Polarization and nonlinear effects in the beam-beam interaction

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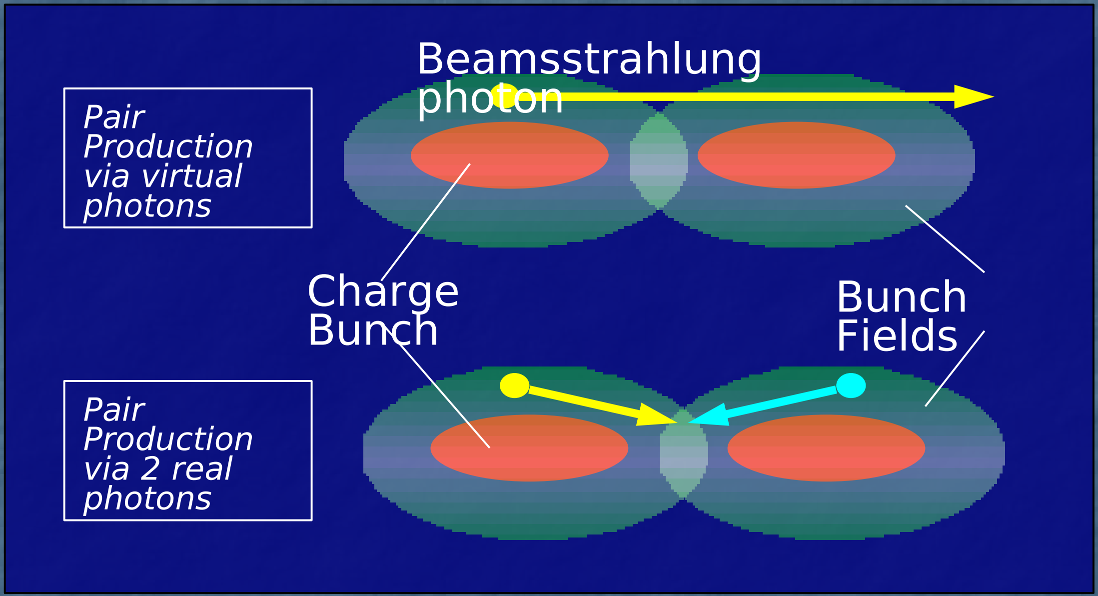
- Review beam-beam incoherent processes and the treatment of polarization
- develop expressions for, virtual photon polarization, Breit-Wheeler cross-section with full polarization, polarization of final states
- simulation runs to find pair numbers, E, P_t and luminosity
- Coherent processes – resonant cross-sections in the second order processes

E^+E^- beam-beam source of background pairs



Incoherent pair processes

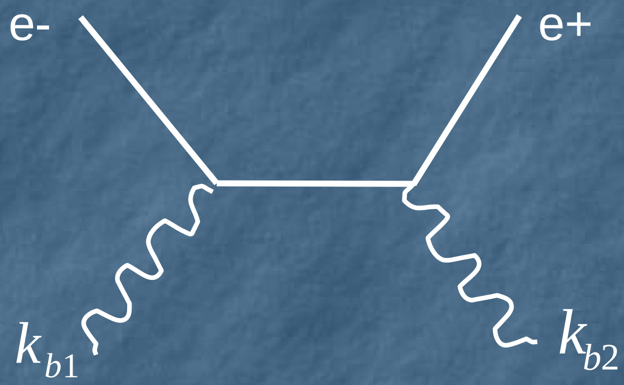
- $\gamma_{virtual} + \gamma_{real} \rightarrow E^+ + E^-$ (~71%)
- $\gamma_{virtual} + \gamma_{virtual} \rightarrow E^+ + E^-$ (~23%)
- $\gamma_{real} + \gamma_{real} \rightarrow E^+ + E^-$ (~6%)

UNKNOWNS

– Polarised beams
– Virtual photon polarisation
– Virtual photon flux (inaccurate at low energies)
– Second order non-linear processes (contains resonant cross-sections)

IS THERE VARIATION IN: pair #'s, P_t ,luminosity?

Beam-beam simulators: CAIN mod polarizations

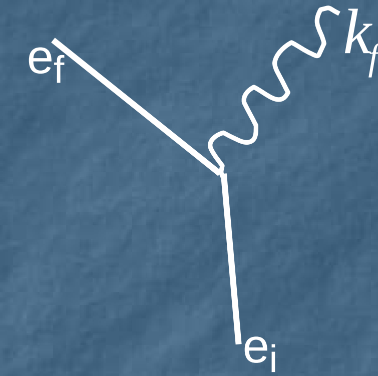


2 real γ's - Breit-Wheeler

1 real and 1 virtual γ – Bethe Heitler

2 virtual γ's – Landau Lifshitz

same cross-section: $k_{b1} + k_{b2} \rightarrow e^- + e^+$



1 virtual γ and 1 electron - Bremstrahlung

	Initial Polarization			Final Polarization		
	ξ(x)	ξ(y)	ξ(z)	ζ(x)	ζ(y)	ζ(z)
Breit_Wheeler	✓	●	✓	✓	✓	✓
Bethe-Heitler	✓	✓	✓	✓	✓	✓
Landau-Lifshitz	✓	✓	✓	✓	✓	✓
Bremstrahlung	?	?	?	?	?	?

