Non-perturbative effects on jet shapes

LHCphenOnet

IFIC
INSTITUT DE FÍSICA CORPUSCULAR

Vicent Mateu
MIT Cambridge – US

Massachusetts Institute of Technology

Boost 2012, Valencia

24 - 07 - 2012

In collaboration with Jesse Thaler and Iain Stewart

Builds on work by	Salam and Wicke	JHEP 0105 (2001) 061
	Lee and Sterman	Phys.Rev. D75 (2007) 014022

Outline

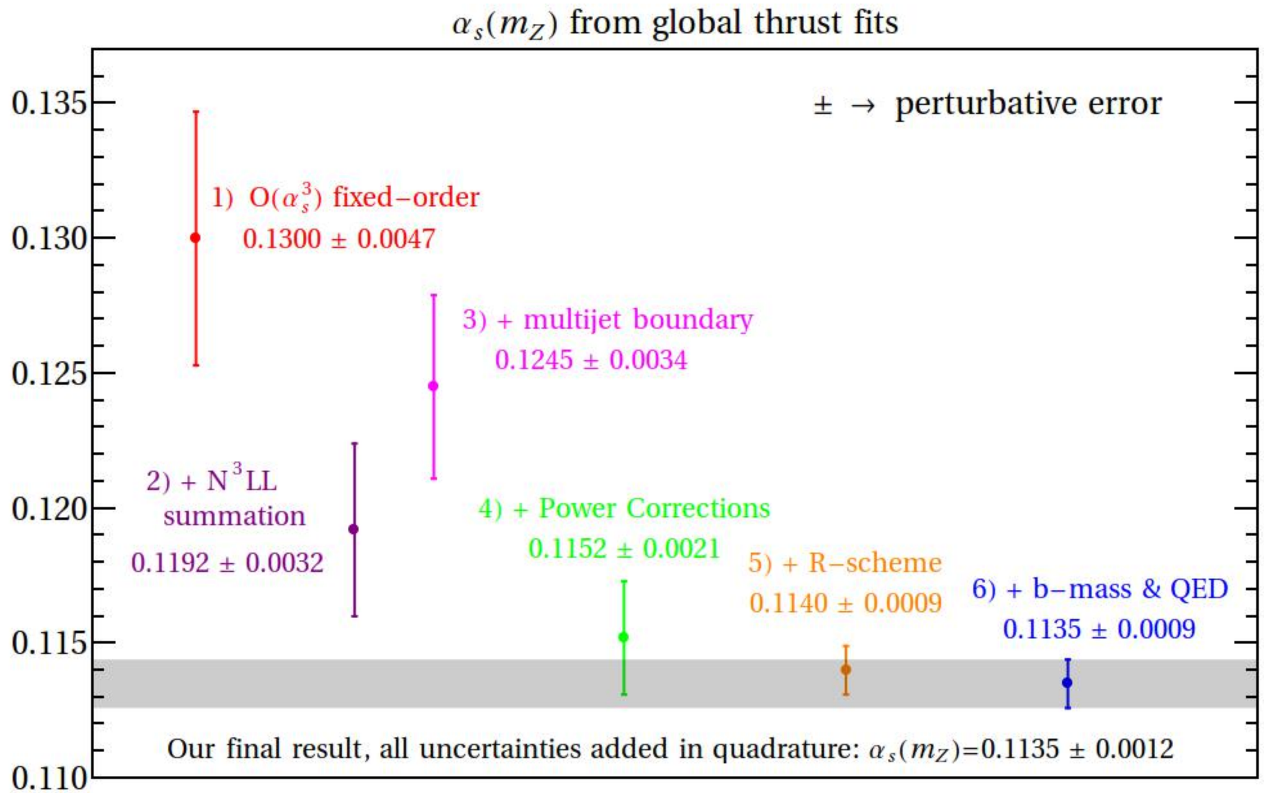
- Introduction
- Power Corrections
- Mass effects in Power Corrections
- Anomalous dimension of Power Correction
- Comparisons to Pythia 8

Introduction

Why are Power Corrections Important?

- Important effects in Jet substructure [e.g. in talk by J. Thaler]
- Important effects in e^+e^- event shapes, and in determinations of $\alpha_s(M_Z)$

  [Dokshitzer, Webber; Korchemsky, Sterman; Zaharov, Akhouri; etc...]



[R. Abbate, M. Fickinger, A.Hoang, VM and I. Stewart]

arXiv:1006.3080

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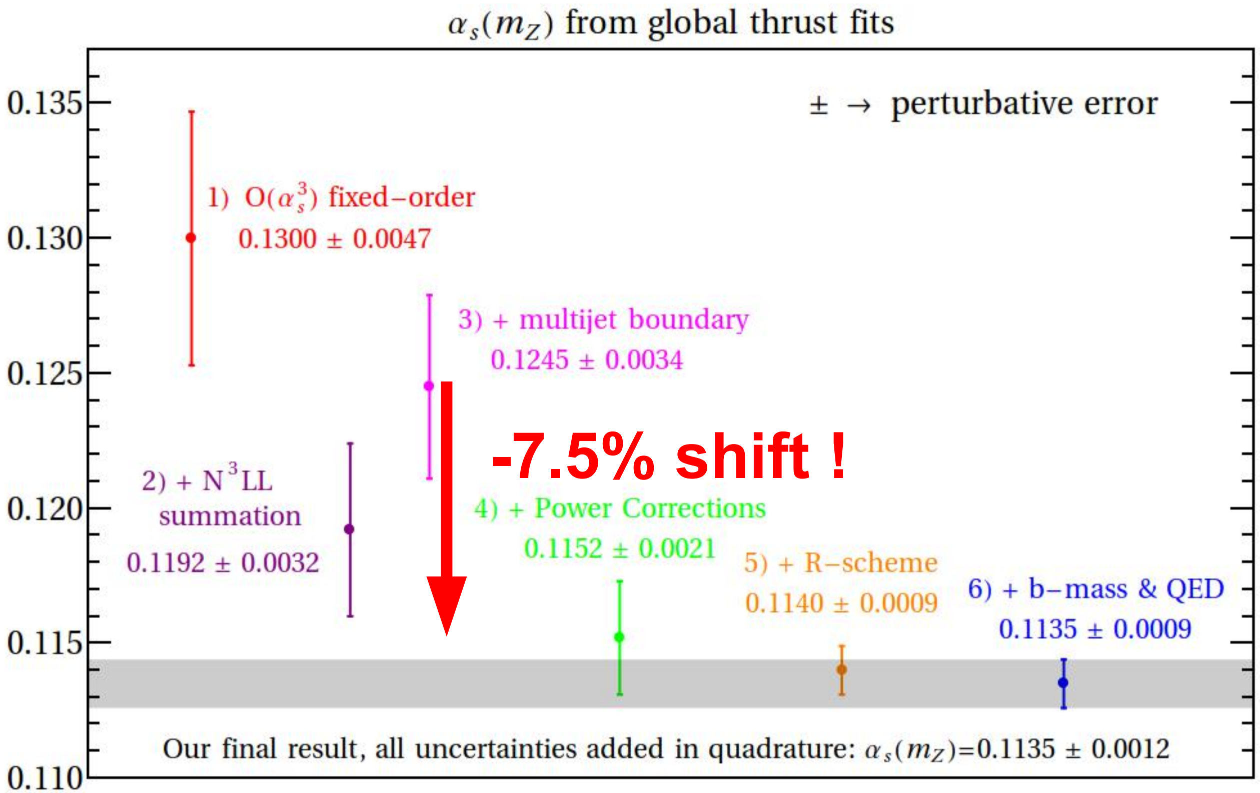

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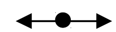
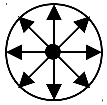
Event shapes

$$e^+ e^- \to \text{jets}$$

- Event shapes **e** characterize the distribution of hadrons in the final state.
- In the cases we study **e** $\longrightarrow$ 0 for a dijet configuration.
- Thrust is the most commonly used event shape.

