Herwiri2: CEEX EW Corrections in a Hadronic MC

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Preface

- QCDXQED Amplitude-Based Resummation

$$d\sigma^{\exp} = e^{\Xi(QCED)_{IR}} \sum_{m,n=0}^{\infty} \frac{1}{m!}\frac{1}{n!} \int \prod_{j=1}^{m} \frac{d^3 k_j}{k_j^0} \prod_{l=1}^{n} \frac{d^3 k'_l}{k'^0_l} e^{i(p_1+p_2-q_1-q_2-\sum_j k_j - \sum_l k'_l)}$$
$$e^{D(QCED)} \hat{\tilde{\tilde{\beta}}}_{m,n}(k_1,\ldots,k_m; k'_1,\ldots,k'_n) \frac{d^3 q_1}{q_1^0}\frac{d^3 q_2}{q_2^0}$$

where we have introduced

$$\Xi(QCED)_{IR} = 2\alpha_s \Re B_{QCED} + 2\alpha_s \tilde{B}_{QCED}(K_{max}),$$

$$2\alpha_s \tilde{B}_{QCED}(K_{max}) = \int d^3\frac{k}{k^0}\tilde{S}_{QCED}(k)\,\theta(K_{max}-k^0),$$

$$D(QCED) = \int d^3\frac{k}{k^0}\left[e^{-iyk} - \theta(K_{max}-k^0)\right]\tilde{S}_{QCED}(k)$$

$$\tilde{S}_{QCED}(k) = \frac{-\alpha_s C_F}{8\pi^2}\left(\frac{p_1}{kp_1} - \frac{p_2}{kp_2}\right)^2 \Big|_{DGLAP-CS\ synthesized} + \cdots,$$

with a corresponding definition of $\Re B_{QCED}$

HERWIRI2: CEEX EW in HERWIG6.5

- HERWIRI2
  • The success of YFS exponentiation in the precision event generator KKMC (S. Jadach, B.F.L. Ward, and Z. Was) for $e^+ e^- \to Z\gamma^* \to f \bar{f}$, provides a natural starting point for incorporating EWK corrections to the parton-level process.
  • HERWIRI2 is a hybrid of KK MC with a with hadronic event generator,HERWIG in its present incarnation.
  • The two programs function largely independently:
  ◦ HERWIG generates the parton momenta and shower.
  ◦ KK MC provides a more precise calculation of the hard process and generates multiple ISR and FSR photon emission.
  • KK MC was designed to be upgradable to processes beyond just the $e^+ e^-$ scattering of interest at LEP: thus, the ability to select incoming quarks already exists.
  • See S. Yost, V. Halyo, M. Hejna, and B.F.L. Ward, PoS (RADCOR 2011), 017 [arXiv:1201.0515]

ElectroWeak Corrections in a Hadronic Context

Some other projects that have combined EWK and Hadronic Physics in MC generators:

- PHOTOS (E. Barberio, B. van Eijk, P. Golonka, Z. Wąs) is an add-on generator which adds multi-photon emission to charged final state particles, using YFS-inspired exponentiation.
- ZINHAC and WINHAC describe single W or Z production at hadron colliders with $\mathcal{O}(\alpha)$ YFS exponentiation (EEX) (S. Jadach, W. Płaczek, A. Siódmok)
- HORACE combines exact $\mathcal{O}(\alpha)$ EWK corrections with a QED DGLAP shower. (C.M. Carloni Calame, G. Montagna, O. Nicrosini, M. Treccani, A. Vicini.) ;G.Ballossini et al.(QCDXQED);L.Barge et al.(Powheg-Box).....
- MRST 2004 (A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne) included PDFs with QED corrections. +......

HERWIRI2

- The success of YFS exponentiation in the precision event generator $\mathcal{KK}$ MC (S. Jadach, B.F.L. Ward, and Z. Wąs) for $e^+e^- \to Z\gamma^* \to f\overline{f}$ provides a natural starting point for incorporating EWK corrections to the parton-level process.
- HERWIRI2 is a hybrid of $\mathcal{KK}$ MC with a with hadronic event generator, HERWIG in its present incarnation. MC@NLO/HERWIRI2 seamless.
- The two programs function largely independently:
  - HERWIG generates the parton momenta and shower.
  - $\mathcal{KK}$ MC provides a more precise calculation of the hard process and generates multiple ISR and FSR photon emission.
- $\mathcal{KK}$ MC was designed to be upgradable to processes beyond just the e^+e^- scattering of interest at LEP: thus, the ability to select incoming quarks already exists.
- See S. Yost, V. Halyo, M. Hejna, and B.F.L. Ward, PoS (RADCOR 2011), 017 [arXiv:1201.0515].