● Already existing ✓added to CAIN ? yet to implement

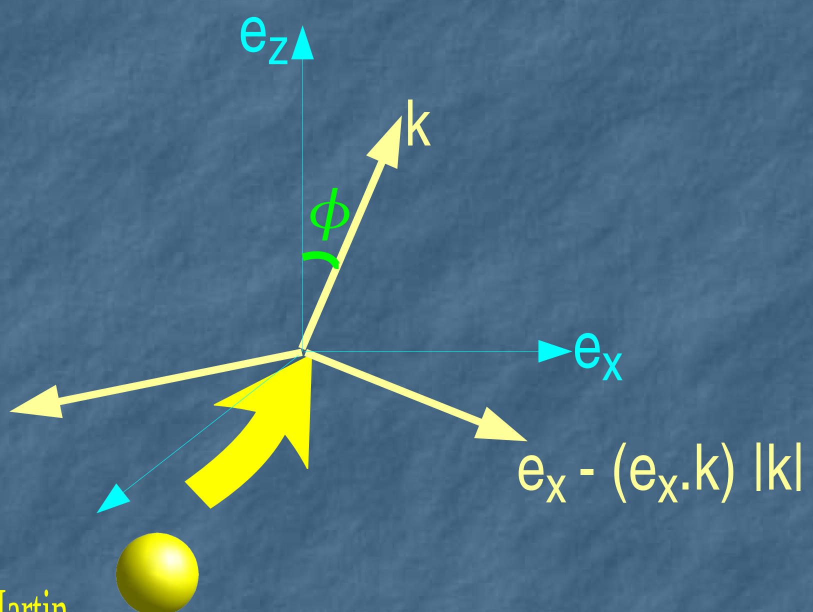
Existing CAIN structure: Beamstrahlung photon polarization

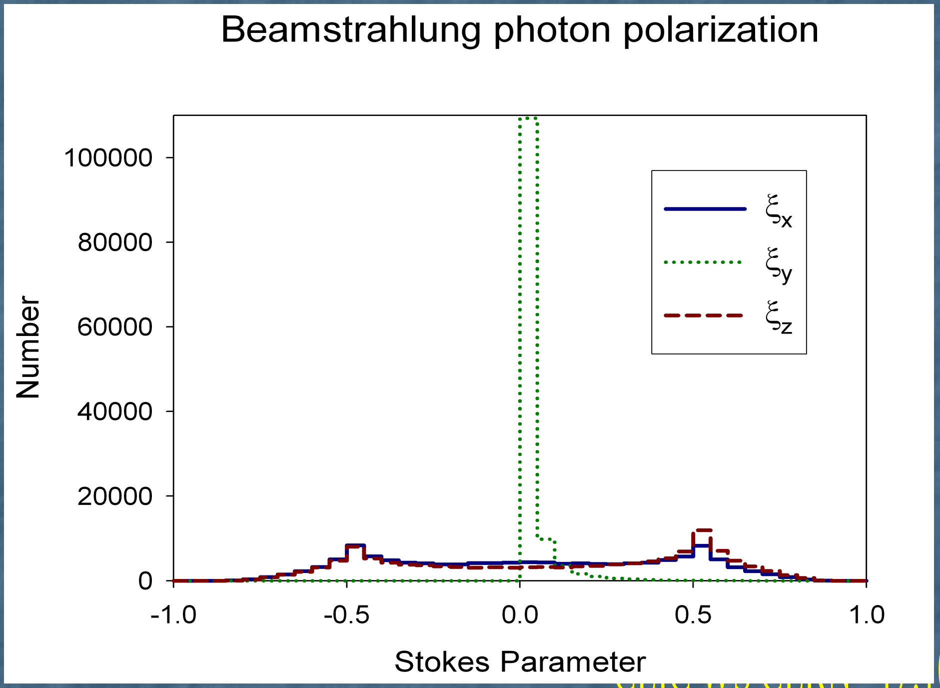
Energy spectra given by Sokolov-Ternov equation

$$dW = -i\,\frac{\alpha m}{\sqrt{3}\pi\gamma}\left[\int_z^{\infty} K_{5/3}(z)\,dz + \frac{x^2}{1-x}K_{2/3}(z)\right]dx \quad where \quad z=\frac{2B_{sch}}{3B\gamma}\,\frac{x}{1-x}$$

polarization first calculated with individual basis vectors then rotated to the same basis to be used as input to pair processes

