Finite coupling correction to heavy quark potential and jet quenching parameter from AdS/CFT

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Zhang,Hou, Ren,Yin , JHEP1107:035(2011);
Zhang,Hou, Ren, arxiv1210.5187, JHEP
Chu, Hou , Ren JHEP08: 004 (2009)

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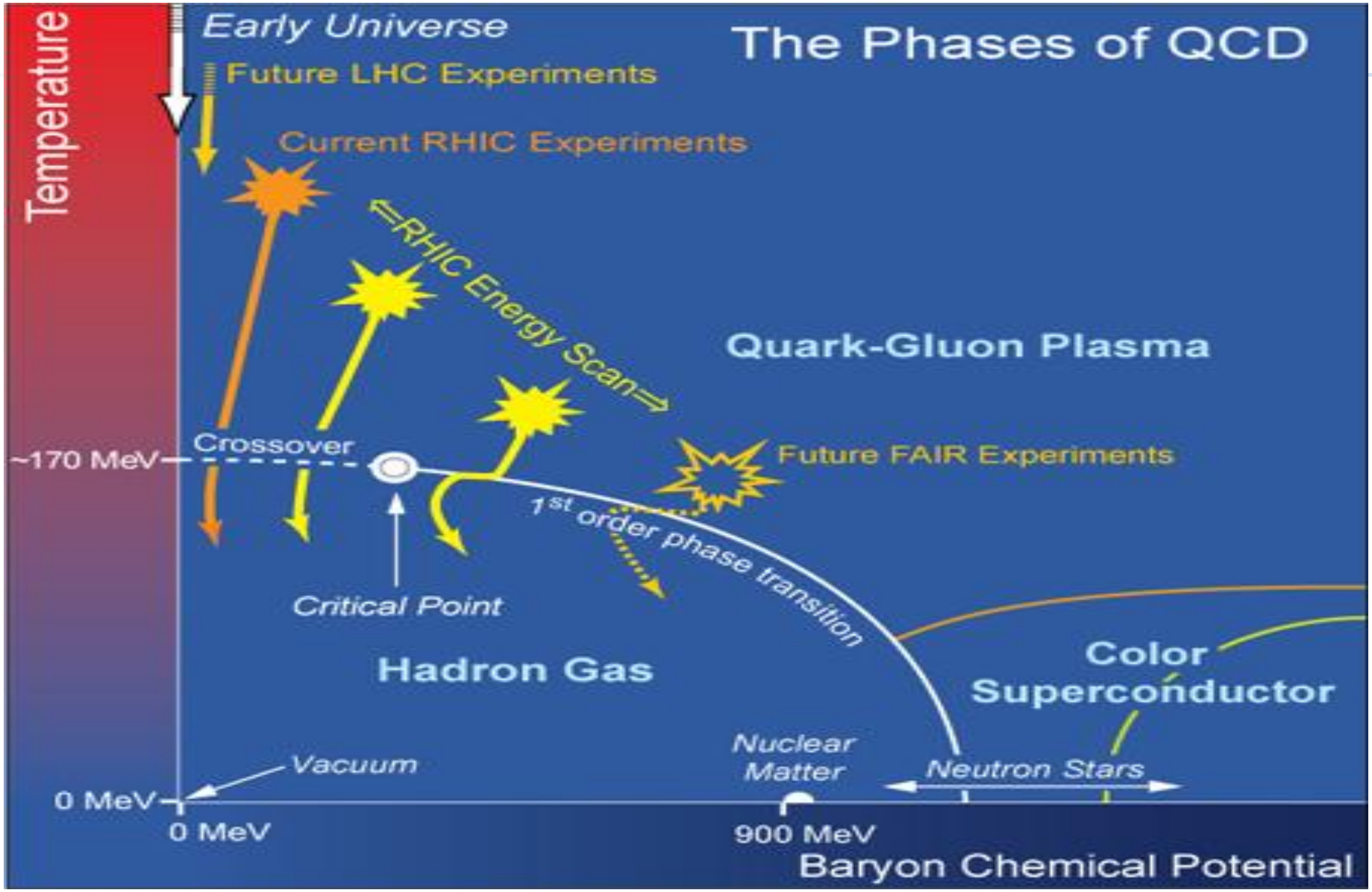
OUTLINES

- **Introduction**
- **NL Holographic heavy quark potential**
- **NL correction to jet quenching**
- **Summary**

Motivations



Many interesting phenomena in QCD lie in the strongly coupled region

New theoretical techniques needed!

Lattice QCD: a first-principle computation.

However, there are technical difficulties in the computations if the system has

1. Finite baryon chemical potential
2. Real time dynamics

…

AdS/CFT corerspondence

4dim. Large-Nc strongly coupled SU(Nc) N=4 SYM (finite T).