CEEX Formalism

The CEEX cross section for $q\overline{q} \rightarrow f\overline{f}$ has the form

$$\sigma = \frac{1}{\text{flux}} \sum_{n=0}^{\infty} \int d\text{PS}\ \rho^{(n)}_{\text{CEEX}}(\vec{p}, \vec{k})$$

where

$$\rho^{(n)}_{\text{CEEX}} = \frac{1}{n!} e^{Y(\vec{p}, E_{\min})} \frac{1}{4} \sum_{\text{hel.}} \left| \mathcal{M} \begin{pmatrix} \vec{p} & \vec{k} \\ \vec{\lambda} & \vec{\mu} \end{pmatrix} \right|^2$$

The YFS form factor is

$$Y(\vec{p}, E_{\min}) = Q_i^2 Y(p_1, p_2, E_{\min}) + Q_f^2 Y(p_3, p_4, E_{\min}) + Q_i Q_f Y(p_1, p_3, E_{\min})$$

$$+ Q_i Q_f Y(p_2, p_4, E_{\min}) - Q_i Q_f Y(p_1, p_4, E_{\min}) - Q_i Q_f Y(p_2, p_3, E_{\min})$$

$$Y(p_i, p_j, E_{\min}) = 2\alpha \widetilde{B}(p_i, p_j, E_{\min}) + 2\alpha \text{Re}\ B(p_i, p_j)$$

$$\widetilde{B} = -\int_{k^0 < E_{\min}} \frac{d^3\vec{k}}{8\pi^2 k^0} \left(\frac{p_i}{p_i \cdot k} - \frac{p_j}{p_j \cdot k} \right)^2, \quad B = \frac{i}{(2\pi)^3} \int \frac{d^4 k}{k^2} \left(\frac{2p_i + k}{2p_i \cdot k + k^2} - \frac{2p_j - k}{2p_j \cdot k - k^2} \right).$$

CEEX Formalism

The n-photon helicity-spinor amplitude can be expanded in terms of order α^r having the form

$$\mathcal{M}_n^{(r)} = \sum_{\mathcal{P}} \prod_{i=1}^{n} \mathcal{S}_i^{(\mathcal{P}_j)} \left[\beta_0^{(r)} \begin{pmatrix} \vec{p} & \\ \vec{\lambda} & \end{pmatrix} ; X_{\mathcal{P}} \right) + \sum_{j=1}^{n} \frac{\beta_1^{(r)} \begin{pmatrix} \vec{p} & k & \\ \vec{\lambda} & \mu & ; X_{\mathcal{P}} \end{pmatrix}}{\mathcal{S}_j^{(\mathcal{P}_j)}} \right.$$

$$\left. + \cdots + \sum_{1<j_1<\cdots<j_n} \frac{\beta_n^{(r)} \begin{pmatrix} \vec{p} & \vec{k} & \\ \vec{\lambda} & \vec{\mu} & ; X_{\mathcal{P}} \end{pmatrix}}{\mathcal{S}_{j_1}^{(\mathcal{P}_{j_1})} \cdots \mathcal{S}_{j_n}^{(\mathcal{P}_{j_n})}} \right]$$

with residual spinor amplitudes $\beta_i^{(r)}$ and complex soft photon factors $\mathcal{S}_j$ with the property

$$\left| \mathcal{S}_j^{(\mathcal{P}_j)} \right| = -2\pi\alpha Q^2 \left(\frac{p_a}{p_a \cdot k_j} - \frac{p_b}{p_b \cdot k_j} \right)^2$$

where Q, p_a, p_b belong to the initial or final fermions depending on the partition $\mathcal{P}_j$.

ElectroWeak Corrections

$\mathcal{KK}$ MC incorporates the DIZET library (version 6.2) from the semi-analytical program ZFITTER by A. Akhundov, A. Arbuzov, M. Awramik, D. Bardin, M. Bilenky, P. Christova, M. Czakon, A. Frietas, M. Gruenewald, L. Kalinovskaya, A. Olchevsky, S. Riemann, T. Riemann.

- The γ and Z propagators are multiplied by vacuum polarization factors:

$$H_\gamma = \frac{1}{2 - \Pi_\gamma}, \qquad H_Z = 4\sin^2(2\theta_{\mathrm{W}})\frac{\rho_{\mathrm{EW}} G_\mu M_Z^2}{8\pi\alpha\sqrt{2}}.$$

- Vertex corrections are incorporated into the coupling of Z to f via form factors in the vector coupling:

$$g_V^{(Z,f)} = \frac{T_3^{(f)}}{\sin(2\theta_W)} - Q_f F_{\mathrm{v}}^{(f)}(s)\tan\theta_W.$$

- Box diagrams contain these plus a new angle-dependent form-factor in the doubly-vector component:

$$g_V^{(Z,i)} g_V^{(Z,f)} = \frac{T_3^{(i)} T_3^{(f)} - 2T_3^{(i)} Q_f F_{\mathrm{v}}^{(f)}(s) - 2Q_i T_3^{(f)} F_{\mathrm{v}}^{(i)}(s) + 4Q_i Q_f F_{\mathrm{box}}^{(i,f)}(s,t)}{\sin^2(2\theta_W)}.$$

The correction factors are calculated at the beginning of a run and stored in tables.