A version of the Sokolov-Ternov equation with the polarizations written explicitly gives beamstrahlung polarizations





Virtual photon polarization I

Spectral component of bunch electric field as a function of transverse position

$$E_\omega^{x,y} = -\frac{ie}{\pi v}\iint \frac{q_{x,y}}{q_x^2+q_y^2} F(q)\, e^{i x q_x} e^{i y q_y}\, dq_x\, dq_y$$

where the form factor is

$$F(q) = N \exp\left[-\frac{1}{2}(q_x\sigma_x)^2 - \frac{1}{2}(q_y\sigma_y)^2\right]$$

and the polarization vector of virtual photons

$$e_{x,y} = \frac{E_\omega^{x,y}}{\left|E_\omega^{x,y}\right|}$$

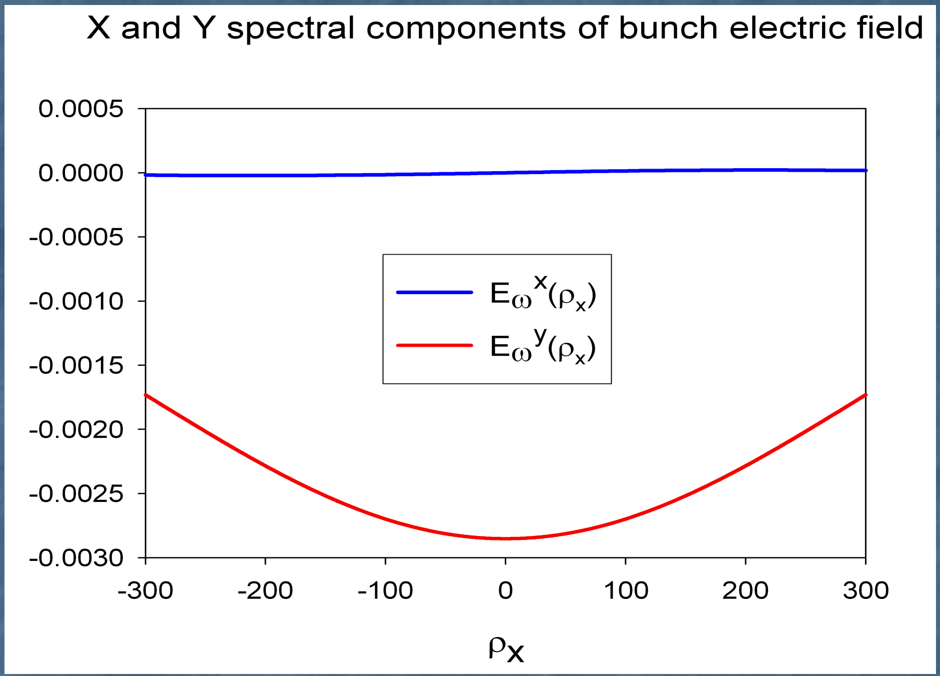
see Engel, Schiller & Serbo Z Phys C 71, 665 (1996)

integration is performed by expanding in a taylor series and using the limit $\sigma_x >> \sigma_y$ for flat beams

$$E_\omega^x = -i\frac{x}{\sigma_x^3}\exp\left(-x^2/2\sigma_x^2\right)\left[\sigma_y \exp\left(-y^2/2\sigma_y^2\right) + \sqrt{\frac{\pi}{2}}\, y\, Erf\left(y/\sqrt{2}\sigma_y\right)\right]$$

and similar for E_ω^y

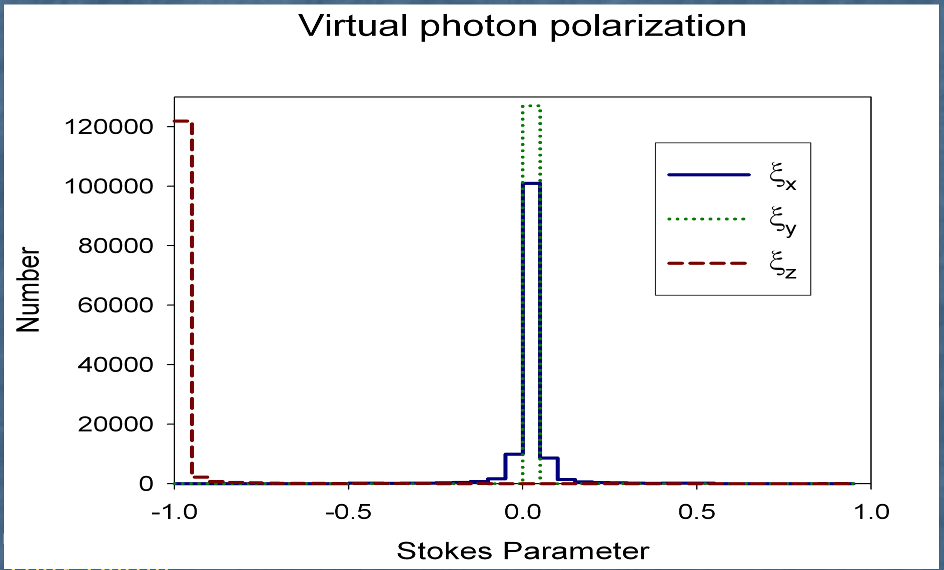
Virtual photon polarization II



Magnitude of y component of spectral electric field is much greater than x component. Has consequences for stokes parameters since

