Wigner Distributions

and

Quark Orbital Angular Momentum

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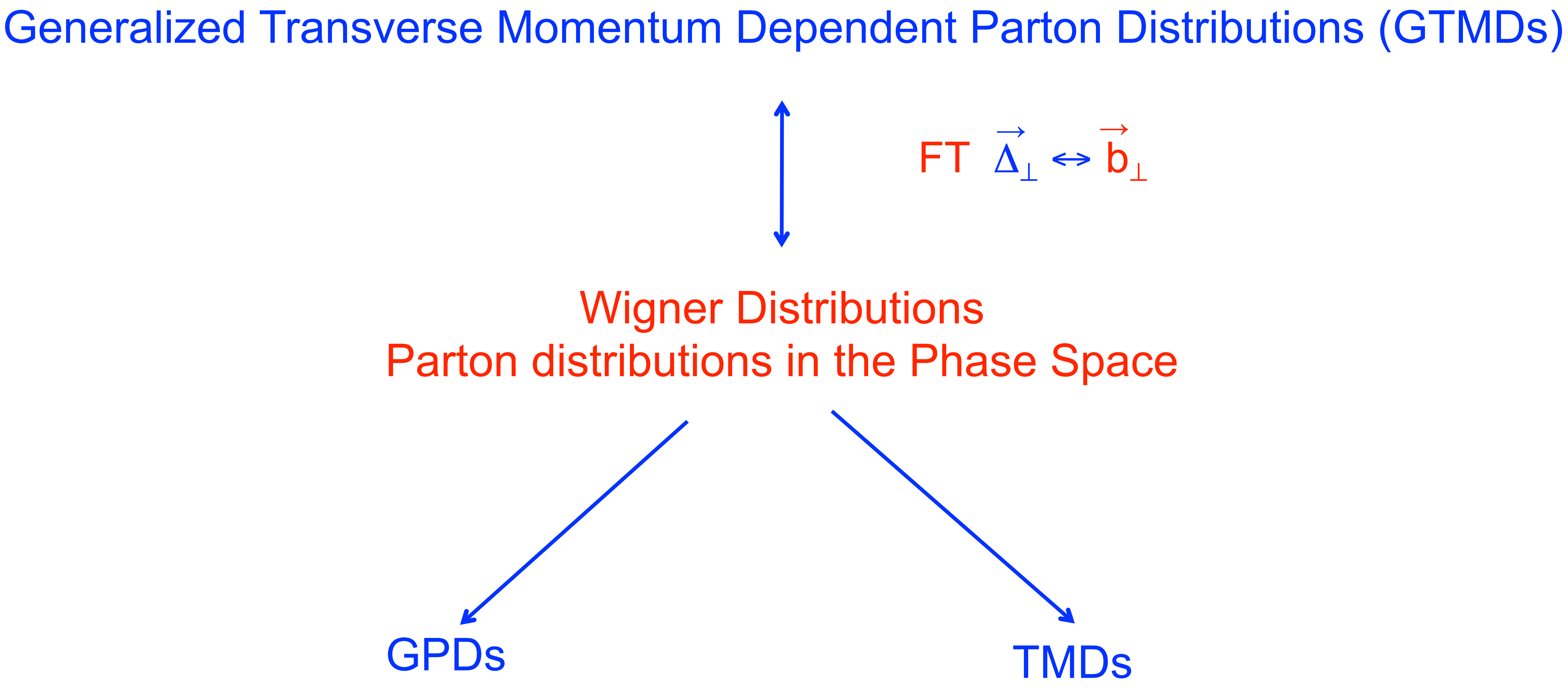
Outline



Generalized Transverse Momentum Dependent Parton Distributions (GTMDs)

FT $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Wigner Distributions
Parton distributions in the Phase Space

GPDs

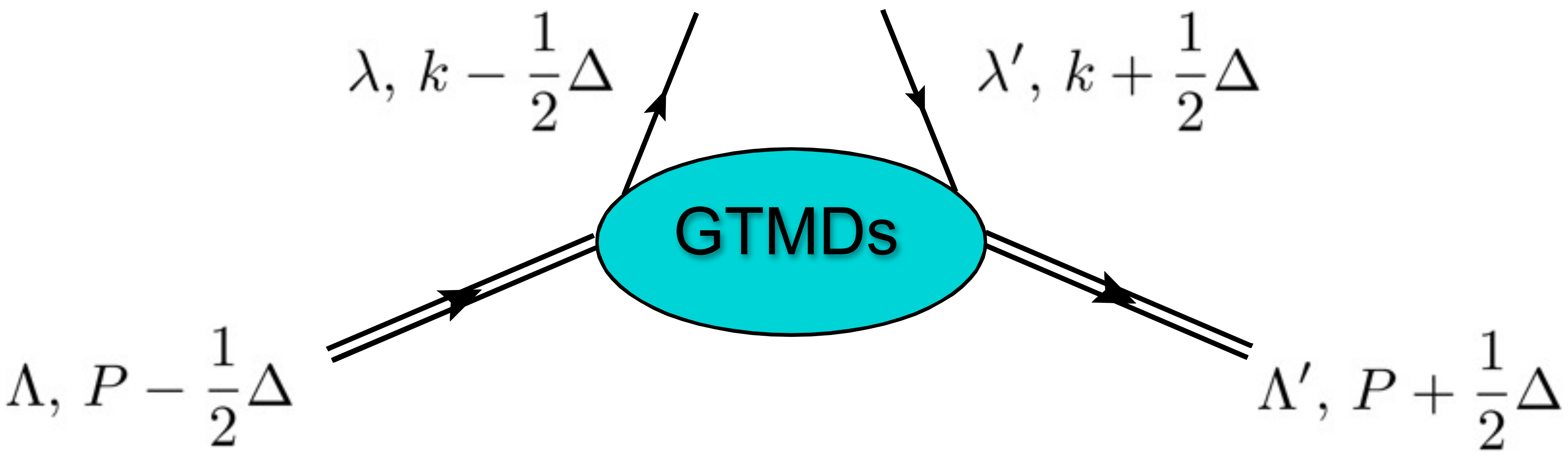
TMDs

spin and orbital angular momentum structure of the nucleon

insights from model calculations

Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2 z_\perp}{(2\pi)^3}\langle P',\Lambda'|\,\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\ \Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\,\psi^q_{\lambda}(\frac{z}{2})\,|P,\Lambda\rangle e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations ↔ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

x: average fraction of quark longitudinal momentum

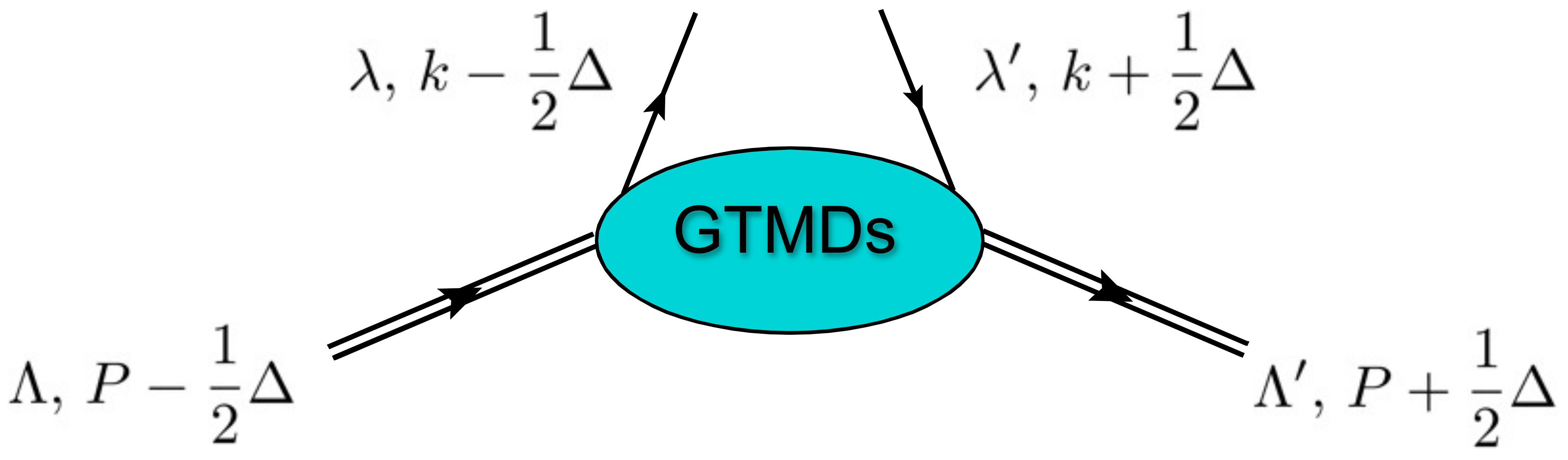
ξ: fraction of longitudinal momentum transfer

$\vec{k}_\perp$: average quark transverse momentum

$\vec{\Delta}_\perp$: nucleon transverse-momentum transfer

Generalized TMDs and Wigner Distributions

[Meißner, Metz, Schlegel (2009)]



Quark polarization

$$\boxed{W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} \langle P', \Lambda' |\, \bar{\psi}^q_{\lambda'}(-\frac{z}{2})\; \Gamma\, \mathcal{W}(-\frac{z}{2}, \frac{z}{2}|n)\, \psi^q_\lambda(\frac{z}{2})\, | P, \Lambda \rangle e^{i(xP^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)}}$$

Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$\begin{tikzcd} W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp) \arrow[d, "\text{Fourier transform } \vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp"] \\ \tilde{W}^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{b}_\perp) \end{tikzcd}$$

16 real Wigner distributions

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

●

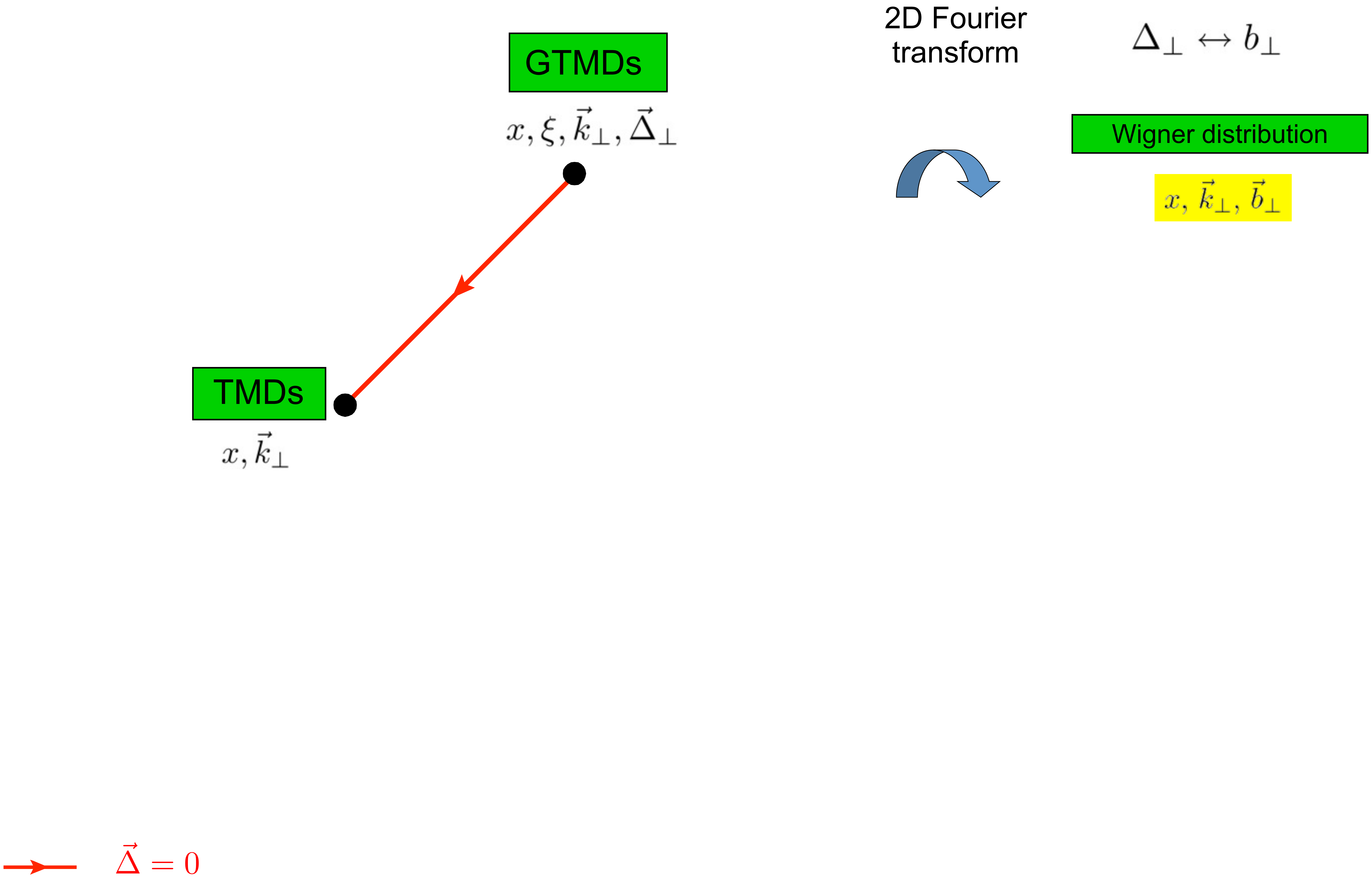
2D Fourier
transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

2D Fourier transform

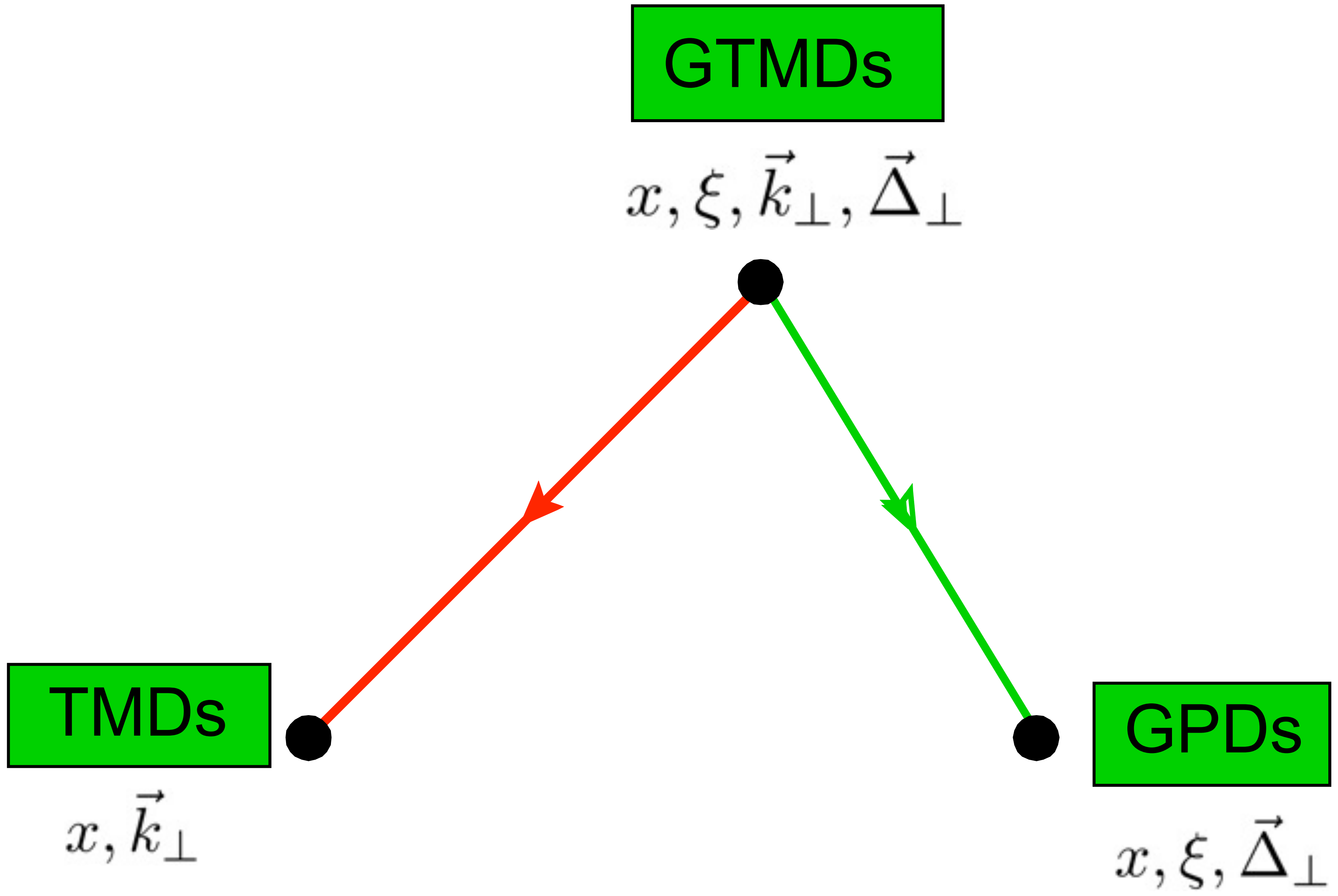
$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

$\vec{\Delta} = 0$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

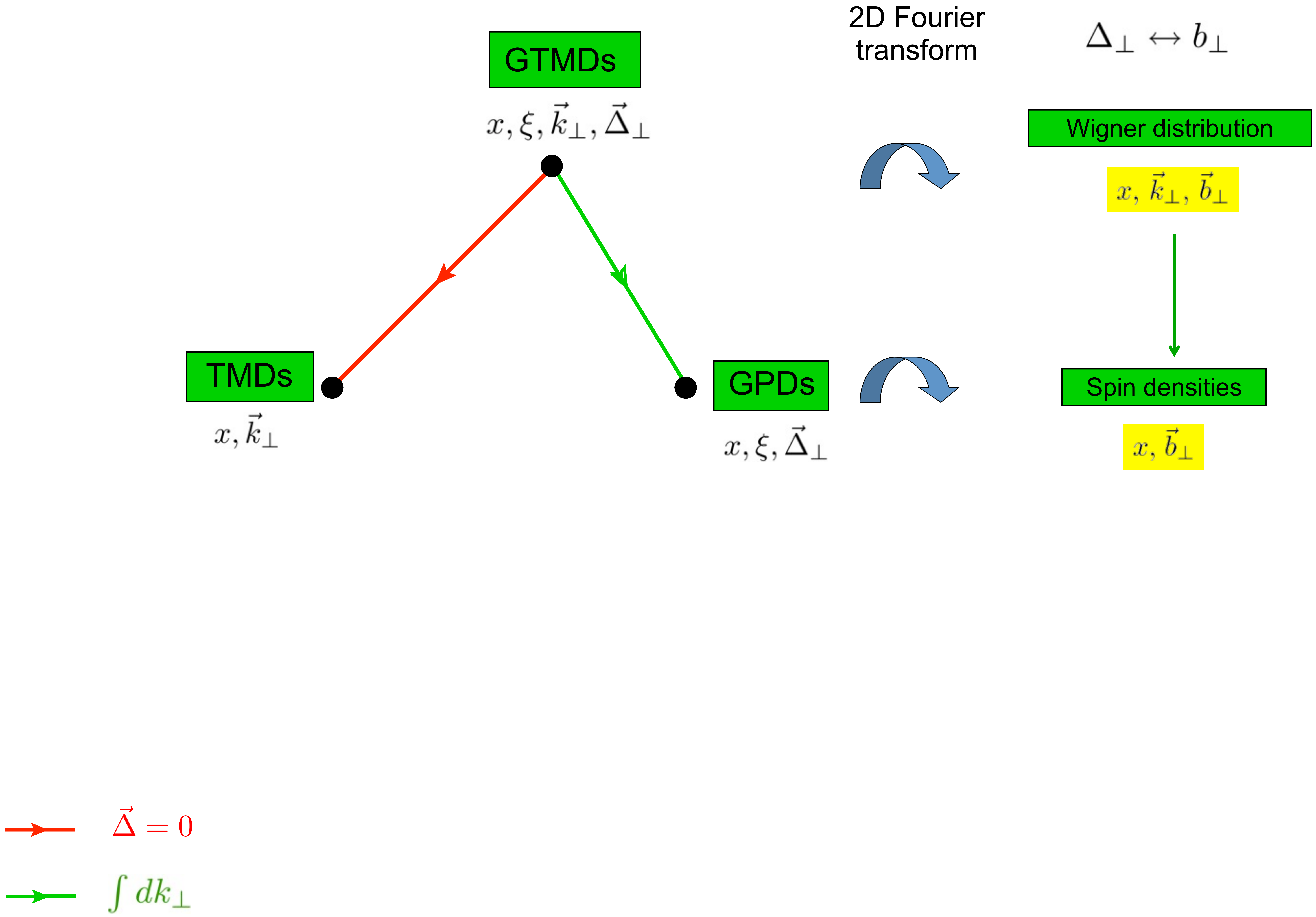
Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