Combining $\mathcal{KK}$ MC with a Shower

- The Drell-Yan cross section with multiple-photon emission can be expressed as an integral over the parton-level process $q_i(p_1)\overline{q}_i(p_2) \rightarrow f(p_3)\overline{f}(p_4) + n\gamma(k)$, integrated over phase space and summed over photons.

- The parton momenta p_1, p_2 are generated using parton distribution functions giving a process at CMS energy q and momentum fractions x_1, x_2 such that $q^2 = x_1 x_2 s$:

$$\sigma_{\rm DY} = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\sum_i f_i(q, x_1) f_{\bar{i}}(q, x_2)\sigma_i(q^2)\delta(q^2 - x_1 x_2 s),$$

where the final state phase space includes p_3, p_4 and k_i, $i = 1, \cdots, n$ and multiple gluon radiation + hadronization is included through a shower.

Combining $\mathcal{KK}$ MC with a Shower

- HERWIG generates the hard process at Born level and passes it through the shower routines. The event record is passed to HERWIRI2.
- HERWIRI2 finds the Z/γ^* and the partons interacting with it in the event record. The initial partons define p_1, p_2, which are transformed to the CM frame and projected on-shell to create a starting point for $\mathcal{KK}$ MC.
- $\mathcal{KK}$ MC generates the final fermion momenta p_3, p_4 and photons k_i (both ISR and FSR.) The generated particles are transformed back to the lab frame and placed in the event record.
- This HERWIRI2 weight is a product of the HERWIG and $\mathcal{KK}$ MC weights with a common factor removed and appropriate additional factors required because the scale and incoming fermion vary in $\mathcal{KK}$ MC.

MC Weights

The DY cross section in HERWIG can be expressed as

$$\begin{aligned}\sigma_{\mathrm{DY}} &= \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\sum_i f_i(q,x_1)f_{\bar{i}}(q,x_2)\sigma_i(q^2)\delta(q^2 - x_1 x_2 s) \\ &= \int_{q_{\min}}^{q_{\max}} dq P(q)\int_{q^2/s}^{1}\frac{dx_1}{x_1}\sum_i P_i \quad W_{\mathrm{HW}}^{(i)}(q^2,x_1) \\ &= \langle W_{\mathrm{HW}}\rangle\end{aligned}$$

where $P(q)$ is a normalized, integrable, crude probability distribution for q, P_i is the crude probability of generating parton i, and W_{HW} is the HERWIG event weight. This weight depends only on the hard Born cross section and is not altered by the shower.

$$P(q) = \frac{1}{2}[P_\gamma(q) + P_Z(q)], \qquad P_\gamma(q) = \frac{N_\gamma}{q^4}, \qquad P_Z(q) = \frac{N_2 q}{(q^2 - M_Z^2) + \Gamma_Z^2 M_Z^2}$$

The HERWIG Event Weight

- The HERWIG event weight is

$$W_{\mathrm{HW}} = \sum_i W_{\mathrm{HW}}^{(i)}, \qquad W_{\mathrm{HW}}^{(i)} = \frac{1}{P(q)} f_i(q, x_1) f_{\bar{i}}(q, x_2) \ln\left(\frac{s}{q^2}\right) \sigma_{\mathrm{HW}}^{(i)}(q^2)$$

  and the corresponding probability for selecting parton i is

$$P_i = W_{\mathrm{HW}}^{(i)} / W_{\mathrm{HW}}$$

- We have chosen to introduce EW corrections in a “minimally invasive” way, incorporating them in a form factor

$$F_{EW}^{(i)}(q^2) = \frac{\sigma_i(q^2)}{\sigma_{\mathrm{Born}}^{(i)}(q^2)}$$

- $\mathcal{KK}$ MC will calculate the EW form factor, and multiply it by the Herwig Born cross section.

$$\sigma_{\mathrm{HW+EW}} = \langle W_{\mathrm{Tot}} \rangle, \qquad W_{\mathrm{Tot}} = F_{EW}^{(i)}(q^2) W_{\mathrm{HW}} = W_{\mathrm{HW}} \frac{\sigma_{\mathrm{KK}}^{(i)}(q^2)}{\sigma_{\mathrm{Born}}^{(i)}(q^2)}.$$

$\mathcal{KK}$ MC Generator Structure

- The $\mathcal{KK}$ MC cross section is calculated using a "primary distribution"

$$\frac{d\sigma^{(i)}_{\mathrm{Pri}}(s,v)}{dv} = \sigma^{(i)}_{\mathrm{Born}}(s(1-v))\frac{1}{2}\left(1+\frac{1}{\sqrt{1-v}}\right)\overline{\gamma}_i v^{\overline{\gamma}_i - 1} v_{\mathrm{min}}^{\gamma_i - \overline{\gamma}_i}$$

  with

$$\gamma_i = \frac{2\alpha}{\pi}Q_i^2\left[\ln\left(\frac{s}{m_i^2}\right) - 1\right], \qquad \overline{\gamma}_i = \frac{2\alpha}{\pi}Q_i^2 \ln\left(\frac{s}{m_i^2}\right)$$

  to generate the factor v giving the fraction of s remaining after ISR photon emission, $s_X = s(1-v)$.