$$\xi_1 = \hat{E}_\omega^x \hat{E}_\omega^y{}^* + \hat{E}_\omega^y \hat{E}_\omega^x{}^* \quad ; \quad \xi_3 = \hat{E}_\omega^x \hat{E}_\omega^x{}^* - \hat{E}_\omega^y \hat{E}_\omega^y{}^*$$

$$\xi_2 = \Im\left[\hat{E}_\omega^y \hat{E}_\omega^x{}^* - \hat{E}_\omega^x \hat{E}_\omega^y{}^*\right] = 0$$



No circular polarization **BUT** processes occur in bunches undergoing pinch effect and other disruption, could use a more realistic form factor for the bunch field

Modified cross-section

- Breit-Wheeler cross-section, CAIN original:

$$\sigma_{orig} \propto 2\left(1-h+\frac{2\epsilon^2-1}{2\epsilon^4}\right)\sinh^{-1}p + \frac{p}{\epsilon}\left(3h-1-\frac{1}{\epsilon^2}\right)$$

where
p = electron momentum
ϵ = electron energy
$h = \xi_2 \xi'_2$

full treatment due to Baier & Grozin hep-ph/0209361

$$\frac{d\sigma}{d\cos(\theta)\,d\phi} = \frac{\alpha^2}{4s^2x^2y^2}\sum_{ii'jj'} F^{ii'}_{jj'}\,\xi_j\xi'_{j'}\,\zeta_i\zeta'_{i'}$$

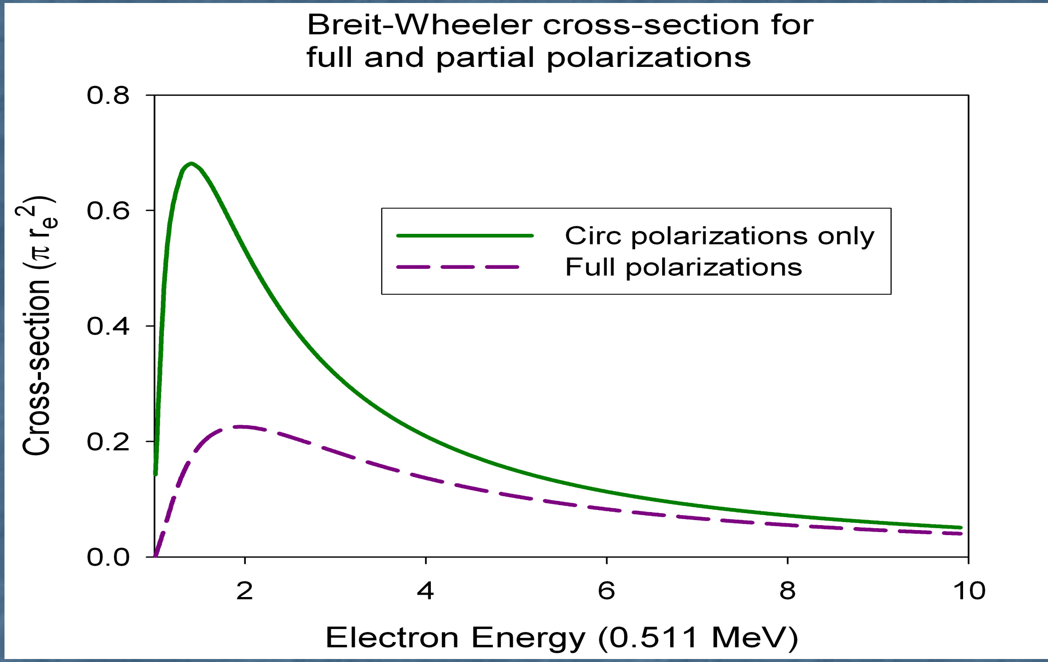
F are functions of scalar products of 4-momenta

$$\sigma_{new} \propto 2\left(1-h+\frac{2}{\epsilon^2}(ha+\xi_1\xi'_1)-\frac{ha}{\epsilon^4}\right)\sinh^{-1}p + \frac{p}{\epsilon}\left(3h-1-\xi_1\xi'_1-\xi_3\xi'_3-\frac{ha}{\epsilon^2}\right)$$

where $ha = 1+\xi_3+\xi'_3+\xi_3\xi'_3$

Full expression has similar structure to original CAIN form, so can utilise existing monte-carlo methods

