Could the Higgs Boson be the Inflaton?

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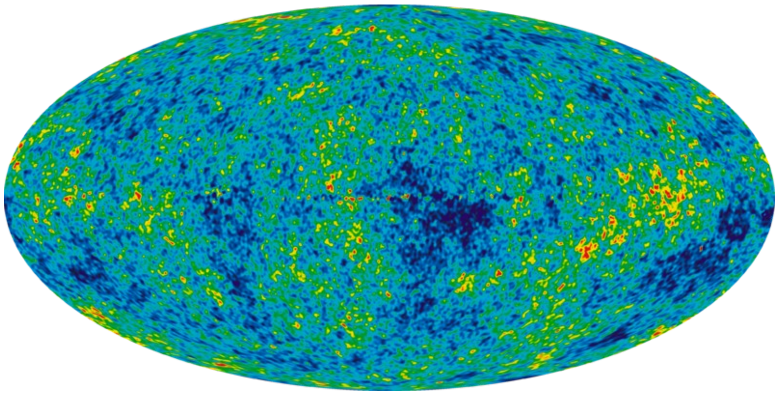
Outline

- Why inflation?
- The Higgs as the inflaton
- Unitarity and Higgs inflation
- SM false vacuum inflation

Why Inflation?

•Why does the universe appear flat, homogeneous and isotropic?



The CMB temperature fluctuations
from the 7-year WMAP data

Temperature of CMB is
2.725K ± 0.0002K –
extremely uniform!

•Can be explained if the universe went through a very early period of exponential expansion - inflation.

•Inflation also explains the origin of the large-scale structure of the cosmos.

Slow Roll Inflation

- Inflation is driven by a negative-pressure vacuum energy density.

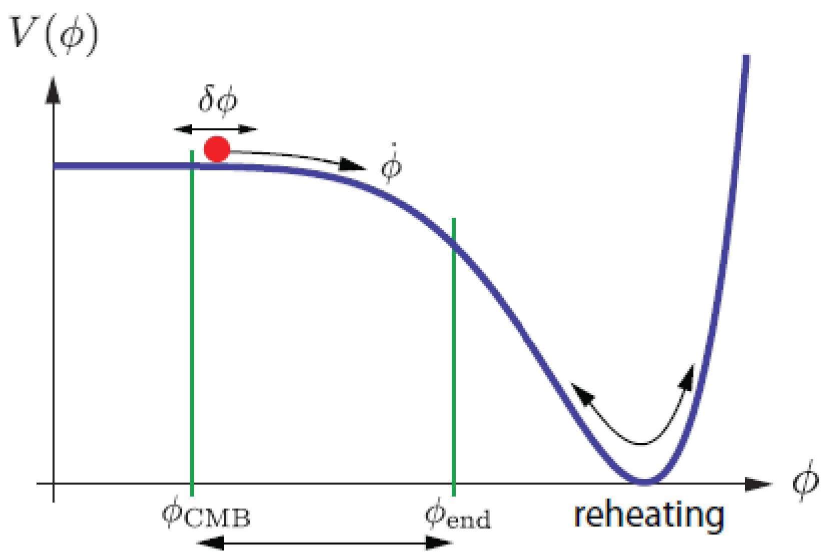
- Example: slowly rolling scalar field

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$
$$\Rightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases} \Rightarrow \quad \rho + 3p = 2(\dot{\phi}^2 - V(\phi))$$

$$\text{if} \quad \dot{\phi}^2 < V(\phi) \quad \Longrightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

$\boxed{\text{accelerated expansion}}$



[Source - arXiv:0907.5424]

CMB fluctuations are created by quantum fluctuations *δφ* about 60 *e*-folds before the end of inflation.

Higgs Inflation

The standard model Higgs potential is not flat $V = \lambda_H \left(\mathcal{H}^\dagger \mathcal{H} - \frac{v_H^2}{2} \right)^2$

However, scalar fields can (should?) be non-minimally coupled to gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{\bar{M}_P^2}{2} R - \xi \mathcal{H}^\dagger \mathcal{H} R + \mathcal{L}_{SM} \right]$$

Can transform to the Einstein frame

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \ , \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2} \qquad\qquad \frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2 / M_P^2}{\Omega^4}}$$

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2} \hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\}$$

where the potential is $U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left(1 + \exp\left(-\frac{2\chi}{\sqrt{6} M_P} \right) \right)^{-2}$