$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$\tau = 0$ dijet 

$\tau = \frac{1}{2}$ spherical 

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We will concentrate on event shapes that are **not recoil sensitive** and that in the **dijet limit** can be written as

$$e(N) = \frac{1}{Q} \sum_{i \in N} m_i^{\perp} f_e(r_i, y_i)$$

$$y = \frac{1}{2} \log\left(\frac{E + p_z}{E - p_z}\right)$$ rapidity

$$r \equiv \frac{p^{\perp}}{m^{\perp}}$$ transverse velocity

All event shapes can be expressed as a function of these two variables

$$m^{\perp} = \sqrt{p_T^2 + m^2}$$ transverse mass

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Useful relations

Event shapes $e^+ e^- \to \text{jets}$

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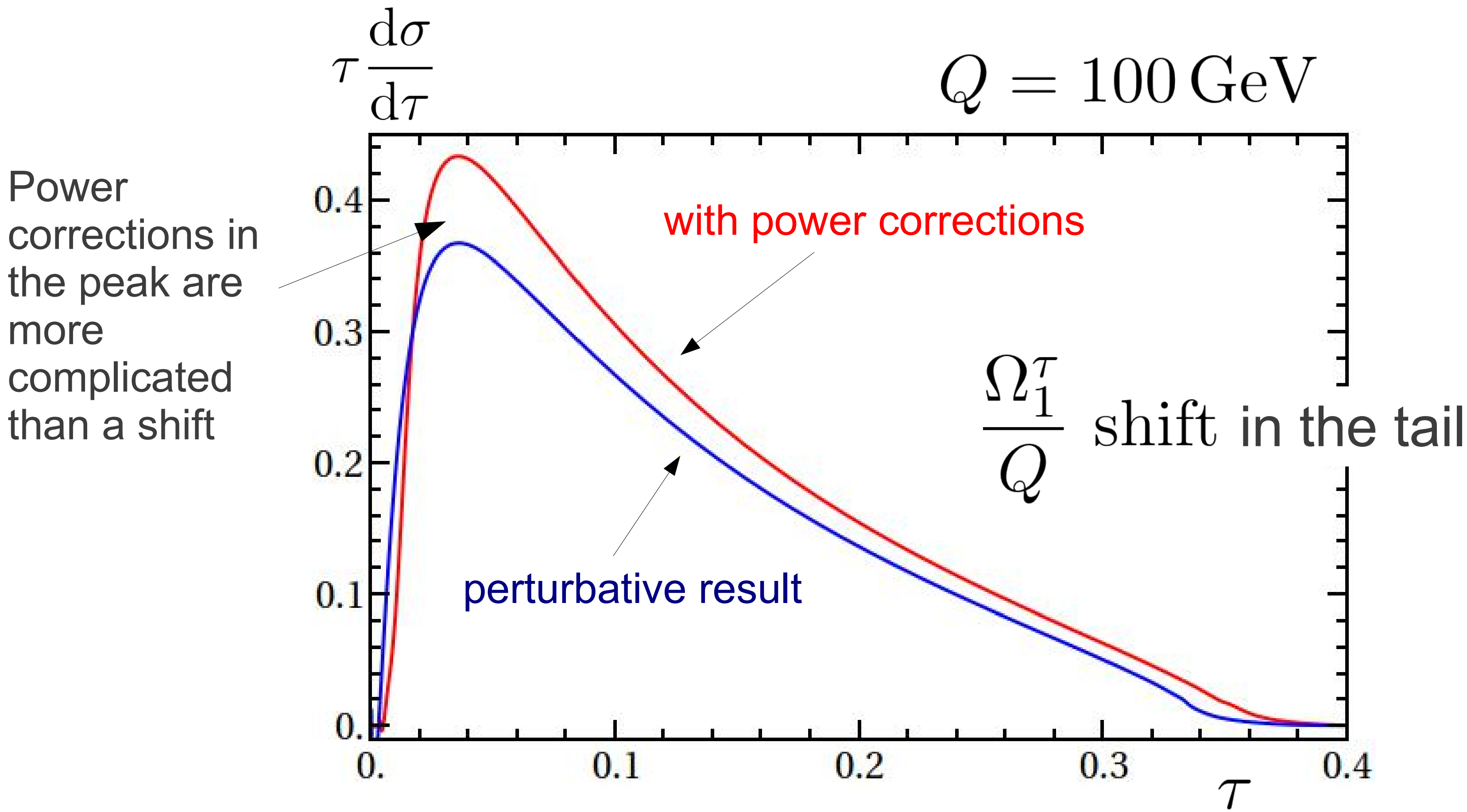
massless limit

$$v = r = 1$$
$$y = \eta$$
$$m^{\perp} = p^{\perp}$$

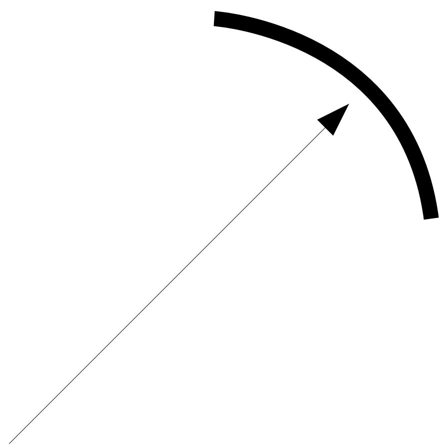
Power corrections for event shapes

Power corrections for thrust



The main effect of the power correction is shifting the distribution to the right. The shift is proportional to 1/Q.

Hadron masses and schemes



What can be measured when a particle hits the detector?

Ideally we would like **energy and momentum separately measured**, but that is not always possible.

If a **particle** is **not identified**, mass is not known, no information on magnitude of momentum.

One can assume all particles are pions [**default scheme**]

Alternatively one can use only energy and directions [**E scheme**] $|\vec{p}| \to E$

This assumptions are irrelevant in perturbation theory, but have important consequences in power corrections!

Approaches to Power Corrections

- Monte Carlo generators

Pythia, Ariadne, Herwig, Powheg, ...