- The $\mathcal{KK}$ MC cross section is

$$\sigma(q^2) = \int d\sigma_{\mathrm{Pri}} \frac{d\sigma_{\mathrm{Cru}}}{d\sigma_{\mathrm{Pri}}} \frac{d\sigma_{\mathrm{Mod}}}{d\sigma_{\mathrm{Cru}}} = \sigma_{\mathrm{Pri}} \left\langle W_{\mathrm{Cru}} W_{\mathrm{Mod}} \right\rangle .$$

  W_{Cru} is calculated during ISR generation and W_{Mod} is generated after s_X is available.

Combined Generator HERWIRI2

- We want to use HERWIG and $\mathcal{KK}$ MC together to calculate

$$\sigma_{\rm Tot} = \left\langle W_{\rm HW} \frac{\sigma_i(q^2)}{\sigma^{(i)\star}_{\rm Born}(q^2)} \right\rangle = \left\langle W_{\rm HW} \sigma^{(i)}_{\rm Pri}(q^2) \frac{W^{(i)}_{\rm Cru} W^{(i)}_{\rm Mod}}{\sigma^{(i)\star}_{\rm Born}(q^2)} \right\rangle ,$$

- This average *could* be calculated using a joint probability distribution for q and v, $D(q,v) = P(q) d\sigma_{\rm Pri}/dv$, with $P(q)$ from HERWIG.
- An adaptive MC (S. Jadach's FOAM) could calculate the normalization of the distribution at the beginning of the run, in a similar manner to how $\mathcal{KK}$ MC presently integrates the one-dimensional primary distribution. In fact, to account for beamsstrahlung, $\mathcal{KK}$ MC permits such a distribution, in up to three variables, to be introduced by the user.
- As a first step, we have tried to run HERWIRI2 using $\mathcal{KK}$ MC's one-dimensional primary distribution. This requires fixing an overall scale q_0 to initialize $\mathcal{KK}$ MC (*e.g.*, $q_0 = M_Z$).

Combined Generator HERWIRI2

- The built-in primary distribution for electrons at scale q_0 can be used for the low-level generation of v. The transformation from this distribution to a distribution at HERWIG's generated scale q for quark i is then obtained by a change of variables.

$$\sigma_{\rm Tot} = \sigma^{(e)}_{\rm Pri} \left\langle W_{\rm HW} \left(\frac{d\sigma^{(i)}_{\rm Pri}(q^2, v)}{d\sigma^{(e)}_{\rm Pri}(q_0^2, v)} \right) \left(\frac{W^{(i)}_{\rm Crud} W^{(i)}_{\rm Mod}}{\sigma^{(i)\star}_{\rm Born}(q^2)} \right) \right\rangle$$

with

$$\frac{d\sigma^{(i)}_{\rm Pri}(q^2, v)}{d\sigma^{(e)}_{\rm Pri}(q_0^2, v)} = W^{(i)}_\gamma \frac{\sigma^{(i)}_{\rm Born}(q^2(1-v))}{\sigma^{(e)}_{\rm Born}(q_0^2(1-v))} ,$$

where

$$W_\gamma = \frac{\overline{\gamma}_i}{\overline{\gamma}_e} \left(\frac{v}{v_{\rm min}} \right)^{\overline{\gamma}_i - \overline{\gamma}_e} v_{\rm min}^{\gamma_i - \gamma_e} .$$

The γ factors are calculated using q^2/m_i^2 for parton i and q_0^2/m_e^2 for the electron.

Combined Weight

- Shuffling the numerators and denominators about gives the expression used in HERWIRI2:

$$\sigma_{\mathrm{Tot}} = \langle W_{\mathrm{HW}} W_{\mathrm{Mod}} W_{\mathrm{Karl}} W_{\mathrm{FF}} W_{\gamma} \rangle$$

  with two new weights

$$W_{\mathrm{Karl}} = \frac{\sigma^{(e)}_{\mathrm{Pri}} W^{(i)}_{\mathrm{Crud}}}{\sigma^{(e)}_{\mathrm{Born}}(q_0^2(1-v))}, \qquad W_{\mathrm{FF}} = \frac{\sigma^{(i)}_{\mathrm{Born}}(q^2(1-v))}{\sigma^{(i)\star}_{\mathrm{Born}}(q^2)}.$$

- The weights can be calculated by insuring that each subroutine is initialized for either an electron or parton i, as appropriate.
- Since σ_{Pri} is calculated before generation begins, $\mathcal{KK}$ MC can be pre-initialized with its standard values. The primary distribution is, in fact, hard-wired, and cannot be changed. The weight W_{Crud} for a quark requires a little rewriting.
- W_{Mod} requires only passing the correct variables, since the CEEX module that calculates it already anticipated being called with different fermions.