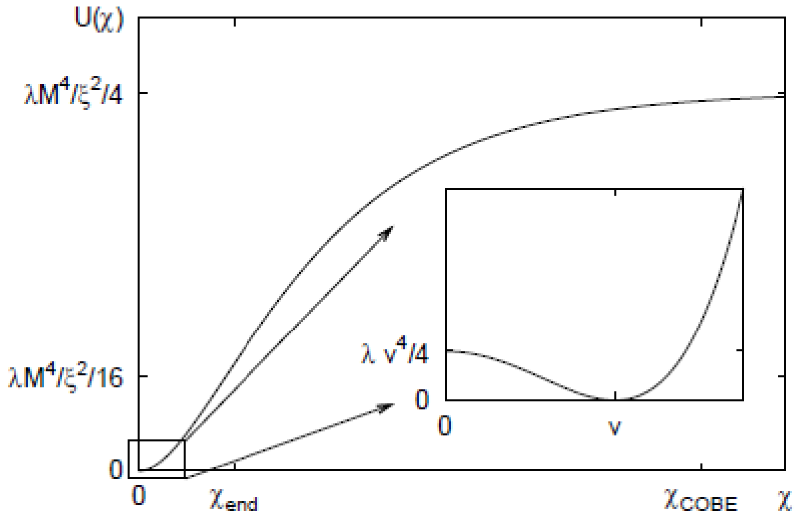
[F.L. Bezrukov & Mikhail Shaposhnikov]

Higgs Inflation

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1+\exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)\right)^{-2}$$

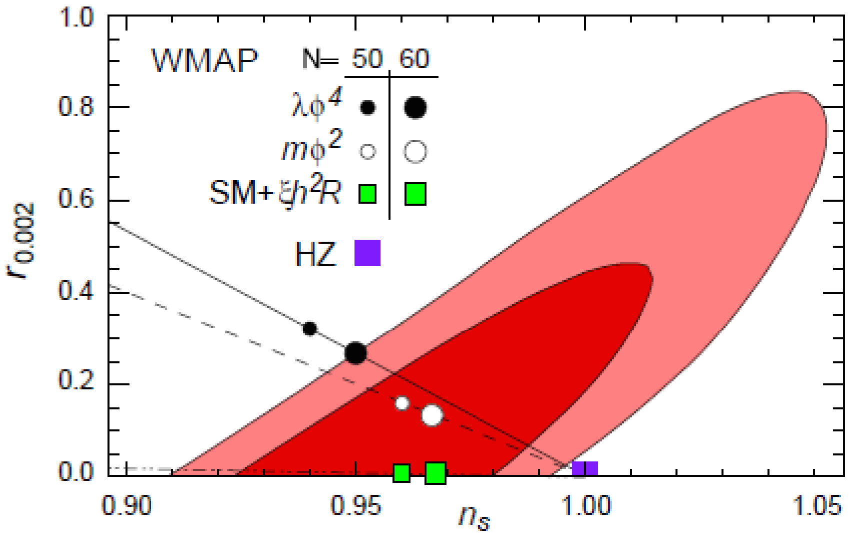
When $\chi \gg M_P \ (h \gg M_P/\sqrt{\xi})$ the potential is flat and slow roll inflation can occur.

However it is found that we need $\xi \sim 10^4$ to obtain the correct amplitude of density fluctuations.



Potential in the Einstein frame.

[Source - arXiv:0710.3715]



The allowed WMAP region for inflationary parameters spectral index *n*, and the tensor to scalar ratio *r*.

Full 2-loop analysis finds we need mh > 125.7 (136.7) GeV to produce inflation.