- Uses hadronization models
- Hard to separate perturbative vs non-perturbative effects

- Renormalon based

Effective coupling model [Dokshitzer and Webber]
Dressed gluon [Gardi and Gruenberg]

- Residual model dependence

- Shape functions

factorization based [Korchemski, Sterman, Tafat]
SCET based [Hoang, Stewart; Lee, sterman]

- Derived directly from QCD
- Operator definition
- Systematically improvable

Studies of Universality

- **Dispersive approach:** [Dokshitzer and Webber 1995]
  - Predicts universality for a bunch of event shapes, including those which are recoil sensitive.
  - They are based on a model, and on the one-gluon approximation. Modification of strong coupling constant (effective coupling) below a cutoff scale.
  - Milan factor takes into account two-gluon effects [Dokshitzer, Webber, Salam]
- **SCET-CSS approach:** [Lee and Sterman 2006]
  - Predicts universality for no-recoil-sensitve event shapes.
  - They are model-independent, formulated in terms of QCD matrix elements.
  - Do not rely on one-gluon approximation.

Non-perturbative | perturbative

$$\frac{d\sigma}{de} = \frac{d\hat{\sigma}}{de} - \frac{\Omega_1^e}{Q}\frac{d}{de}\frac{d\hat{\sigma}}{de} + O(Q^{-2})$$

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Both approaches assume all particles are massless !

Massless predictions for universality

$$\Omega_1^e = c_e\, \Omega_1^\rho \qquad c_e = \int \mathrm{d}y f_e(r=1,y)$$

thrust $\quad \tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum |\vec{p}_i\,|} \quad c_\tau = 2$

2-Jettiness $\quad \tau_{12} = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q} \quad c_{\tau_{12}} = 2$

C-parameter $\quad C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j|\sin^2(\theta_{ij})}{(\sum_i |\vec{p}_i|)^2} \quad c_C = 3\pi$

angularities $\quad \tau_a = \frac{1}{Q}\sum_i E_i\,(\sin\theta_i)^a(1-|\cos\theta_i|)^{1-a} \quad c_{\tau_a} = \frac{2}{1-a}$

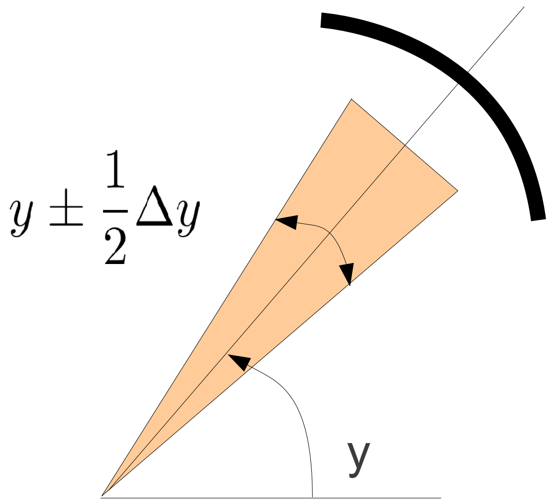
Jet masses $\quad \rho_\pm = \frac{1}{Q^2}\Big(\sum_{i\in\pm} p_i\Big)^2 \quad c_\rho = 1$

Massless universality in SCET-CSS

In massless limit one has $$e(N) = \frac{1}{Q}\sum_{i\in N} p_i^\perp f_e(1,y_i)$$

Transverse energy flow operator $$\mathcal{E}_T(y)\,|N\rangle = \sum_{i\in N} p_i^\perp \delta(y-y_i)\,|N\rangle$$

[Lee Sterman, Korchemsky Oderda Sterman, Sveshnikov and F. V. Tkachov
Ore Sterman



measures all momenta flowing in a given rapidity

$$\hat{e} = \int \mathrm{d}y\, f_e(1,y)\mathcal{E}_T(y) \longrightarrow \hat{e}\,|N\rangle = e(N)\,|N\rangle$$ event-shape operator

$$\mathcal{E}_T(y) = \frac{1}{\cosh^3 y}\int_0^{2\pi} \mathrm{d}\phi \lim_{R\to\infty}\int_0^\infty \mathrm{d}t\, \hat{n}_i T_{0i}(t, R\hat{n})$$

[Bauer, Fleming, Lee, Sterman]

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Universal power correction

$$\Omega_1^e = \int \mathrm{d}y \, f_e(1, y) \, \langle 0 | \, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(y) Y_n \overline{Y}_{\bar{n}} \, |0\rangle = \boxed{c_e} \times \Omega_1^E$$

Boost invariance requires that this terms is **y-indepenent**

Calculable coefficient, depends on event shape

$$\Omega_1^E = \langle 0 | \, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(0) Y_n \overline{Y}_{\bar{n}} \, |0\rangle$$

Operator definition of power correction

Mass effects on Power Corrections

Mass effects in Power Corrections

Salam and Wicke have studied mass effects on power corrections

- Use the Flux-tube model (later refined with QCD effects)
- Predict that mass effects **break universality**
- They find a privileged scheme (**E-scheme**) in which **universality is recovered**
- They predict that hadron multiplicity translate into **log(Q) effects** on the Power Correction

$$\Omega_1 \to \Omega_1 + K \left(\log \frac{Q}{\Lambda} \right)^{\frac{4C_A}{\beta_0}}$$

[Salam Wicke 2001]

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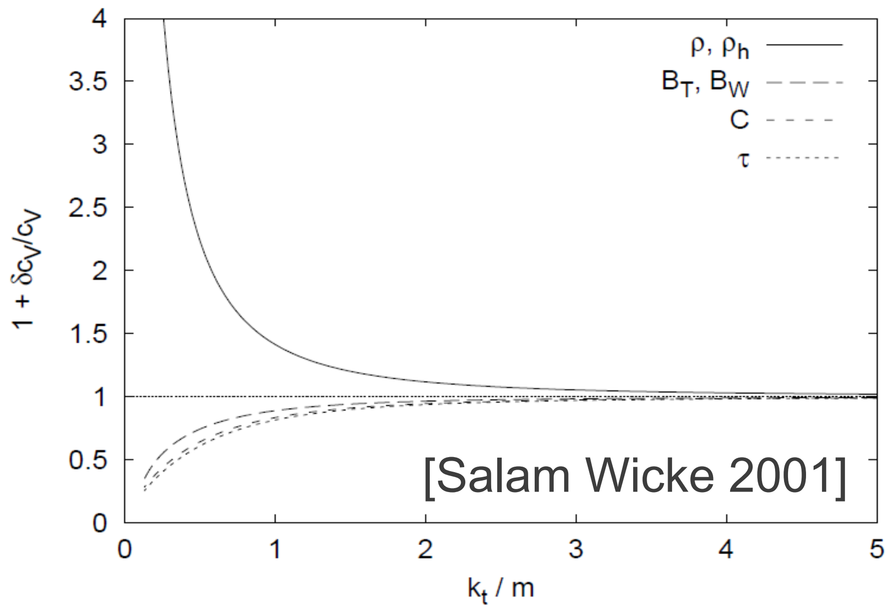

Similar curves correspond to similar power corrections. HJM has very different PC than the rest.

Mass effects treated as a correction to massless prediction.