Cross-section plots

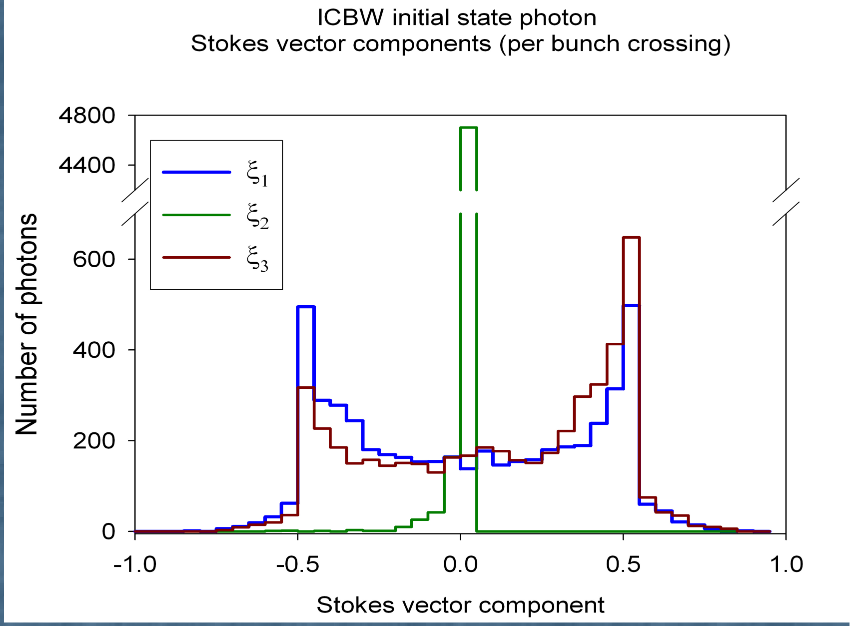


Both Breit-Wheeler cross-sections peak near the electron rest mass, but Breit-Wheeler with full polarizations shows substantially lower values at low pair energies – suggests we are currently overestimating pair numbers

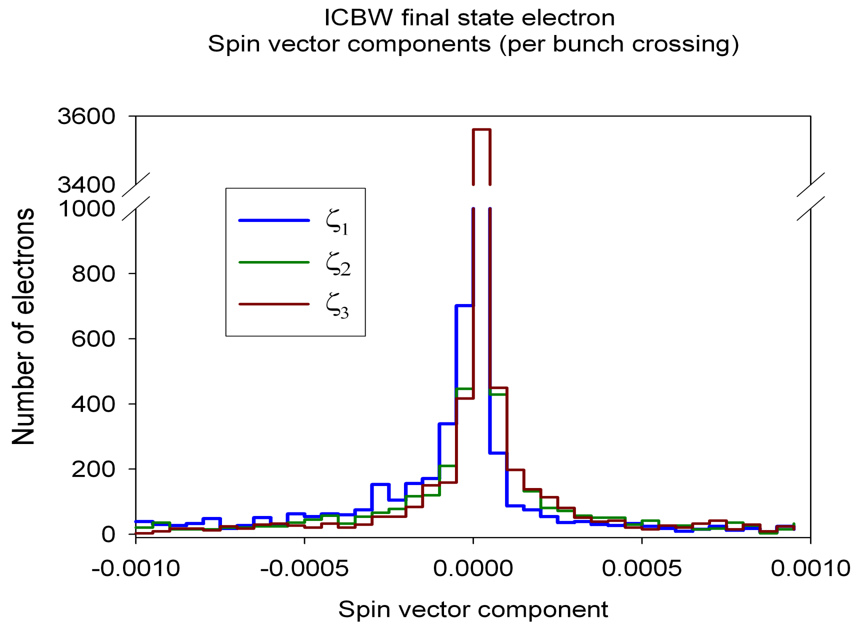
- **Bremstrahlung** calculation has been changed to allow full polarizations, **but**
- CAIN cross-section only terms containing products of one or two polarizations (there should be products of 3 and 4 as well)
- terms of up to products of two polarizations depend only on the second stokes parameter which is zero for virtual photons $\therefore$ no change
- full cross-section is necessary (Grozin, Proc VIII workshop on HEP & QFT, 93)

Final pair polarizations $\zeta^{(f)}$

$$\zeta_i^{(f)} = \frac{1}{F} \sum_{ii'jj'} F^{i0}_{jj'} \xi_j \xi'_{j'} \quad where \quad F = \sum_{jj'} F^{00}_{jj'} \xi_j \xi'_{j'}$$



- Beamstrahlung photons have almost no circular polarization component – due to beam field having constant crossed field vectors
- Breit-Wheeler pair polarization depends heavily on the photon circular polarization component, therefore pairs almost completely depolarized