Unitarity in Quantum Field Theory

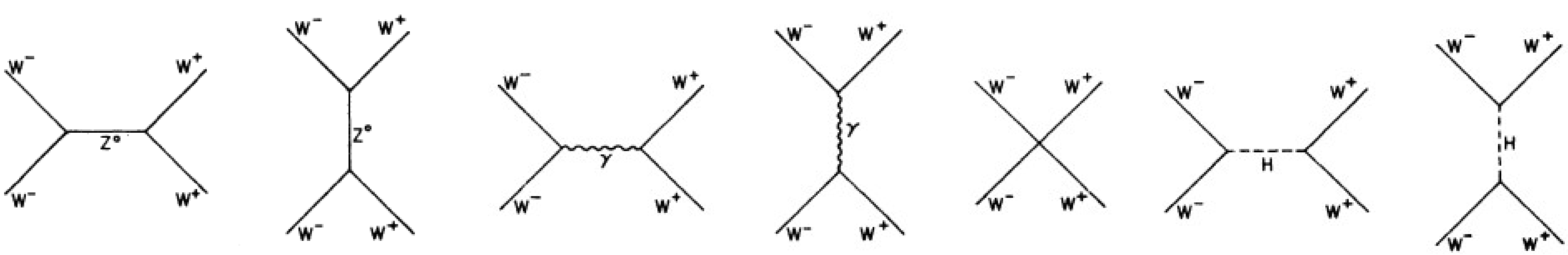
- Follows from the conservation of probability, i.e. unitarity of the S-matrix: $S^\dagger S = 1$

- Implies that amplitudes do not grow too fast with energy.

- Can derive a bound on the size of the partial wave amplitudes:

$$\mathcal{A} = 16\pi \sum_j (2j+1) P_j(\cos\theta)\, a_j \qquad\qquad |\mathrm{Re}\, a_j| \leq \frac{1}{2}$$

- Well known example is the bound on the Higgs boson mass in the Standard Model, $m_H \lesssim 790\ \mathrm{GeV}$



Breakdown of Perturbative Unitarity

$$|\mathrm{Re}\, a_j| \leq \frac{1}{2}$$

- Applying the bound to tree level amplitudes in an effective field theory can provide a bound for the cutoff (Λ) for the theory.

- Effective theory contains higher order terms suppressed by powers of the cutoff:

$$\frac{(H^\dagger H)^n}{\Lambda_0^{2n-4}}$$

- At energies above the cutoff all these operators become relevant and perturbation theory breaks down.

$\Rightarrow$ strong coupling or new physics ???

Unitarity in Higgs Inflation

The large value of $\xi \sim 10^4$ might make one concerned from a particle physics perspective.

Let us consider gravitational scattering of Higgs bosons (we impose different in and out states – *s*-channel only) in the Jordan frame

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_P}$$

$$a_0 = \frac{\pi}{3}\frac{s}{M_P^2}(1+12\xi)^2 \sim \frac{\xi^2}{M_P^2}s \leq \frac{1}{2} \qquad \text{for } \xi \gg 1$$

$$\boxed{\Rightarrow \Lambda \lesssim \frac{M_P}{\xi}}$$

[MA & X. Calmet] [Burgess, Lee & Trott] [Barbon & Espinosa]

But remember inflation takes place for $h \gg M_P/\sqrt{\xi}$ which is therefore above the regime of validity for the effective theory!

Einstein vs. Jordan Frame [Hertzberg] [Burgess, Lee, Trott]

The cut off (Λ) should be the same in both frames. But if we look at the Einstein frame action again

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right\}$$

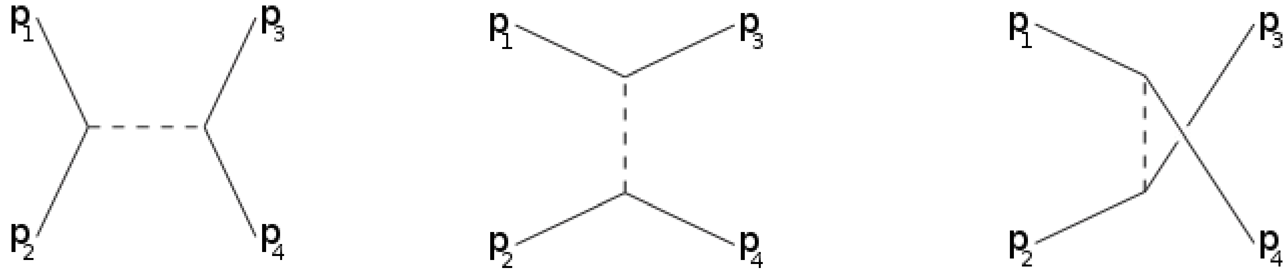
we see that the cut off is just the usual gravitational cut off ($\Lambda = M_P$).

Einstein Frame

Cannot canonically normalise all the fields of the Higgs doublet so cannot actually get Einstein frame potential with multiple scalars.

Jordan Frame

If only have a single field need to include *s, t* and *u* channels. Then we find a cancellation between the three diagrams leaving ($\Lambda = M_P$).