→ $\vec{\Delta} = 0$

→ $\int dk_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

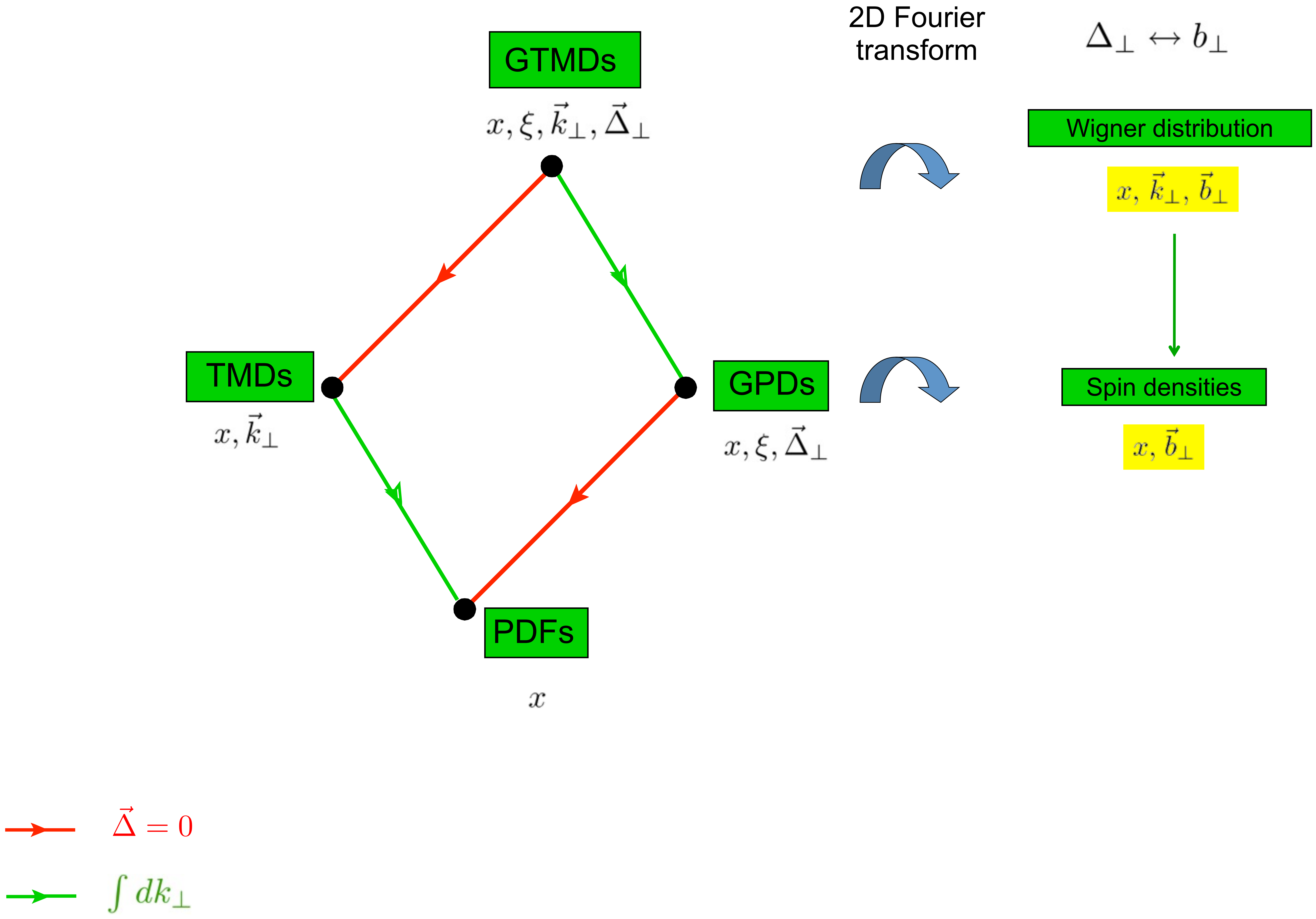
Spin densities

$x, \vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

PDFs

x

Wigner distribution

$x,\, \vec{k}_\perp,\, \vec{b}_\perp$

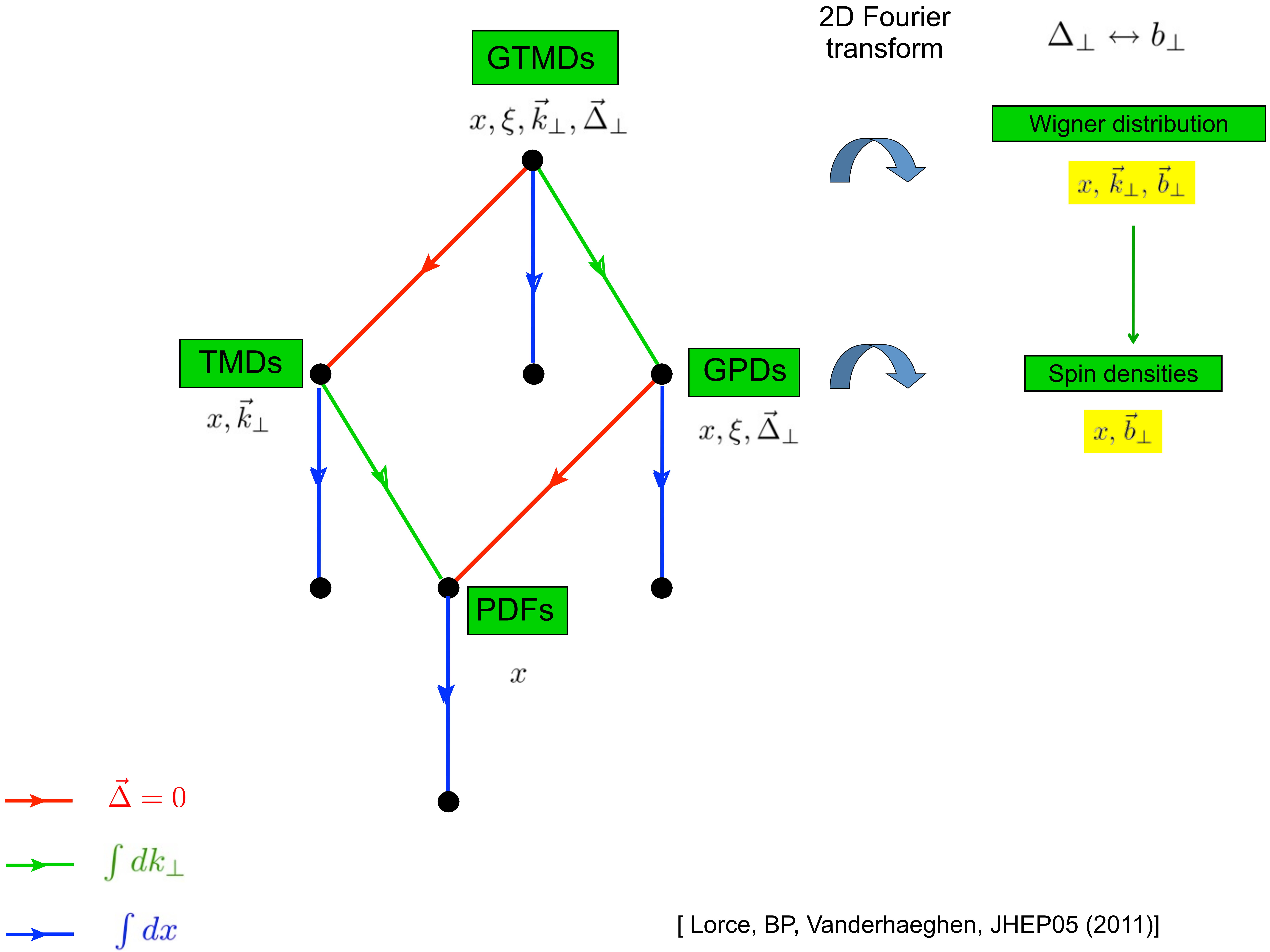
Spin densities

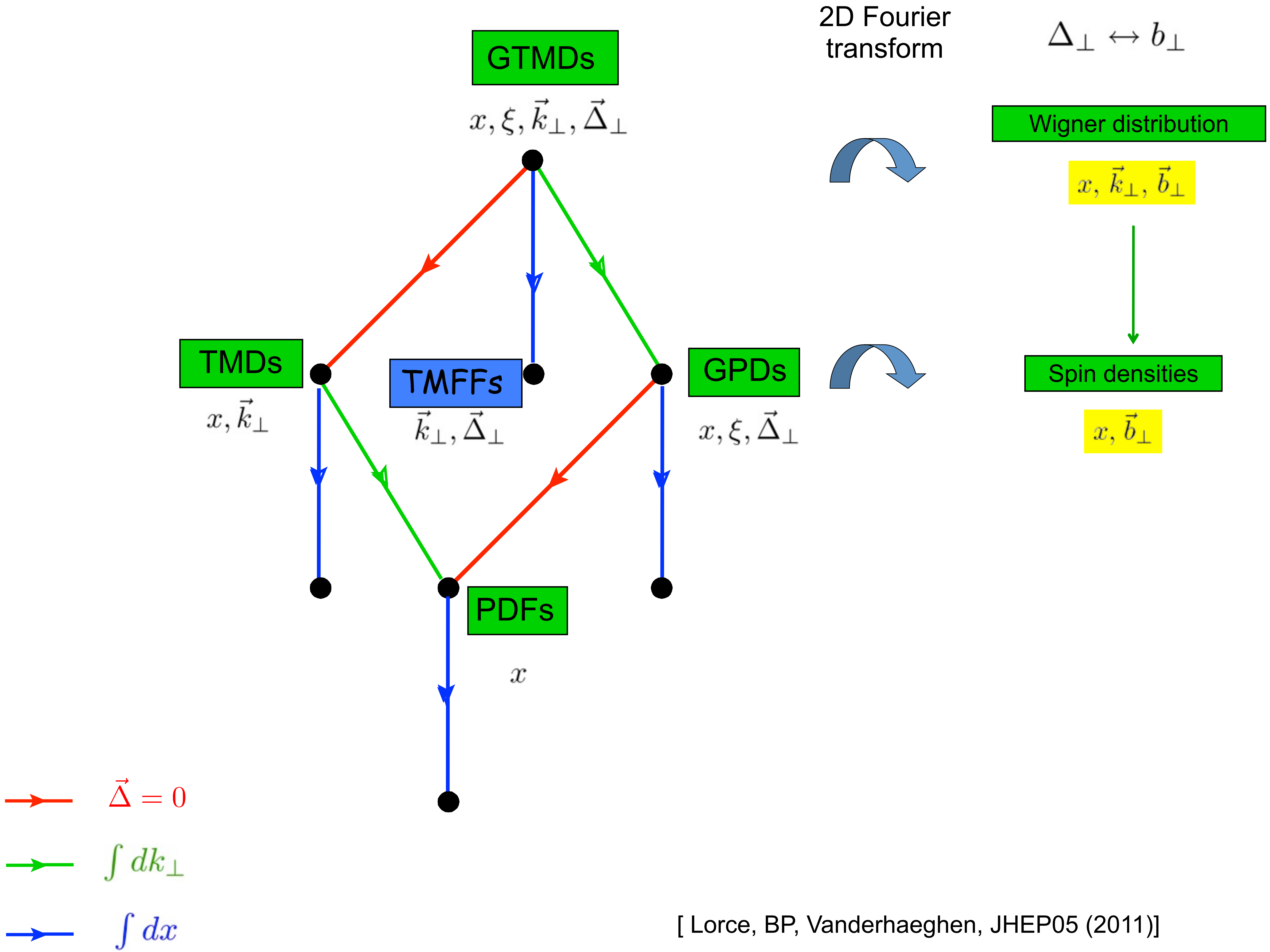
$x,\, \vec{b}_\perp$

🟥 $\vec{\Delta} = 0$

🟩 $\int dk_\perp$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]





GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

PDFs

x

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

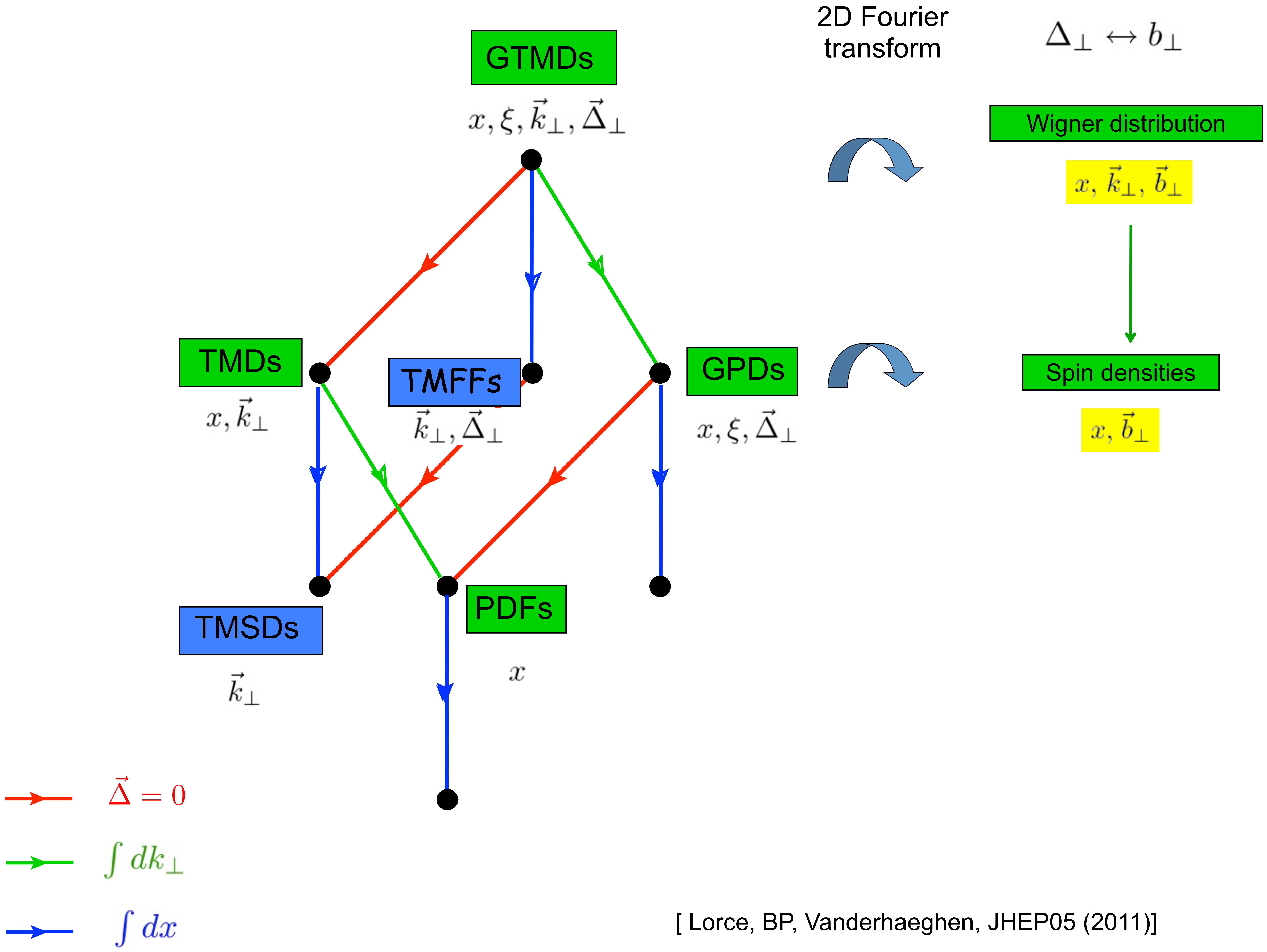
$x, \vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

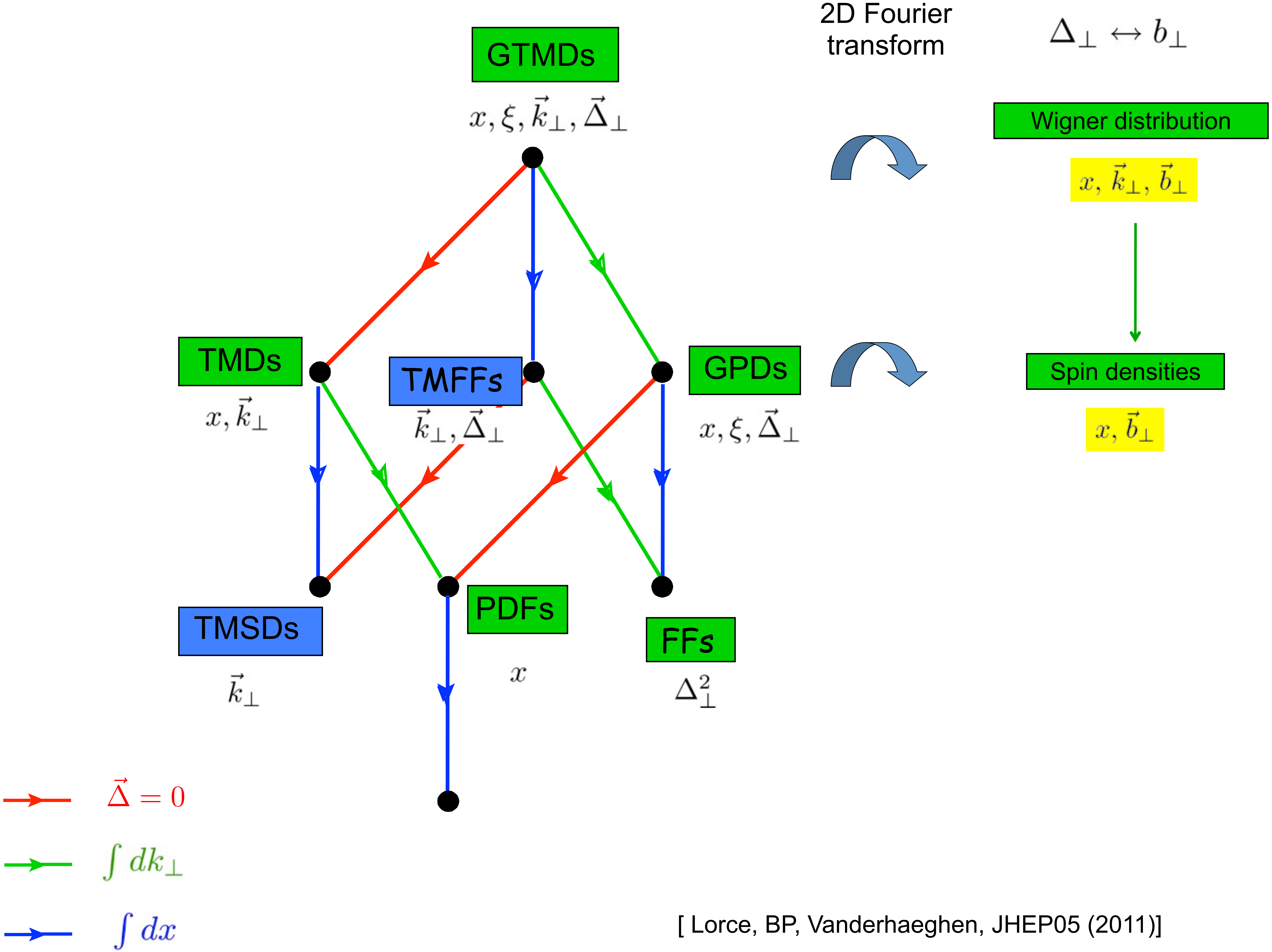
$x, \vec{b}_\perp$

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

$x, \vec{b}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

FFs

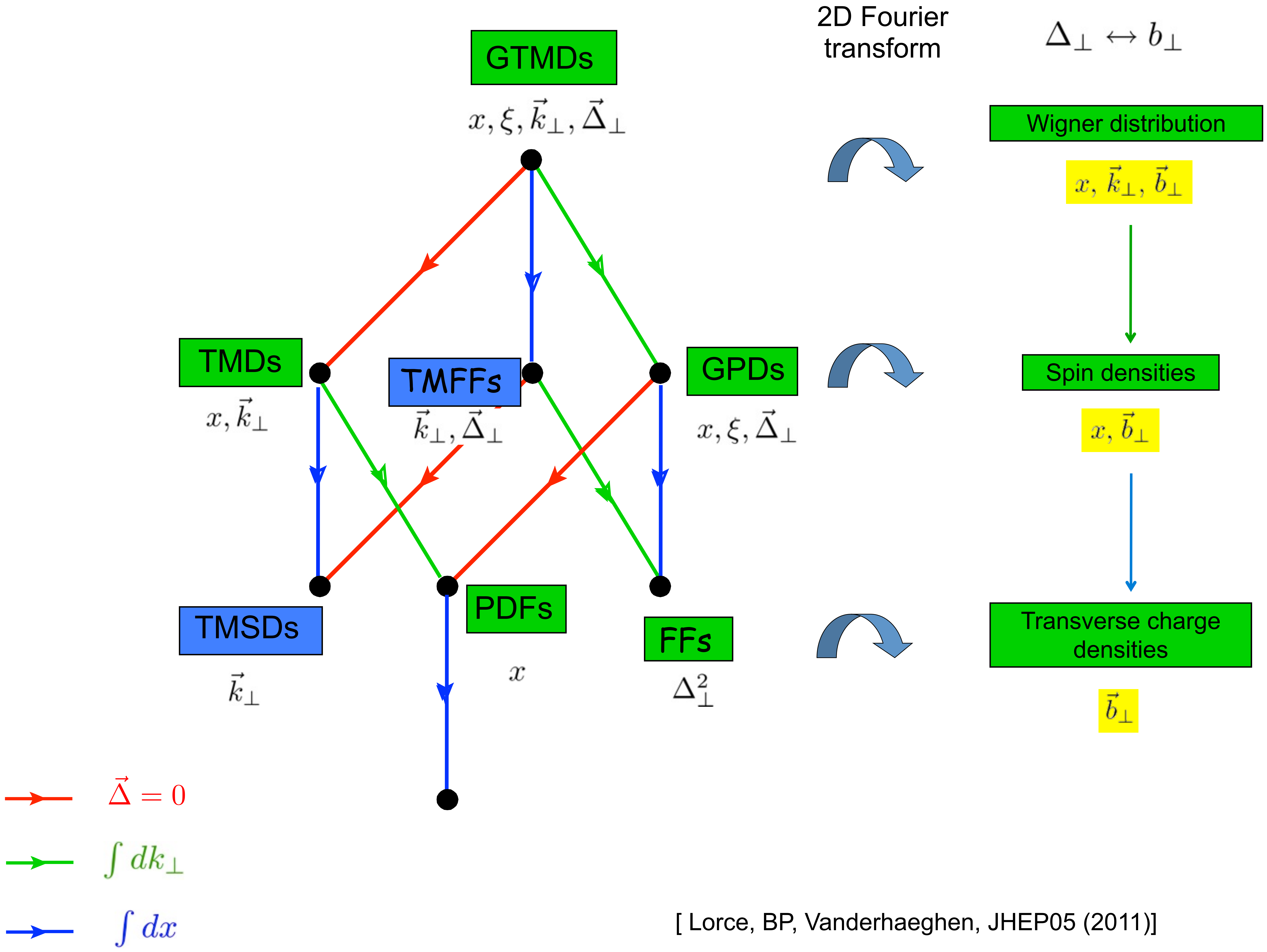
$\Delta_\perp^2$

$\vec{\Delta} = 0$

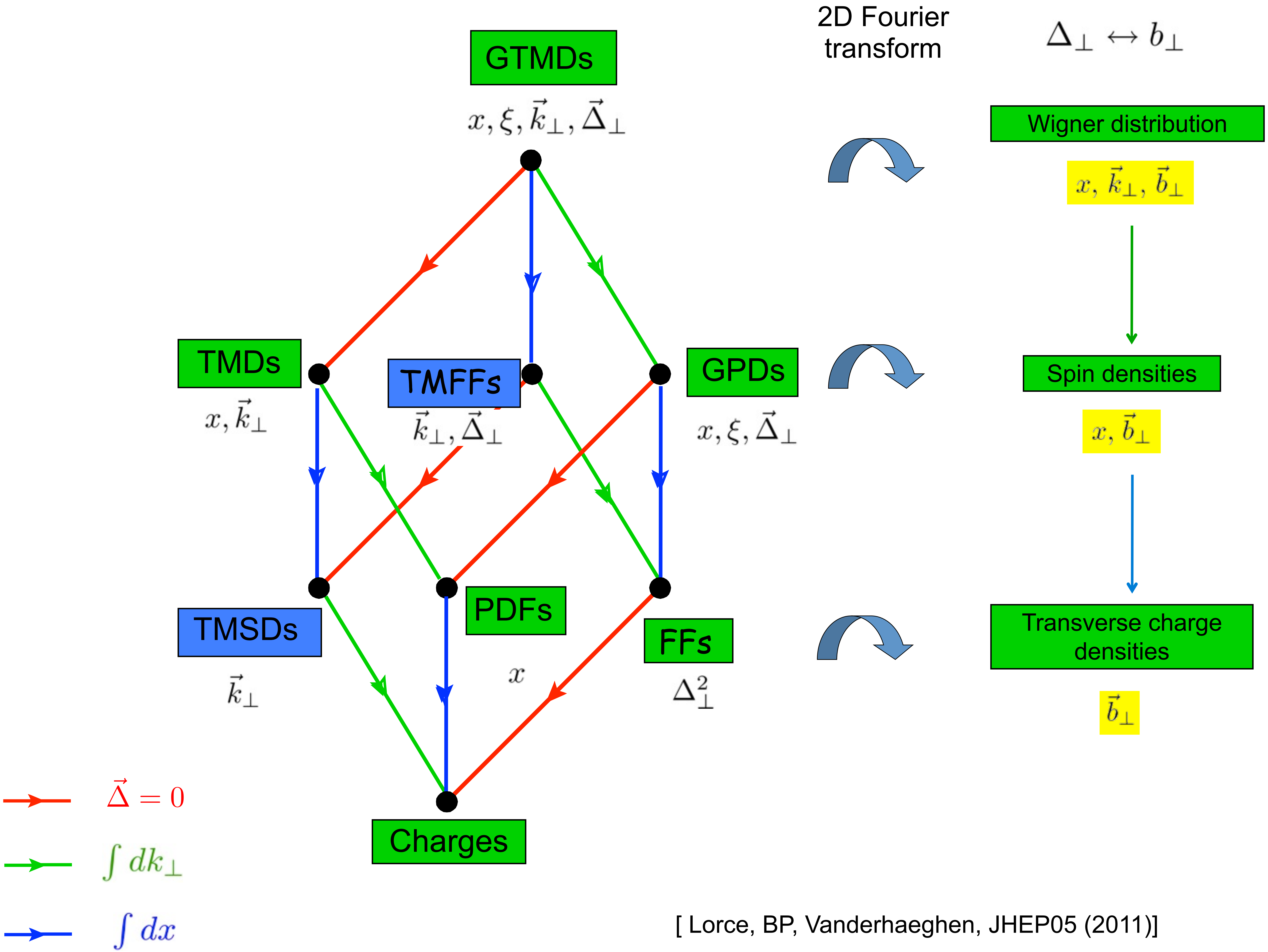
$\int dk_\perp$

$\int dx$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]