All curves equal one for massless hadrons

P-scheme $\quad E \to |\vec{p}\,|$

In the P-scheme all curves are very similar, approximate universality

E-scheme $\quad \vec{p} \to \dfrac{E}{|\vec{p}\,|}\,\vec{p}$

In the E-scheme all curves are equal to each other, restores universality

Mass effects in Power Corrections

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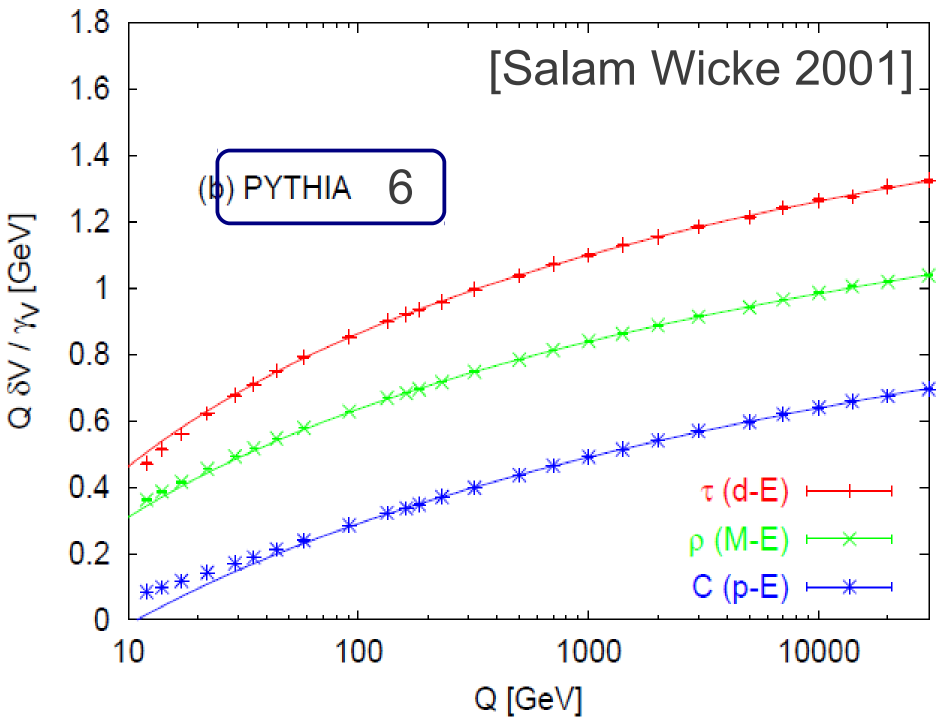
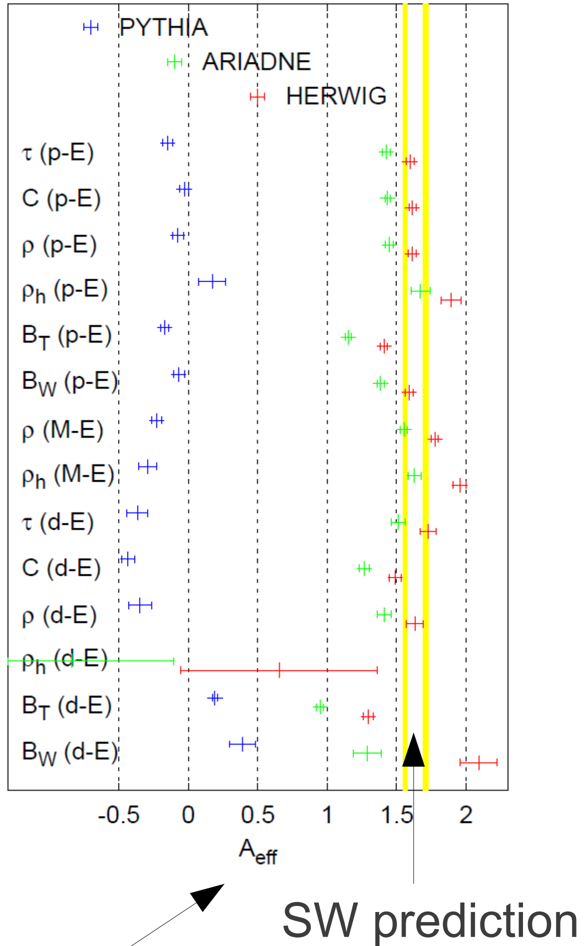




$\log^{A_{\rm eff}}(Q)$

MC generator predictions for differences of the mean value in two schemes, for three event shapes.

MC predict a log(Q) behavior, but different MC see different corrections.

Mass effects in SCET

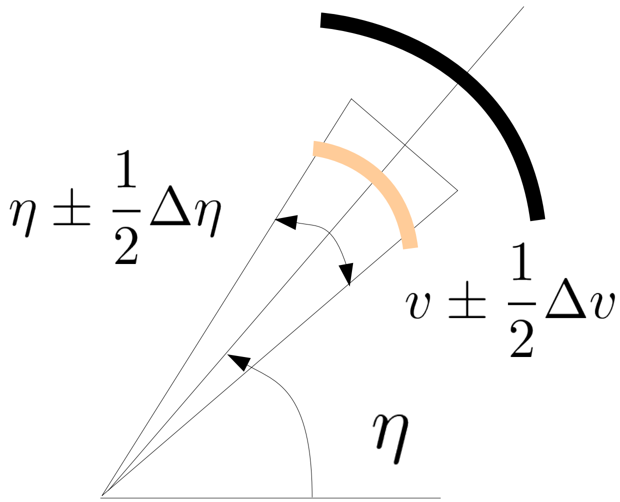
VM
I. Stewart
J Thaler, w.i.p.

$$e(N) = \frac{1}{Q} \sum_{i \in N} m_i^{\perp} f_e(r_i, y_i)$$

One has to generalize the transverse energy flow operator

Transverse mass flow operator

$$\mathcal{E}_T(r,y)\,|N\rangle = \sum_{i \in N} m_i^{\perp} \delta(r - r_i)\delta(y - y_i)\,|N\rangle$$



$$v = v(r,y)$$

$$\eta = \eta(r,y)$$

measures momenta of particles with given velocity flowing at a given pseudo-rapidity

$$\mathcal{E}_T(v,\eta) = -\frac{v(1 - v^2 \tanh^2 y)^{\frac{3}{2}}}{\cosh \eta} \lim_{R \to \infty} R^3 \int_0^{2\pi} \mathrm{d}\phi\, \hat{n}_i\, T_{0i}(R, v\, R\, \hat{n})$$

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Using same boost invariance argument one finds

$$\Omega_1^e = c_e \int \mathrm{d}r\, g_e(r)\, \Omega_1(r)$$

c_e **same as massless case**

$g_e(r)$ **encodes all mass effects**

each g(r) defines a universality class of event with same power correction

$$g_e(r) = \frac{1}{c_e} \int \mathrm{d}y\, f_e(r, y)$$

Operator definition of power correction

$$\Omega_1(r) = \langle 0|\, \overline{Y}_{\bar{n}}^{\dagger} Y_n^{\dagger} \mathcal{E}_T(r, 0) Y_n \overline{Y}_{\bar{n}}\, |0\rangle$$