Background Dependence

[Bezrukov, Magnin, Shaposhnikov & Sibiryakov]

For the original Higgs inflation model we expanded around *φ=0*.
We could expand around inflating background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \, , \qquad \phi = \bar{\phi} + \delta\phi \, .$$

Then we find an interaction term

$$\frac{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}(\delta\hat{\phi})^2 \Box \hat{h}$$

Leading to a $\bar{\phi}$ dependent cut-off

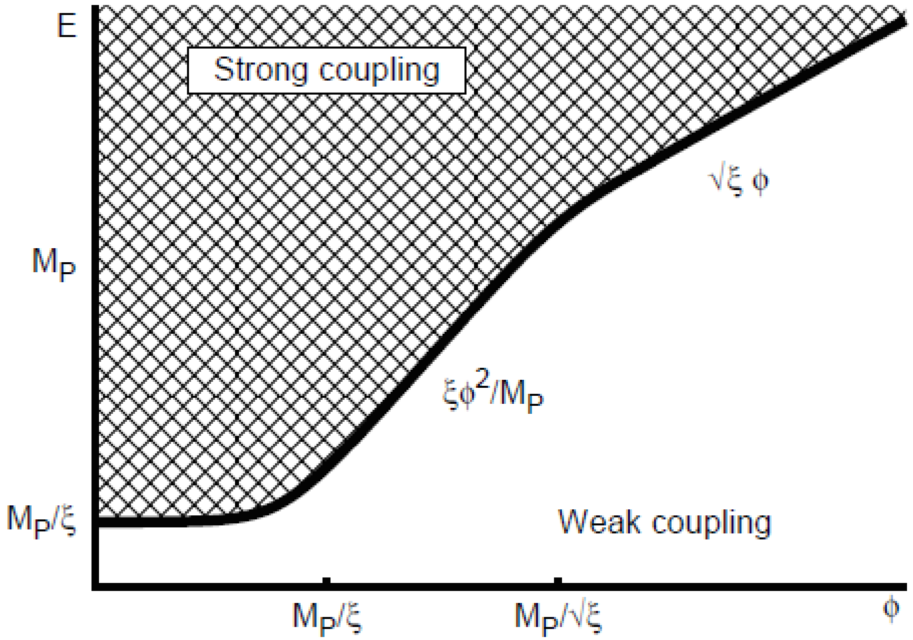
$$\Lambda^J(\bar{\phi}) = \frac{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}$$

Small field: $\bar{\phi} \ll M_P/\xi \qquad \Lambda^J \simeq \frac{M_P}{\xi}$

Re-heating: $M_P/\xi \ll \bar{\phi} \ll M_P/\sqrt{\xi} \qquad \Lambda^J \simeq \frac{\xi\bar{\phi}^2}{M_P}$

Inflation: $\bar{\phi} \gg M_P/\sqrt{\xi} \qquad \Lambda^J \simeq \sqrt{\xi}\bar{\phi}$

New Physics or Strong Coupling?

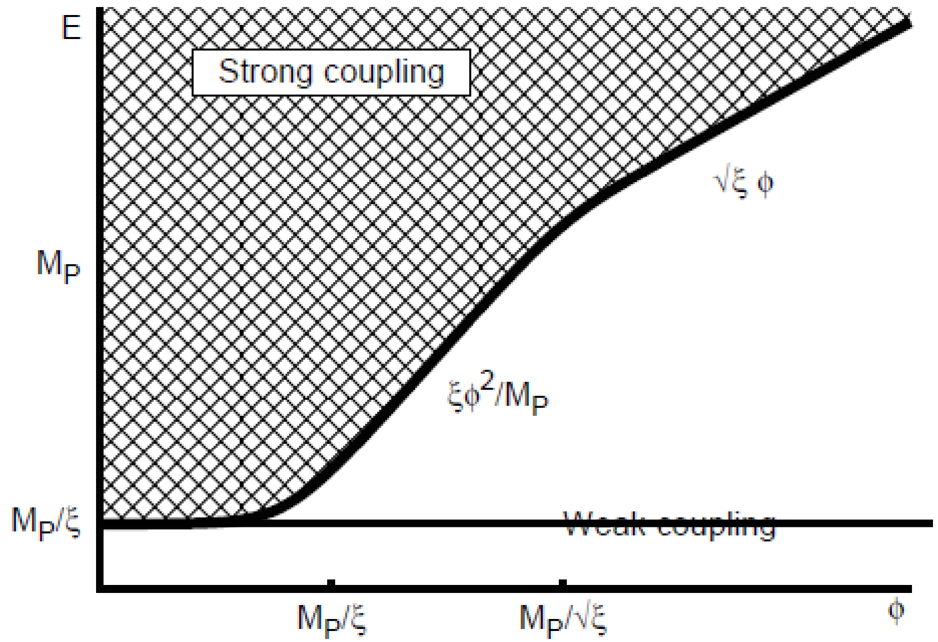


Cut-off as a function of the background value of Higgs field.