2D Fourier transform

$\Delta_\perp \leftrightarrow b_\perp$

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

Charges

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

Spin densities

$x, \vec{b}_\perp$

Transverse charge densities

$\vec{b}_\perp$

$\vec{\Delta} = 0$

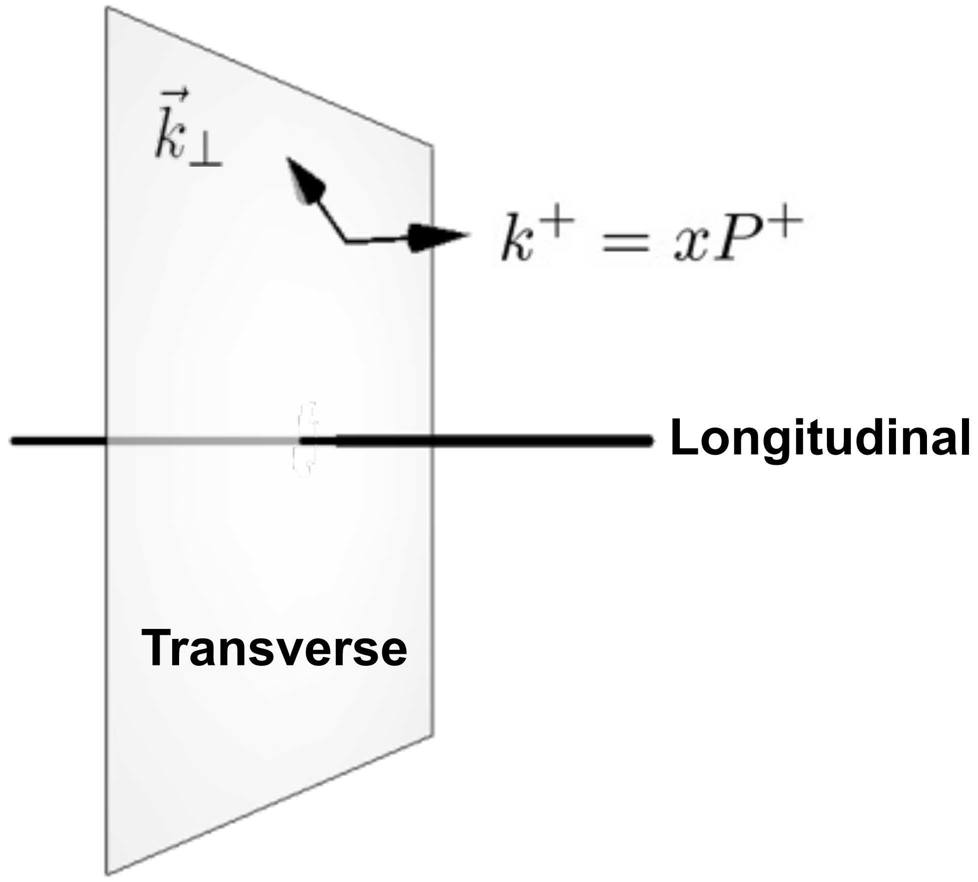
$\int dk_\perp$

$\int dx$

[Lorce, BP, Vanderhaeghen, JHEP05 (2011)]

TMDs

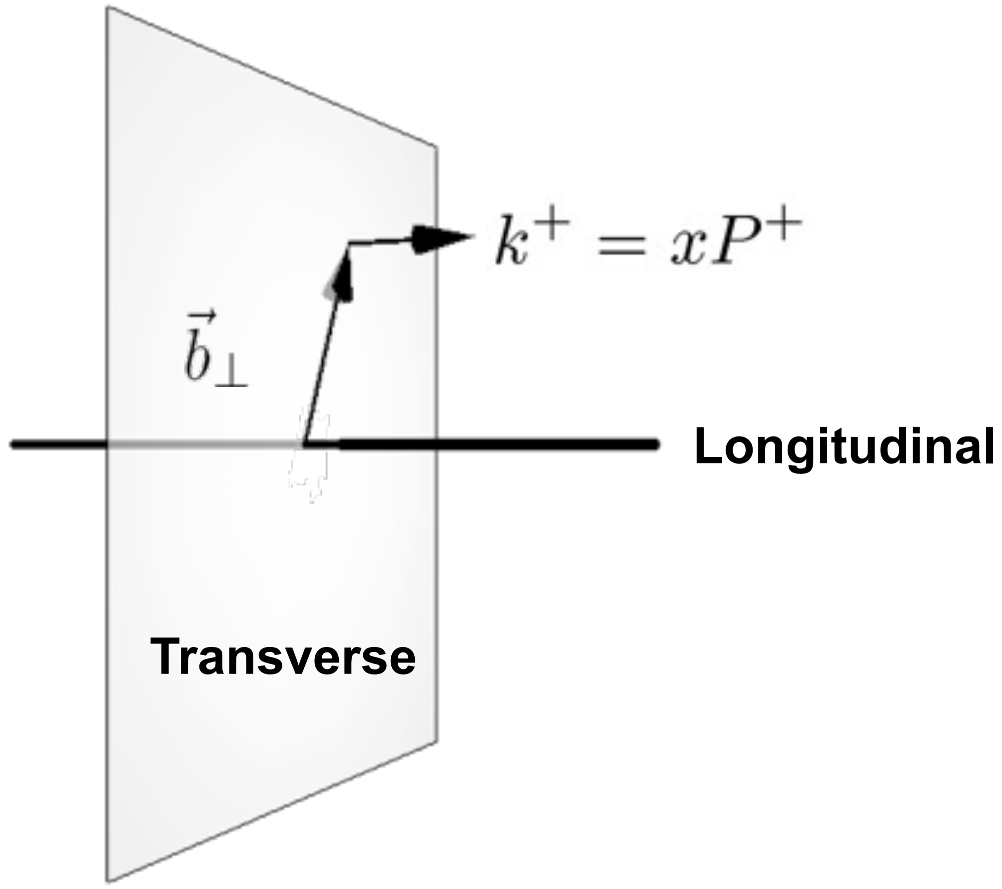
$\rho(x,\vec{k}_\perp)$



probabilistic interpretation

GPDs

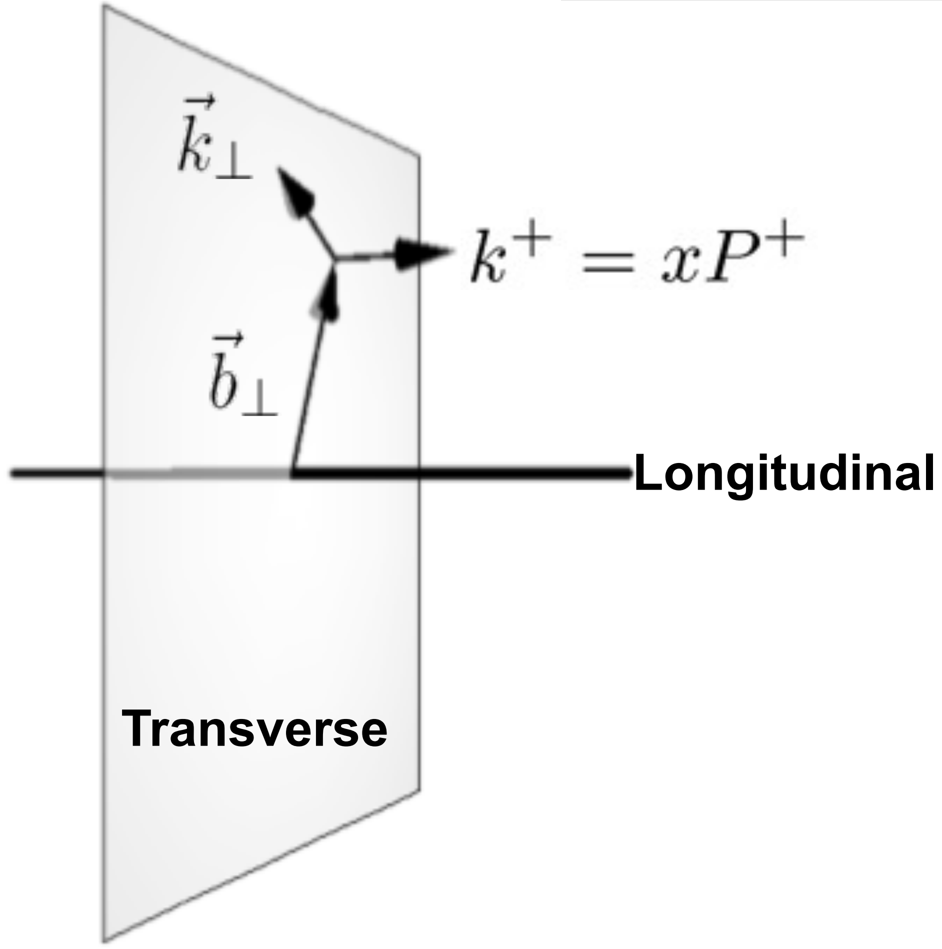
$\rho(x,\vec{b}_\perp)$



Wigner Distributions

$\rho(x,\vec{b}_\perp,\vec{k}_\perp)$

Heisenberg's uncertainty relations → quasi-probabilistic interpretation

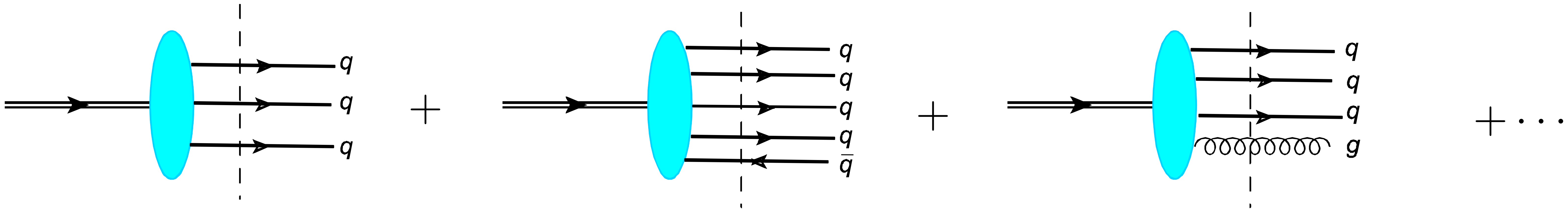


Wigner (1932)
Belitsky, Ji, Yuan (04)
Lorce', BP (11)

Quark Wigner Distributions

✦ Light-cone Fock expansion of Nucleon state:

$$|N\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q\,q\bar{q}}|3q\,q\bar{q}\rangle + \Psi_{3q\,g}|qqqg\rangle + \cdots$$



fixed light-cone time (x^+=0)

Quark Wigner Distributions

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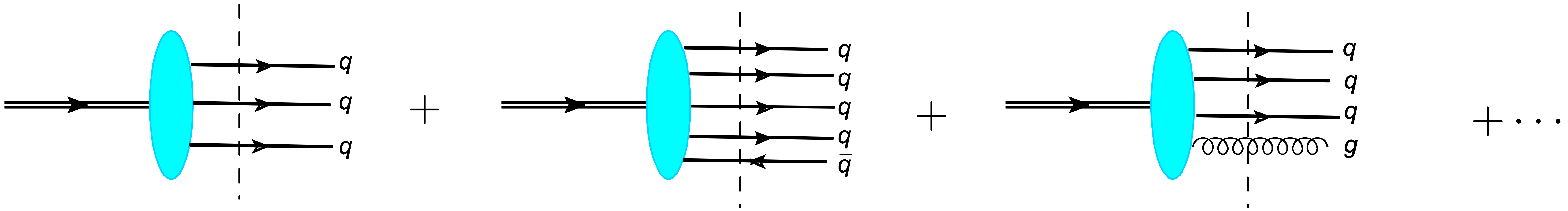
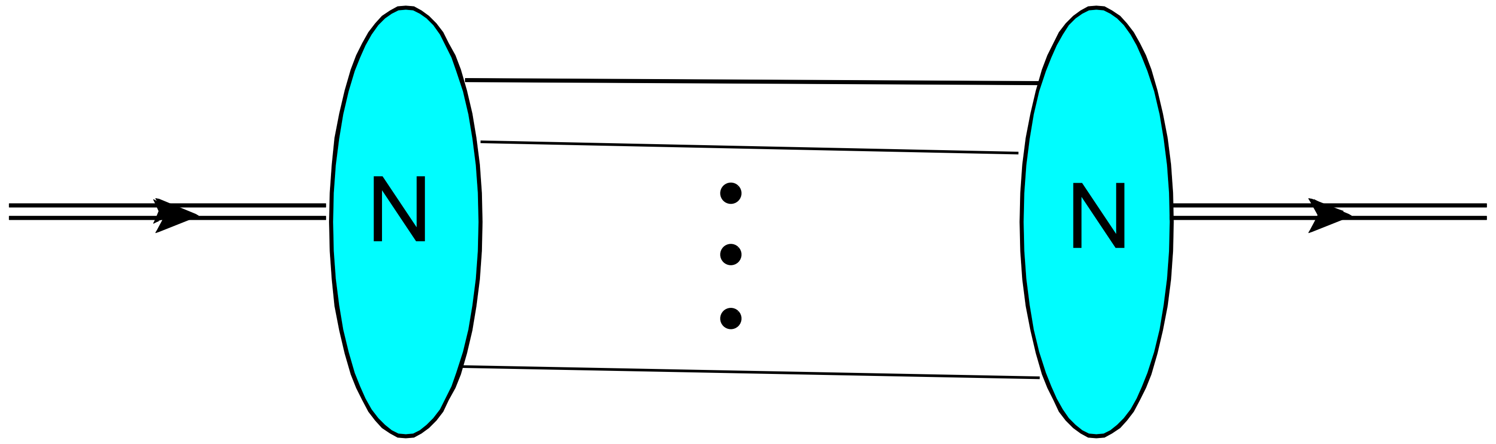
fixed light-cone time (x^+=0)



✦ Light-cone wave function representation of Wigner Distributions:

in the A^+=0 gauge and at ξ=0 ⇒ diagonal in the Fock-space



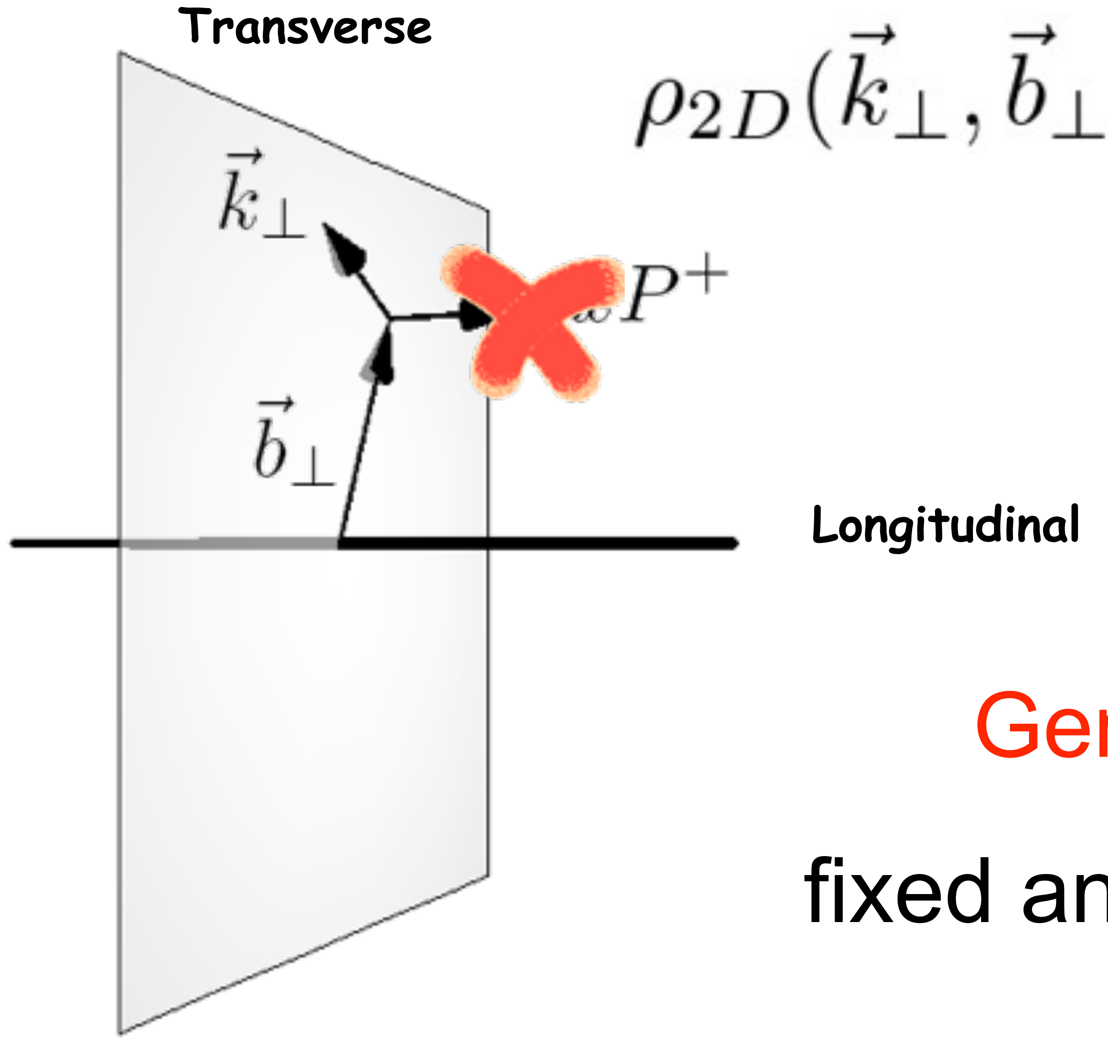
N=3: overlap of quark light-cone wave-functions

✦ real functions, but in general not-positive definite

- ➡ not probabilistic interpretation
- ➡ correlations of quark momentum and position in the transverse plane as function of quark and nucleon polarizations

Unpol. up Quark in Unpol. Proton

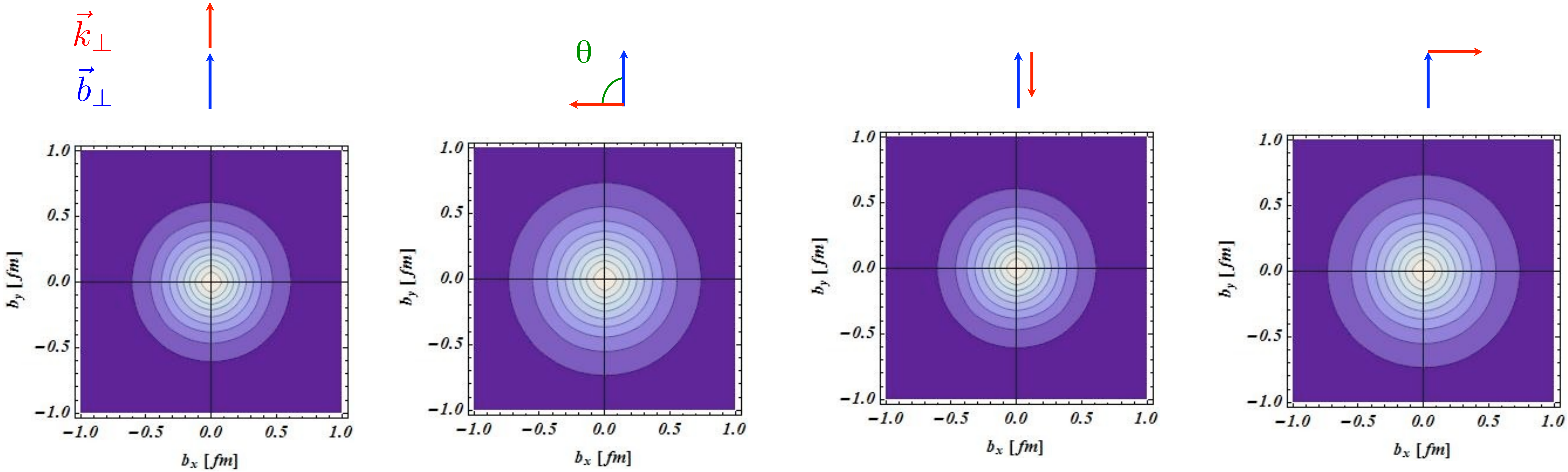
[Lorce', BP, PRD84 (2011)]



$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$

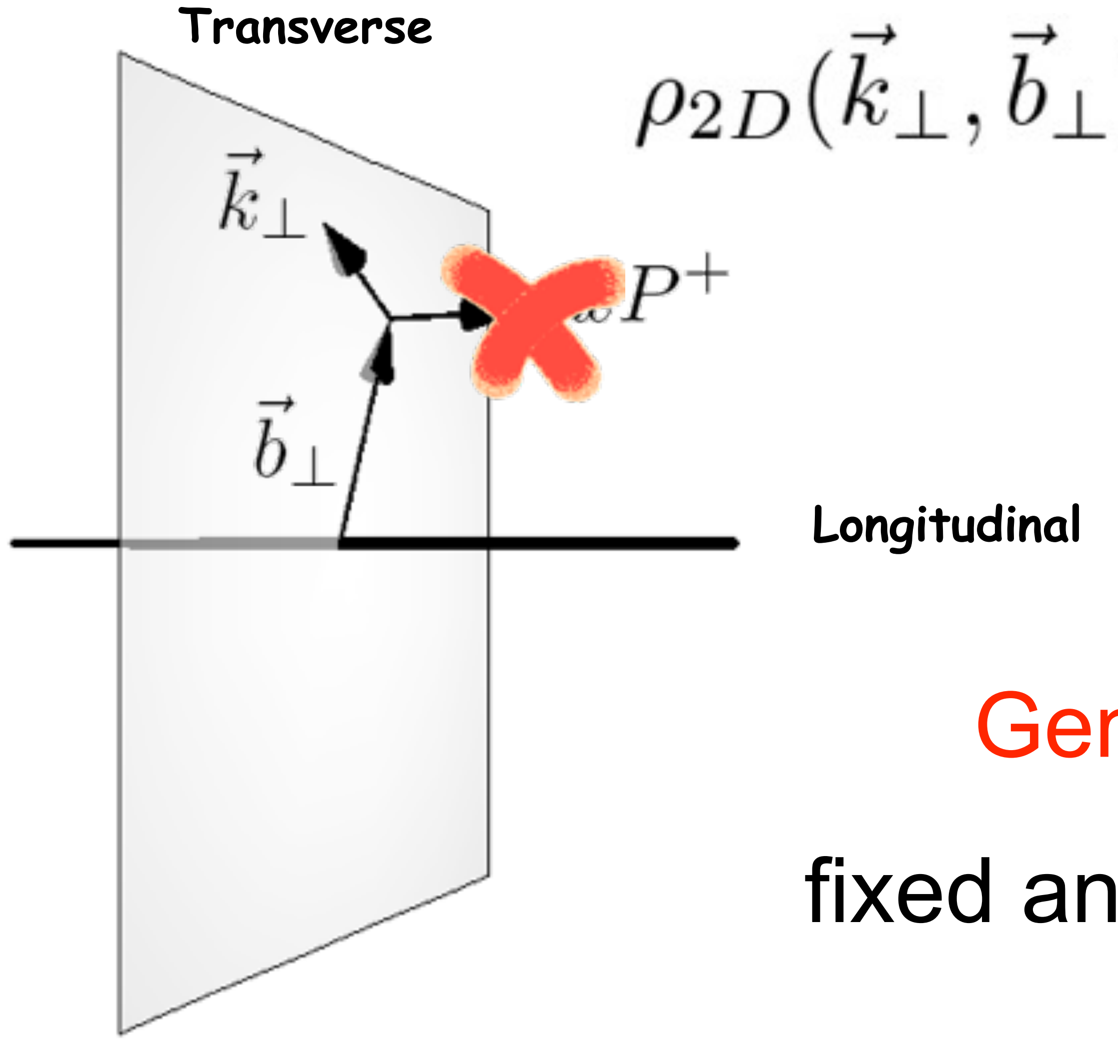
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up Quark in Unpol. Proton

