IBS and Polarization

in the regime of strong intrabeam scattering where the CLIC e- (or e+) damping rings operate, we might encounter effects related to polarization

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many thanks to Hans Braun for encouragement & discussions

IBS effect for CLIC

equilibrium emittances in the CLIC damping ring are determined by balance of radiation damping and intrabeam scattering

(PhD Thesis M. Korostelev 2006)

→ new regime of storage ring operation

M. Korostelev, PhD Thesis Parameters* of the extracted beam.

CLIC Damping Ring

Parameter	Symbol	RING 1	RING 2	RING 3	Unit
Bunch population	N_{bp}	2.56	2.56	2.56	$\times 10^9$
Bunches per train	k_{bt}	110	110	110	
Maximum number of bunch trains	N_{trains}^{max}	14	14	12	
Minimum number of bunch trains	N_{trains}^{min}	4	4	4	
Norm. horizontal emittance w/o IBS	$\gamma\epsilon_{x0}$	131	79	95	nm
Norm. horizontal emittance with IBS	$\gamma\epsilon_x$	540	380	430	nm
Norm. vertical emittance with IBS	$\gamma\epsilon_y$	3.4*	2.4*	2.7*	nm
Norm. longitudinal emittance** with IBS	ϵ_t	4990	4985	5000	eVm
RMS bunch length w/o IBS	σ_{s0}	1.21	1.25	1.21	mm
RMS energy spread w/o IBS	$\sigma_{\delta 0}$	0.915	0.113	0.111	%
RMS bunch length with IBS	σ_s	1.65	1.51	1.5	mm
RMS energy spread with IBS	σ_δ	0.125	0.136	0.137	%
Horizontal IBS growth time	T_x	3.89	1.88	2.34	ms
Longitudinal IBS growth time	T_p	5.57	4.403	4.83	ms

* Note that the parameters in this table were computed for the betatron coupling $\epsilon_{y0}/\epsilon_{x0} = 0.0063$ and zero vertical dispersion.
** Note that $\epsilon_t = \gamma\sigma_s\sigma_\delta m_0c^2$.

emittance with IBS is ~4x larger than w/o IBS!

Parameter	Symbol	RING 1	RING 2	RING 3	Unit
Energy	E	2.42	2.42	2.42	GeV
Circumference	C	364.96	364.96	300.48	m
Revolution time	T_0	1216.53	1216.53	1001.6	ns
Total length of wigglers	L_w	152	152	96	m
Number of wigglers	N_w	76	76	48	
Length of wiggler	L_{ID}	2	2	2	m
Wiggler peak field	B_w	1.7	2.52	2.52	T
Wiggler period length	λ_w	10	4.5	4.5	cm

polarization for CLIC

CLIC plans to operate with polarized e- beams and possibly also with polarized e+ beams (Compton scheme);

a ***study of spin transport and polarization issues in CLIC*** from the sources to the collision point was performed by R. Assmann and F. Zimmermann for Snowmass 2001 (***CLIC Note 501***) ; however, possible interplays of polarization with IBS were not addressed at the time

2 (or 3) issues

- change of IBS rate for polarized beams
- depolarization due to IBS:
  - rotation of polarization vector
  - shrinkage of polarization vector
    ("spin flip")

work in progress!

relativistic or non-relativistic?

this depends on (local) horizontal momentum

$$p_{x,rms} \approx \sqrt{\frac{\gamma\varepsilon_x}{\gamma\beta_x}}\, p_{\|} \approx \sqrt{\frac{6\times 10^{-7}\,\mathrm{m}}{5\times 10^{3}\ 5\,\mathrm{m}}}\, 2.42\ \mathrm{GeV/c} \approx 12\ \mathrm{keV/c}$$