[Source - arXiv:1008.5157]

During inflation still in perturbative regime.

New Physics or Strong Coupling?



Cut-off as a function of the background value of Higgs field.

[Source - arXiv:1008.5157]

During inflation still in perturbative regime.

However if new physics is required to unitarise the theory at small background field values, potential must include the operators

$$\frac{(H^\dagger H)^n}{\Lambda_0^{2n-4}}$$

Appearing at $\Lambda_0 = \frac{M_P}{\xi}$. and spoiling the flat potential.

[MA & X Calmet]

New Model of Higgs Inflation

To get around the unitarity problems a new model of Higgs inflation was proposed [Germani & Kehagias]

$$S = \int d^4x \sqrt{-g} \left[\frac{\bar{M}_P^2}{2} R - \frac{1}{2}(g^{\mu\nu} - w^2 G^{\mu\nu}) \partial_\mu \Phi \partial_\nu \Phi - \frac{\lambda}{4} \Phi^4 \right]$$

where $G^{\mu\nu} = R^{\mu\nu} - \frac{R}{2} g^{\mu\nu}$ is the Einstein tensor.

Expanding around the inflating background we find an interaction

$$I \simeq \frac{1}{2H^2 \bar{M}_P} \partial^2 h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi.$$

Which gives a cut-off

$$\Lambda \simeq (2H^2 \bar{M}_P)^{1/3}. \simeq 2 \times 10^{-3} \bar{M}_P.$$

But during inflation we have $2.1 \times 10^{-2} \bar{M}_P < \Phi_0 < 2.7 \times 10^{-2} \bar{M}_P$

and so again the inflationary scale exceeds the realm of validity of the effective theory. [MA & X Calmet]

Three More Scenarios

1. Asymptotic Safety [MA & X Calmet]

The theory is non-perturbatively renormalisable and approaches a non trivial fixed point in the UV, no new physics is required.

2. Unitarising Higgs Inflation [Giudice & Lee]

Can remove unitarity problem by introducing a massive σ field in analogy with the non-linear sigma model. But in reality it is this new field that then drives inflation.

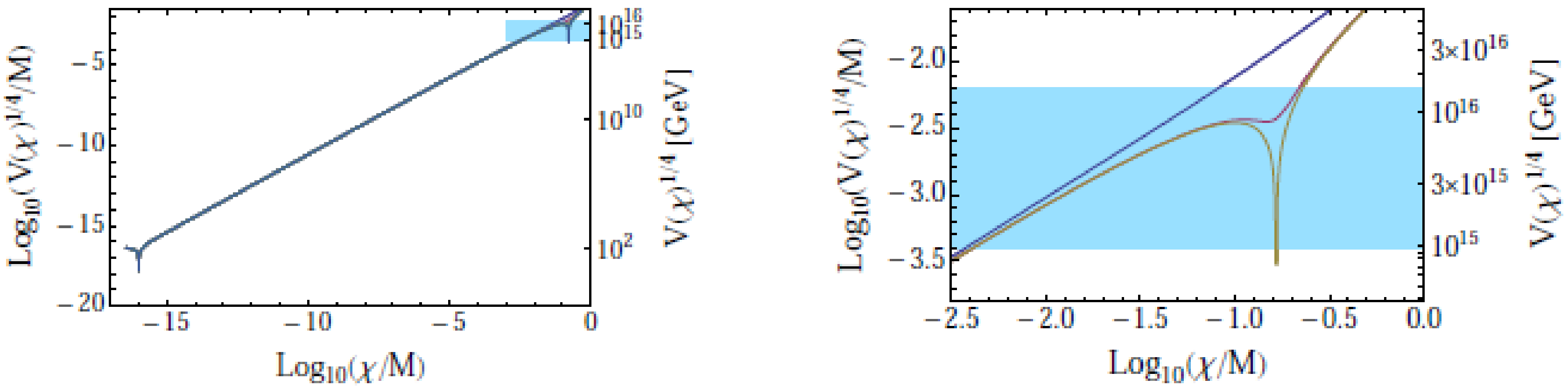
3. Composite Inflation [Channuie, Jørgensen, Sannino]