Event shapes considered

Thrust

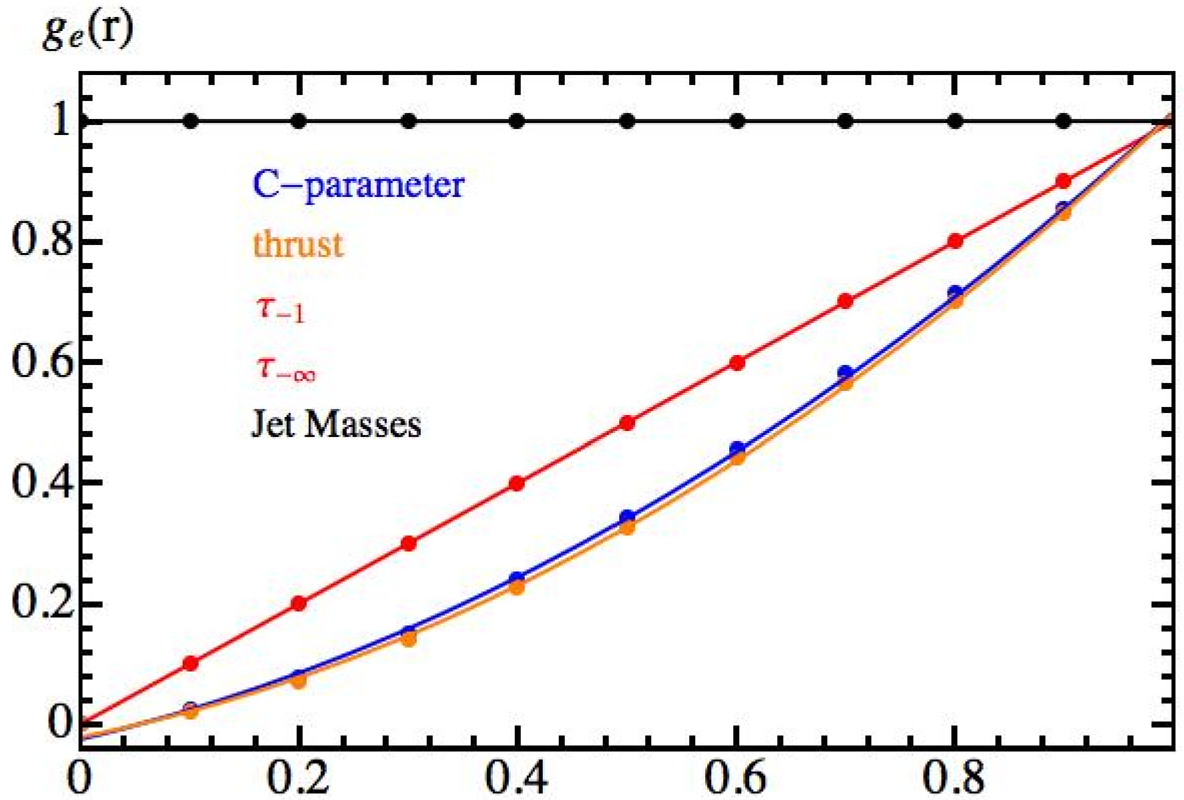
Jet masses

C-parameter

Angularities

2-Jettiness

Mass scheme (default definition)



$$r = \frac{p^\perp}{m^\perp}$$

Same **color** means same **power correction**

Event shapes considered

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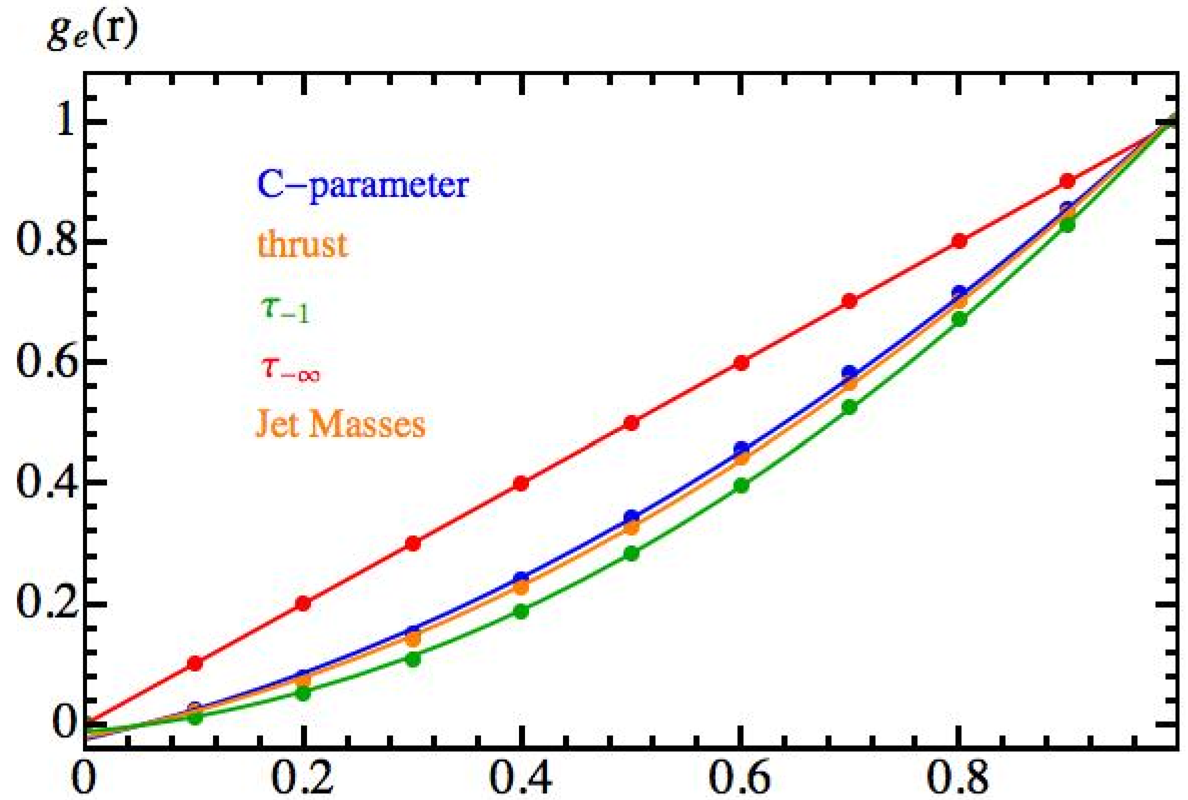
2-Jettiness

P-scheme



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Scheme changes event shape definition