Maldacena '97 $\updownarrow$ **conjecture** Witten '98

Type II B Super String theory on AdS5-BH $\times$ S5

Maldacena conjecture: *Maldacena, Witten*

$N=4$ SUSY YM on the boundary $\Leftrightarrow$ Type IIB string theory in the bulk

't Hooft coupling $\lambda \equiv N_c g_{YM}^2 = \frac{1}{\alpha'^2}$ (string tension $= \frac{1}{2\pi\alpha'}$)

$$\frac{\lambda}{N_c} = 4\pi g_s$$

$$< e^{\int d^4x \phi_0(x) O(x)} > = Z_{\text{string}}[\phi(x,0)=\phi_0(\text{x})]$$

In the limit $N_c \to \infty$ and $\lambda \to \infty$

$$Z_{\text{string}}[\phi(x,0)=\phi_0(x)] = e^{-I_{\text{sugra}}[\phi]} |_{\phi(x,0)=\phi_0(x)}$$

$$I_{\text{sugra}}[\phi] = \text{classical supergravity action}$$

AdS/CFT applied to RHIC physics

- **Viscosity ratio, η/s.** $$\frac{\eta}{s}=\frac{1}{4\pi}$$ Policastro, Son and Starinet
- **Thermodynamics.** $$s=\frac{3}{4}s^{(0)}$$ Gubser
- **Jet quenching** $$\hat{q}=\pi^{\frac{3}{2}}\frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\sqrt{\lambda}T^{3}$$ Liu, Rajagopal and Wiederman
- **Photon production ,** Yaffe et al
- **Heavy quarkonium (hard probe)** Maldacena
- **Hardron spectrum (AdS/QCD)**
- **AdS/CDM**

Heavy quark potential

The gravity dual of a Wilson loop at large N_c and large λ

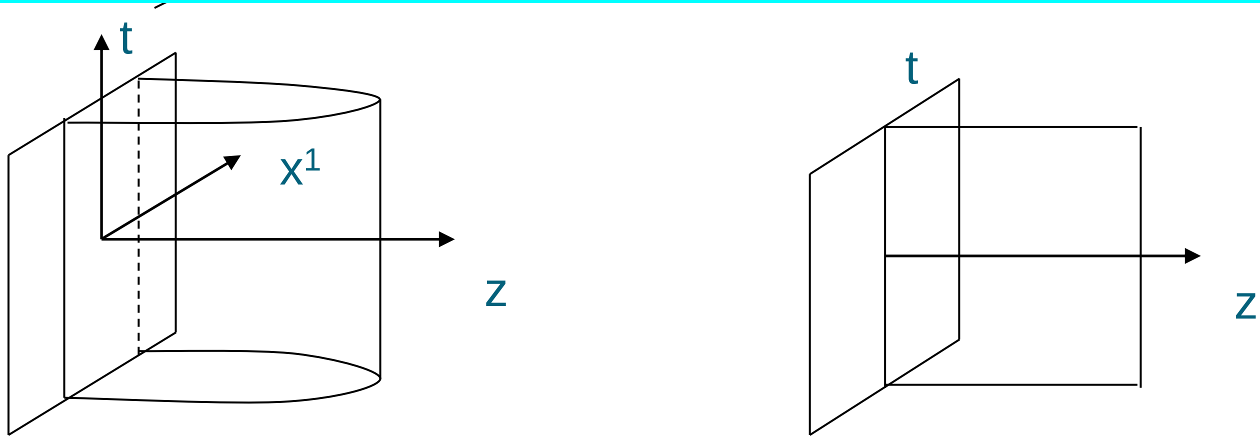
$$\mathrm{tr} < W(C) >= e^{-\sqrt{\lambda} S_{\min}[C]}$$

the min.area of string world sheet in the AdS_5

$$W(C) = Pe^{-i\oint_C dx^{\mu} A_{\mu}(x)}$$

Heavy quark potential probes confinement hadronic phase and meson melting in plasma

$$F(r,T) = T\left(S_{\min}[\text{parallel lines}] - 2S_{\min}[\text{single line}]\right)$$

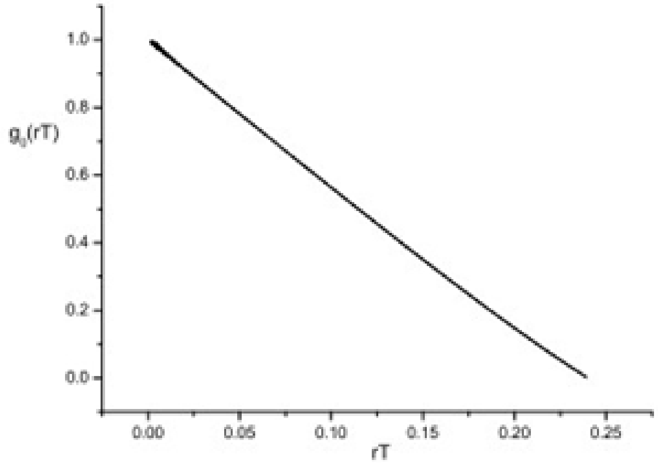


Heavy quark potential leading order from AdS/CFT

$$V(r) \simeq -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\; g_0(rT)$$

Maldacena PRL.80, (1998) 4859

where $g_0(rT)$ is a monotonic decreasing function corresponds to the **leading order**