[Lorce', BP, PRD84 (2011)]



$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$

Generalized Transverse Charge Density

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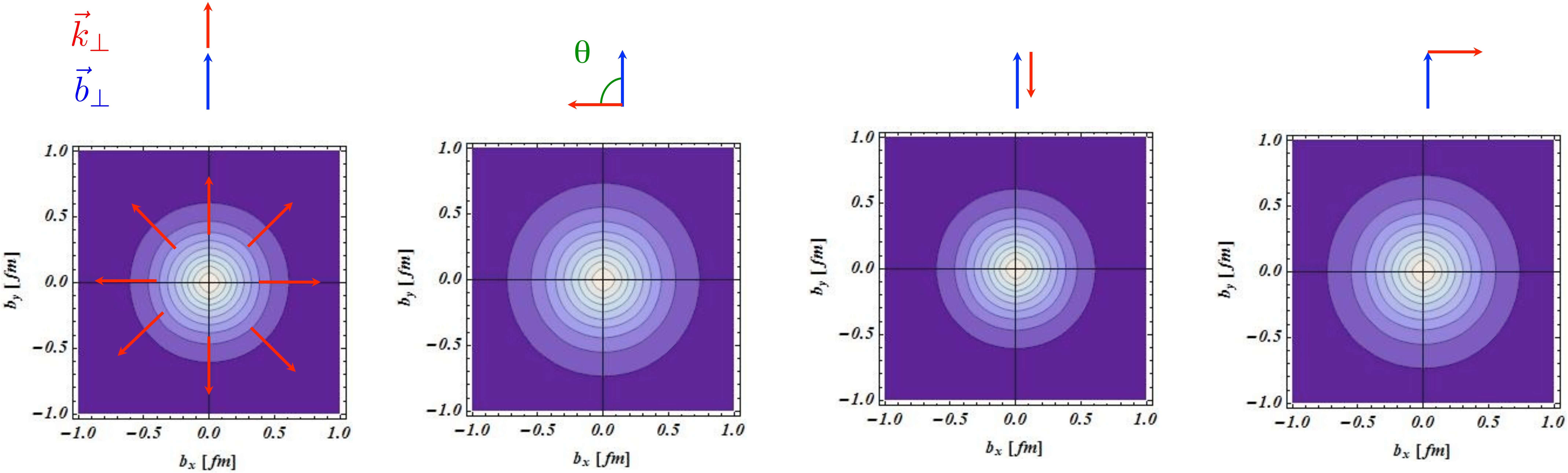

Unpol. up Quark in Unpol. Proton

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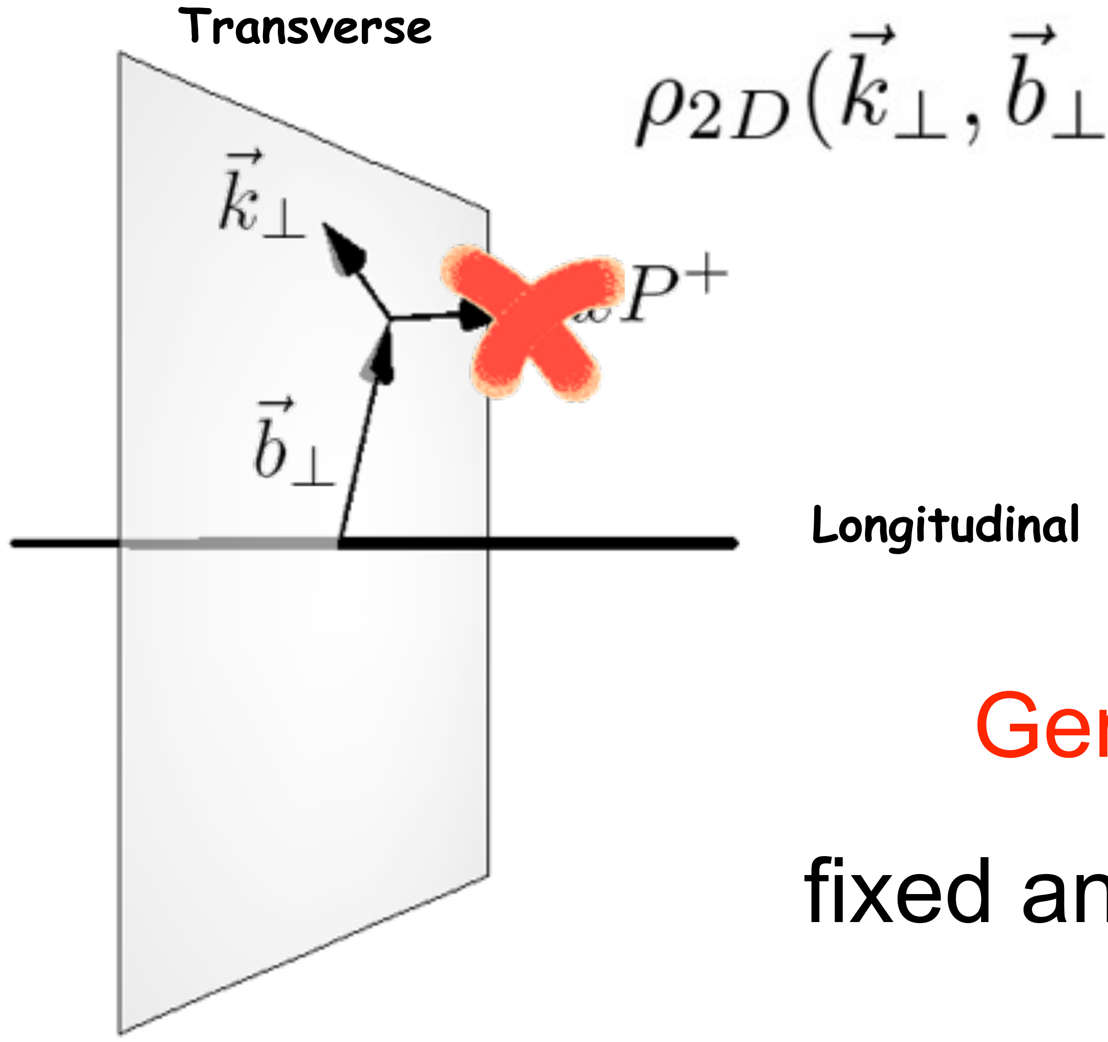
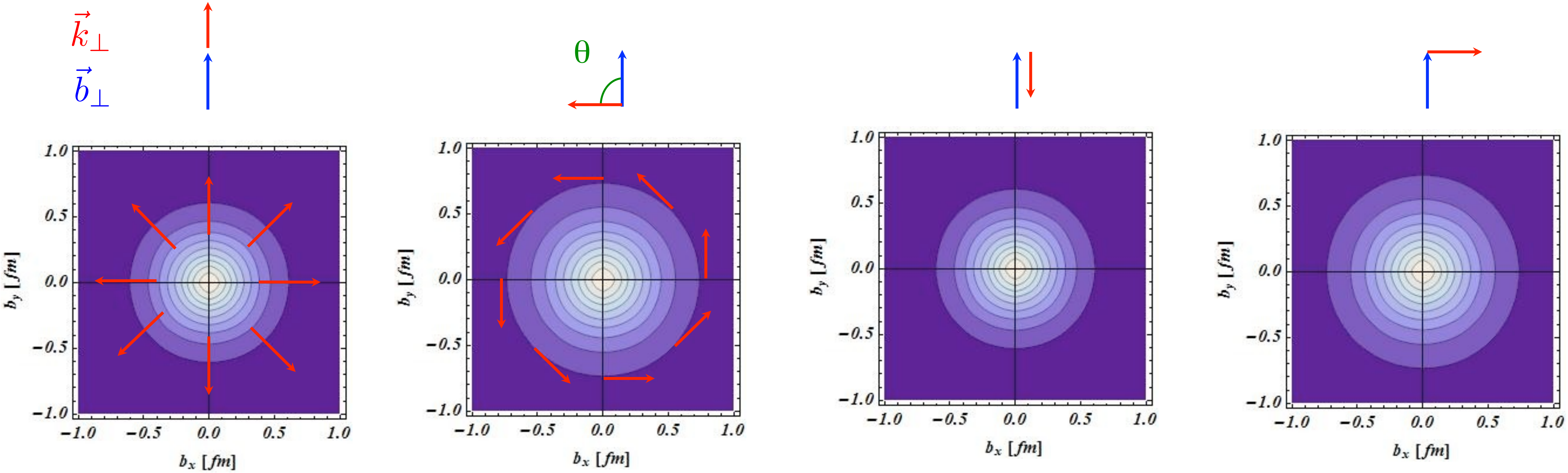


$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$

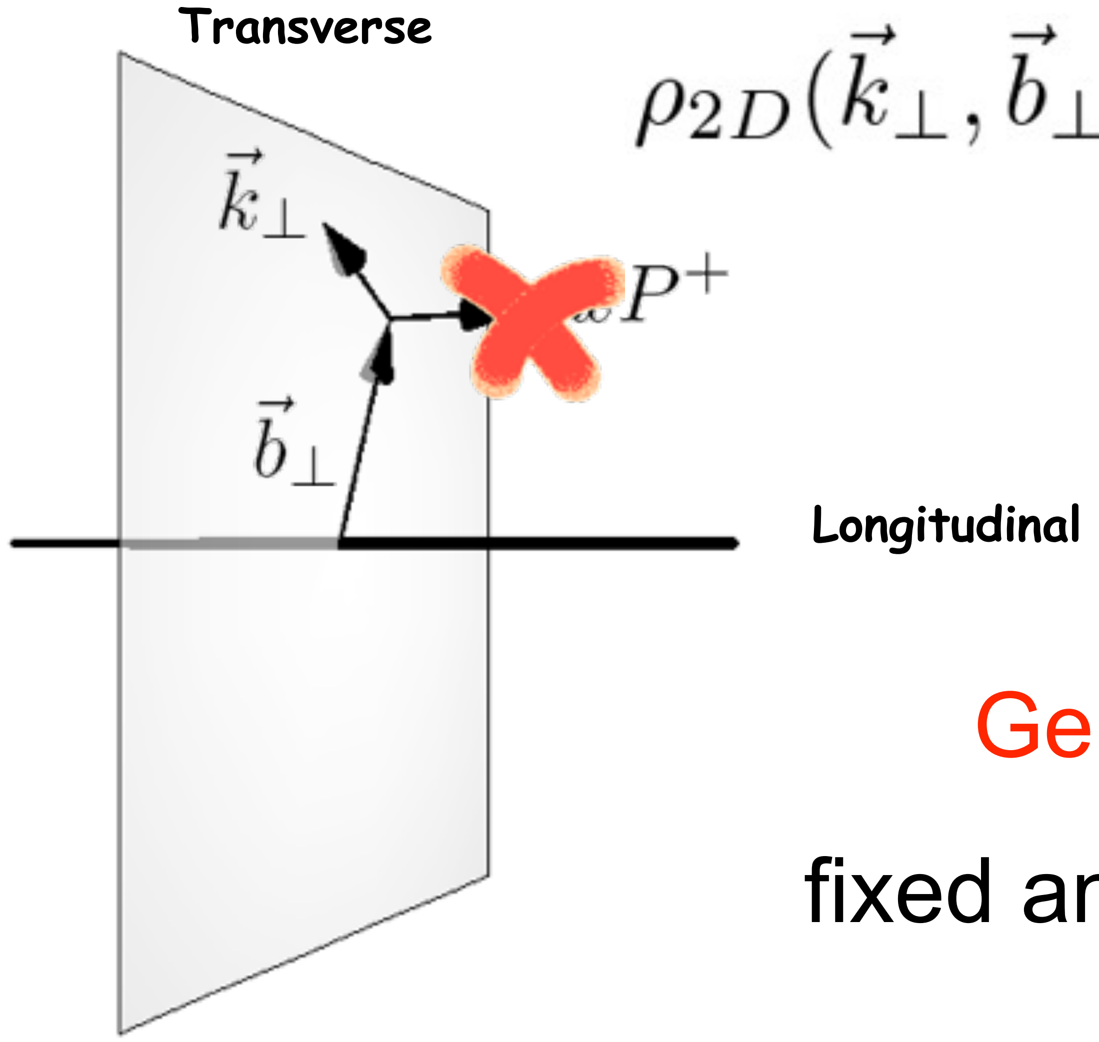
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up Quark in Unpol. Proton

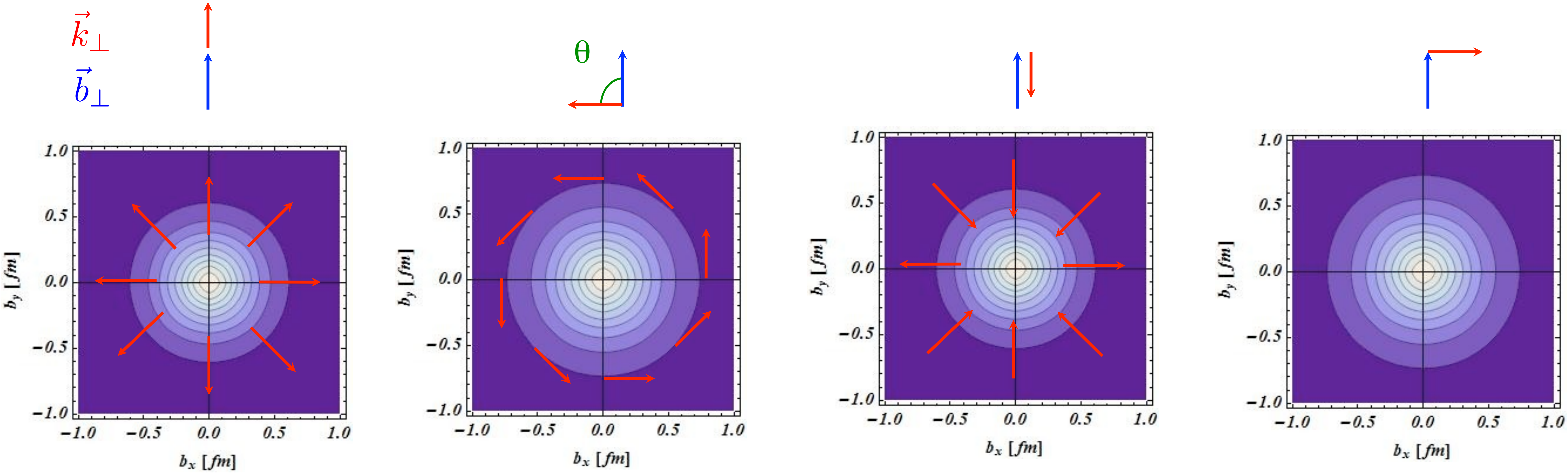
[Lorce', BP, PRD84 (2011)]



$$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$$

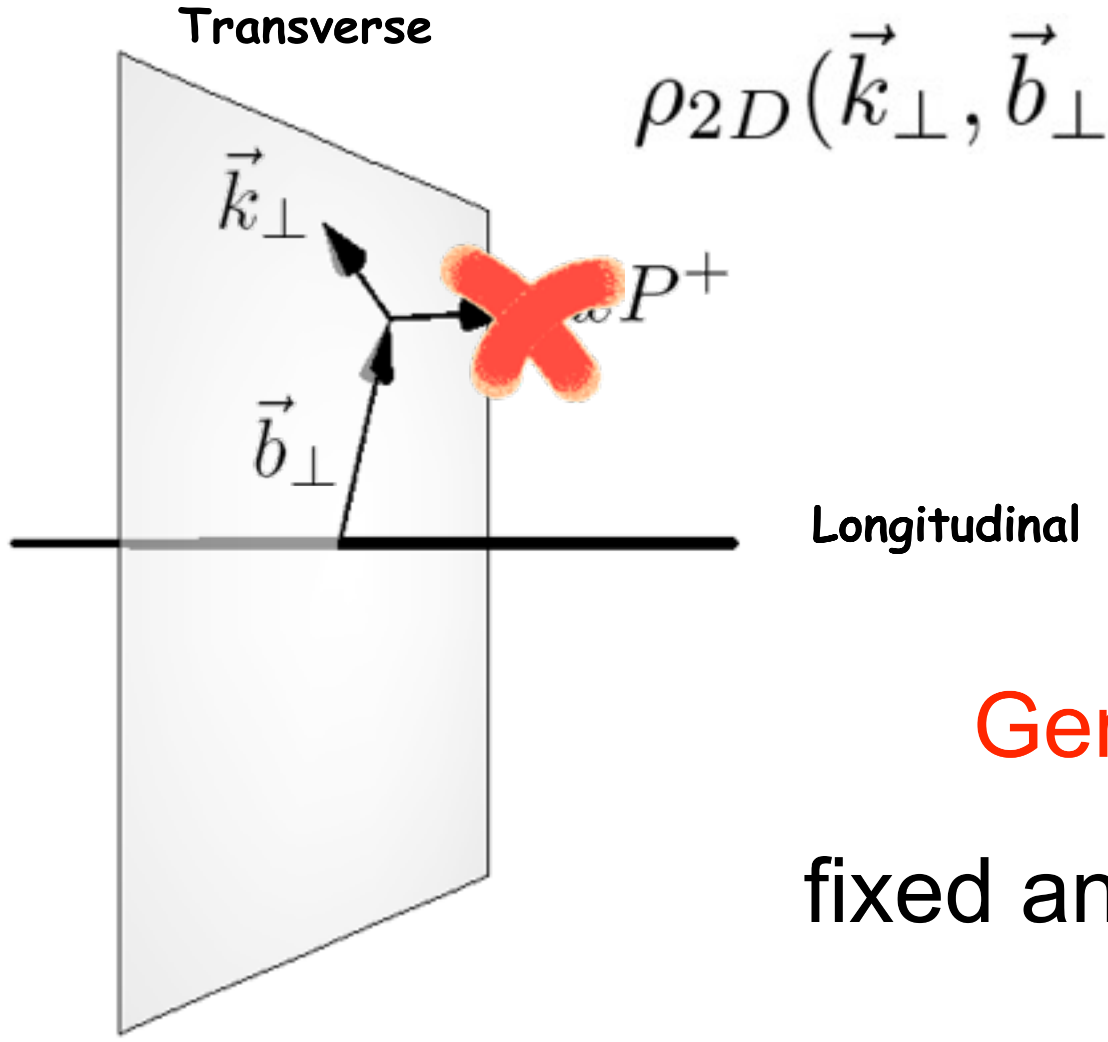
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$



Unpol. up Quark in Unpol. Proton

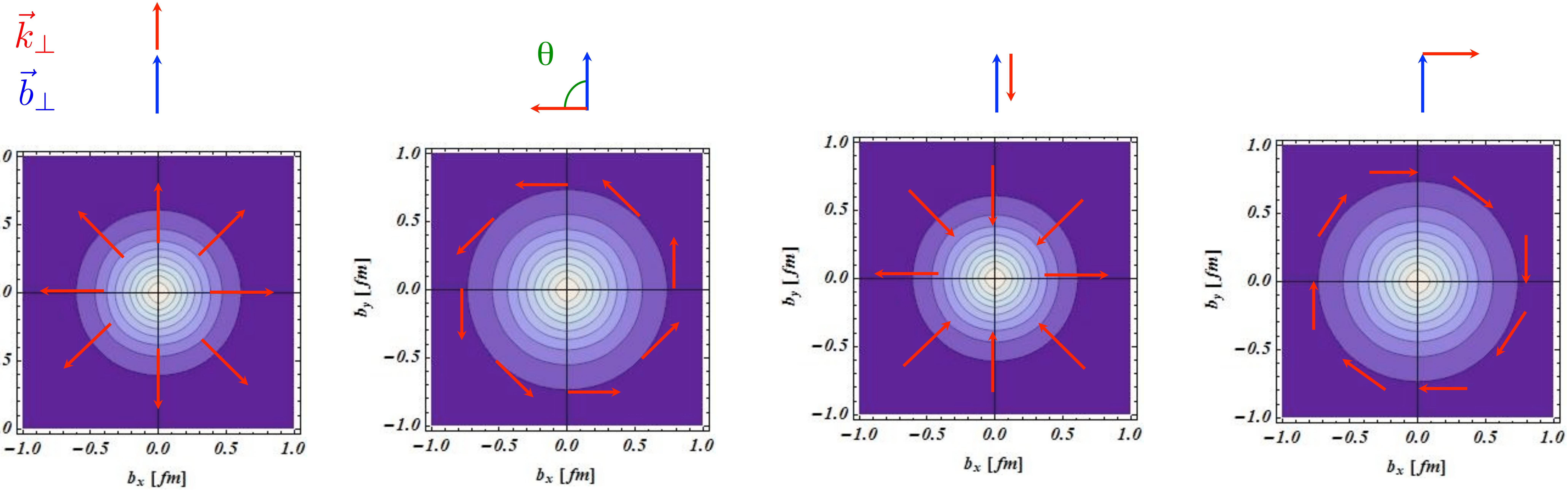
[Lorce', BP, PRD84 (2011)]



$\rho_{2D}(\vec{k}_\perp, \vec{b}_\perp)$

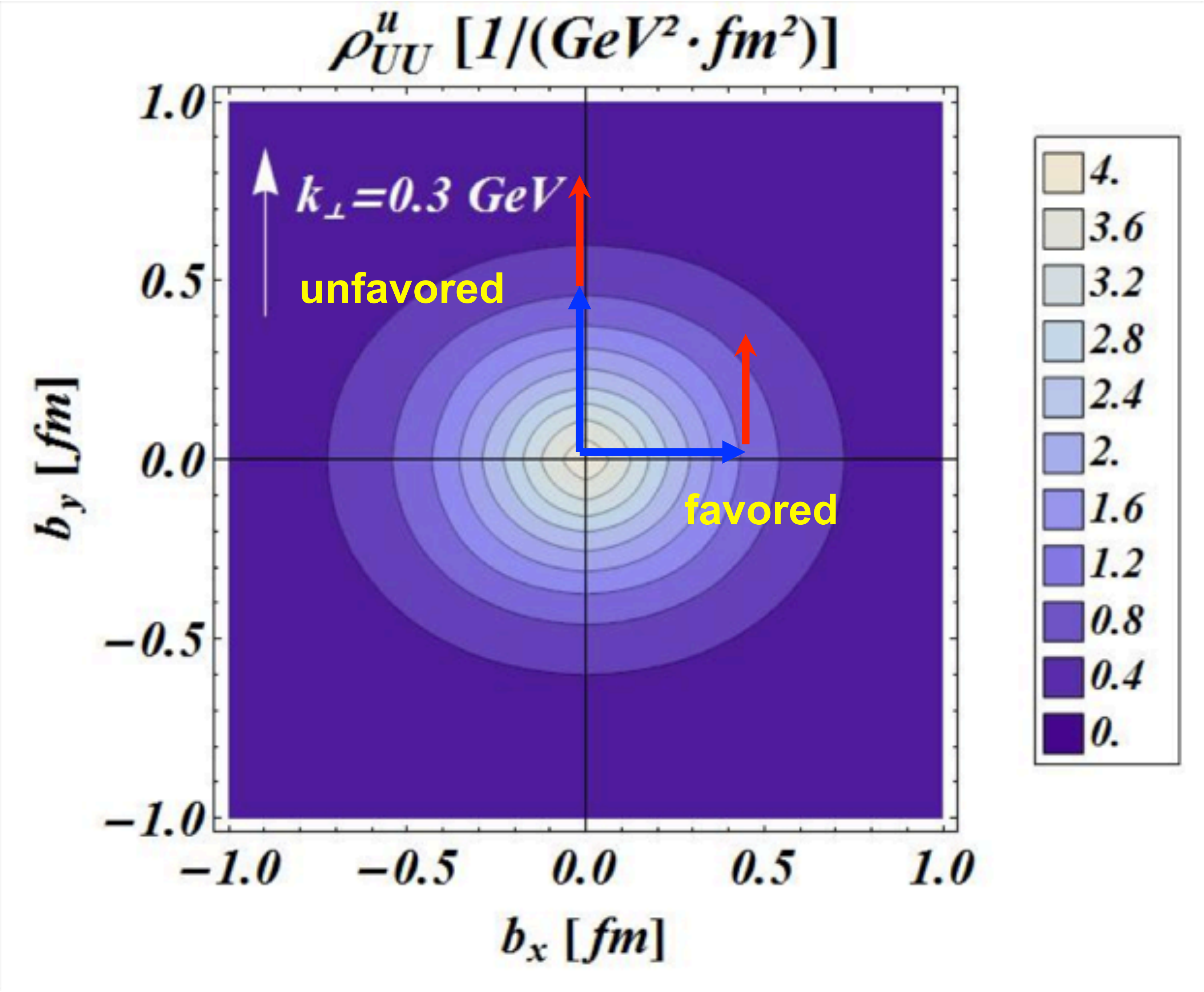
Generalized Transverse Charge Density

fixed angle between $\vec{k}_\perp$ and $\vec{b}_\perp$ and fixed value of $|\vec{k}_\perp|$