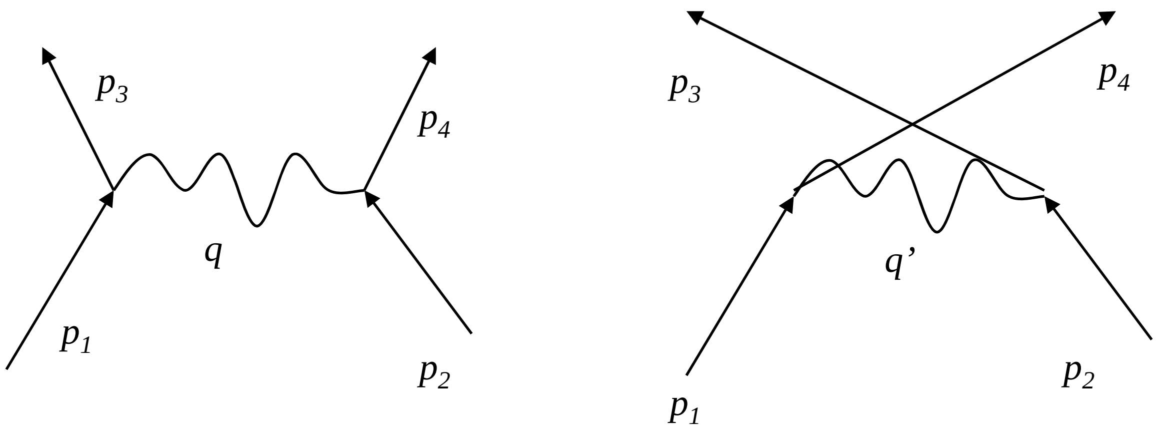
CLIC damping ring parameters

or

$$\beta \approx 0.023$$

→ **we can use non-relativistic approximation**

two Feynman diagrams for Moller scattering in lowest order:



$$M_{fi} = ie^2 \frac{1}{V^2}\left[\frac{1}{q^2}\overline{u}(p_3)\gamma_\mu u(p_1)\overline{u}(p_4)\gamma^\mu u(p_2) - \frac{1}{q'^2}\overline{u}(p_4)\gamma_\mu u(p_1)\overline{u}(p_3)\gamma^\mu u(p_2)\right]$$

general differential cross section in cm system:

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2}\frac{1}{W^2}\frac{p'}{p}\left(\overline{|M_{fi}|^2} V^4\right)$$

Fermi's golden rule

$$W = \sqrt{s^2} = \sqrt{(p_1+p_2)^2}, q^2 \approx -2p_1p_3 \approx -W^2\sin^2\left(\frac{\theta}{2}\right), q'^2 \approx -2p_1p_4 \approx -W^2\cos^2\left(\frac{\theta}{2}\right)$$

$$p_1p_2 = p_3p_4 \approx \frac{1}{2}W^2$$

unpolarized cross section

average over the initial spin directions and sum over the final ones;
this simplifies the calculation of the squared matrix element:

$$\overline{\left|M_{fi}\right|^2} = e^4 \frac{1}{V^4}\frac{1}{4}\sum_{s_1,s_2,s_3,s_4} \left[\begin{array}{l} \frac{1}{q^4}\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_1)\gamma_\nu u(p_3)\bar{u}(p_4)\gamma^\mu u(p_2)\bar{u}(p_2)\gamma^\nu u(p_4) \\ +\frac{1}{q'^4}\bar{u}(p_4)\gamma_\mu u(p_1)\bar{u}(p_1)\gamma_\nu u(p_4)\bar{u}(p_3)\gamma^\mu u(p_2)\bar{u}(p_2)\gamma^\nu u(p_3) \\ -\frac{1}{q^2 q'^2}\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_1)\gamma_\nu u(p_4)\bar{u}(p_4)\gamma^\mu u(p_2)\bar{u}(p_2)\gamma^\nu u(p_3) \\ -\frac{1}{q^2 q'^2}\bar{u}(p_4)\gamma_\mu u(p_1)\bar{u}(p_1)\gamma_\nu u(p_3)\bar{u}(p_3)\gamma^\mu u(p_2)\bar{u}(p_2)\gamma^\nu u(p_4) \end{array} \right]$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{st^2u^2}\left\{\left(s-2m^2\right)^2\left(t^2+u^2\right)+ut\left(-4m^2 s+12m^4+ut\right)\right\}$$

$$s = \left(p_1+p_2\right)^2$$

$$t = \left(p_1-p_3\right)^2 = -4\left|\vec{p}\right|\sin^2\left(\frac{\theta}{2}\right)$$

$$u = \left(p_1-p_4\right)^2 = -4\left|\vec{p}\right|\cos^2\left(\frac{\theta}{2}\right)$$