S.J.Rey , Nucl.Phys.B 527,171(1998)

Higher order corrections

Leading orders are strictly valid when $N_c \to \infty$, $\lambda \to \infty$

- **For real QCD. The t'Hooft coupling is not infinity**

$$5.5 < \lambda < 6\pi.$$

- **The super gravity correction to the AdS-Schwarschild metric is of order** $O(\lambda^{-\frac{3}{2}})$

,

- **The fluctuation around the minimum world sheet presents at all T, and is of order** $O(\lambda^{-\frac{1}{2}})$ **(more important)**

Semi-classical expansion	Strong coupling expansion
The type Ⅱ B superstring	N=4 SYM
The path integral of the string-sigma action	Wilson loop expectation
Classical limit	Leading order
One loop correction	Sub-leading order

Gravity dual of a Wilson loop at finite coupling

$$W[C] \equiv \left< \exp\left(i\oint_C dx^\mu A_\mu \right) \right> = \int [dX][d\theta] \exp\left[\frac{i}{2\pi\alpha'} S(X,\theta) \right]$$

Strong coupling expansion ⇄ Semi-classical expansion

$$\ln W[C] = i\sqrt{\lambda}\left[S\left(\bar{X},0\right) + \frac{b[C]}{\sqrt{\lambda}} + \ldots \right]$$

$\bar{X}$ = the solution of the classical equation of motion;

b[C] comes from the fluctuation of the string world sheet around $\bar{X}$

more significant than α'^3 -correction for Wilson loops.

$$\frac{1}{2\pi\alpha'} S(X,\theta) = \text{the superstring action in } AdS_5 \times S^5$$

Metsaev and Tseytlin

With fluctuations:

$$X^\mu = \bar{X}^\mu + \delta X^\mu, \qquad \theta \neq 0 \qquad g_{ij} = \bar{g}_{ij} + \delta g_{ij}$$

$$S(X,\theta) = S(\bar{X},0) + S_B^{(2)}(\delta X) + S_F^{(2)}(\theta) + \ldots$$

Bosonic and fermionic fluctuations decouple.

$$W[C] = e^{iS(\bar{X},0)} Z \qquad Z = Z_B Z_F$$

Partition function at finite T with fluctuations underlying the potential

Hou, Liu, Ren, PRD80,2009

Straight line：

$$Z = Z_B Z_F = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{3}{2}}\left(-\nabla^2 + \frac{8}{3} + \frac{1}{2}R^{(2)}\right)\det^{\frac{5}{2}}(-\nabla^2)}$$

Parallel lines：

$$Z = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{1}{2}}\left(-\nabla^2 + 4 + R^{(2)} - 2\delta\right)\det(-\nabla^2 + 2 + \delta)\det^{\frac{5}{2}}(-\nabla^2)}$$

Partition function at finite T with fluctuation underlying the jet quenching parameter.

Zhang, Hou, Ren, arxiv1210.5187

$$Z=\frac{\det^2\mathcal{A}_+\det^2\mathcal{A}_-}{\det^{\frac{1}{2}}\mathcal{A}_{\xi\eta}\det^{\frac{1}{2}}\mathcal{A}_\zeta\det^{\frac{5}{2}}(-\nabla^2)}$$

where $$\mathcal{A}_\zeta=-e^{-2\phi}\left(\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial\rho^2}\right)+M_1^2$$

The other A refer to the correspondence operators too

The one loop effective action

$$S_{eff}=S[\bar{X},0]-i\ln Z$$

Computing of the determinant ratio

$$\frac{\det H_2}{\det H_1} = \frac{\Lambda[u_2, v_2]}{\Lambda[u_1, v_1]}$$

I.M.Gelfand, et, al, J.Math.Phys.,1,48(1960)

(u_i, v_i) *are 2 independent solutions* 。

$$\Lambda[u_i, v_i] = \frac{u_i(a)v_i(b) - u_i(b)v_i(a)}{W[u_i, v_i]}$$

Wronskian determinant

Reduce evaluating functional determinants to a set of 2nd order ordinary differential equations, which are solved numerically

Next leading order Results

$$V(r) \approx -\frac{4\pi^2}{\Gamma^4(\frac{1}{4})}\frac{\sqrt{\lambda}}{r}[1-\frac{1.33460}{\sqrt{\lambda}}+O(\frac{1}{\lambda})] \qquad \text{for } \lambda \gg 1$$

Chu, Hou, Ren, JHEP0908, (2009)

$$V_{\text{ladder}}(r) = -\frac{\sqrt{\lambda}}{\pi r}\left(1-\frac{\pi}{\sqrt{\lambda}}\right).$$

Erickson etc. NPB582, 2000

$$-\frac{\lambda}{4\pi r}\left[1-\frac{\lambda}{2\pi^2}\left(\ln\frac{2\pi}{\lambda}-\gamma_E+1\right)+O(\lambda^2)\right] \qquad \text{for } \lambda \ll 1$$