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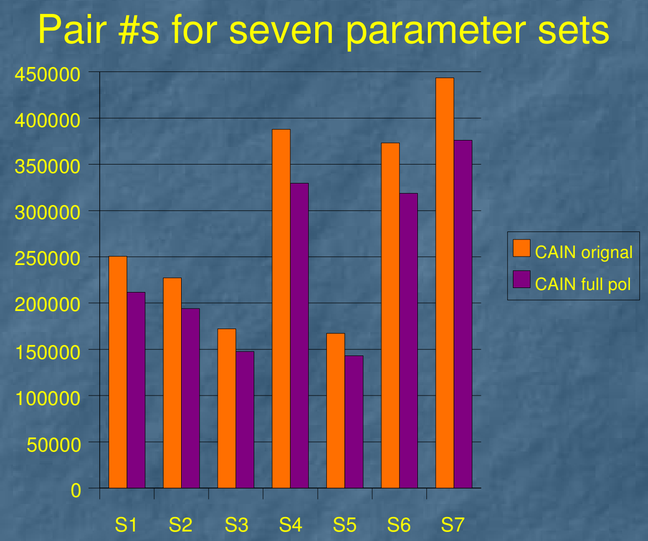
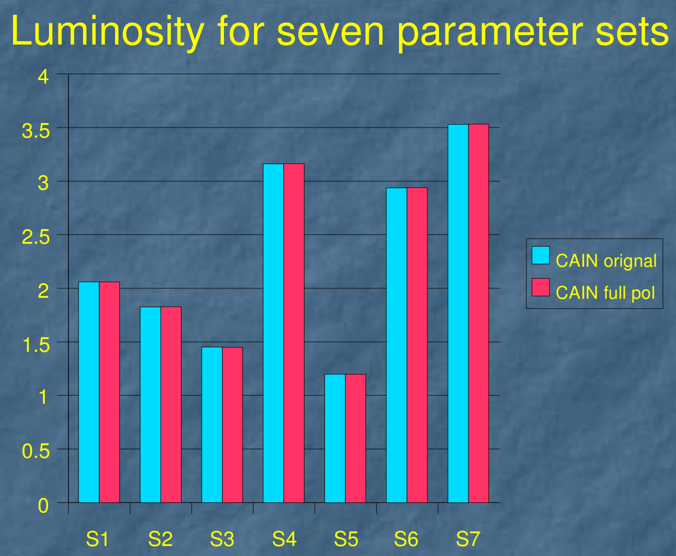
CAIN original & full pol – ILC data





- Full polarizations show 10-20% less pair particles aross all parameter schemes
- Would be good to compare with GUINEA-PIG inclusive of polarization effects
- Usually lower pair numbers indicates lower luminosity, so important to check the effect on luminosty of full polarizations
- However luminosity remains unchanged across all schemes

CLIC 3TeV pair background results

N=4x10^9, σ_x/σ_y=53/1 nm, $\gamma\varepsilon_x/\gamma\varepsilon_y$=660/20 nm rad

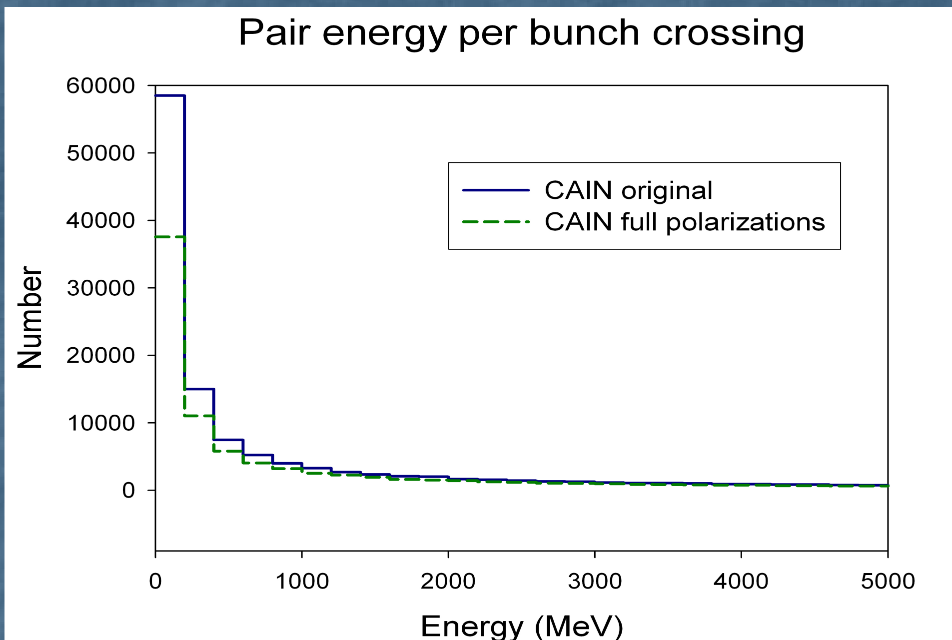
Background generator	CAIN 2.35	Guinea-Pig++	CAIN Full Pol
Bethe-Heitler pairs	216214	238105	186830
Landau-Lifshitz pairs	101838	128445	48392
Bremsstrahlung pairs	149545	152649	149580
Breit-Wheeler pairs	1164	1252	1358
INCOHERENT PAIRS TOTAL	468761	520451	386160
COHERENT PAIRS	5.126 x10^8	5 x10^8	5.126 x10^8