non-relativistic limit: $\left|\vec{p}\right| << m$

$$\left(\frac{d\sigma}{d\Omega}\right)_{unpol} = \frac{\alpha^2 m^2}{16\left|\vec{p}\right|^4}\left\{\frac{1}{\sin^4\left(\frac{\theta}{2}\right)}+\frac{1}{\cos^4\left(\frac{\theta}{2}\right)}-\frac{1}{\sin^2\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)}\right\} = \frac{\alpha^2 m^2}{4\left|\vec{p}\right|^4}\left\{\frac{4}{\sin^4\theta}-\frac{3}{\sin^2\theta}\right\}$$

[1] P. Schmuser, Feynman-Graphen und Eichtheorien fur Experimentalphysiker, Springer-Verlag, Berlin (1998)
[2] O. Nachtmann, Phaenomene und Konzepte der Elementarteilchenphysik, Vieweg Braunschweig (1986).

polarized cross section

calculated by A.A. Kresnin, L.N. Rosentsveig, Soviet Physics JETP 5, 2 (1957);
in the nonrelativistic limit:

$$\left(\frac{d\sigma}{d\Omega}\right)_{pol} \approx \left(\frac{d\sigma}{d\Omega}\right)_{unpol}\left[1-\frac{\sin^2\theta}{1+3\cos^2\theta}|S|^2\right]$$

polarization vector

$$\rightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{pol} \approx \frac{\alpha^2 m^2}{4|\vec{p}|^4}\left\{\frac{4}{\sin^4\theta}-\frac{3+|S|^2}{\sin^2\theta}\right\}$$

non-relativistic & small-angle cross section for polarized e-e- scattering

Bjorken-Mtingwa and Piwinski kept only ~this term

these terms reduce the effect of IBS

polarization → 25% change of neglected term

spin-flip cross section

project initial and final spin states onto opposite polarity using projection operator Σ; keep only dominant direct scattering term in scattering matrix

$$\overline{|M_{fi}|^2} = e^4 \frac{1}{V^4} \sum_{s_1,s_2,s_3,s_4} \left[\frac{1}{q^4} \bar{u}(p_3)\gamma_\mu \Sigma(s_1) u(p_1) \bar{u}(p_1)\gamma_\nu \Sigma(s_3) u(p_3) \bar{u}(p_4)\gamma^\mu \Sigma(s_1) u(p_2) \bar{u}(p_2)\gamma^\nu u(p_4) \right]$$

$\Sigma(s) = \frac{1+\gamma_5 \not{s}}{2}$ covariant spin projection operator where $\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

rename: $p_3 \rightarrow p_1'$, $p_4 \rightarrow p_2'$

$$\overline{|M_{fi}|^2} = e^4 \frac{1}{V^4} \frac{1}{q^4} Tr\left[\frac{\not{p}_1' + m_0}{2m_0} \gamma_\mu \frac{1+\gamma_5 \not{s}_1}{2} \frac{\not{p}_1 + m_0}{2m_0} \gamma_\nu \frac{1+\gamma_5 \not{s}_3}{2} \right] \times Tr\left[\frac{\not{p}_2' + m_0}{2m_0} \gamma_\mu \frac{1+\gamma_5 \not{s}_1}{2} \frac{\not{p}_2 + m_0}{2m_0} \gamma_\nu \right]$$

drop terms which contain single $\gamma_5 \not{s}$ factor (perhaps not correct?!?)

$$\overline{|M_{fi}|^2} = e^4 \frac{1}{V^4} \frac{1}{q^4} \left(\frac{1}{4m_0^2} \left(p'_{1,\mu} p_{1,\nu} + p_{1,\mu} p'_{1,\nu} - g_{\mu\nu} \left(p_1 \cdot p_1' - m_0^2 \right) \right) + Tr\left[\frac{\not{p}_1' + m_0}{2m_0} \gamma_\mu \frac{\gamma_5 \not{s}_1}{2} \frac{\not{p}_1 + m_0}{2m_0} \gamma_\nu \frac{\gamma_5 \not{s}_3}{2} \right] \right)$$

$$\times \left(\frac{1}{2m_0^2} \left(p_2^{',\mu} p_2^\nu + p_2^\mu p_2^{'\nu} - g^{\mu\nu} \left(p_2 \cdot p_2' - m_0^2 \right) \right) \right)$$