Confirmed by Forini JHEP 1011 (2010) 079 arXiv:1009.3939

$$V_{q\bar{q}}(\lambda, L) = -\frac{c(\lambda)}{L} \ ,$$

$$c(\lambda) = \begin{cases} \frac{\lambda}{4\pi}\left[1 - \frac{\lambda}{2\pi^2}\left(\ln\frac{2\pi}{\lambda} - \gamma_E + 1\right) + \mathcal{O}(\lambda^2)\right] & \lambda \ll 1 \ , \\ \frac{\sqrt{\lambda}\,\pi}{4\,\mathbb{K}^2}\left[1 + \frac{a_1}{\sqrt{\lambda}} + \mathcal{O}\left(\frac{1}{(\sqrt{\lambda})^2}\right)\right] & \lambda \gg 1 \ . \end{cases}$$

$$\begin{aligned} a_1 &= \frac{5\pi}{12} - 3\ln 2 + \frac{2\mathbb{K}}{\pi}\left(\mathbb{K} - \sqrt{2}(\pi + \ln 2) + \mathcal{I}^{\rm num}\right) \\ &= -1.334595305280600773 64\ldots \ , \end{aligned}$$

-1.33460

Next leading order potential at finite T

Zhang,Hou, Ren,Yin JHEP07:035（2011）

$$V(r) \simeq -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\left[g_0(rT) - \frac{1.33460 g_1(rT)}{\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right)\right]$$

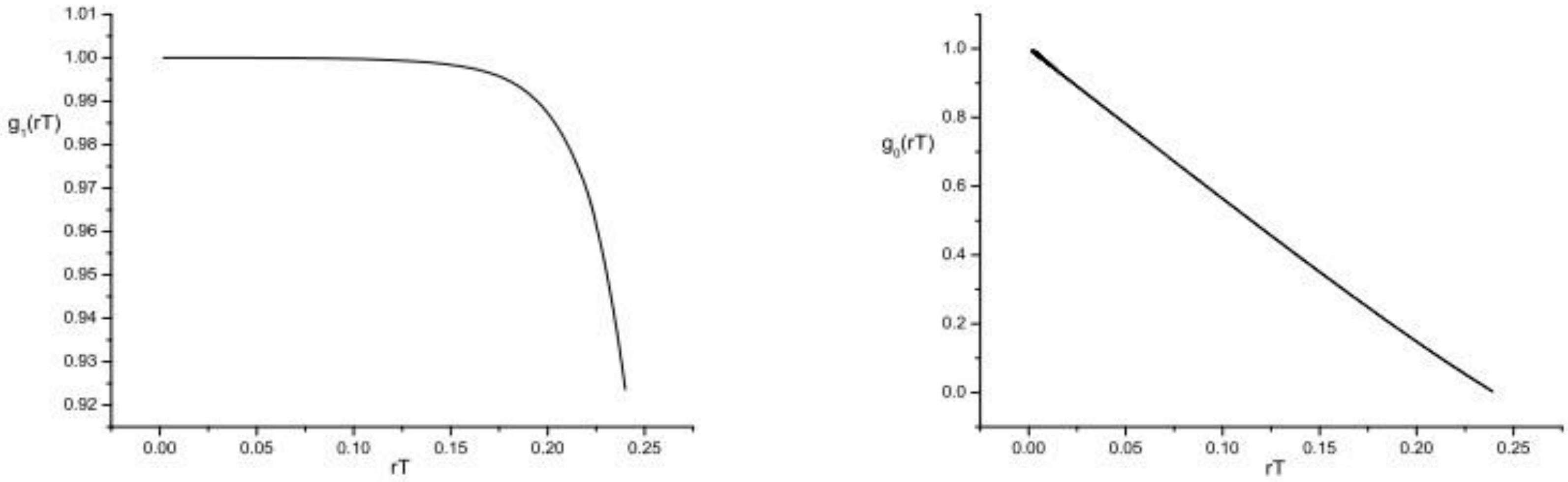


Figure 3. The left curve represents $g_1(rT)$, while the right represents $g_0(rT)$.