Event shapes considered

Thrust

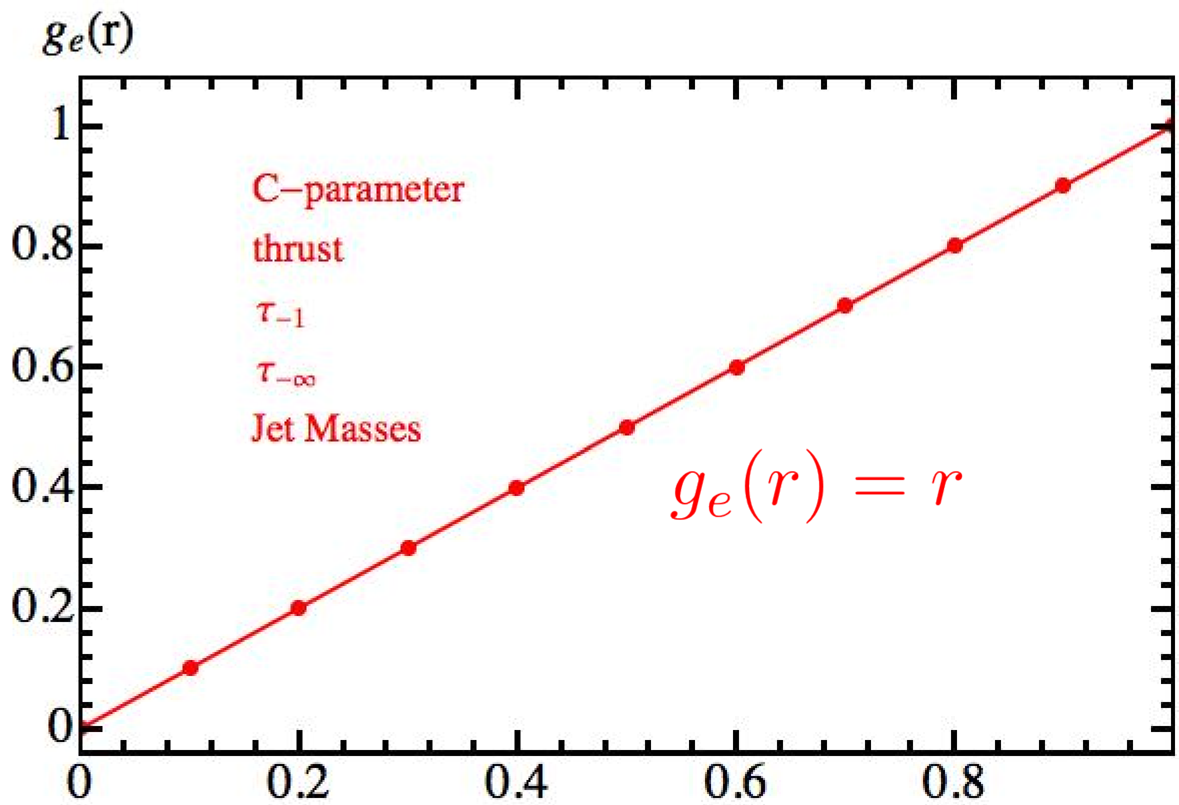
Jet masses

C-parameter

Angularities

2-Jettiness

E-scheme



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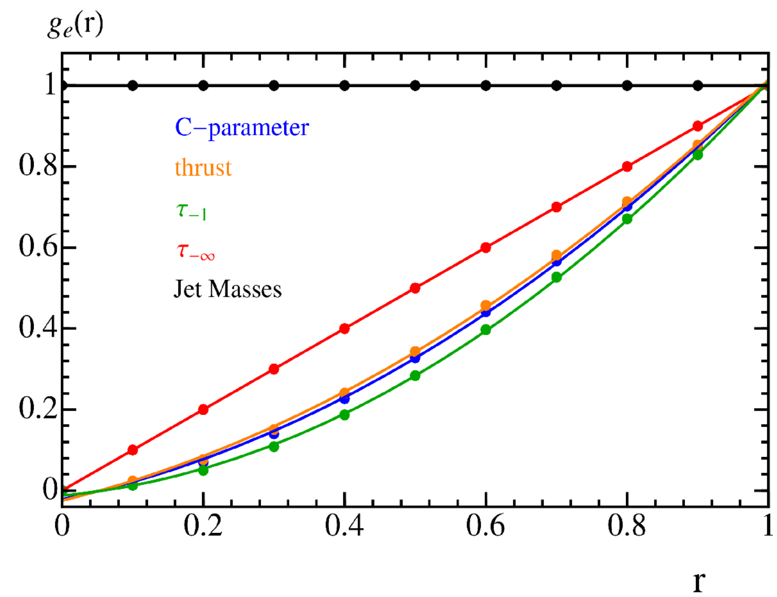
Scheme changes
event shape definition

Effective parametrization



g(r) functions are different, but it seems they could be approximated well by some suitable set of orthogonal polynomial

$$h_n(r) = \sqrt{2n+1}\,P_n(2x+1)$$

$$g_e(r) = \sum_{n=0}^{\infty} b_n^e\,h_n(r)$$

$\Omega_1(r)$ can be expanded as well

$$\Omega_1(r) = \Omega_1^{\rho}\,h_0(r) + \sqrt{3}(2\Omega_1^E - \Omega_1^{\rho})h_1(r) + \Omega_1^{\delta}h_2(r) + \cdots$$

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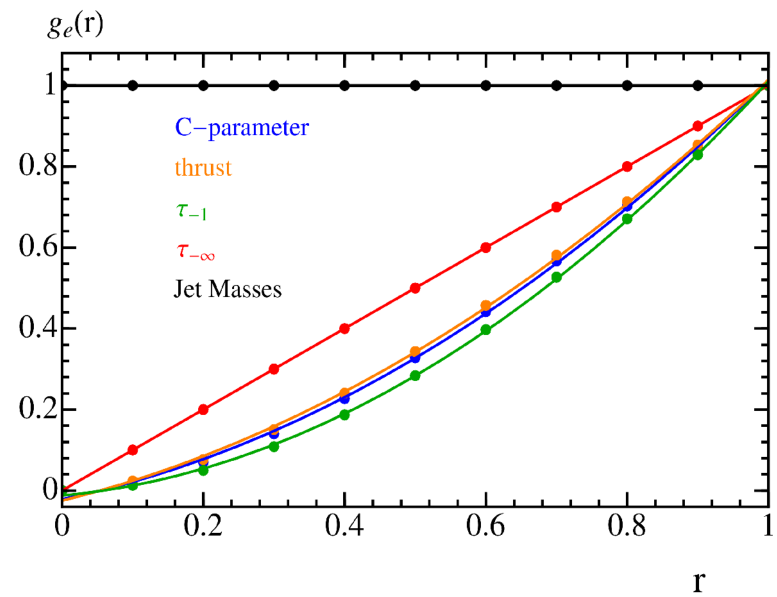

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$$\Omega_1^\tau = 1.034\,\Omega_1^E - 0.135\,\Omega_1^\rho + 0.050\,\Omega_1^\delta$$

$$\Omega_1^C = 1.039\,\Omega_1^E - 0.127\,\Omega_1^\rho + 0.046\,\Omega_1^\delta$$

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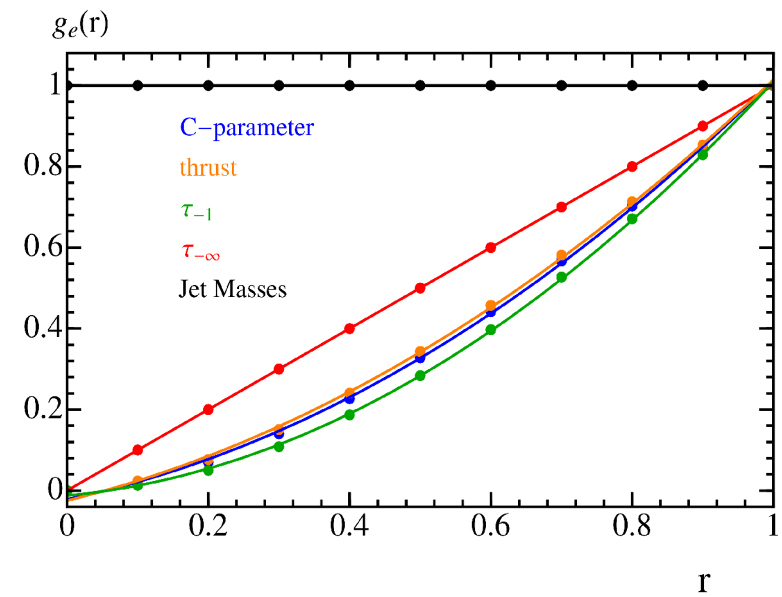

