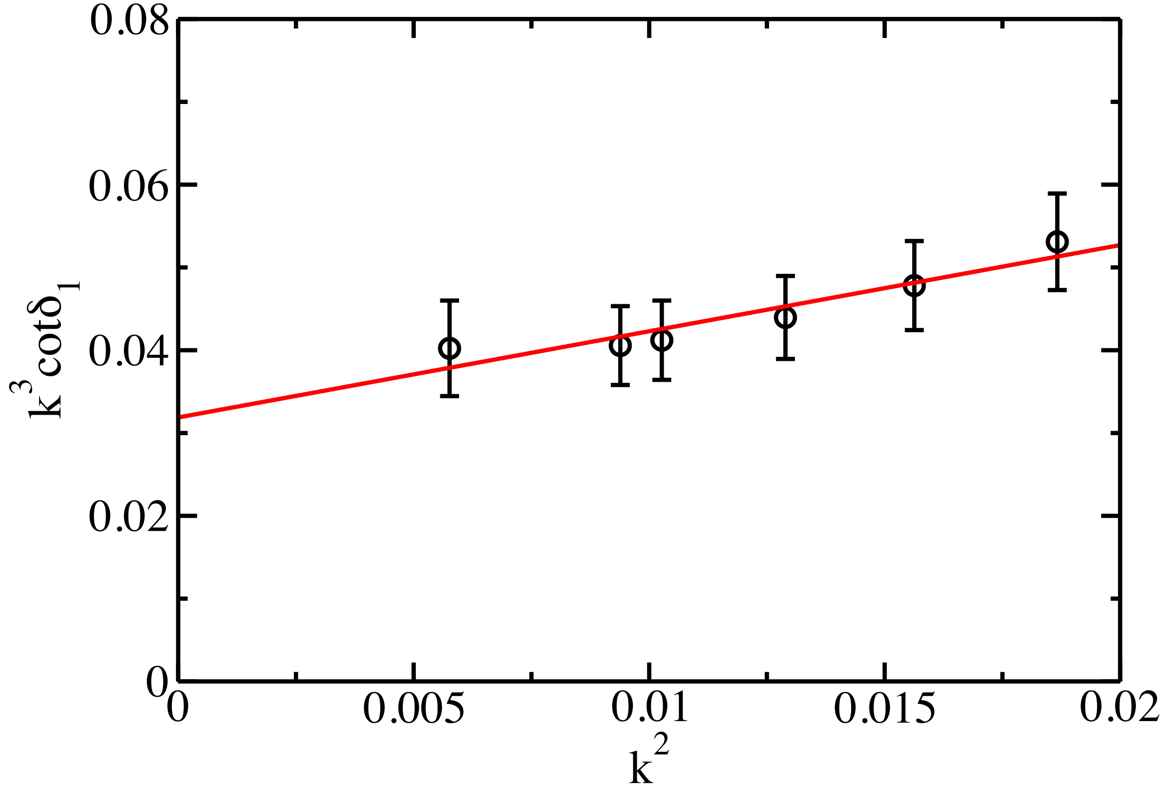
κ_{charm}	ν	r_s	c_B	c_E
0.1082	1.2153	1.2131	2.0268	1.7911