Meson Melting Temperature

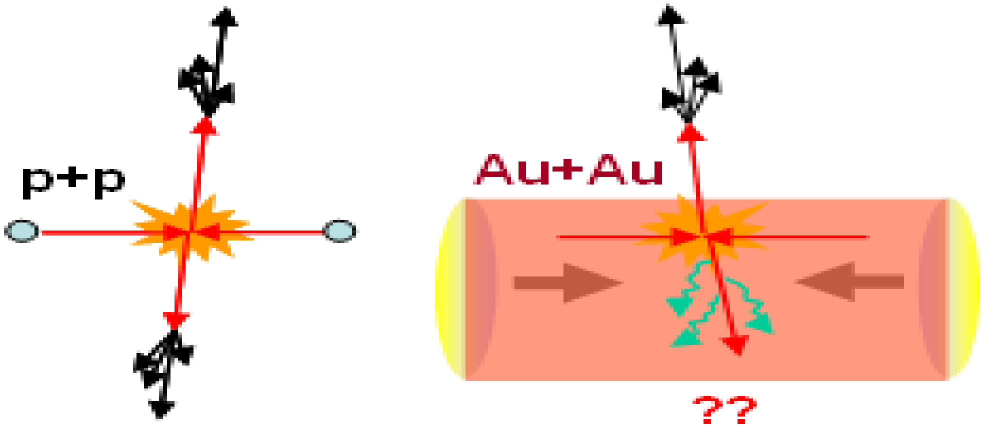
Hou and Ren, JHEP0801:029

ansatz	J/ψ	Υ
F	NA	235-385
U	219-322	459-780

ansatz	J/ψ(holographic)	J/ψ(lattice)	Υ(holographic)	Υ(lattice)
F	NA	1.1	1.3-2.1	2.3
U	1.2-1.7	2.0	2.5-4.2	4.5

T_d/T_c

Jet quenching in QGP



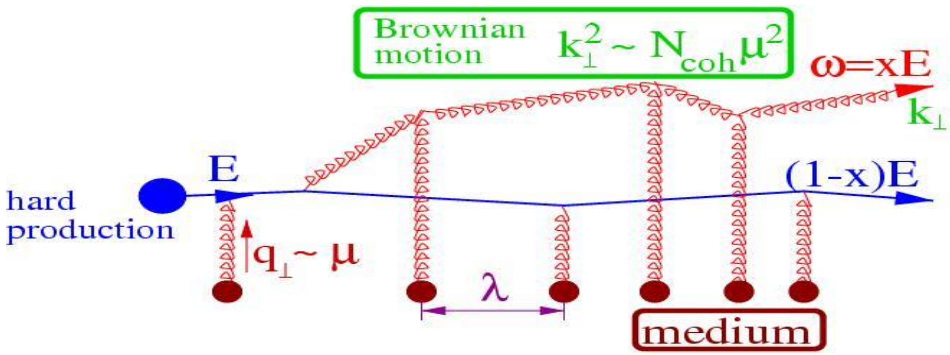
$$\Delta E \approx -\frac{\alpha_S}{2\pi} N_C \, \hat{q} \, L^2$$

Baier, Dokshitzer, Mueller, Peigne, Schiff (1996):

$\hat{q}$ reflects the ability of the medium to “quench” jets.

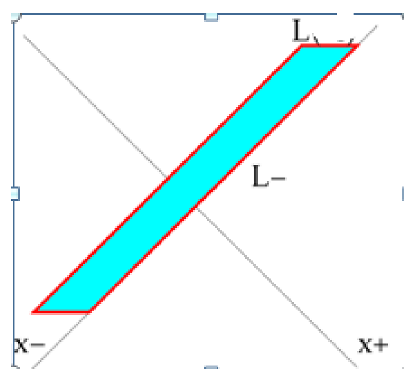
A non-perturbative definition of $\hat{q}$

Wiedemann 2000



$$\int_{\mathbf{y}=0=\mathbf{r}(y_l)}^{\mathbf{u}=\mathbf{r}(\bar{y}_l)} \mathcal{D}\mathbf{r}\, \exp\left[i\int_{y_l}^{\bar{y}_l} d\xi \frac{\omega}{2}\left(\dot{\mathbf{r}}^2 - \frac{\hat{q}(\xi)\mathrm{r}^2}{2i\,\omega}\right)\right] \longrightarrow \exp\left[-\frac{1}{4}\hat{q}(y_l-\bar{y}_l)\mathrm{r}^2\right] = \langle W^A(\mathcal{C})\rangle_T$$

$$W^A[C] = \exp\left(-\frac{\hat{q}\,L_- L^2}{4\sqrt{2}}\right)$$



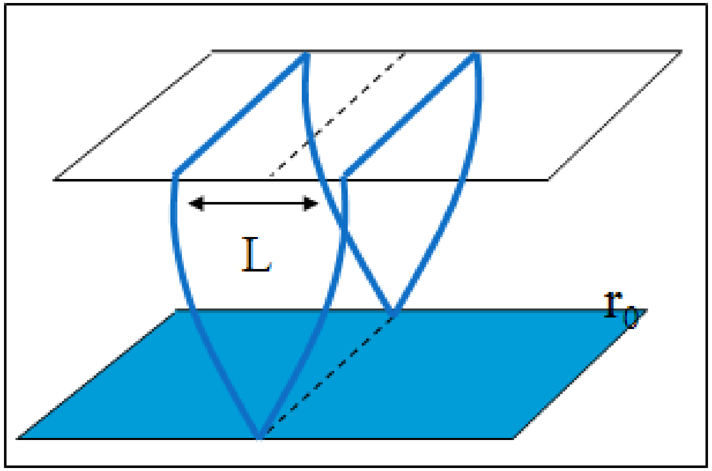
$$L^- \gg 1/T \gg L$$

Leading order jet quenching parameter from AdS/CFT

Liu, Rajagopal & Wiedemann, PRL,97,182301(2006)

$$\hat{q}_0 = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3$$

Dipole amplitude: two parallel Wilson lines in the light cone:



NL correction to jet quenching parameter

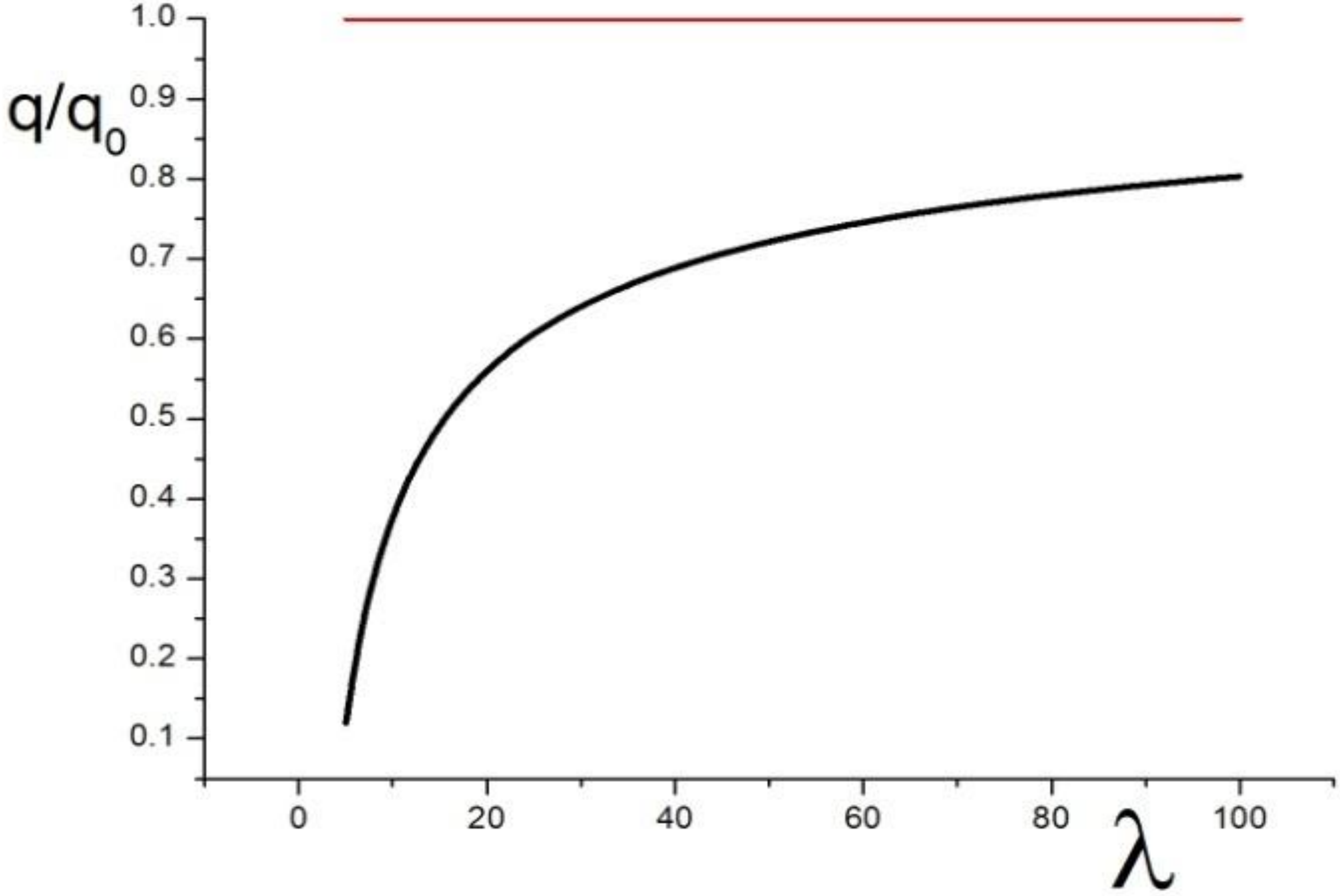
Zhang, Hou, Ren, Arxiv 1210.5187 appear @JHEP

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3[1-1.97\lambda^{-1/2}+O(\lambda^{-1})]$$

dominant

$$1 - 1.765\lambda^{-3/2}.$$

N. Armesto et al JHEP09 (06)



q/q$_0$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
40
60
80
100
λ

Jet quenching parameter discussion

$$\hat{q}_{\exp} = 1 \rightarrow 15 GeV^2 / fm$$ Nestor Armesto ,et, al, JHEP 0609 (2006) 039

Take $N_c = 3, \alpha_{SYM} = \frac{1}{2} \Longleftrightarrow \lambda = 6\pi$

Choose T=300MeV

$$\hat{q}_0 = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3 = 4.48 GeV^2 / fm$$

1.Sub-leading order gives rise to 32% reduction of $\hat{q}$ from the leading order amount in this case.