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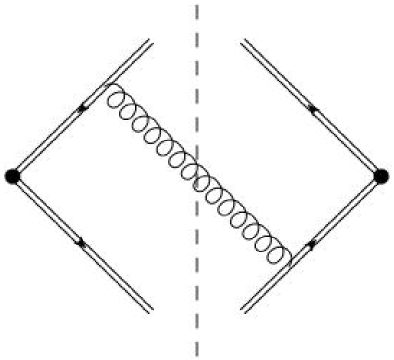
small correction

Anomalous dimension of power correction

Anomalous dimension computation

One needs to compute diagrams that **probe the operator**. We choose

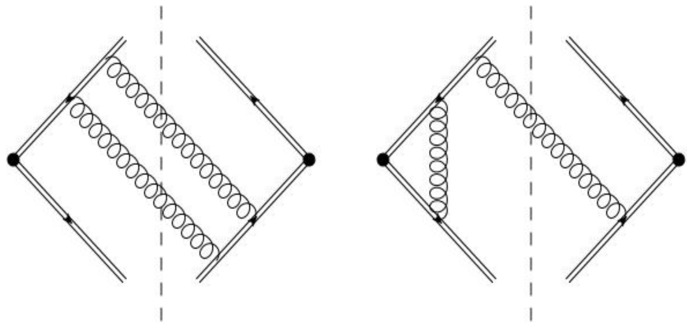


$$\Omega_1(r) = \langle 0| \, \overline{Y}^{\dagger}_{\bar{n}} Y^{\dagger}_{n} \mathcal{E}_T(r,0) Y_n \overline{Y}_{\bar{n}} \, |0\rangle$$

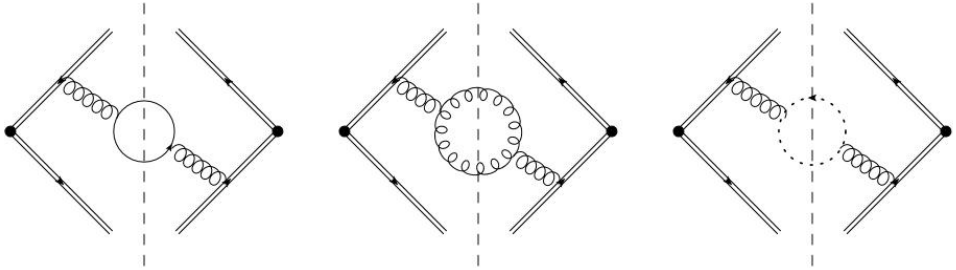
The measured gluon is **off-shell**

This probes values of **r away from 1**

One loop corrections



Abelian diagrams exactly **cancel** each other



Self energy diagrams **do not contribute** to the anomalous dimension of the operator

(Crossed diagrams and complex conjugate diagrams not shown)

Anomalous dimension computation

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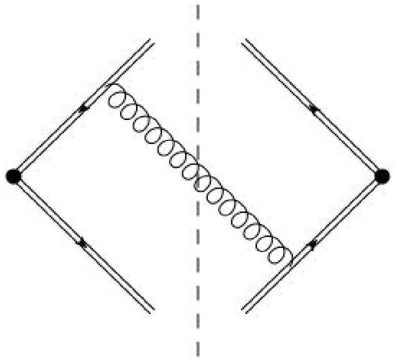

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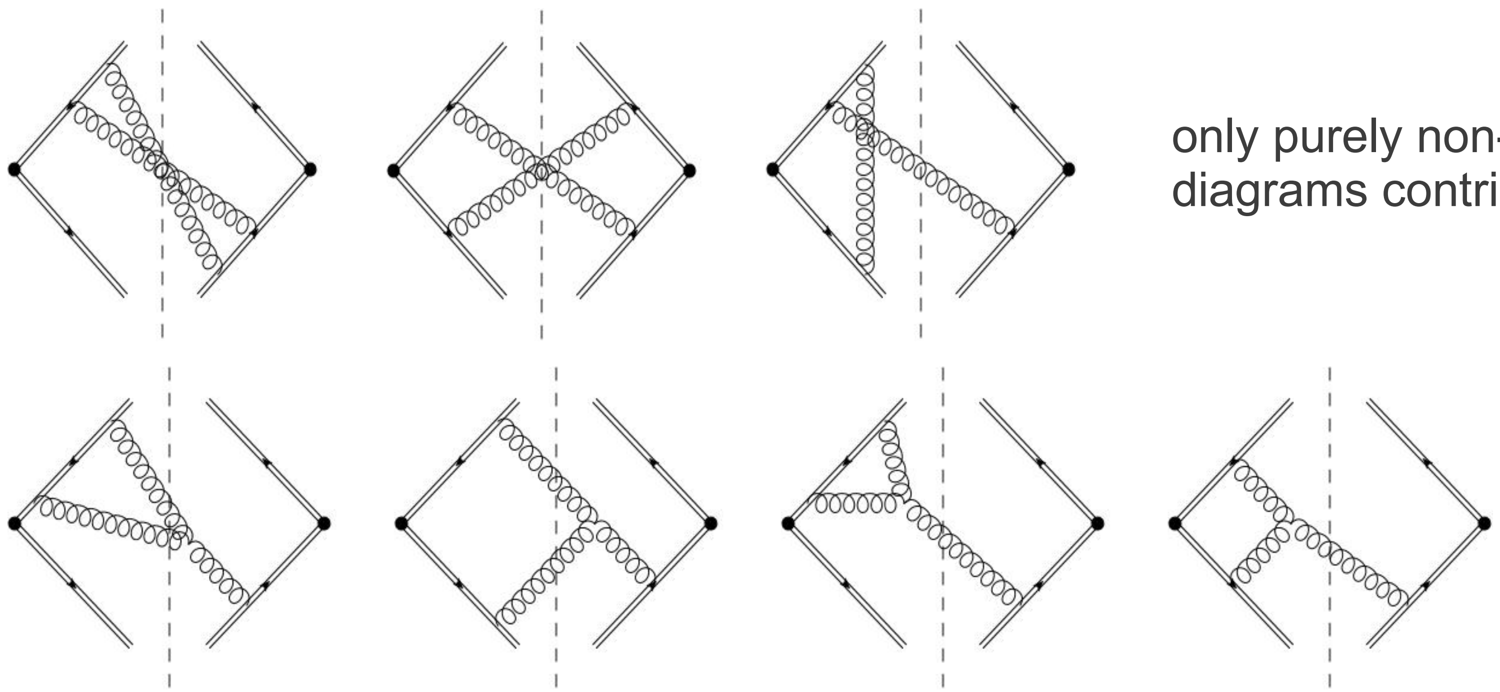

only purely non-abelian diagrams contribute

(Crossed diagrams and complex conjugate diagrams not shown)