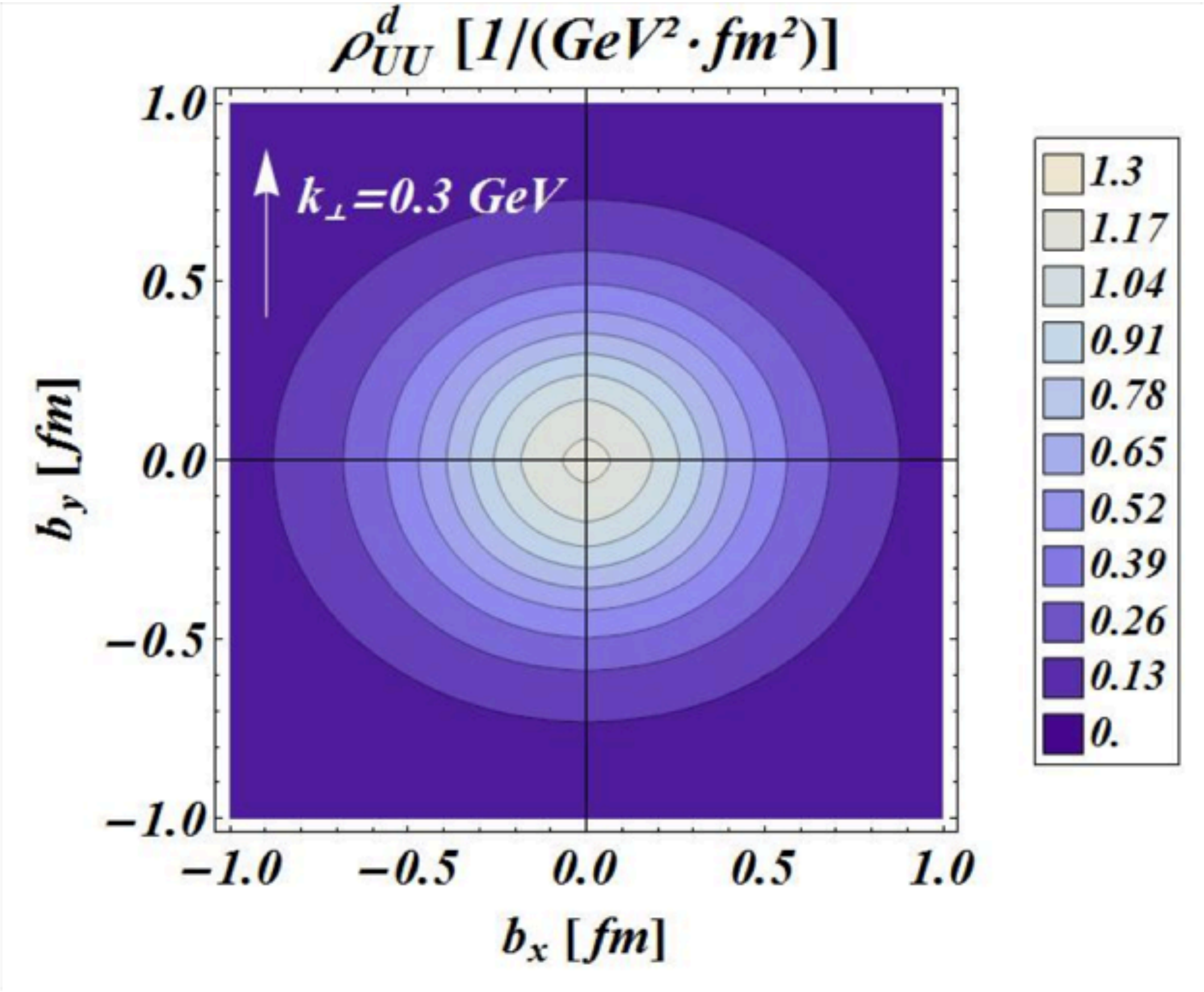


fixed $\vec{k}_\perp$: ↑

up quark



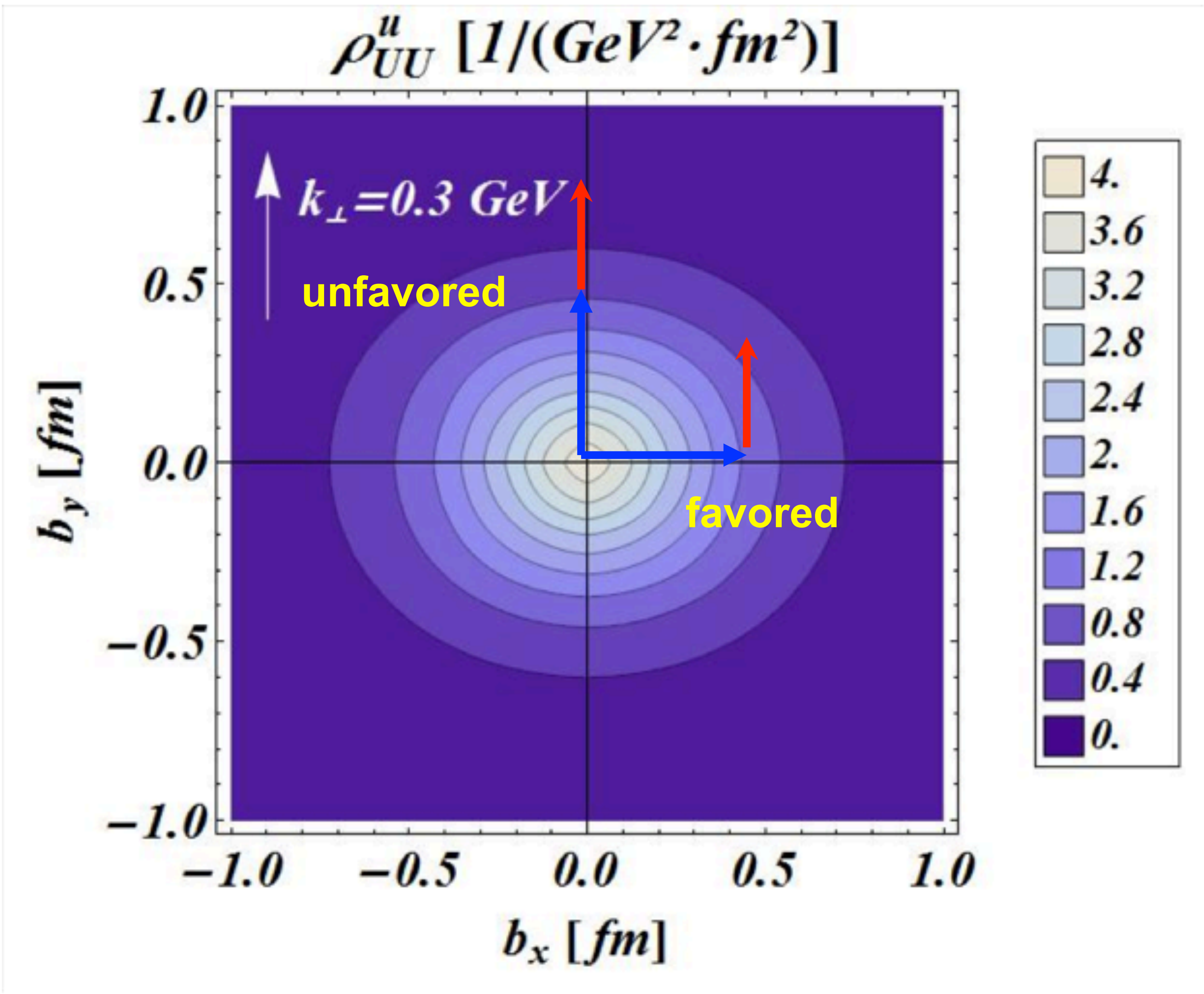
down quark



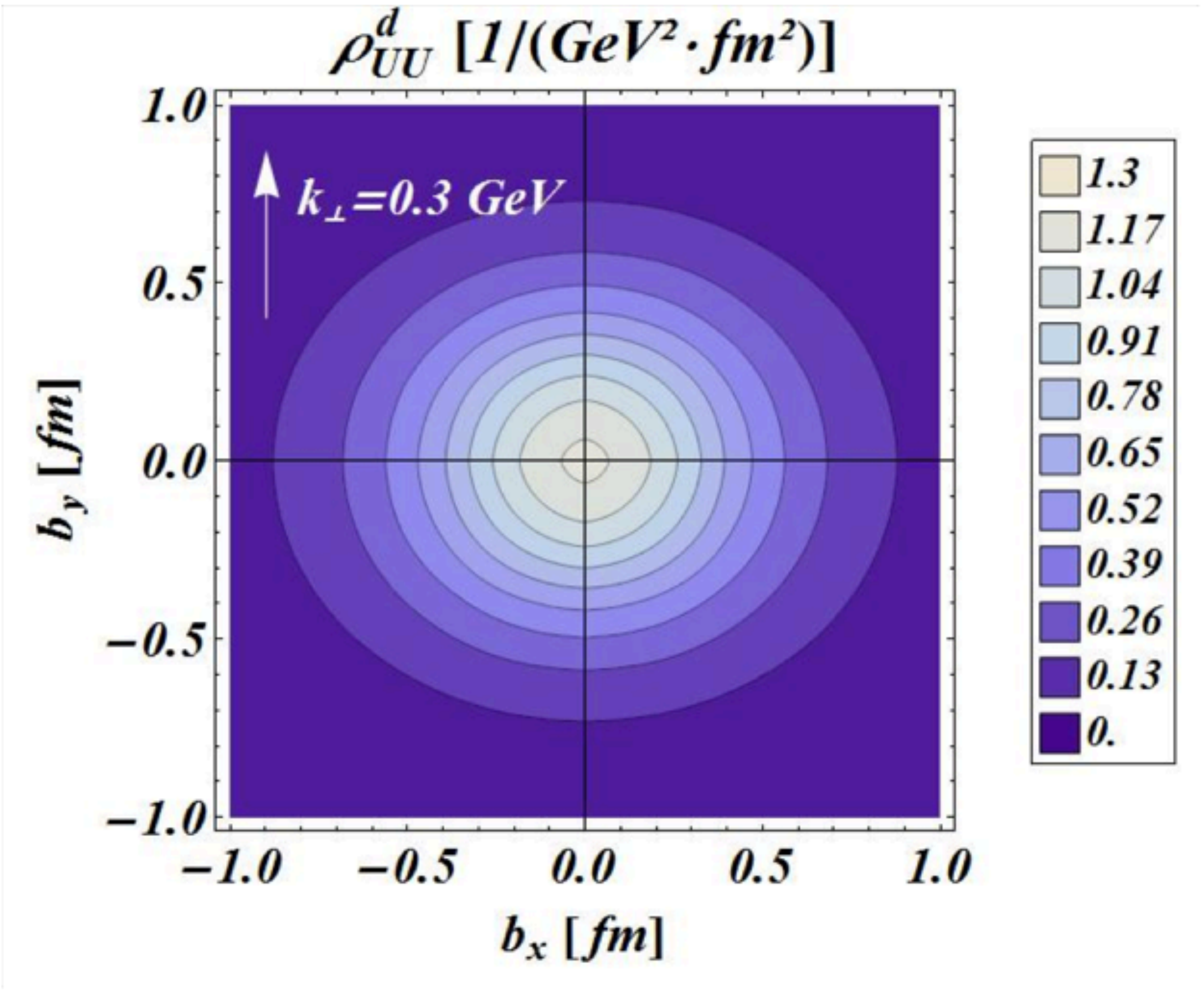
left-right symmetry of distributions → no net orbital angular momentum

quarks are as likely to rotate clockwise as to rotate anticlockwise fixed $\vec{k}_\perp$: ↑

up quark



down quark



left-right symmetry of distributions ➡ no net orbital angular momentum
quarks are as likely to rotate clockwise as to rotate anticlockwise

✦ integrating over $\vec{b}_\perp$ ➡ transverse-momentum density

$$f_1^q(k_\perp^2) = \int \mathrm{d}x f_1^q(x, k_\perp^2)$$

✦ integrating over $\vec{k}_\perp$ ➡ charge density in the transverse plane $\vec{b}_\perp$

[Miller (2007); Burkardt (2007)]

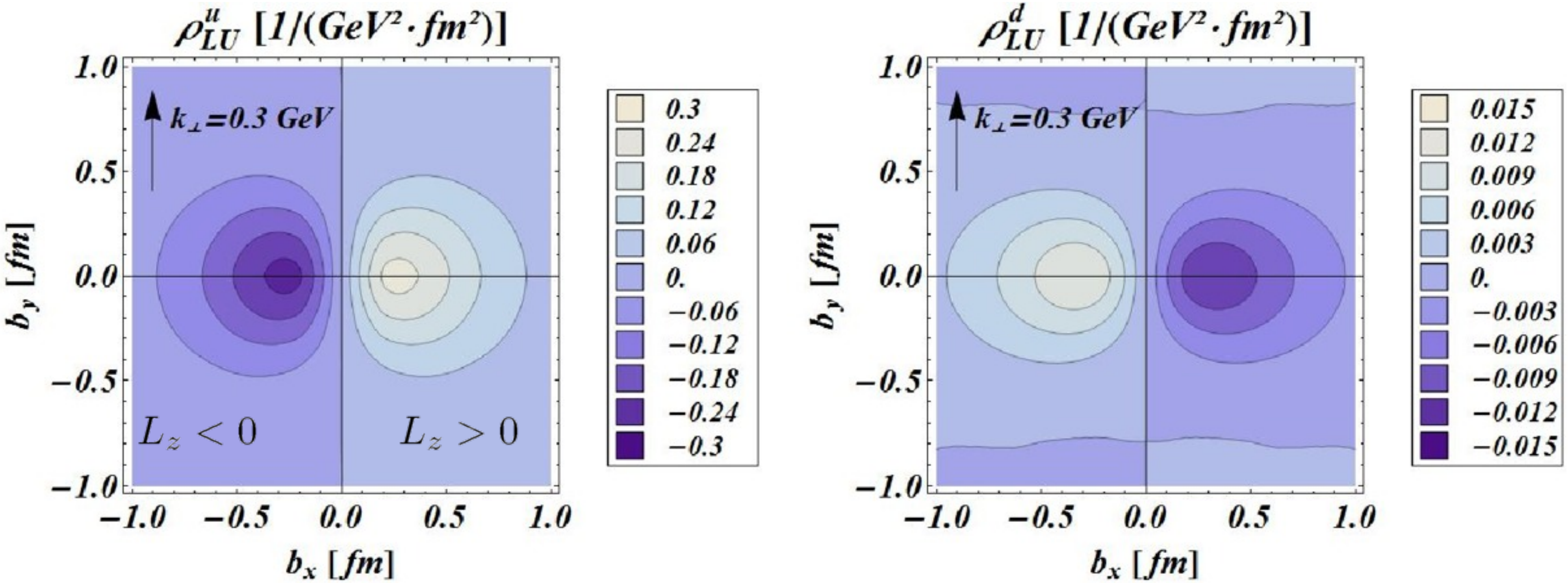
$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$

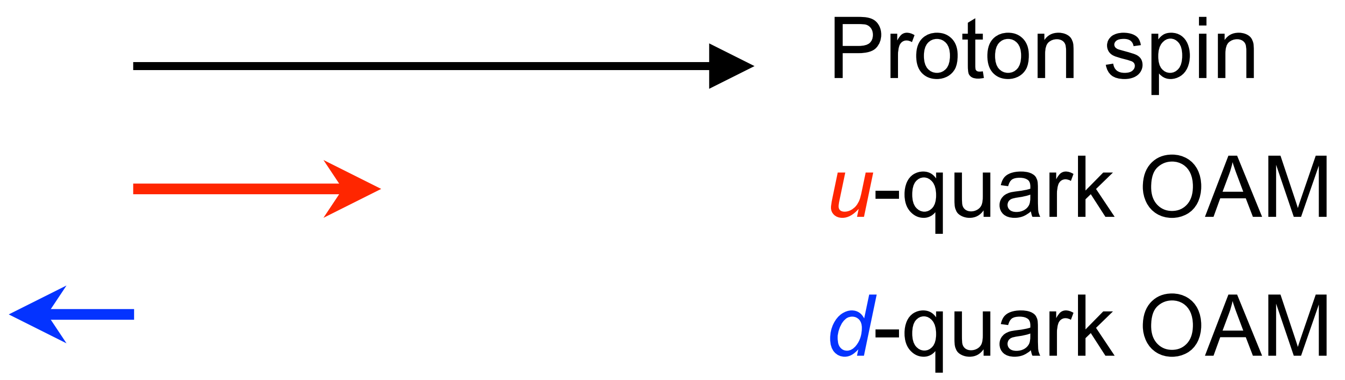
Monopole

Distributions

Unpol. quark in long. pol. proton

fixed $\vec{k}_\perp$ ↑





Proton spin

u-quark OAM

d-quark OAM

✦ projection to GPD and TMD is vanishing

➡ unique information on OAM from Wigner distributions

Quark Orbital Angular Momentum

$$\mathcal{L}_z^q = \int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

Lorce', BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)

⬇

Wigner distribution
for Unpolarized quark in a Longitudinally pol. nucleon

Quark Orbital Angular Momentum

Lorce', BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)

$$\mathcal{L}_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

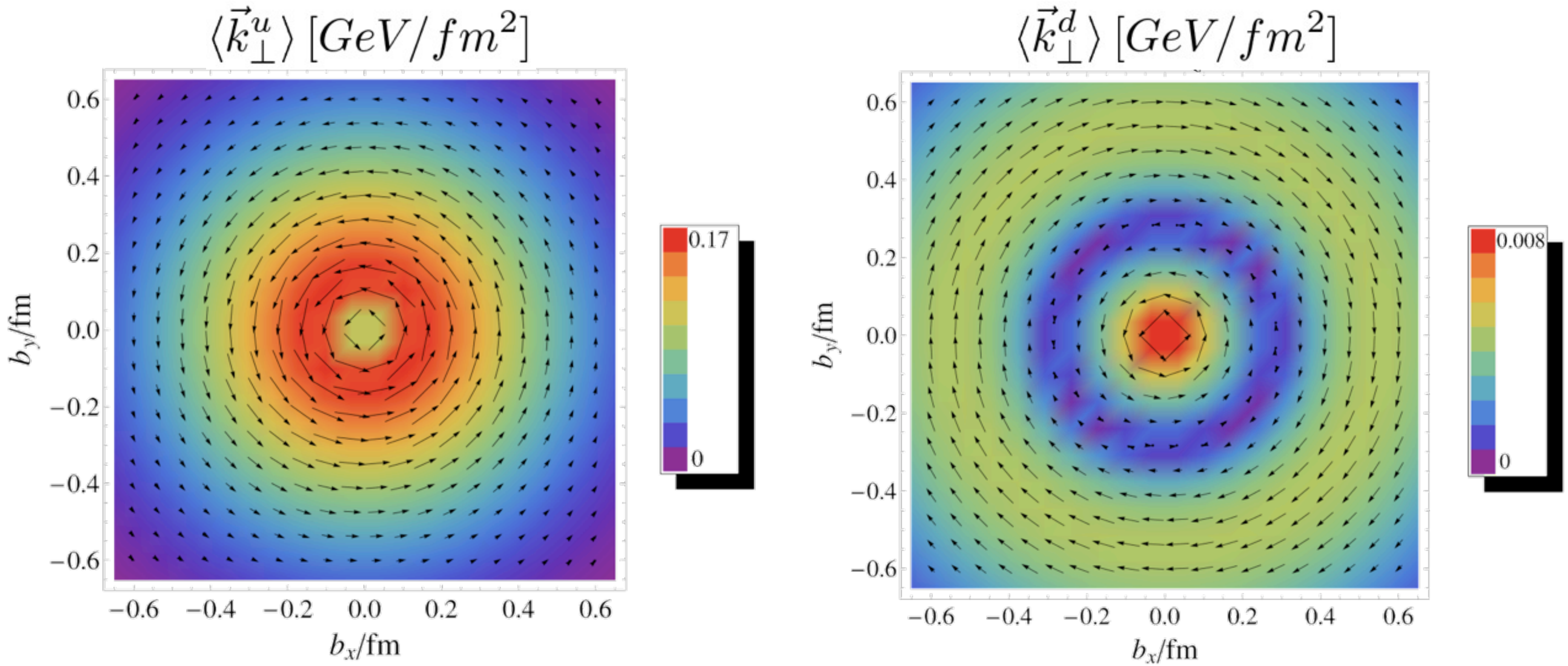
$$= \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}^q_\perp \rangle \longrightarrow \quad \langle \vec{k}^q_\perp \rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp\, \vec{k}_\perp \rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

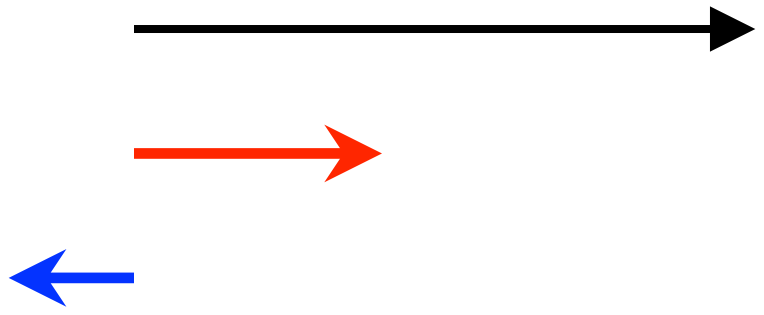
Quark Orbital Angular Momentum

$$\mathcal{L}_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp\mathrm{d}^2\vec{b}_\perp(\vec{b}_\perp \times \vec{k}_\perp)\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$= \int \mathrm{d}^2\vec{b}_\perp\vec{b}_\perp \times \langle\vec{k}^q_\perp\rangle \longrightarrow \langle\vec{k}^q_\perp\rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp\,\vec{k}_\perp\rho^q_{LU}(\vec{b}_\perp, \vec{k}_\perp, x)$$

Lorce', BP (11)
Hatta (12)
Ji, Xiong, Yuan (12)