The inflaton emerges as a composite field of a strongly interacting gauge theory.

$$\frac{\xi}{2}\frac{(QQ)^{\dagger}QQ}{\Lambda_{ECI}^{4}}R$$

SM False Vacuum Inflation [Masina & Notari]

• Stability of the Higgs effective potential is extremely sensitive to the value of the Higgs and top quark masses. Can obtain a local minima at large field values.



Higgs potential as function of Higgs field value, mt=171.8 GeV, mh=125.2, 125.158, 125.1577 GeV

• Higgs sitting in this false minima would provide exponential inflation and could then tunnel at end of inflation.

• To match amplitude of density perturbations with mt=173.2 GeV find

$$m_H = (126.0 \pm 3.5)\,\mathrm{GeV}\ .$$

SM False Vacuum Inflation

- To end inflation need tunnelling rate $\Gamma \simeq H^4$ But if both parameters are constant then inflation ends too quickly.

- Solution – introduce a Brans-Dicke scalar which provides a time dependent Planck mass.

$$-S = \int d^4x\sqrt{-g}\left[\mathcal{L}_{SM} + \frac{(\partial_\mu\phi\partial^\mu\phi)}{2} - \frac{M^2}{2}f(\phi)R\right] \qquad f(\phi) \simeq 1 + \beta\left(\frac{\phi}{M}\right)^2 + \gamma\left(\frac{\phi}{M}\right)^4 + \ldots ,$$

- If $f(\phi)$ is a monotonic increasing function it will grow during inflation and the effective Planck mass grows. Gravity becomes weaker and so the Hubble parameter decreases with time until $\Gamma \simeq H^4$ and the Higgs field tunnels efficiently.

- BSM physics can change the Higgs effective potential. I have found that inflation is still viable in the presence of seesaw neutrino masses with reasonable bounds on the neutrino mass parameters, although the false minimum moves closer towards the Planck mass (work in progress....)

Conclusions

- Inflation explains why the universe appears flat, homogeneous and isotropic
- With a large non-minimal coupling the Higgs boson could drive inflation which agrees with CMB data.
- The Higgs inflation models (old and new) suffer from unitarity problems.
- With the addition of a Brans-Dicke scalar a false minimum in the Higgs potential can produce inflation.

RG Higgs Self Coupling [arXiv:1112.3022]