since γ_5 anticommutes with all γ matrices and $\gamma_5^2=1$:

$$Tr\left[\frac{\not{p}_1'+m_0}{2m_0}\gamma_\mu\frac{\gamma_5\not{s}_1}{2}\frac{\not{p}_1+m_0}{2m_0}\gamma_\nu\frac{\gamma_5\not{s}_3}{2}\right]=Tr\left[\frac{\not{p}_1'+m_0}{2m_0}\gamma_\mu\frac{\not{s}_1}{2}\frac{(-\not{p}_1+m_0)}{2m_0}\gamma_\nu\frac{\not{s}_3}{2}\right]$$

since traces of an odd number of γ matrices vanish

$$Tr\left[\frac{\not{p}_1'+m_0}{2m_0}\gamma_\mu\frac{\not{s}_1}{2}\frac{(-\not{p}_1+m_0)}{2m_0}\gamma_\nu\frac{\not{s}_3}{2}\right]=-\frac{1}{16m_0^2}Tr\left[\not{p}_1'\gamma_\mu\not{s}_1\not{p}_1\gamma_\nu\not{s}_3\right]+\frac{1}{16}Tr\left[\gamma_\mu\not{s}_1\gamma_\nu\not{s}_3\right]$$

for the sake of definiteness choose $\not{s}_1=\gamma_3$ **spin vertically up**

$\not{s}_3=-\gamma_3$ **spin vertically down**

assuming the initial momentum to be in the horizontal plane

$$\frac{1}{16}Tr\left[\gamma_\mu\not{s}_1\gamma_\nu\not{s}_3\right]=-\frac{1}{4}\left(2g_{3\mu}g_{3\nu}+g_{\mu\nu}\right)$$

$$-\frac{1}{16m_0^2}Tr\left[\not{p}_1'\gamma_\mu\not{s}_1\not{p}_1\gamma_\nu\not{s}_3\right]=\frac{p_{1\nu}p_{1\mu}'}{m_0^2}$$

$$\overline{\left|M_{fi}\right|^2} = e^4 \frac{1}{V^4}\frac{1}{2}\frac{1}{q^4}\left(\frac{1}{4m_0^2}\left(5p'_{1,\mu}p_{1,\nu} + p_{1,\mu}p'_{1,\nu} - g_{\mu\nu}p_1\cdot p'_1 - 2g_{3\mu}g_{3\nu}m_0^2\right)\right)$$

$$\times\frac{1}{m_0^2}\left(p_2'^{,\mu}p_2^{\nu} + p_2^{\mu}p_2'^{,\nu} - g^{\mu\nu}\left(p_2\cdot p'_2 - m_0^2\right)\right)$$

non-relativistic limit: $\left|\vec{p}\right| << m$ → $\overline{\left|M_{fi}\right|^2}\Big|_{flip} \approx e^4\frac{1}{V^4}\frac{1}{q^4}\frac{1}{4}$

spin flip ~ ¼ of all scattering events!?

no spin flip: $\$_1 = \gamma_3 = \$_3$ → $\overline{\left|M_{fi}\right|^2}\Big|_{no-flip} \approx e^4\frac{1}{V^4}\frac{1}{q^4}\frac{3}{4}$

correct???

to cross check and for comparison we rewrite the total cross section as:

$$\overline{\left|M_{fi}\right|^2}\Big|_{total} \approx e^4\frac{1}{V^4}\frac{1}{4}\frac{1}{q^4}\frac{1}{m_0^2}\left(p'_{1,\mu}p_{1,\nu} + p_{1,\mu}p'_{1,\nu} - g_{\mu\nu}\left(p_1\cdot p'_1 - m_0^2\right)\right)\times\frac{1}{m_0^2}\left(p_2'^{,\mu}p_2^{\nu} + p_2^{\mu}p_2'^{,\nu} - g^{\mu\nu}\left(p_2\cdot p'_2 - m_0^2\right)\right)$$

$$\longrightarrow \quad \overline{\left|M_{fi}\right|^2}\Big|_{total} \approx e^4\frac{1}{V^4}\frac{1}{4}\frac{1}{q^4}\frac{2}{m_0^4}\left(\left(p_1\cdot p_2\right)^2 + \left(p_1\cdot p'_2\right)^2 - 2m_0^2\,p_1\cdot p'_1 + 2m_0^4\right)$$