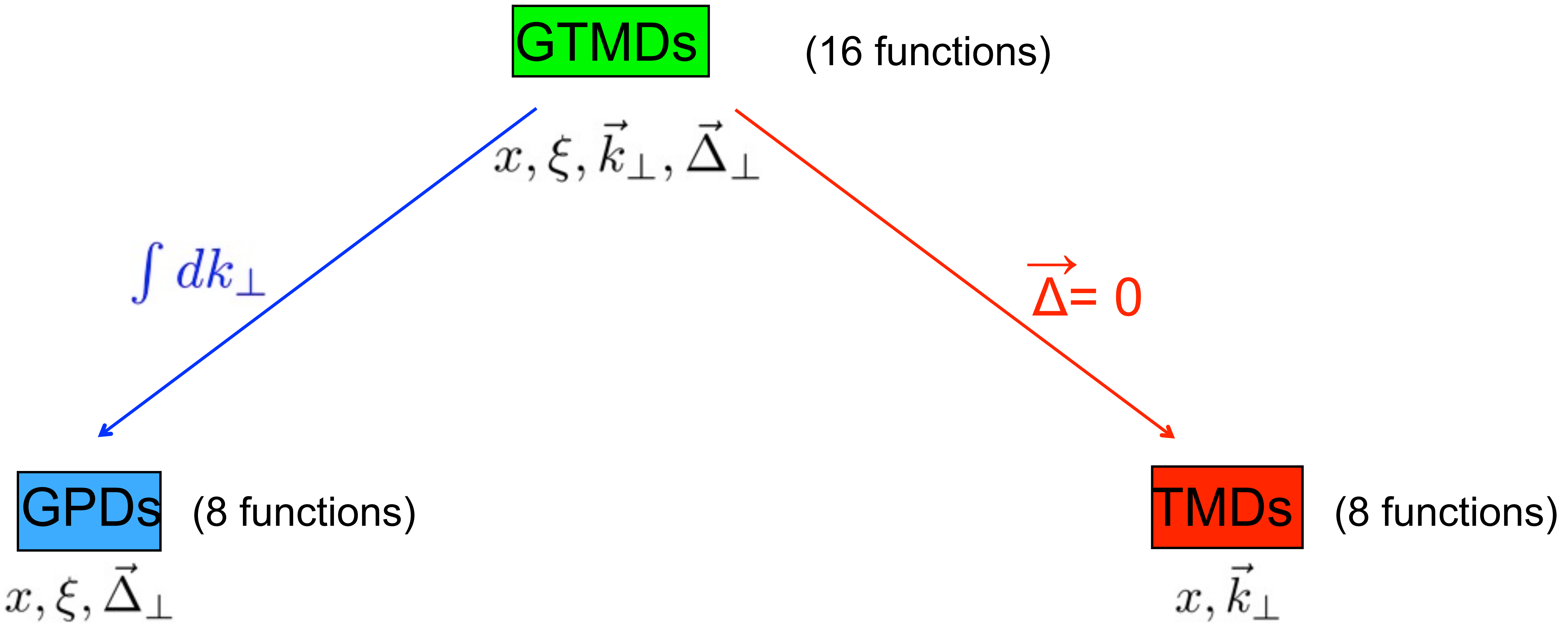


Proton spin

u-quark OAM

d-quark OAM

Results in a light-cone constituent quark model:
Lorce', BP, Xiong, Yuan, arXiv:1111.4827 [hep-ph]



GTMDs (16 functions)

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

$\int dk_\perp$

$\vec{\Delta} = 0$

GPDs (8 functions)

$x, \xi, \vec{\Delta}_\perp$

TMDs (8 functions)

$x, \vec{k}_\perp$

Quark polarization

Nucleon polarization

	U	T	L
U	H	$\mathcal{E}_T$	
T	E	$H_T, \tilde{H}_T$	$\tilde{E}$
L		$\tilde{E}_T$	$\tilde{H}$

Quark polarization

Nucleon polarization

	U	T	L
U	f_1	$h_1^\perp$	
T	$f_{1T}^\perp$	$h_1, h_{1T}^\perp$	g_{1T}
L		$h_{1L}^\perp$	g_{1L}

✦ almost all distributions (in red) vanish if there is no quark orbital angular momentum

✦ quark GPDs (at ξ=0) and TMDs given by the same overlap of LCWFs but in different kinematics

⇒ each distribution contains unique information

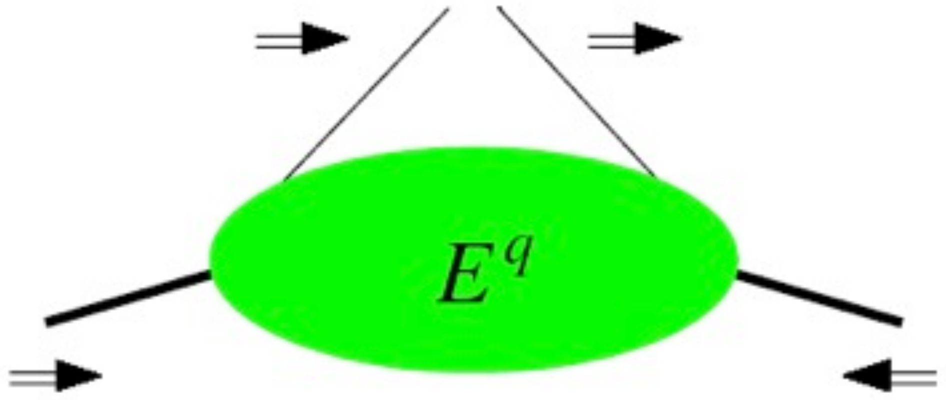
⇒ no model-independent relations between GPDs and TMDs

Angular Momentum Relation ("Ji's Sum Rule")

[X. Ji, Phys. Rev. Lett. 78 (1997)]

- quark and gluon contribution to the nucleon spin

$$J^{q,g} = \frac{1}{2}\int_{-1}^{1} dx\, x(H^{q,g}(x,\xi,t=0) + E^{q,g}(x,\xi,t=0)$$



proton helicity flipped but quark helicity conserved

⬇

"Helicity mismatch" requires orbital angular momentum

Proton spin decomposition

$$\frac{1}{2} = \sum_q J^q + J^g$$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta\Sigma^q + \sum_q L^q + J^g$$

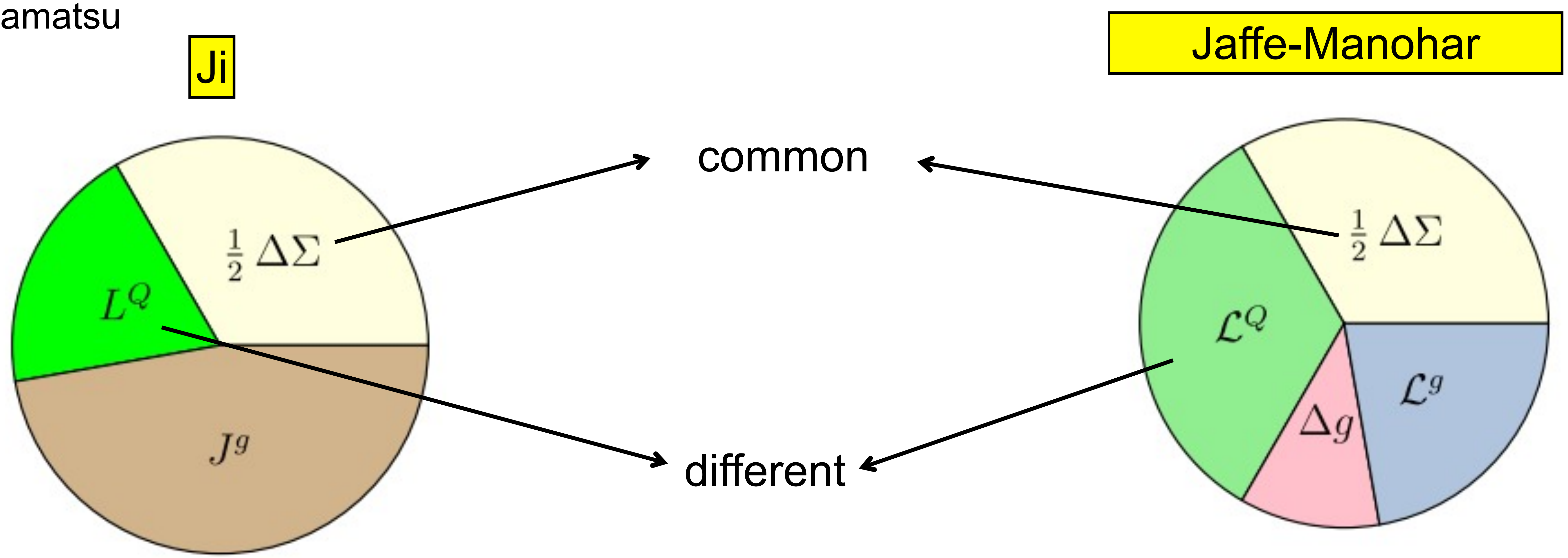
- ☐ inclusive processes (parton densities) $\rightarrow \Delta\Sigma^q$
- ☐ exclusive processes (GPDs) $\rightarrow J^q$

⬇

Quark Orbital Angular Momentum: $L^q = J^q - \frac{1}{2}\Delta\Sigma^q$

Alternative Decompositions of Nucleon Spin

☞ see talk by Wakamatsu



- Each term is gauge invariant
- No decomposition of J^g in spin and orbital part

$$\frac{1}{2} = \sum_q J^q + J^g = \frac{1}{2}\Delta\Sigma + \sum_q L^q + J^g$$

$\Downarrow$

from GPDs

✦ can be calculated in Lattice QCD

☞ see talk by Alexandrou

- Decomposition is gauge dependent $(A^+ = 0)$

$$\frac{1}{2} = \sum_q \mathcal{J}^q + \mathcal{J}^g$$

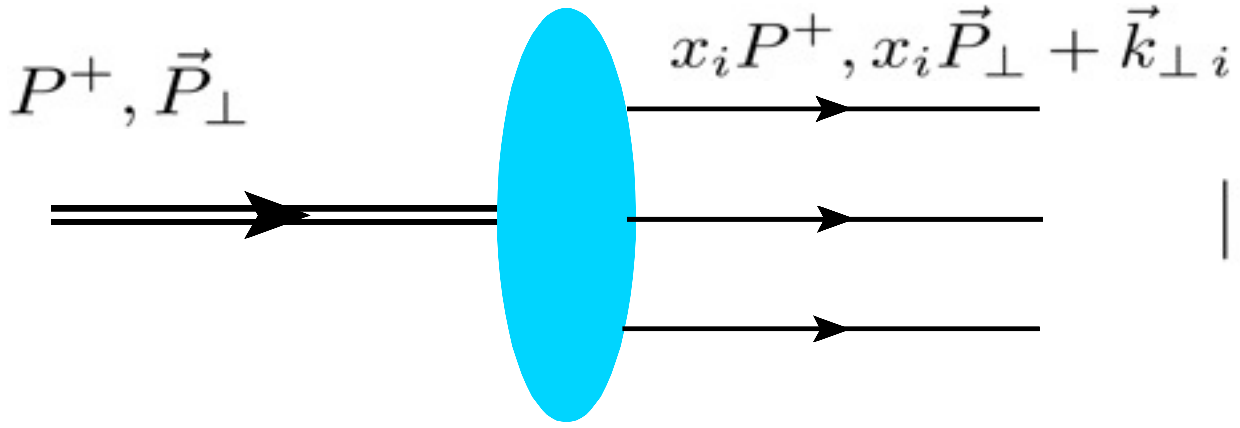
$$= \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}^q + \Delta g + \mathcal{L}^g$$

✦Δg and $\Delta\Sigma$ measured by COMPASS, HERMES , RHIC

✦ no direct connection of $\mathcal{L}^q$ and $\mathcal{L}^g$ with observables

✦ model-dependent information from TMDs

Quark OAM: Partial-Wave Decomposition



$$| \, P, \Lambda \rangle = \int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3] \; \underbrace{\Psi^{\Lambda}_{\lambda_1 \lambda_2 \lambda_3}(x_i, \vec{k}_{\perp,i})}_{\downarrow} \; \frac{\varepsilon^{ijk}}{\sqrt{6}} \, u^\dagger_{i\lambda_1}(1) u^\dagger_{j\lambda_2}(2) d^\dagger_{k\lambda_3}(3) | \, 0 \rangle$$

LCWF: eigenstate of OAM

$J^q_z \longrightarrow$ $(\uparrow\uparrow\uparrow)_{LC} = \frac{3}{2}$ $(\uparrow\uparrow\downarrow)_{LC} = \frac{1}{2}$ $(\uparrow\downarrow\downarrow)_{LC} = -\frac{1}{2}$ $(\downarrow\downarrow\downarrow)_{LC} = -\frac{3}{2}$

$L^q_z = \frac{1}{2} - J^q_z \longrightarrow$ $L^q_z = -1$ $L^q_z = 0$ $L^q_z = 1$ $L^q_z = 2$

${}^{L_z}\langle P, \uparrow | P, \uparrow \rangle^{L_z}$:probability to find the proton in a state with eigenvalue of OAM L_z

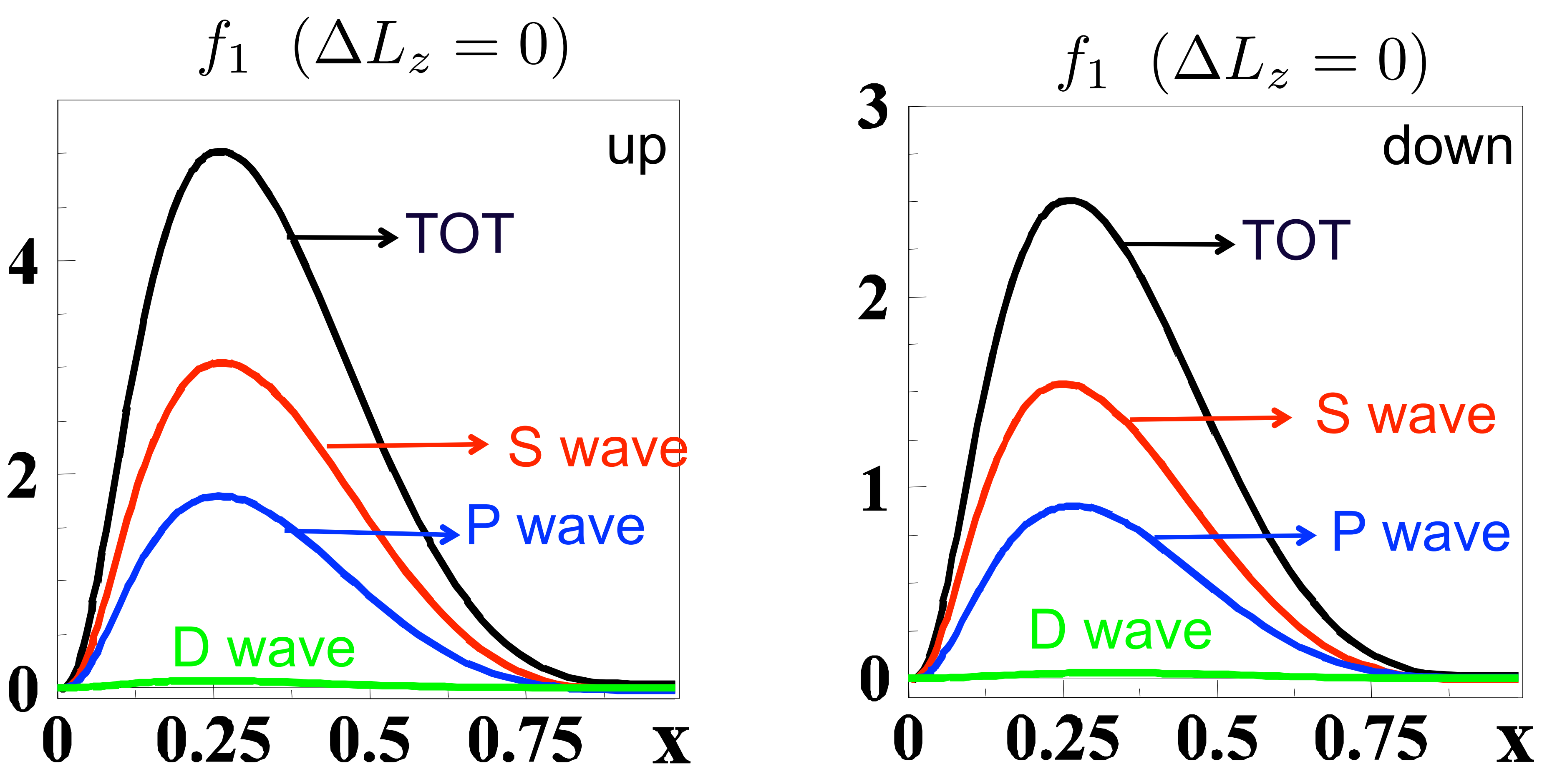
$$\Big\Downarrow$$

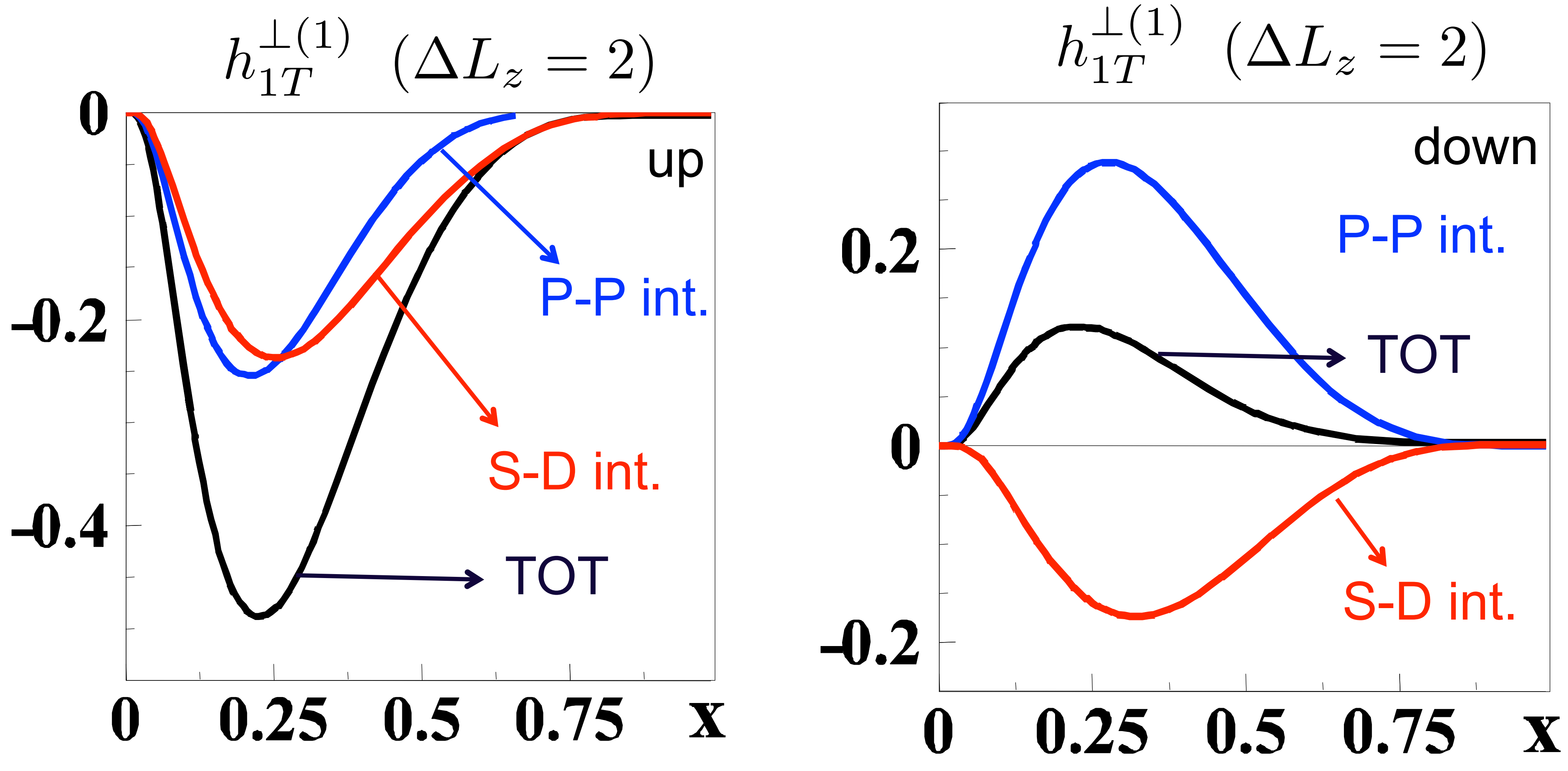
$$\mathcal{L}_z = \sum_{L_z} L_z \; {}^{L_z}\langle P, \uparrow | P, \uparrow \rangle^{L_z}$$

$$\downarrow$$

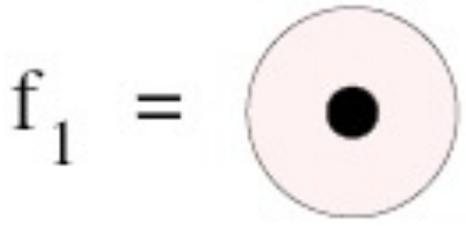
squared of LCWFs

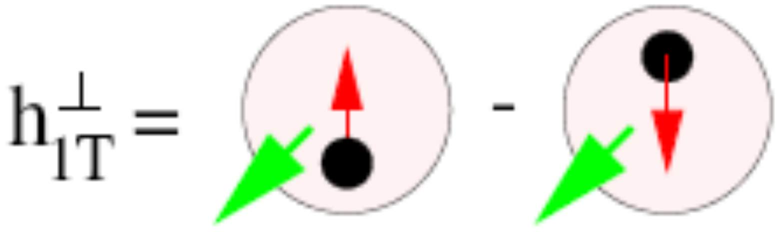
✦ Orbital angular momentum content of TMDs (light-cone constituent quark model)





“pretzelosity”