Result and consequences

$$\gamma^{\Omega_1} = -\frac{\alpha_s C_A}{\pi}\log(1-r^2)$$

Anomalous dimension is **r-dependent**
There is **no mixing** between different r values

The RGE can be easily solved and logs can be resummed at LL

$$\Omega_1(r,\mu) = \Omega_1(r,\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{2\,C_A}{\beta_0}\,\log(1-r^2)}$$

$$\sim \Omega_1(r,\mu_0)\left[1-\frac{\alpha_s(\mu_0)C_A}{\pi}\log\left(\frac{\mu}{\mu_0}\right)\log(1-r^2)\right]$$

Expanded out result

However this does not translate into a resummation formula for Ω_1^e

$$\Omega_1^e(\mu) = \int \mathrm{dr}\, \mathrm{g_e(r)}\, \Omega_1(\mathrm{r},\mu_0)\left(\frac{\alpha_\mathrm{s}(\mu)}{\alpha_\mathrm{s}(\mu_0)}\right)^{\frac{2\,\mathrm{C_A}}{\beta_0}\,\log(1-\mathrm{r}^2)}$$

Unknown function!

However if one uses the expanded out expression

$$\Omega_1^e(\mu) = \Omega_1^e(\mu_0) - \frac{\alpha_s(\mu_0)C_A}{\pi}\log\left(\frac{\mu}{\mu_0}\right)\Omega^e_{\log}(\mu_0)$$

$$\Omega^e_{\log}(\mu_0) = \int \mathrm{dr}\,\log(1-\mathrm{r}^2)\,\mathrm{g_e(r)}\,\Omega_1(\mathrm{r},\mu_0)$$

New non-perturbative parameter

Result and consequences

One can check that the expanded out result is often a reasonable approximation

$$\Omega_1^e(\mu) = \int \mathrm{d}r\, g_e(r)\, \Omega_1(r,\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{2\,C_A}{\beta_0}\,\log(1-r^2)} = \Omega_1^e(\mu_0) - \frac{\alpha_s(\mu_0) C_A}{\pi} \log\left(\frac{\mu}{\mu_0}\right) \Omega_{\log}^e(\mu_0) + \cdots$$

The Q dependence of the power correction comes through scale setting

$$\mu_0 \sim 2\,\mathrm{GeV}$$

$$\mu = \mu_s \propto Q\, e$$

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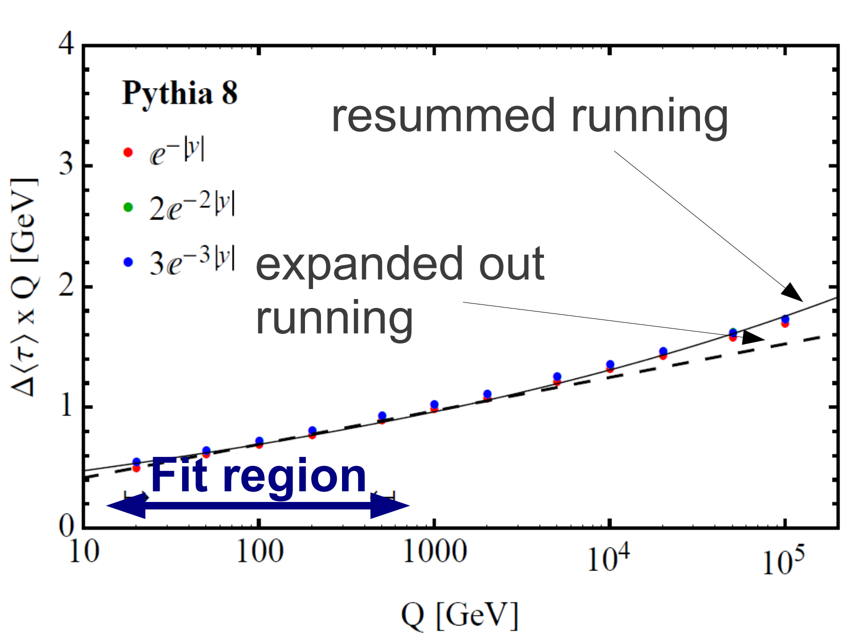
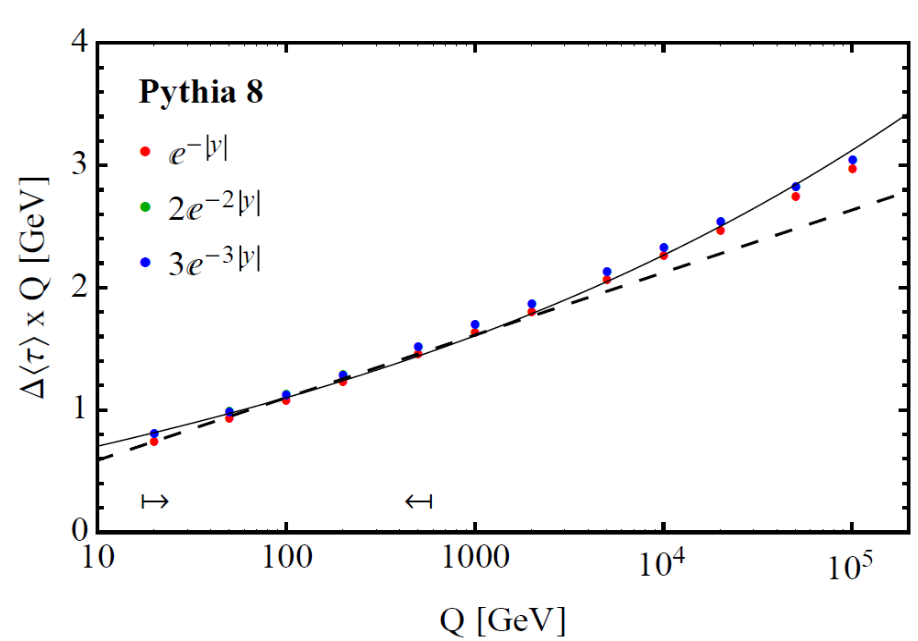
Similar running factor

Comparisons to Pythia

Comparisons to Pythia 8





We fit for the two power corrections

We also fit for $\Omega_1(r,\mu)$ at a fixed scale and evolve it to higher Q values

For this exercise we use first moment

$$\Omega_1^e(\mu) = \boxed{\Omega_1^e(\mu_0)} - \frac{\alpha_s(\mu_0)C_A}{\pi}\log\left(\frac{\mu}{\mu_0}\right)\boxed{\Omega_{\log}^e(\mu_0)}$$

$$\Omega_1(r) = \Big[{\color{red}a} + {\color{red}b}\,r + ({\color{red}c} + {\color{red}d}\,r)\log(1-r^2)\log(Q)\Big] \times \text{evolution}$$

- Pythia reproduces universality for each class
- Our running prediction seems compatible with Pythia in each class

Conclusions

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- Operator description of mass effects in power corrections
- These mass effects break universality and are not simply a correction
- Set of **privileged classes** in which there is universality. **Approximate universality** among classes (agreement with Salam and Wicke)
- Computation of **anomalous dimension** predict log(Q) dependence of power corrections. Systematic way of including those effects.
- Comparisons to **Pythia 8** support our findings