Result

i) kcotδ0 from $(0,0,0)$, $(0,0,\pi)$, $(\pi,\pi,0)$, (π,π,π) data



ii) k^3cotδ1 from (θ, θ, θ) data

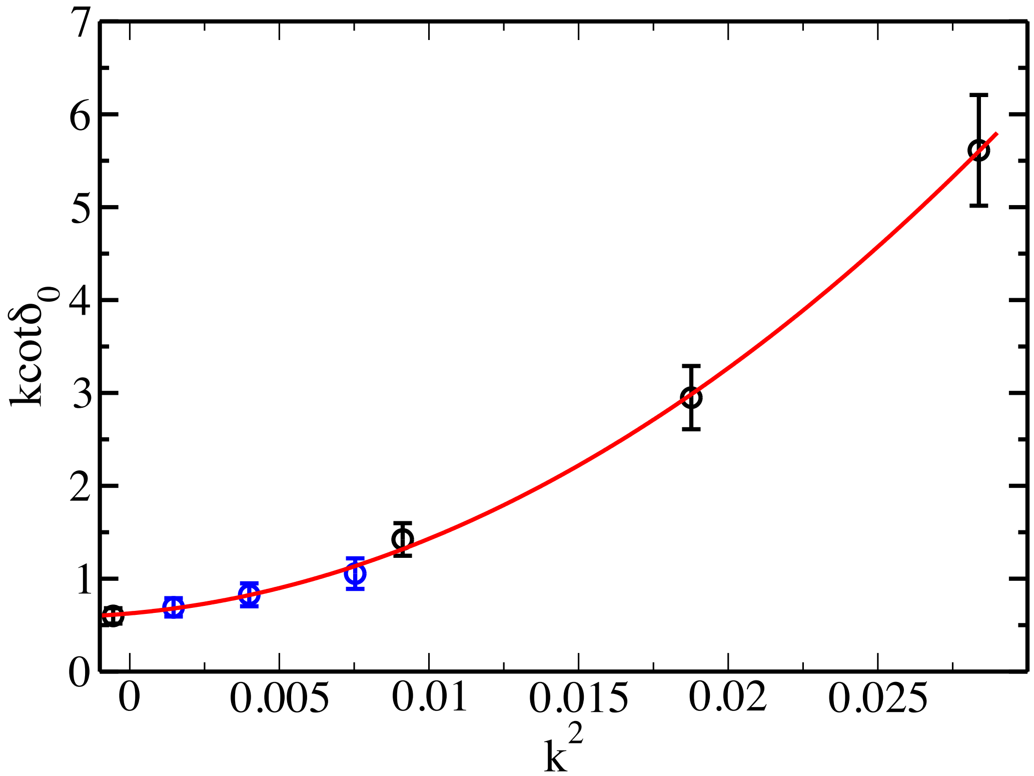


At k=0, we can obtain threshold parameter of p-wave

$$\frac{\tan\delta_1}{k^3}|_{k=0} = a_1$$

$$a_1 = 0.0234 \pm 0.0039 \;[\mathrm{fm}^3]$$

iii) Using previous δ1, we evaluate low energy δ0 from $(0, 0, \theta)$ data



Effective range expansion

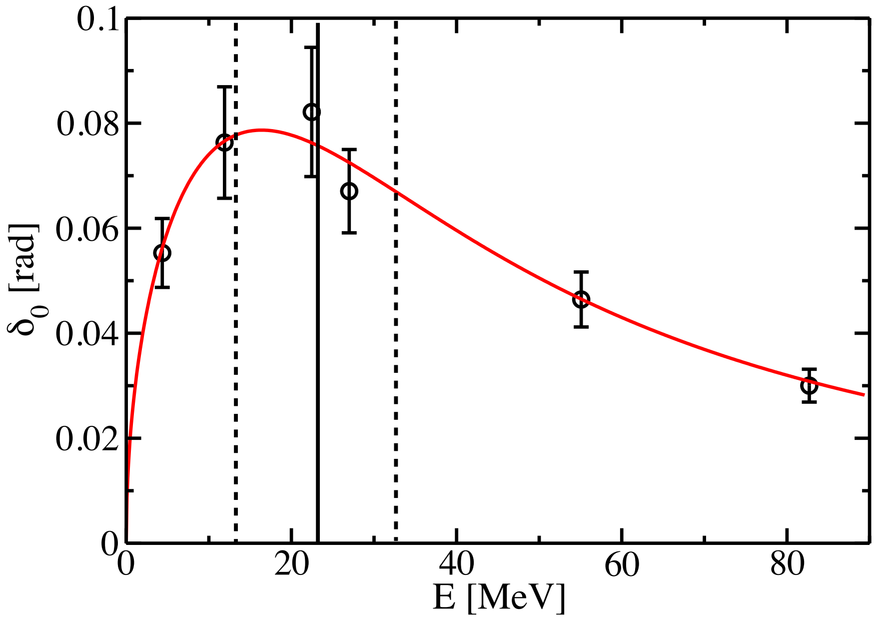
$$k\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 k^2 + p r_0^3 k^4$$

From jack knife fit

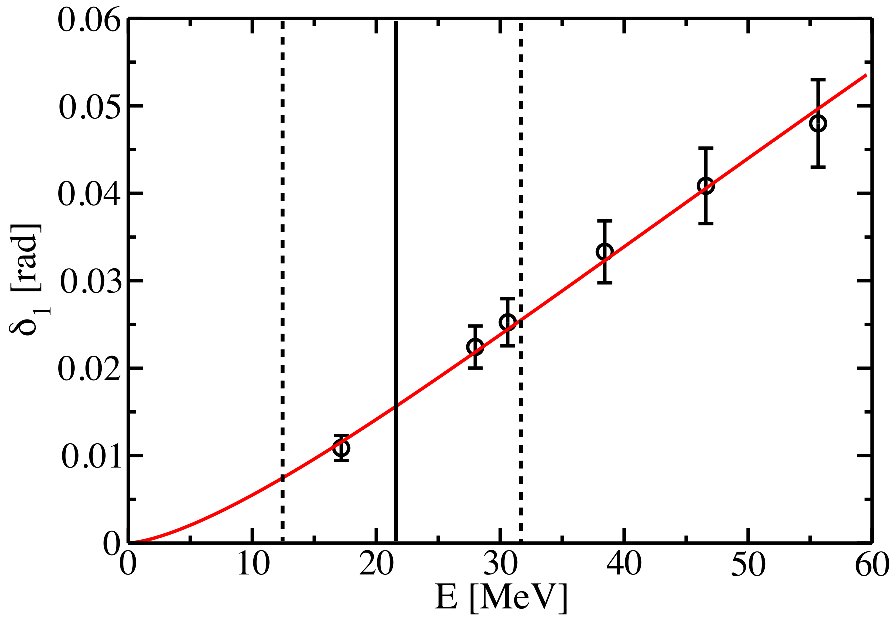
$$a_0 = 0.145 \pm 0.021 \ [\mathrm{fm}]$$
$$r_0 = 5.21 \pm 1.46 \ [\mathrm{fm}]$$
$$p r_0^3 = 3.85 \pm 0.47 \ [\mathrm{fm}^3]$$

Phase shifts near the threshold

s-wave



p-wave



Both figures show typical behaviors of low energy s- and p-wave phase shifts.

No structure in resonance point reported from CDF collaboration.

⟶ Our results prefer Belle and LHCb experiments.

Summary

- We develop an approach to investigate low energy hadron-hadron scatterings from lattice QCD by using finite size formula with twisted boundary condition.

- We apply our approach to low energy J/psi-phi scattering.

- Our result shows typical behaviors of low energy s-wave and p-wave phase shifts, but there is no structure at resonance point reported from CDF collaboration.

  ⟶ This result is consistent with Belle and LHCb experiments.

- We also obtain low energy scattering parameters such as scattering length, effective range and effective volume from effective range expansion.

Prospectives

- Apply to other systems in order to study low energy hadron-hadron scatterings and search for narrow resonances such as
  - D^0-Ds* and Tcc
  - J/ψ-ω and Y(3915)
  - Bottomonium-hadron system
  - D-K and Ds*

- Compare with the results from the potential approach.
  - J/ψ-N system
  - J/ψ-Φ system