non-relativistic limit: $\left|\vec{p}\right| << m$ → $\overline{\left|M_{fi}\right|^2}\Big|_{total} = e^4\frac{1}{V^4}\frac{1}{q^4}$

at least sum of flip and no-flip equals total

spin rotation during the collision

except for small correction, and (only) for small angle scattering, the fraction of the incident polarization vector in the scattering plane follows the scattering angle (C.K. Iddings, G.L. Shaw, Y.S. Tsai, Physical Review 135, 6B, 1964) which for multiple intrabeam scattering could also lead to a loss of polarization

conclusions

- polarization does not affect IBS growth rates
- if no mistake in the above calculation, the IBS spin-flip component could lead to substantial depolarization, perhaps making polarization impossible to attain in IBS dominated damping rings
- IBS spin rotation still needs to be studied; we expect that it is a much smaller effect than the spin flip

Literature:

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A. Raczka and R. Raczka, *Moller Scattering of Arbitrarily Polarized Electrons*, Phys. Rev. 110, 1469 (1958)

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T.-Y. Lee, J. Choi, H.S. Kang, *Simple Determination of Touschek and Beam-Gas Scattering Lifetimes from a Measured Beam Lifetime*, NIM A 554, 85 (2005)

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appendix: other CLIC DR issues

- electron cloud
- space charge
- ion effects
- alternative damping-ring lattice

electron cloud

simulated e- densities in the wiggler vary between 10^{13} m^{-3} and several 10^{14} m^{-3}; single-bunch instability threshold ~$2x10^{12}$ m^{-3},
→ countermeasures are needed
(photon stops, clearing electrodes,…)

D. Schulte, R. Wanzenberg, F. Zimmermann, ECLOUD'04 Napa;
F. Zimmermann, WIGGLE'05 Frascati

space charge

$$\Delta Q_y^{sc} \approx \frac{N_b r_e C}{(2\pi)^{3/2}\gamma^2\sigma_z} \frac{1}{\sqrt{(\gamma\varepsilon_x)(\gamma\varepsilon_y)}} \left\langle \sqrt{\frac{\beta_y}{\beta_x + D_x^2\sigma_\delta^2/\varepsilon_x}} \right\rangle \approx 0.11$$

direct s.c. tune shift close to maximum acceptable value

T. Agoh, M. Korostelev, D. Schulte, K. Yokoya, F. Zimmermann, Collective Effects in the CLIC Damping Rings, CLIC Note 632

ion effects

T. Agoh, M. Korostelev, D. Schulte, K. Yokoya, F. Zimmermann, Collective Effects in the CLIC Damping Rings, CLIC Note 632

ion oscillation frequency ~1 GHz for H^-, lower for others

critical ion mass for trapping (at end of store):

$$A > A_{crit} \equiv \frac{N_b c \Delta t_b r_p Q}{2\sigma_y(\sigma_x + \sigma_y)} \approx 5$$

ion-induced tune shift at end of train (2 Mbarn, 1 nTorr):

$$\Delta Q \approx \frac{n_b N_b r_e C}{\pi\sqrt{(\gamma\varepsilon_x)(\gamma\varepsilon_y)}}\left(\frac{\sigma_{ion} p}{k_B T}\right) \approx 0.014$$

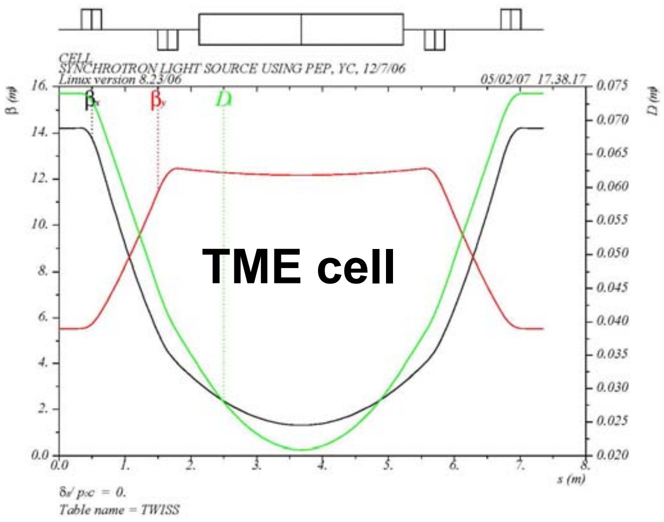
exponential rise time of FBII at end of train (1 nTorr)