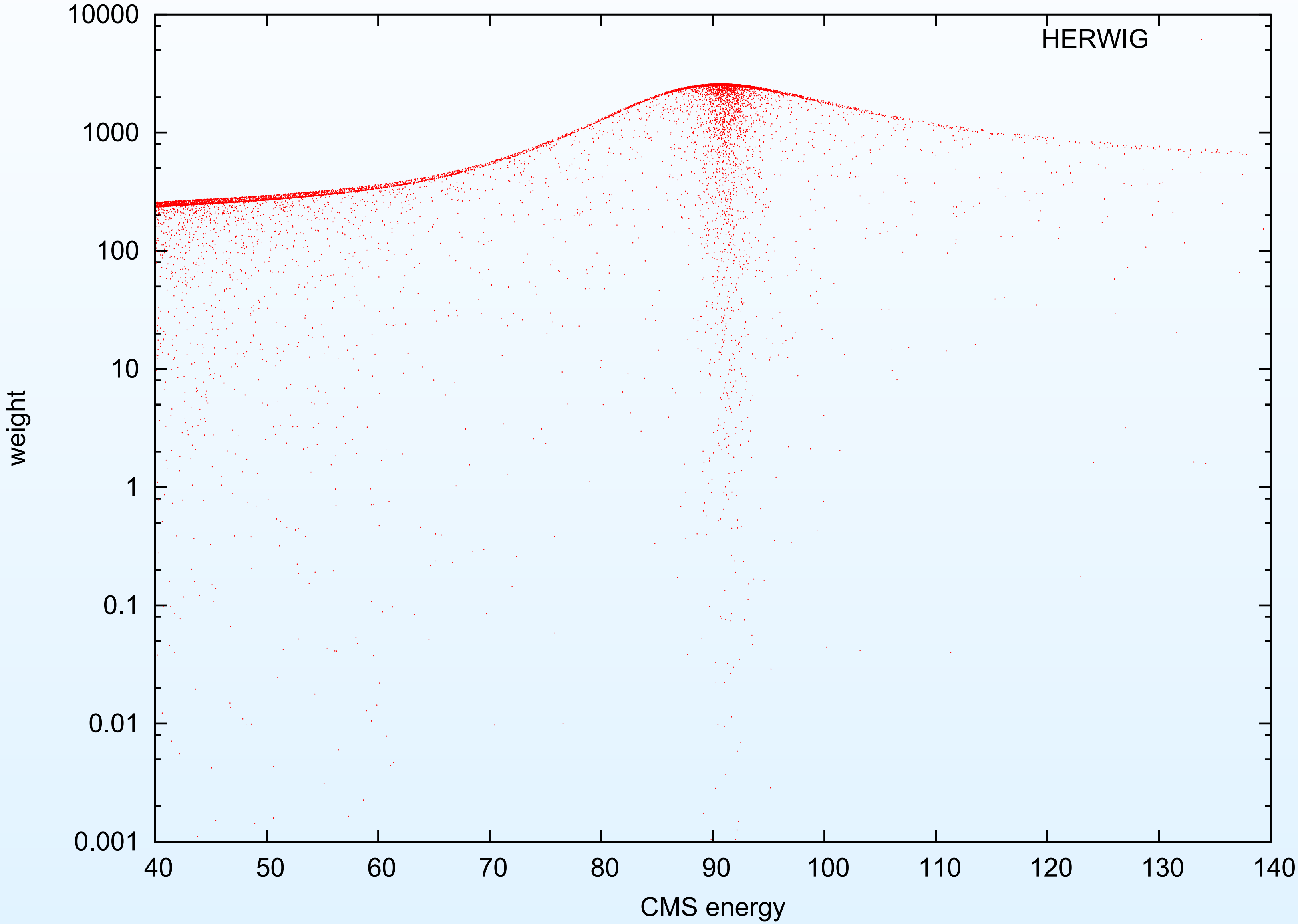
HERWIRI2 Without ISR

- HERWIRI2 is still in testing! All results here are very preliminary.
- Turning off ISR is useful as a first test because the weights simplify: $W_{\mathrm{Karl}} = W_\gamma = 1$. Since $v = 0$, there are no scale shifts in the weights, leading to a much narrower weight distribution.
- A very short test run (10,000 events) with 5 TeV proton beams, ISR off, and CMS energies between 40 and 140 GeV gives cross sections