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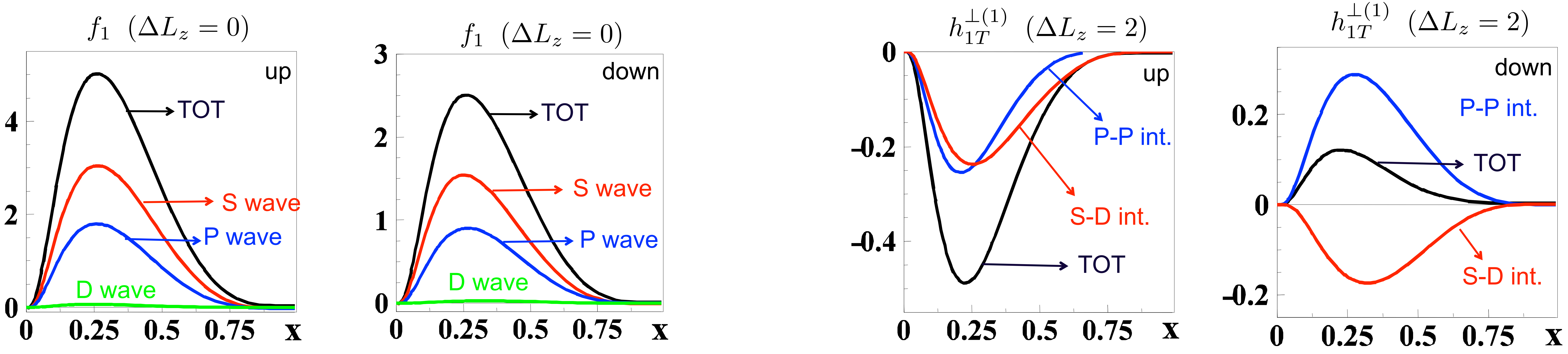
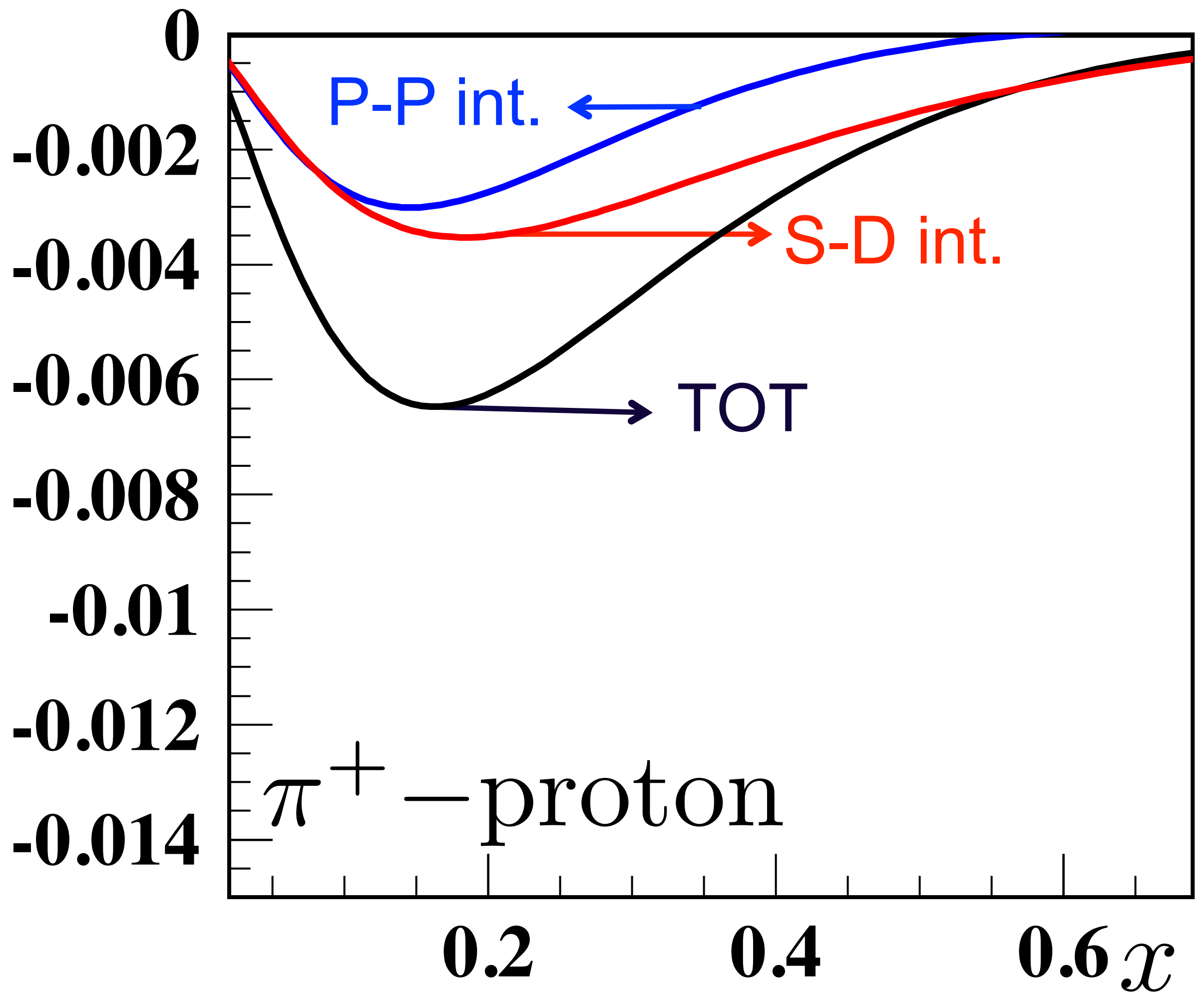
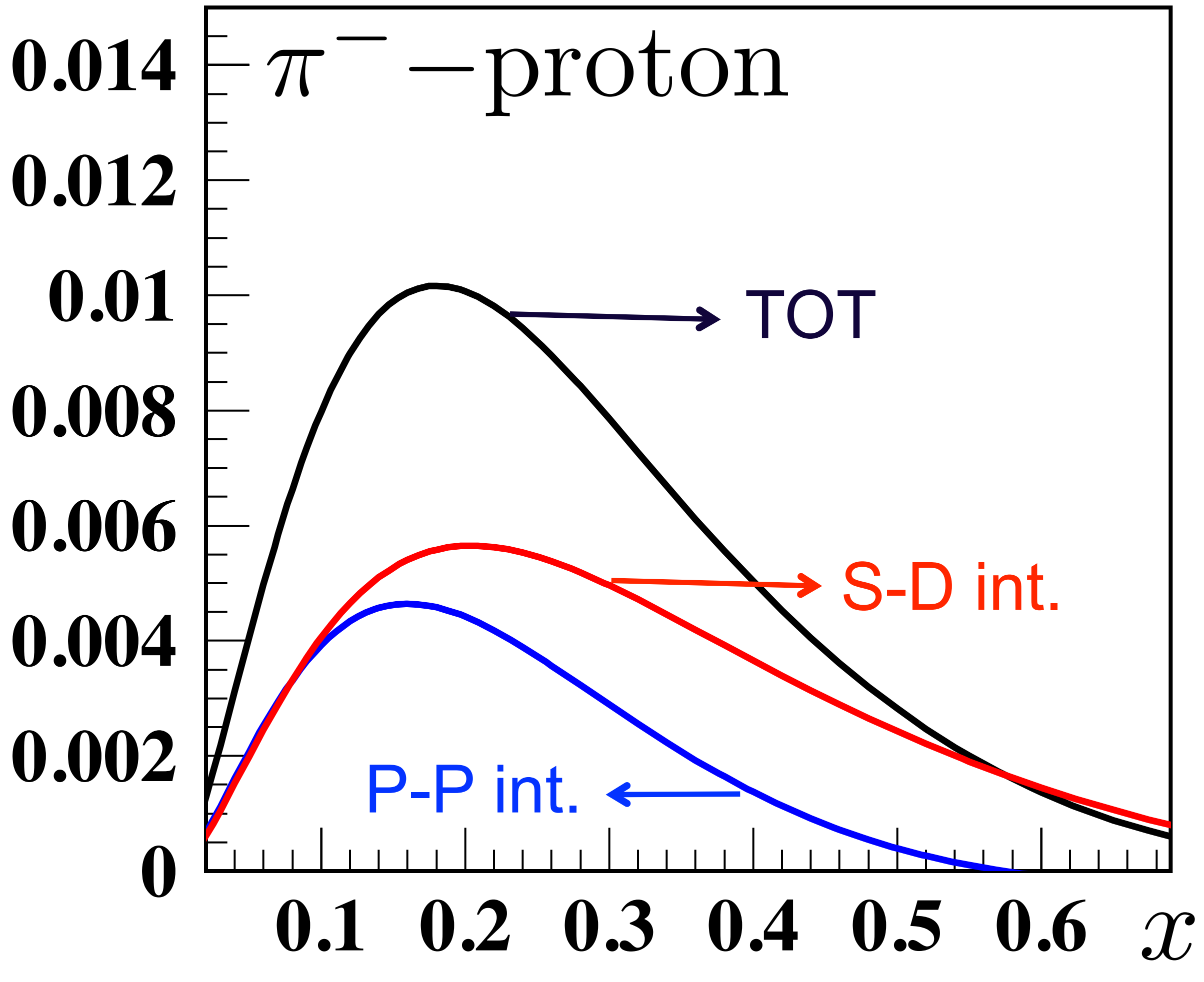


✦ Effects on SIDIS observables

$$A_{UT}^{\sin(3\phi-\phi_S)} \sim \frac{h_{1T}^{\perp} \otimes H_1}{f_1 \otimes D_1}$$



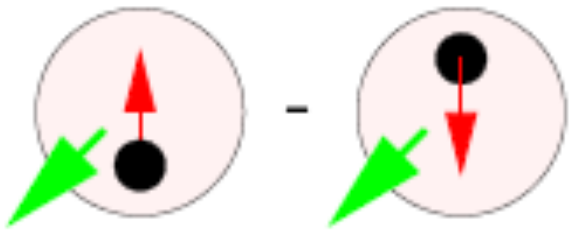
$\langle Q^2 \rangle = 2.5 \ \text{GeV}^2$



Boffi, Efremov, BP, Schweitzer, PRD79(2009)

Quark OAM from Pretzelosity

$h_{1T}^{\perp} =$



"pretzelosity"

model-dependent relation

$$\mathcal{L}_z = -\int \mathrm{d}x \mathrm{d}^2\vec{k}_\perp \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp}(x, k_\perp^2)$$

first derived in LC-diquark model and bag model

[She, Zhu, Ma, 2009; Avakian, Efremov, Schweitzer, Yuan, 2010]

$\mathcal{L}_z$	$h_{1T}^{\perp}$
chiral even and charge even	chiral odd and charge odd
$\Delta L_z = 0$	$\|\Delta L_z\| = 2$

no operator identity
relation at level of matrix elements of operators

$\Downarrow$

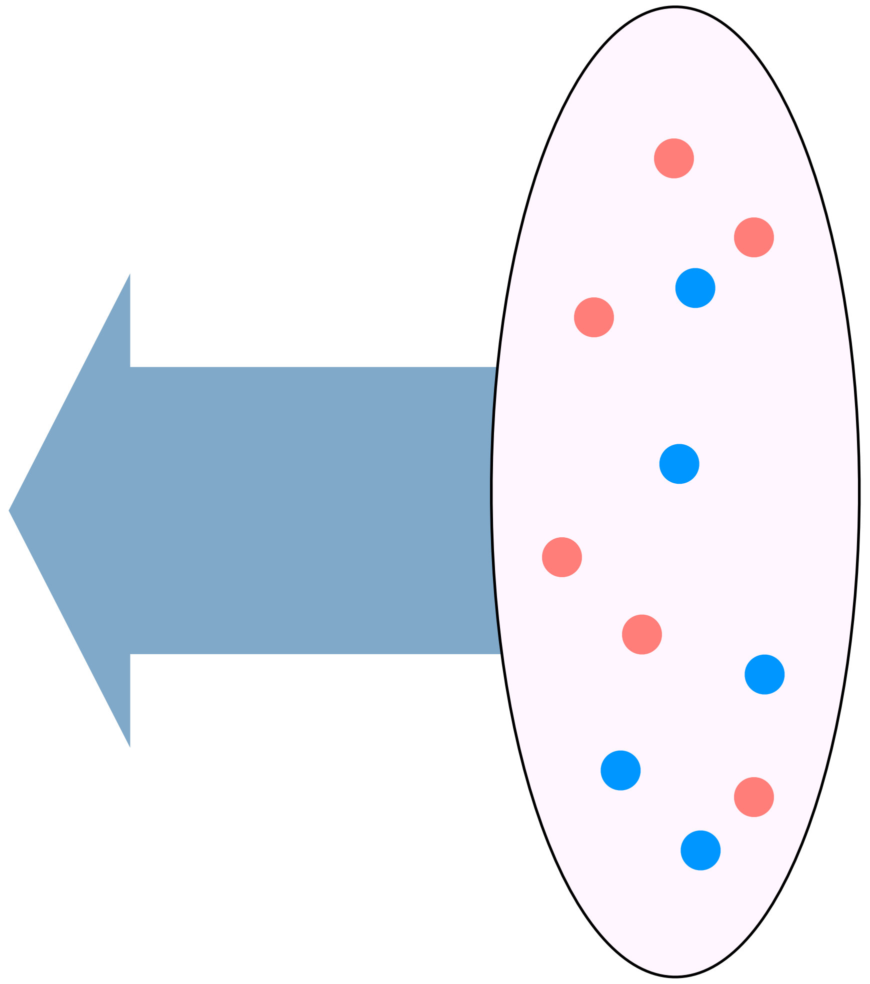
valid in all quark models with spherical symmetry in the rest frame

[Lorce', BP, PLB (2012)]

☞ see talk by C. Lorce'

Constraining quark OAM with Sivers function

unpolarized quark in unpolarized nucleon



Burkardt, PRD66 (02)

A. Bacchetta, DIS2012

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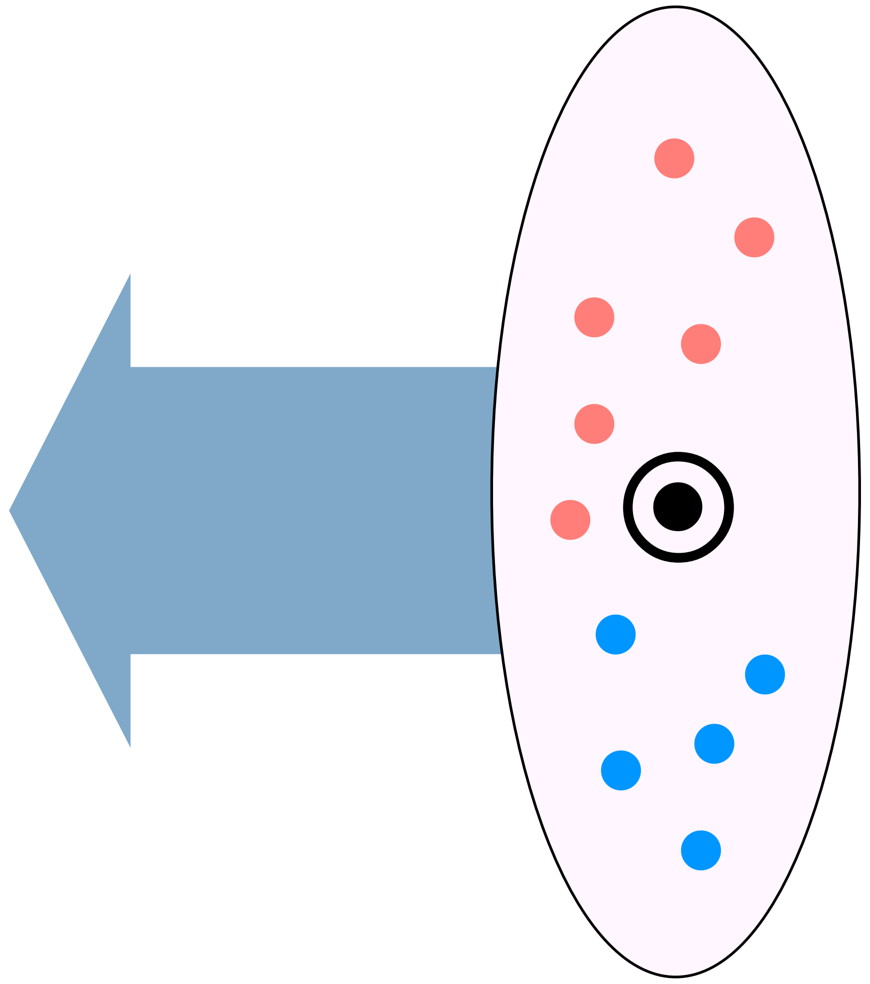

Distortion in impact parameter
(related to GPD E)

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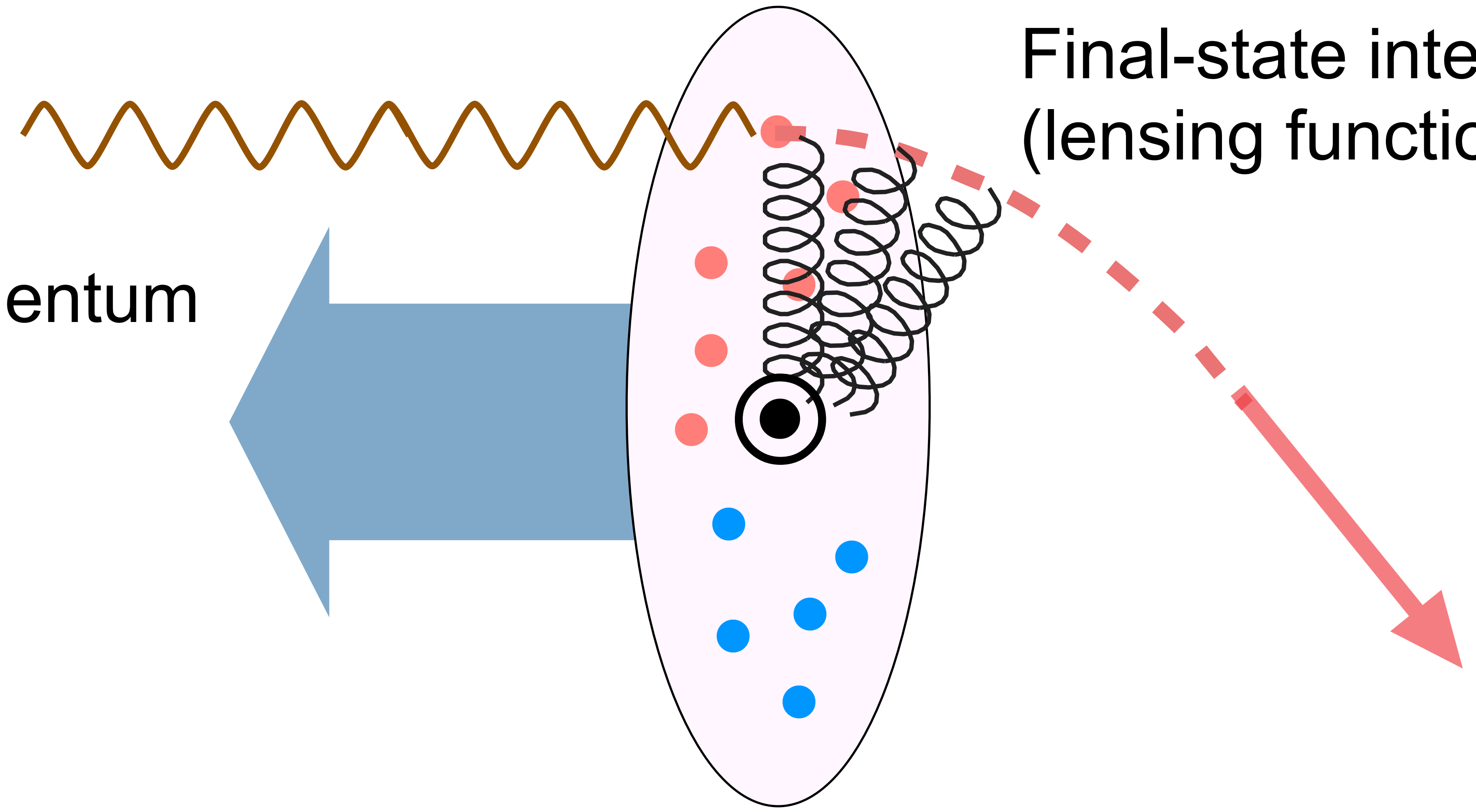

Final-state interaction
(lensing function)

Distortion in transverse momentum
(related to Sivers function)

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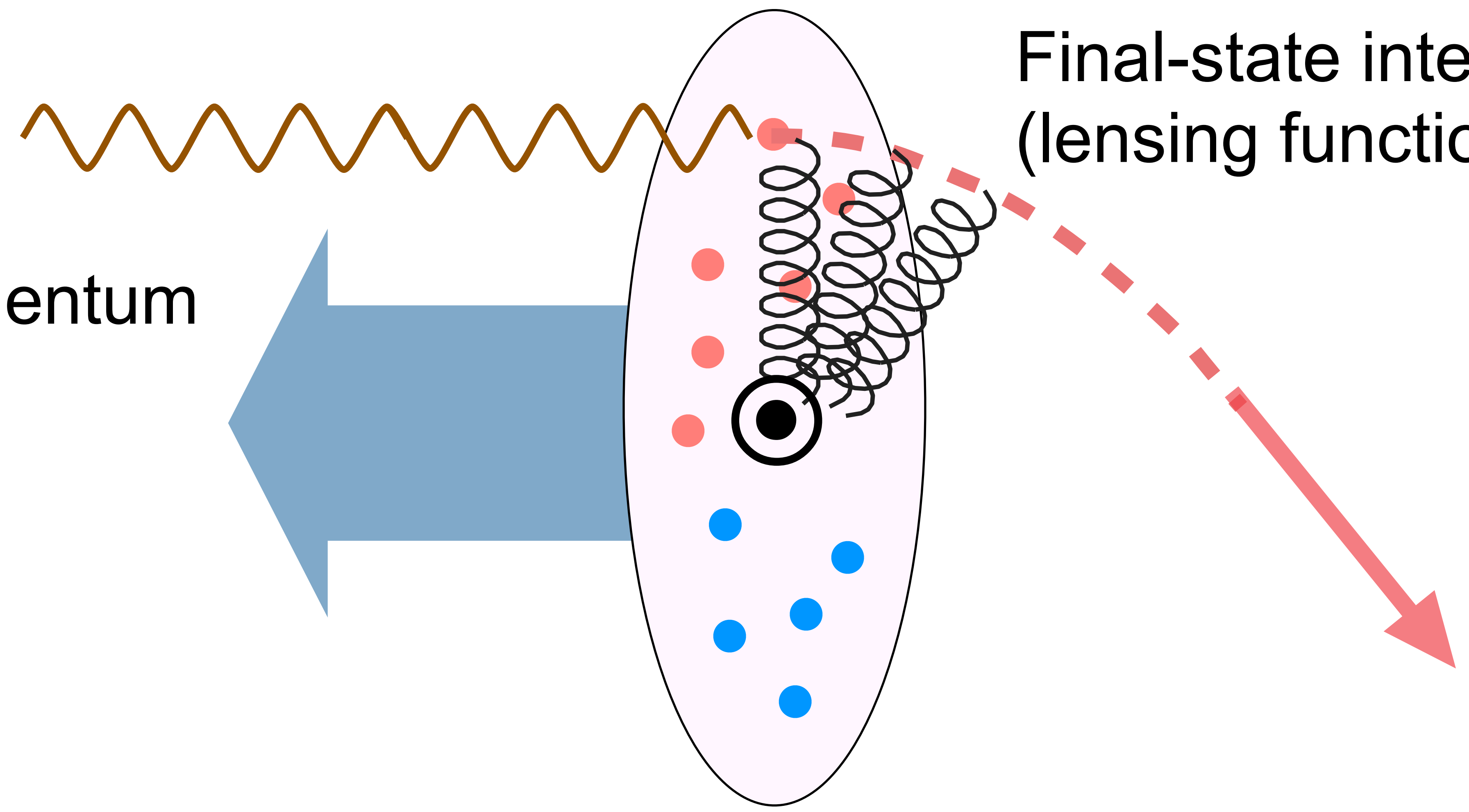

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$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^2) \;\simeq\; \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'$$

Sivers function

Lensing function

F.T. of E(x,0,t)