2.The negative sign of sub-leading order is consistent with a monotonic behavior from strong coupling to weak coupling.

Summary and discussion

AdS/CFT provides a useful way to address the physics at strong coupling .

The partition function of Wilson loop with fluctuations in strongly coupling N=4 SYM plasma are derived.

We computed the jet quenching parameter and heavy quark potential up to sub-leading orders.

The applicability of these AdS/QCD results demands phenomenological work to explain them in a way which can be translated to real QCD.

Thanks

QCD versus
N=4 Super Yang-Mills from gravity dual

	QCD	Super YM
N_c	3	>>1
t'Hooft coupling	5.5-18.8	>>1
Quarks	Fundamental	Adjoint
Conformal symmetry	No	Yes at zero T No at nonzero T
Supersymmetry	No	Yes at zero T No at nonzero T

The operators underlying the determinants at finite T in conformal coordinates

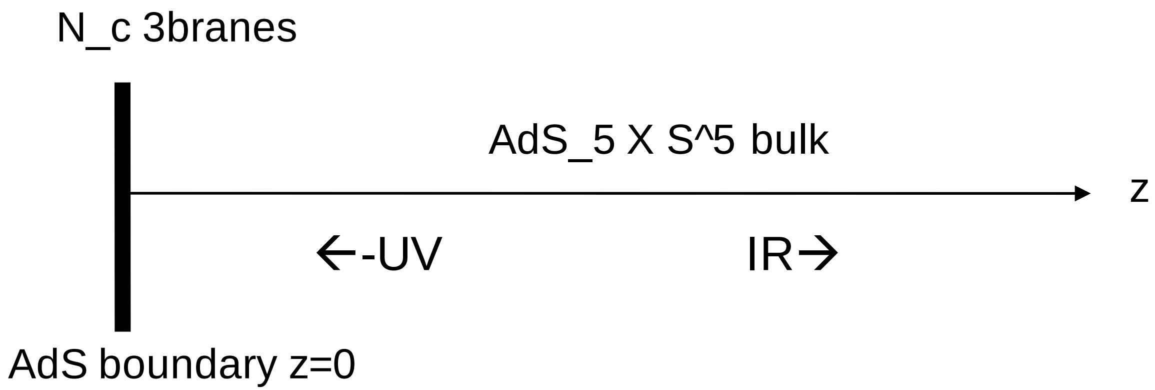
Parallel lines

$$\Delta_0[C_2] = -\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\sigma^2}$$

$$\Delta_1[C_2] = -\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\sigma^2} + e^{2\phi}(2+\delta)$$

$$\Delta_2[C_2] = -\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\sigma^2} + e^{2\phi}(4+R-2\delta)$$

$$D_F[C_2] = -i\left(\frac{\partial}{\partial\sigma} + \frac{1}{2}\frac{d\phi}{d\sigma}\right)\tau_1 - i\frac{\partial}{\partial\tau}\tau_2 + e^{\phi}\tau_3$$



The metric at T=0

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + d\mathbf{x}^2 + dz^2\right) + d\Omega_5^2$$

The metric at T>0

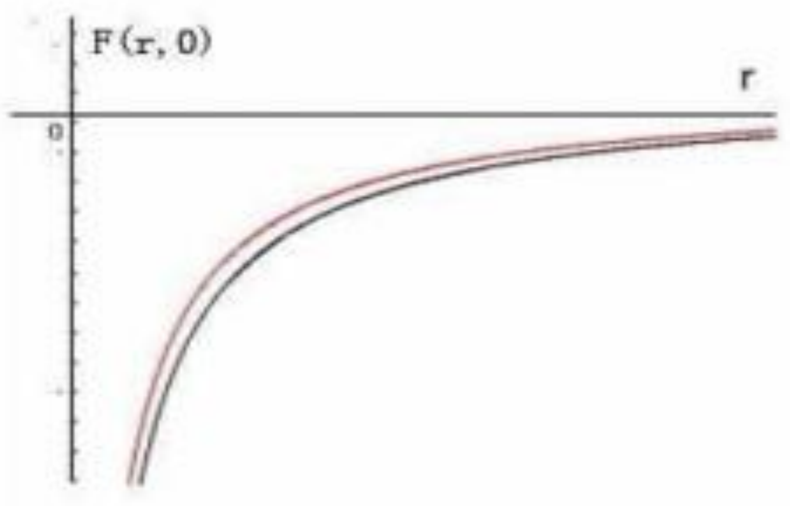
$$ds^2 = \frac{1}{z^2}\left(-fdt^2 + d\mathbf{x}^2 + \frac{dz^2}{f}\right) + d\Omega_5^2$$