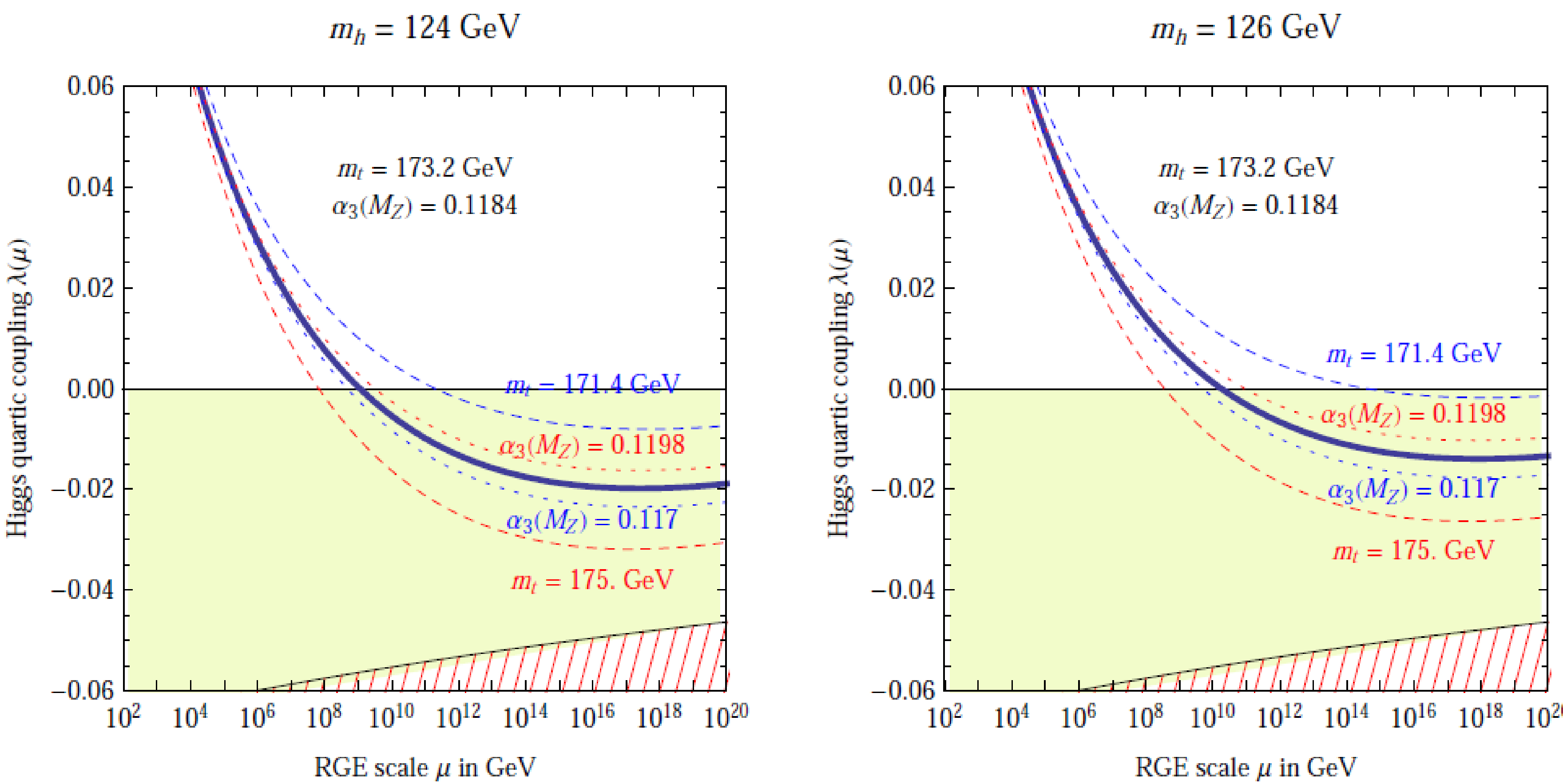


Figure 1: *RG evolution of the Higgs self coupling, for different Higgs masses for the central value of m_t and α_s, as well as for $\pm 2\sigma$ variations of m_t (dashed lines) and α_s (dotted lines). For negative values of λ, the life-time of the SM vacuum due to quantum tunneling at zero temperature is longer than the age of the Universe as long as λ remains above the region shaded in red, which takes into account the finite corrections to the effective bounce action renormalised at the same scale as λ (see [11] for more details).*

Unitarising Higgs Inflation [Giudice & Lee]

Can unitarise Higgs inflation by using an analogy with the non-linear sigma model. Consider the kinetic term in Einstein frame

$$\mathcal{L}_{\rm kin} = -\frac{1}{2(1+\xi_0\vec{\phi}^2/M_P^2)}\left(\delta_{ij} + \frac{6\xi_0^2\phi_i\phi_j/M_P^2}{1+\xi_0\vec{\phi}^2/M_P^2}\right)\partial_\mu\phi_i\partial^\mu\phi_j$$

So we can complete in the UV by introducing a σ field with $\sigma^2 \doteq \Lambda^2 + \vec{\phi}^2$ with $\Lambda^2 \equiv M_P^2/\xi_0$

$$\begin{aligned}\frac{\mathcal{L}_J}{\sqrt{-g_J}} &= \frac{1}{2}\left(\bar{M}^2 + \xi\bar{\sigma}^2 + 2\zeta\mathcal{H}^\dagger\mathcal{H}\right)R - \frac{1}{2}(\partial_\mu\bar{\sigma})^2 - |D_\mu\mathcal{H}|^2 \\ &\quad -\frac{1}{4}\kappa\left(\bar{\sigma}^2 - \bar{\Lambda}^2 - 2\alpha\mathcal{H}^\dagger\mathcal{H}\right)^2 - \lambda\left(\mathcal{H}^\dagger\mathcal{H} - \frac{v^2}{2}\right)^2.\end{aligned}$$

with $\xi \sim \mathcal{O}(10^4)\ ,\ \zeta \sim \mathcal{O}(1)$

Low energy theory is the usual Higgs inflation Lagrangian, but at high energies the sigma field propagates and the cut-off scales with the background to allow control over the potential.