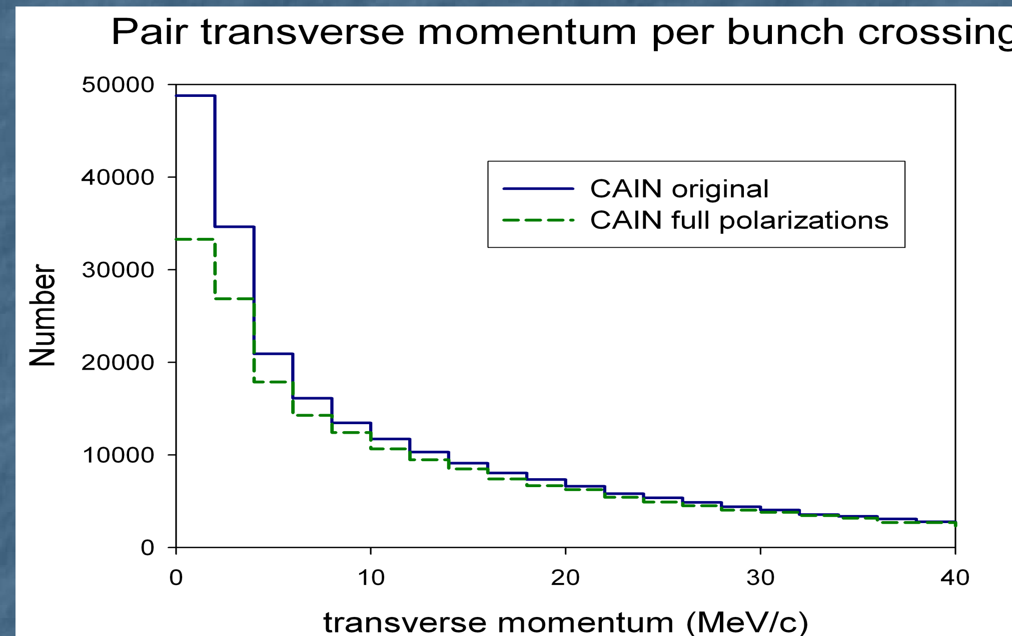
Results

- Landau-Lifshitz pairs reduced by 50%
- Bremsstrahlung yet to be adjusted
- Overall, incoherent pairs reduce by 20%
- Breit-Wheeler pairs increase but from low starting point
- Coherent pairs unchanged