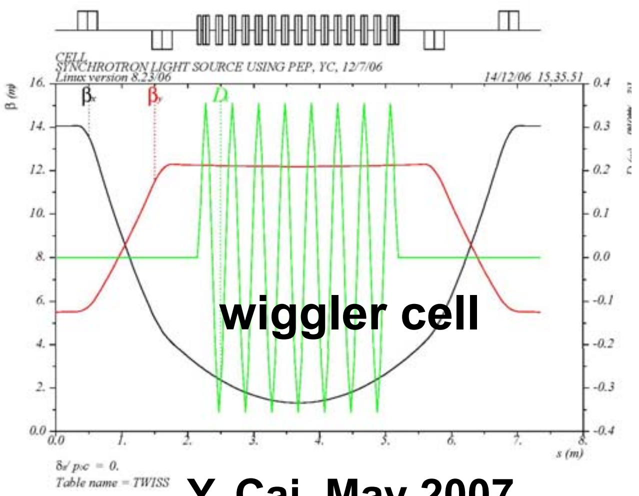
$$\tau_{FBII} \approx \frac{\gamma\sigma_y\sigma_x}{N_b n_b c r_e \beta_y \sigma_{ion}}\left(\frac{k_B T}{p}\right)\sqrt{\frac{8}{\pi}}\left(\frac{\sigma_{f_i}}{f_i}\right) \approx 3.3\mu s$$

~3 turns, controlled by feedback? better vacuum, e.g., 0.1 ntorr?

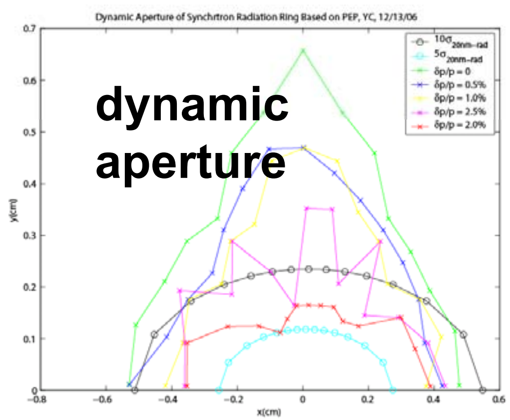
alternative CLIC DR lattice (Y. Cai)

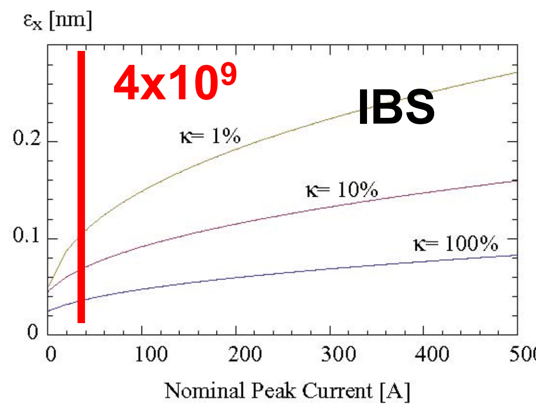
Description	Without wiggler	With wiggler
Energy E(Gev)	4.5	4.5
Circumference (m)	2200	
Norm. horizontal emittance	0.88 μm	0.44 μm
Damping time (ms)	177	15
Tunes, ν_x,ν_y,ν_s	88.57, 38.64, 0.0065	99.57, 39.64, 0.0087
Momentum compaction α_c	6.96×10^{-5}	6.86×10^{-5}
Bunch length σ_z (mm)	1.45	3.13
Energy spread σ_e/E	3.90×10^{-4}	1.14×10^{-3}
Chromaticity ξ_x, ξ_y	-143.4, -62.5	-175.6, -72.4
Energy loss per turn (Mev)	0.37	4.34
RF Voltage (MVolt)	5	10





Y. Cai, May 2007





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