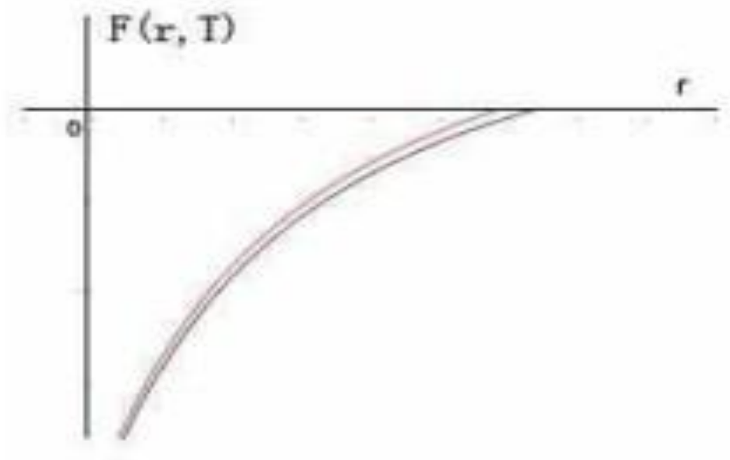
$$f = 1 - \frac{z^4}{z_h^4} \qquad z_h = \frac{1}{\pi T}$$

Heavy quark potential discussion

1.In the weak coupling, the heavy quark potential at T>0 is of Yukawa type, that is non-vanishing for arbitrarily large r.
2.To the leading order of strong coupling, the magnitude of the potential drops to zero at a finite r.

3. The $O\left(\frac{1}{\sqrt{\lambda}}\right)$ term decreases the screening radius.





Chu,Hou, Ren,JHEP0908,(2009)   Zhang,Hou, Ren,Yin JHEP1107:03（2011）

Soft scatterings

Zakharov (1997); Wiedemann (2000)

- Amplitude for a particle propagating in the medium

$$G(x_1,t_1;x_2,t_2) = \int_{x_1}^{x_2} Dx(t)\, P\exp\left[i\int ds\,\left(\frac{1}{2}m\dot{x}^2 + A_\mu(x(s))\dot{x}^\mu\right)\right]$$

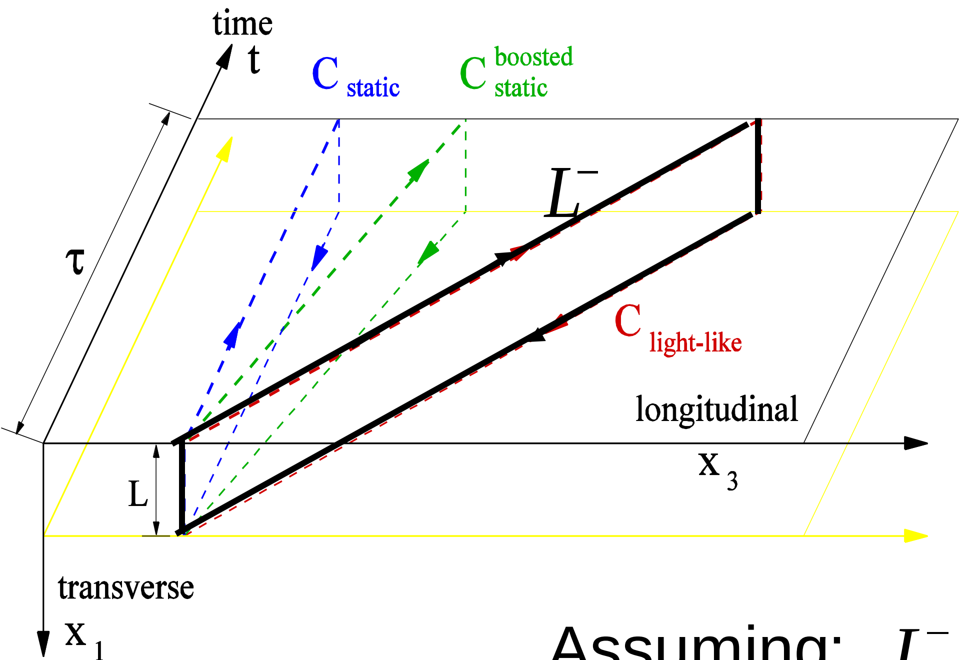
- High energy limit (eikonal approximation):

$$G(x_2^-;x_1^-) = P\exp\left[i\int_{x_1^-}^{x_2^-} dx^-\, A^+(x^-,\mathbf{r}(x^-))\right]$$

Soft scatterings are captured by Light like Wilson lines.

A non-perturbative definition of $\hat{q}$

Wiedemann (2000)



Light-like Wilson loop:

$$W(C) = \mathrm{Tr}\left(P \exp\left[i \oint_C dx^\mu A_\mu(x)\right]\right)$$

L: conjugate to the p_T

L^-: length of the medium

Assuming: $L^- >> 1/T >> L$

$$\langle W(C)\rangle \approx \exp\left[-\frac{1}{8\sqrt{2}}\hat{q}\, L^-\, L^2\right]$$

Thermal average
(Hard to calculate using lattice)

Nonperturbative definition of $\hat{q}$