..but there are changes to coherent processes

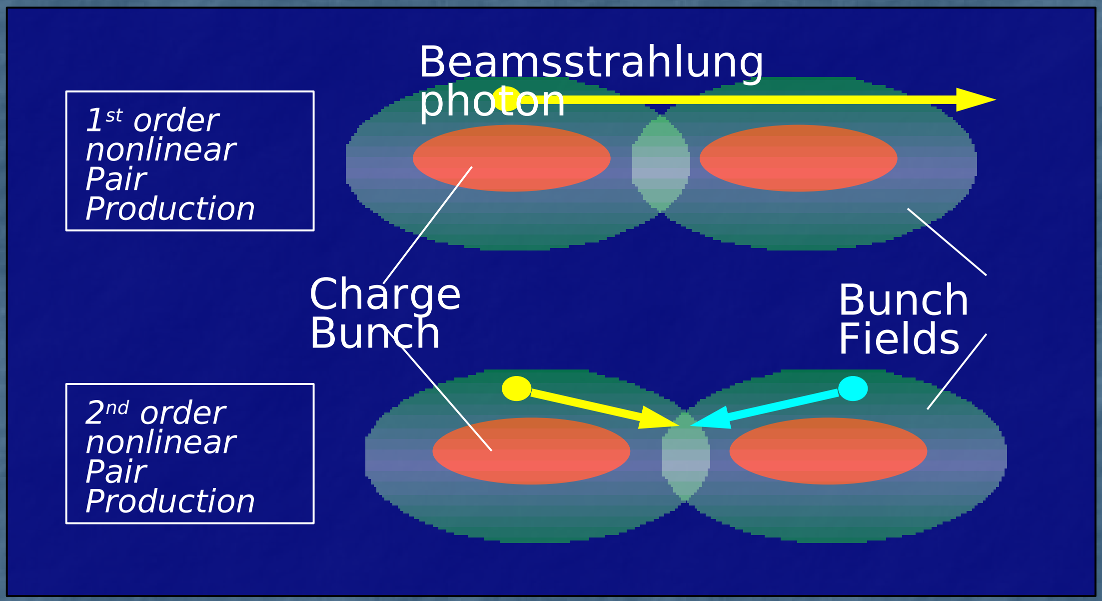
Pair data – Energy and P_t

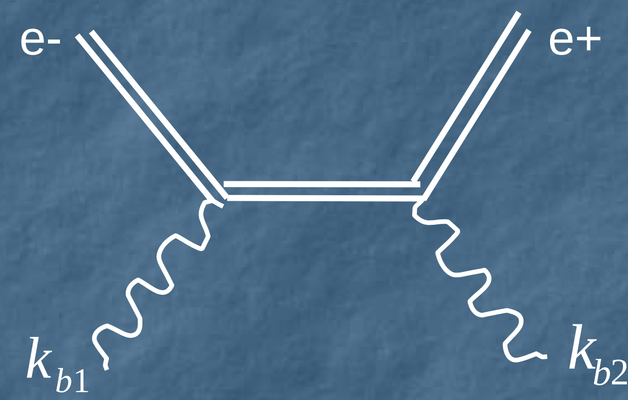




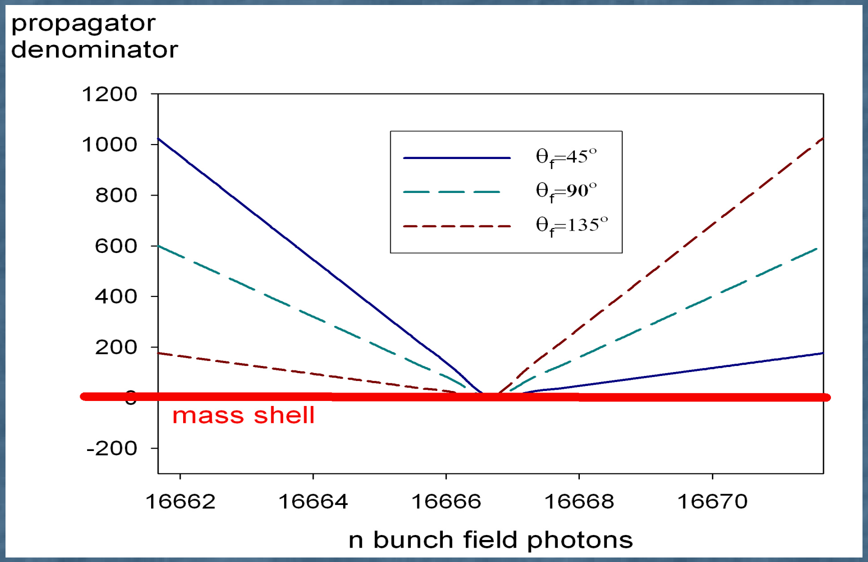
- As expected from the cross-section data there are fewer low energy and low P_t pairs.
- There are no changes at higher energies or higher P_t – so only 'good' surprises, no nasty ones... **SO FAR...**
- **Other effects not yet included could potentially have a large effect on background pair parameters**

Coherent Breit-Wheeler (CBW) process





- 2^{nd} order process contains twice as many Volkov E_p
- Bunch field can act in intermediate virtual state
  - fermions recieve a mass shift due to bunch field and the propagator can reach mass shell whenever $n\omega \sim \omega_b$



CBW cross-section with simplifications

$$\frac{d\sigma_{CBW}}{d\Omega} \approx \frac{d\sigma_{BW}}{d\Omega} \int_{\neg\omega_1/\omega}^{\infty} \frac{dn}{[(n\omega \pm \omega_1)^2 + \Gamma^2]^2} F$$

- Can write CBW diff x-section as the ordinary BW diff x-section times a function F and a resonance
- lower bound of integration is determined physically – c of m energy must be at least 2x0.511 MeV
- F is an integration of products of Airy functions for crossed beam field – numerically difficult
- Γ is a resonance width determined from a self energy calculation
- No resonances for collinear beamstrahlung and bunch momentum, but appear quickly for slightly non-collinear
- More work required

Summary and things to do

- CAIN modified to include virtual photon polarization, Breit-Wheeler x-sect with full polarization and allowance for polarization of initial and final states in the Bremstrahlung process
- 10-20% reduction in low energy and low P_t pair particles across all parameter schemes – due mainly to virtual photon polarization which is related to beam electric field. No change in luminosity
- Circular polarization of initial states very low (beam field is a constant crossed field) consequently final states almost completely depolarised
- Second order non-linear processes may contribute significantly to pair numbers – more work needed