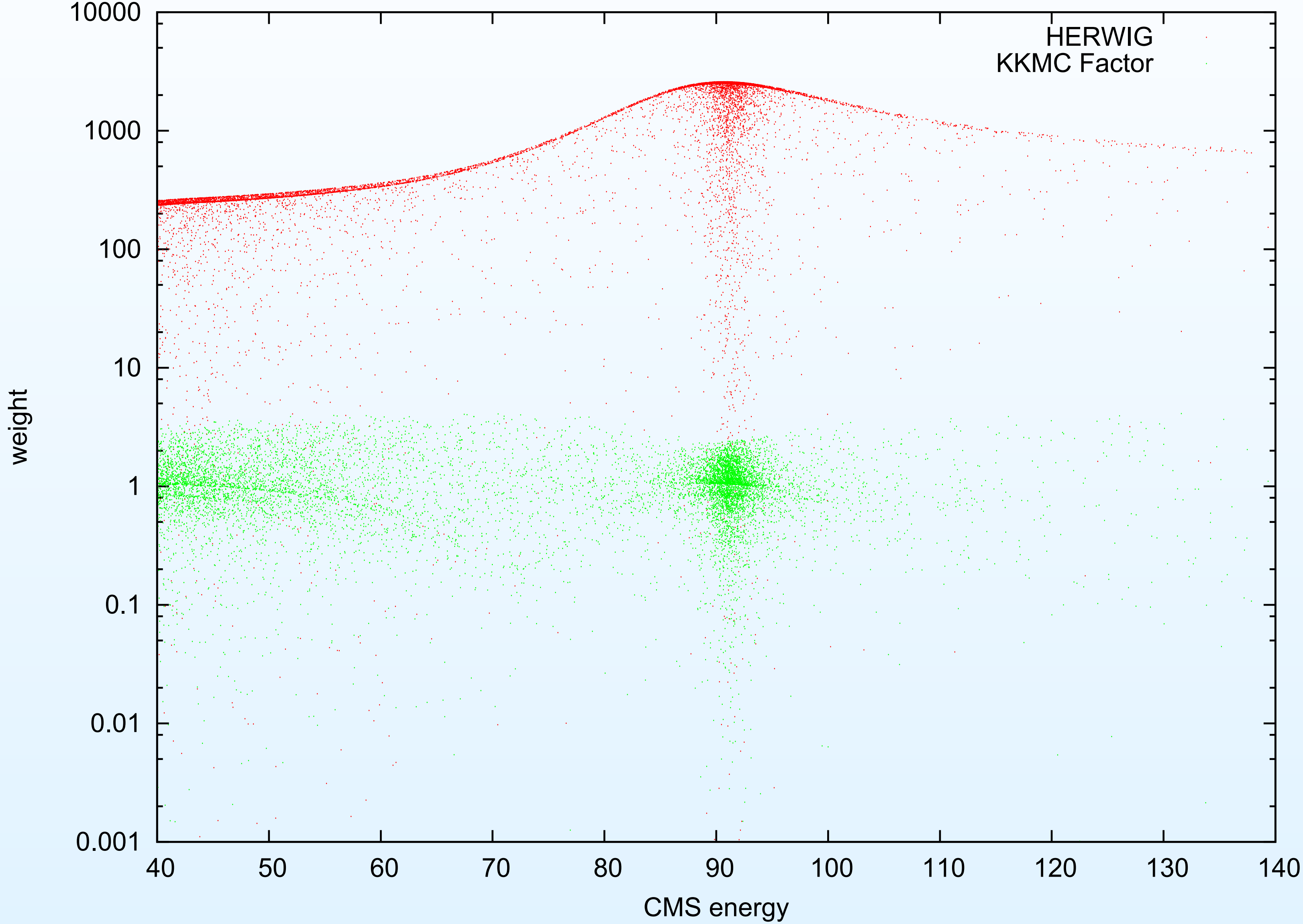
HERWIG CS	1099 ± 1 pb	
HERWIRI2 CS	1184 ± 1 pb	(+7.7%)

- Thus we find an increase of 7.7% from EW corrections.
- An average of 0.6 photons per event is generated, with an average total energy of 1.6 GeV.

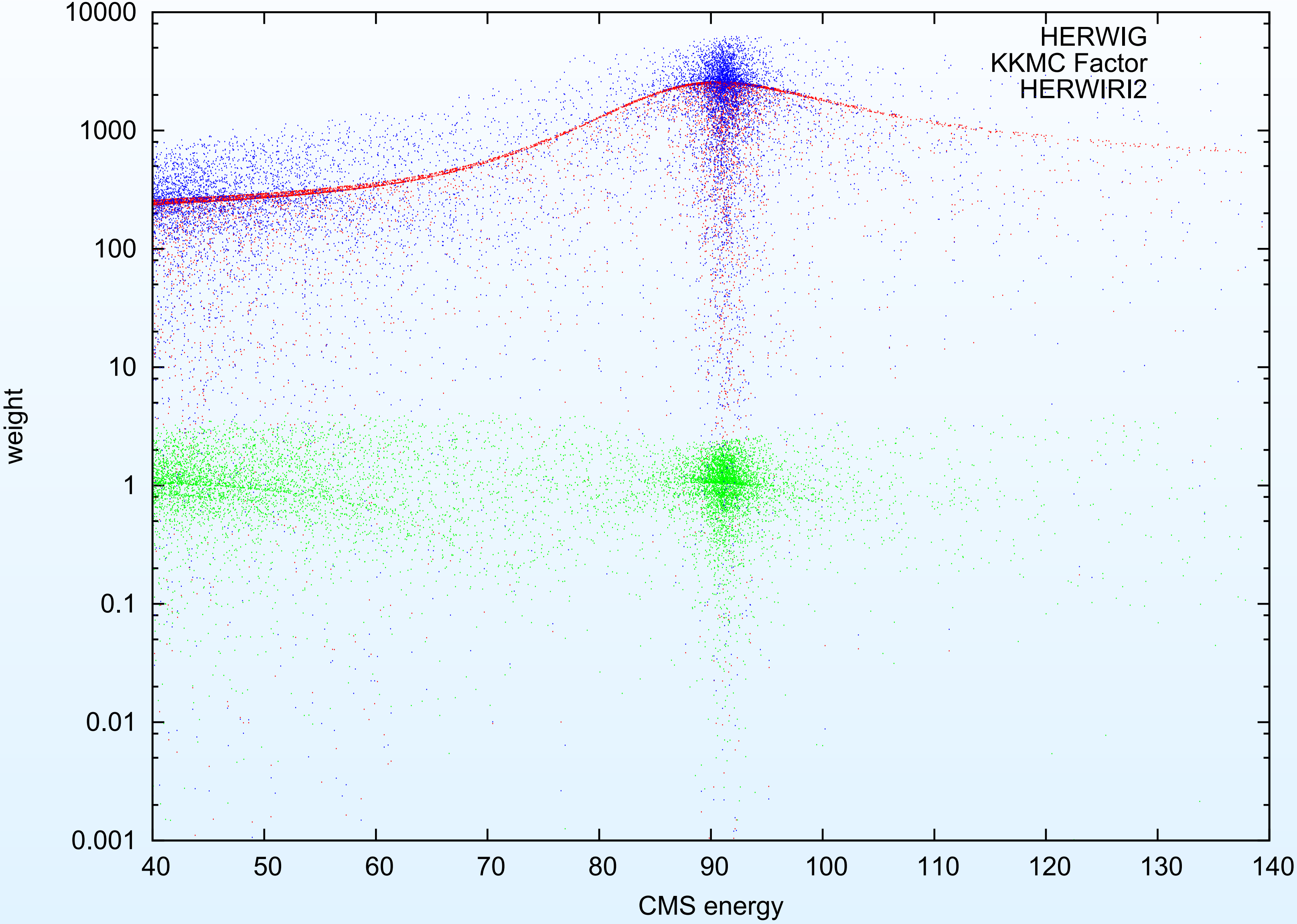
Weight Distributions, No ISR



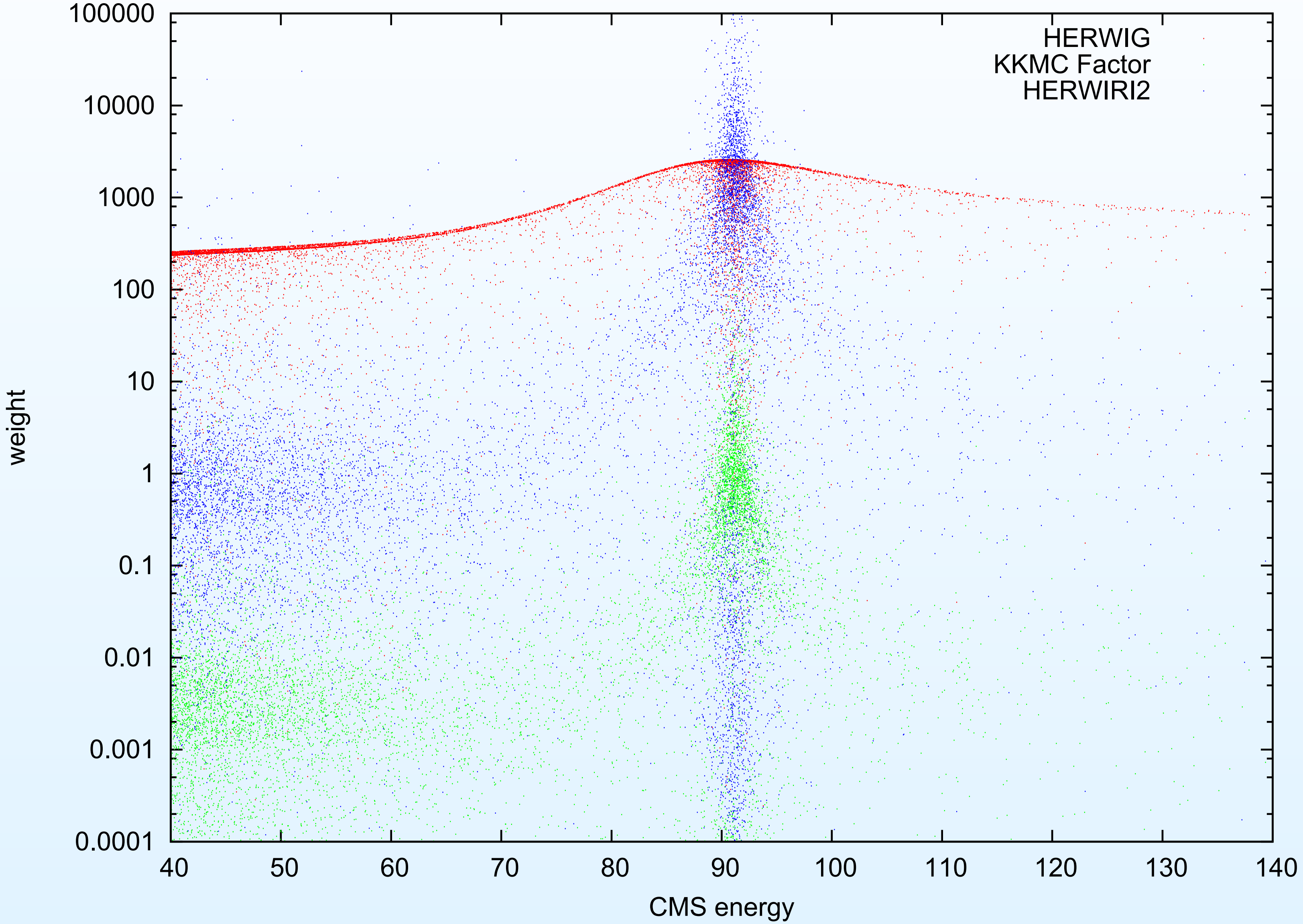
Weight Distributions, No ISR



Weight Distributions, No ISR



Weight Distributions With ISR



HERWIRI2 with ISR

- With ISR turned on, there is a much broader weight distribution.
- The cross section is 1212 ± 109 pb, 11% showing an additional 3.4% effect from ISR.

HERWIG		1099 ± 1 pb	
HERWIRI2	No ISR	1184 ± 1 pb	(+7.7%)
HERWIRI2	With ISR	1212 ± 109 pb	(+11%)

- The large weight variance an issue: The fixed initialization scale creates a wide weight distribution. It appears this will just be a stop-gap until a better primary distribution is introduced.
- W_γ has the broadest distribution, with an average of 3.3 ± 1.2 in this run, and weights ranging up to 7402. It should be possible to improve this with a better initialization scheme taking into account the multiple quark types.

Comparison with Other Approaches

- PHOTOS BENCHMARKED WITH KK MC : Z. Was et al., Nucl.Phys. Proc. Suppl. 181-182(2008) 269 ⇔ .1% or better
- SANC, HORACE, ZGRAD2 CHECKED IN C. Buttar et al.,(0803.0678): < 2 *‰*
- SANC, KORALZ(A. Andonov et al.,0212209): .01% on normalization, …

=> LOOK AT HORACE for LHC

BAYLOR UNIVERSITY

N. Adam, V. Halyo, S.A. Yost,1006.3766:

Electro-Weak Corrections

Z Production

Energy		Born	Born+FSR	Electro-Weak	Difference
7 TeV	$\sigma_{\rm tot}$	906.47 ± 0.40	906.47 ± 0.40	922.14 ± 1.04	$+1.70 \pm 0.12\%$
	$\sigma_{\rm cut}$	356.72 ± 0.46	333.60 ± 0.48	332.82 ± 0.50	$-0.23 \pm 0.21\%$
	A	0.3935 ± 0.0005	0.3680 ± 0.0006	0.3609 ± 0.0007	$+1.96 \pm 0.24\%$
10 TeV	$\sigma_{\rm tot}$	1359.25 ± 0.80	1359.25 ± 0.80	1387.67 ± 1.09	$+2.05 \pm 0.10\%$
	$\sigma_{\rm cut}$	494.58 ± 0.63	462.65 ± 0.66	462.38 ± 0.68	$-0.06 \pm 0.20\%$
	A	0.3639 ± 0.0005	0.3404 ± 0.0005	0.3332 ± 0.0006	$+2.15 \pm 0.23\%$
14 TeV	$\sigma_{\rm tot}$	1964.76 ± 1.13	1964.76 ± 1.13	2001.20 ± 1.79	$+1.82 \pm 0.10\%$
	$\sigma_{\rm cut}$	669.09 ± 0.86	625.66 ± 0.89	625.97 ± 0.89	$+0.05 \pm 0.20\%$
	A	0.3405 ± 0.0005	0.3184 ± 0.0005	0.3128 ± 0.0005	$+1.81 \pm 0.23\%$

- **Stay Tuned**