Constraining quark OAM with Sivers function

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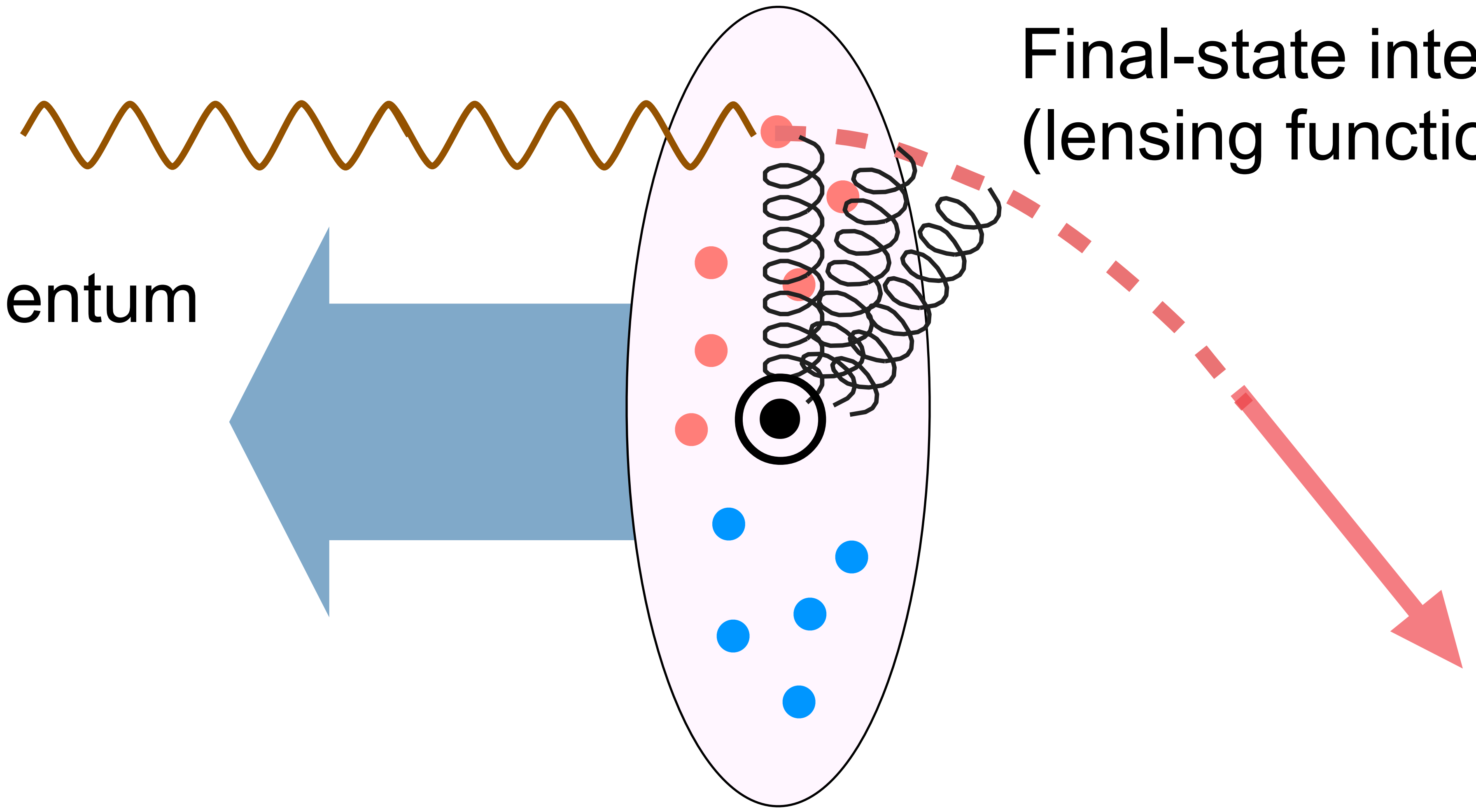

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Sivers function

Lensing function

F.T. of E(x,0,t)

inspired from model results

Bacchetta, Radici, PRL107(2011)

$$f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)\, E^a(x,0,0;Q_L^2)$$

fitted to SIDIS data
(COMPASS,HERMES,JLab)

flavor independent
$\frac{K}{(1-x)^\eta}$

first moment constrained
from anomalous magnetic moment

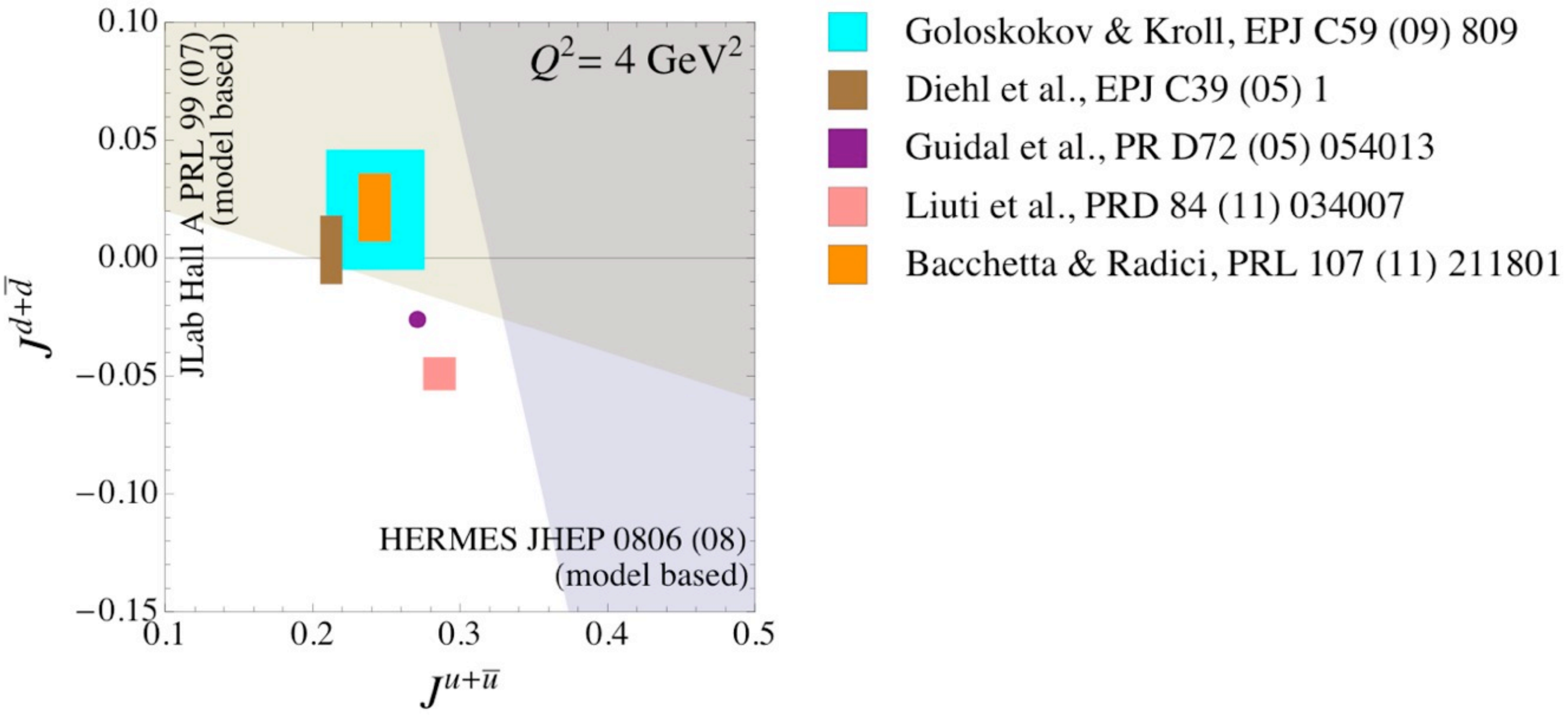
- Results from Sivers ⇦ lensing ⇨ GPD

Bacchetta, Radici, PRL107(2011)

$$J^u = 0.229 \pm 0.002^{+0.008}_{-0.012}, \qquad J^{\bar{u}} = 0.015 \pm 0.003^{+0.001}_{-0.000},$$
$$J^d = -0.007 \pm 0.003^{+0.020}_{-0.005}, \qquad J^{\bar{d}} = 0.022 \pm 0.005^{+0.001}_{-0.000},$$
$$J^s = 0.006^{+0.002}_{-0.006}, \qquad J^{\bar{s}} = 0.006^{+0.000}_{-0.005}.$$

$(Q^2 = 4\ \mathrm{GeV}^2)$

- Comparing with GPD model



Summary

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  - the most complete information on partonic structure of the nucleon

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  - give complementary information useful to reconstruct the nucleon wf
- Orbital Angular Momentum from GPDs (Ji's relations)
- No direct connection between TMDs and OAM ➡ need to use model-inspired connections
  - use LCWF (eigenstate of quark OAM) to quantify amount of OAM in different observables
  - model relation between pretzelosity and OAM
  - OAM from model relation between Sivers function and GPD E

Backup

Ji

$$L^q = \int \mathrm{d}^3 r \psi^\dagger \vec{r} \times (-i\vec{D})\psi$$

Jaffe-Manohar

$$\mathcal{L}^q = \int \mathrm{d}^3 r \psi^\dagger \vec{r} \times (-i\vec{\nabla})\psi$$

no gauge field

$$L^q = \mathcal{L}^q$$

Light-Cone Quark Models

- No gluons
- Independent quarks
- Spherical symmetry in the nucleon rest frame

$$\Psi^{\Lambda;q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}(x_i,\vec{k}_{\perp,i}) = \sum_{s_i}\phi(x_i,\vec{k}_{\perp,i})\,\Phi^{\Lambda;q_1q_2q_3}_{s_1s_2s_3}\prod_i D^{1/2*}_{s_i\lambda_i}(\Theta)$$

symmetric momentum wf

spin-flavor wf

rotation from canonical spin to light-cone spin

$$\mathcal{L}_z{}^q = \Delta q_{\mathrm{NR}}\int[\mathrm{d}x]_3[\mathrm{d}^2k_\perp]_3|\phi|^2\sin^2\frac{\Theta}{2} \qquad -\int \mathrm{d}x\,h_{1T}^{\perp(1)q}(x) = \delta q_{\mathrm{NR}}\int[\mathrm{d}x]_3[\mathrm{d}^2k_\perp]_3|\phi|^2\,\sin^2\frac{\Theta}{2}$$

non-relativistic axial charge

non-relativistic tensor charge

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non-relativistic axial charge

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spherical symmetry in the rest frame

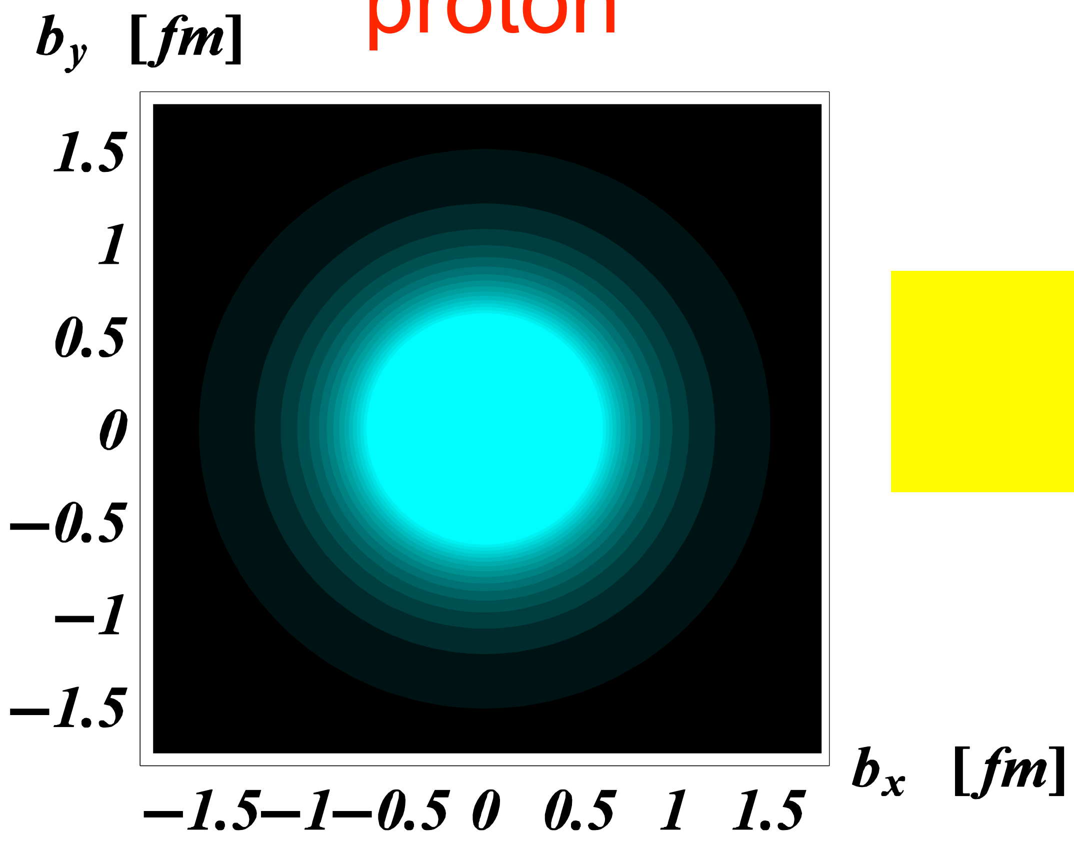
$$\Delta q_{\mathrm{NR}} = \delta q_{\mathrm{NR}}$$

$$\mathcal{L}_z^q = -\int \mathrm{d}x\,h_{1T}^{\perp(1)q}(x)$$

- ❖ Integrating over $b_\perp$ ➡ $f_1^{(1)}(k_\perp^2) = \int \mathrm{d}x\, f_1(x, k_\perp^2)$
- ❖ Integrating over $k_\perp$ ➡ charge density in the transverse plane $b_?$

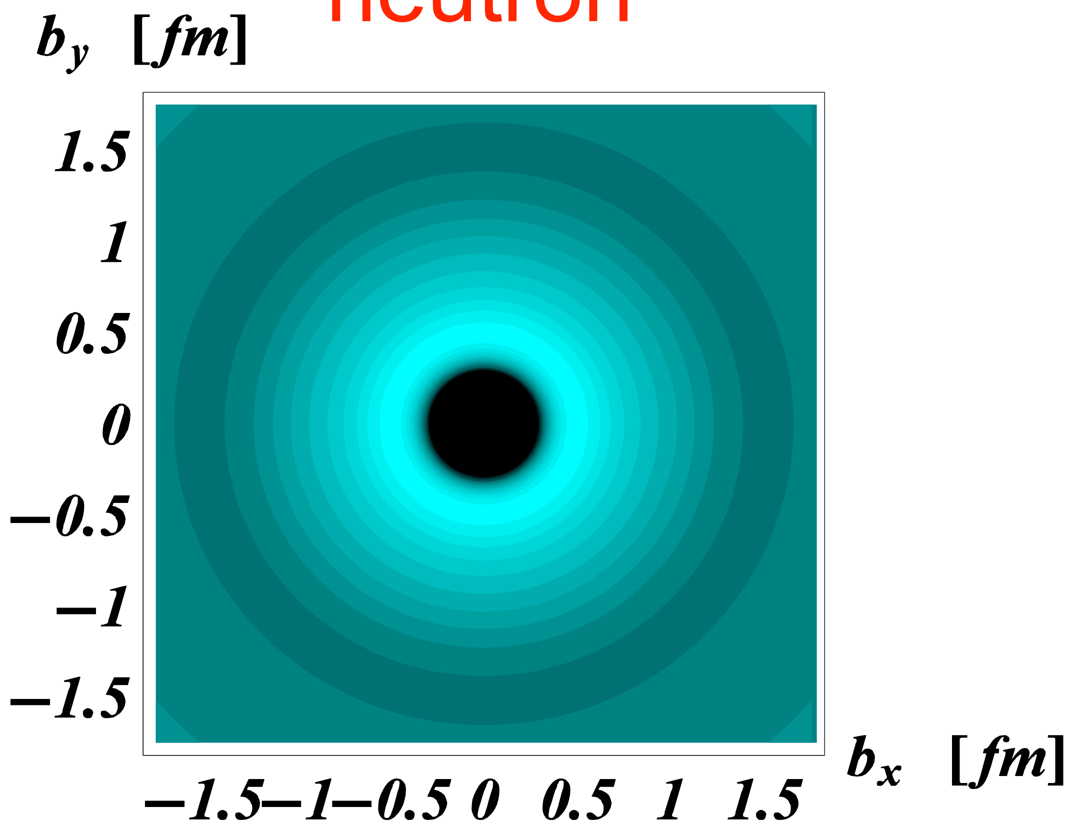
$$\rho^q(b_\perp^2) = e^q \int \mathrm{d}^2\Delta_\perp e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} F_1^q(\Delta_\perp^2)$$

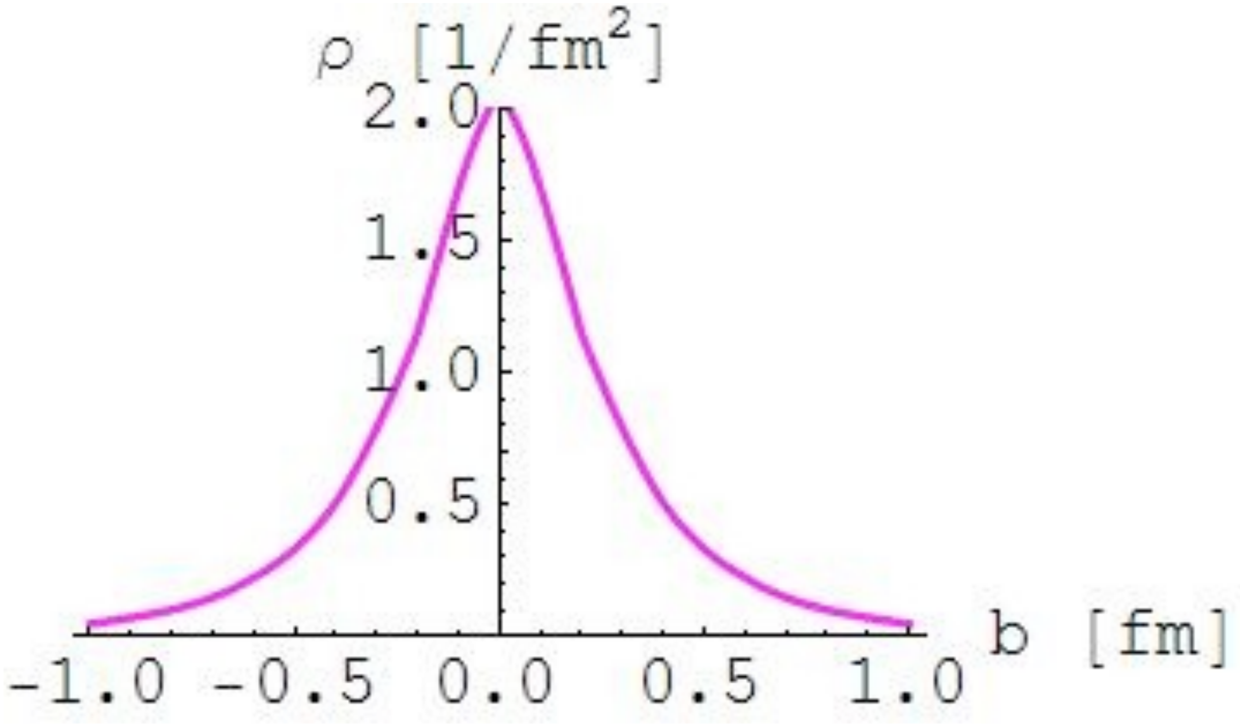
proton

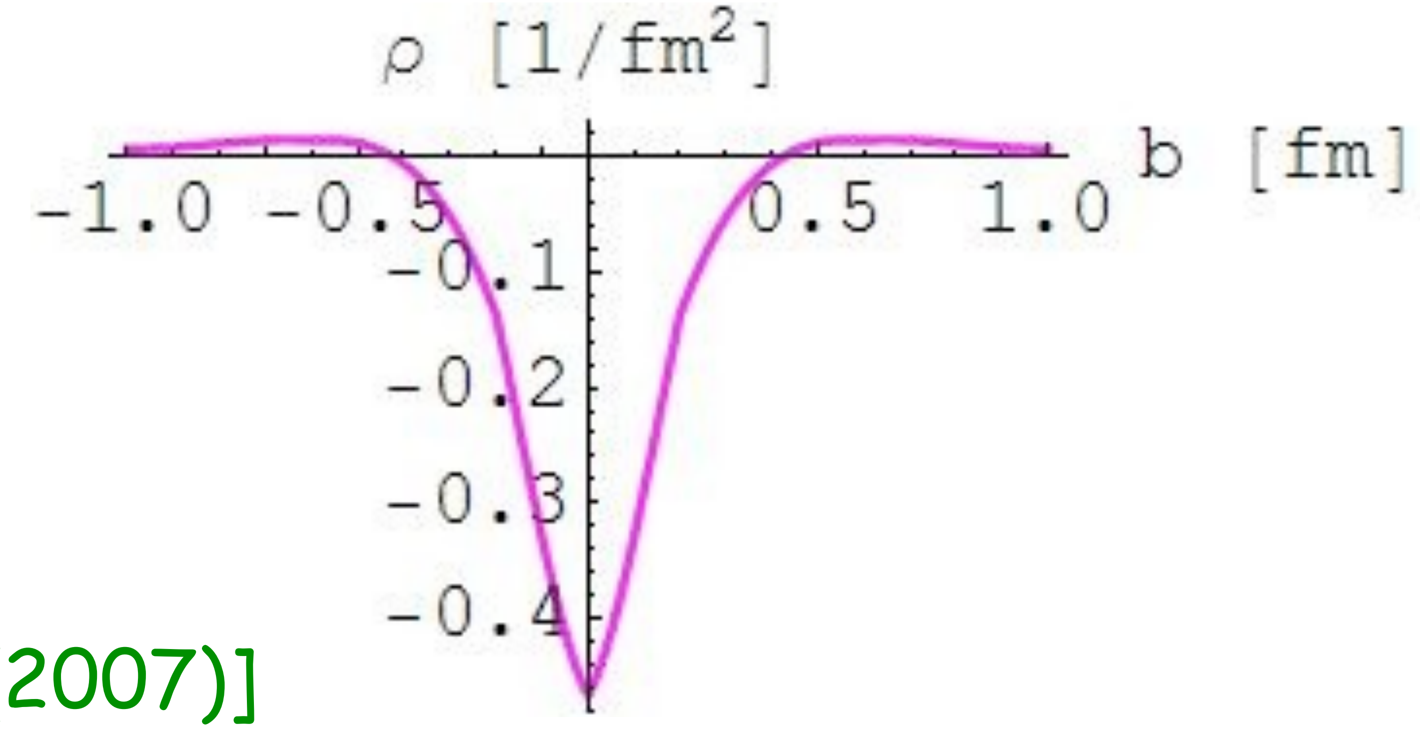


charge distribution in the transverse plane

neutron







[Miller (2007); Burkardt (2007)]

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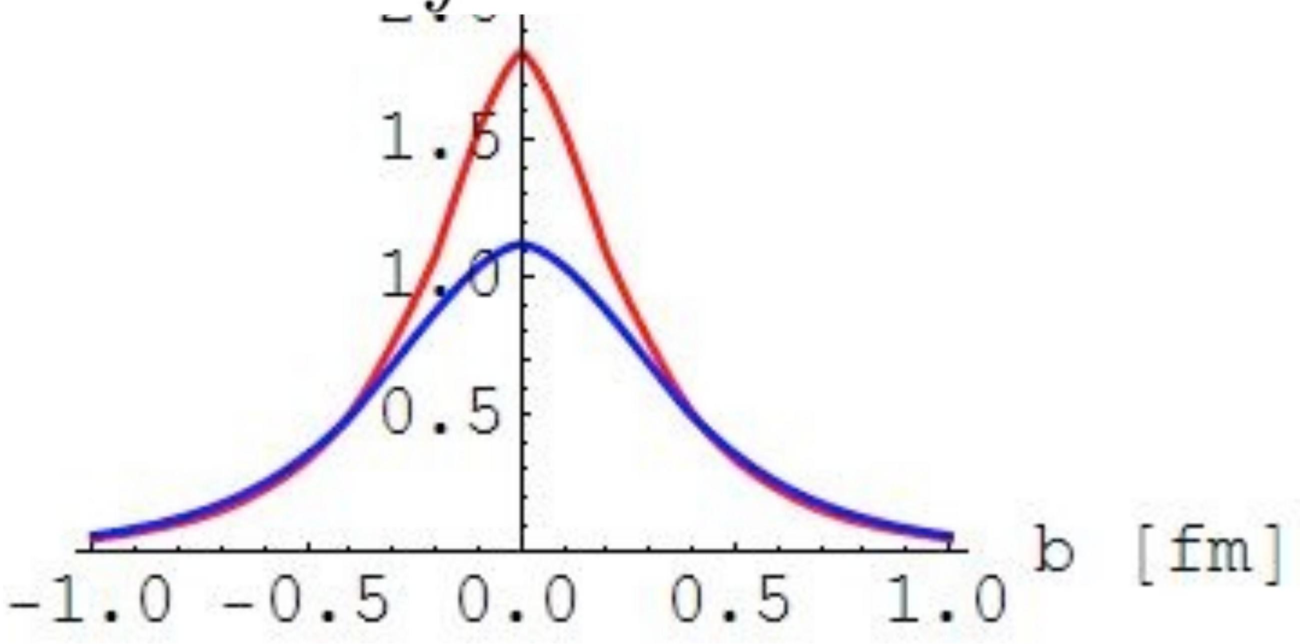
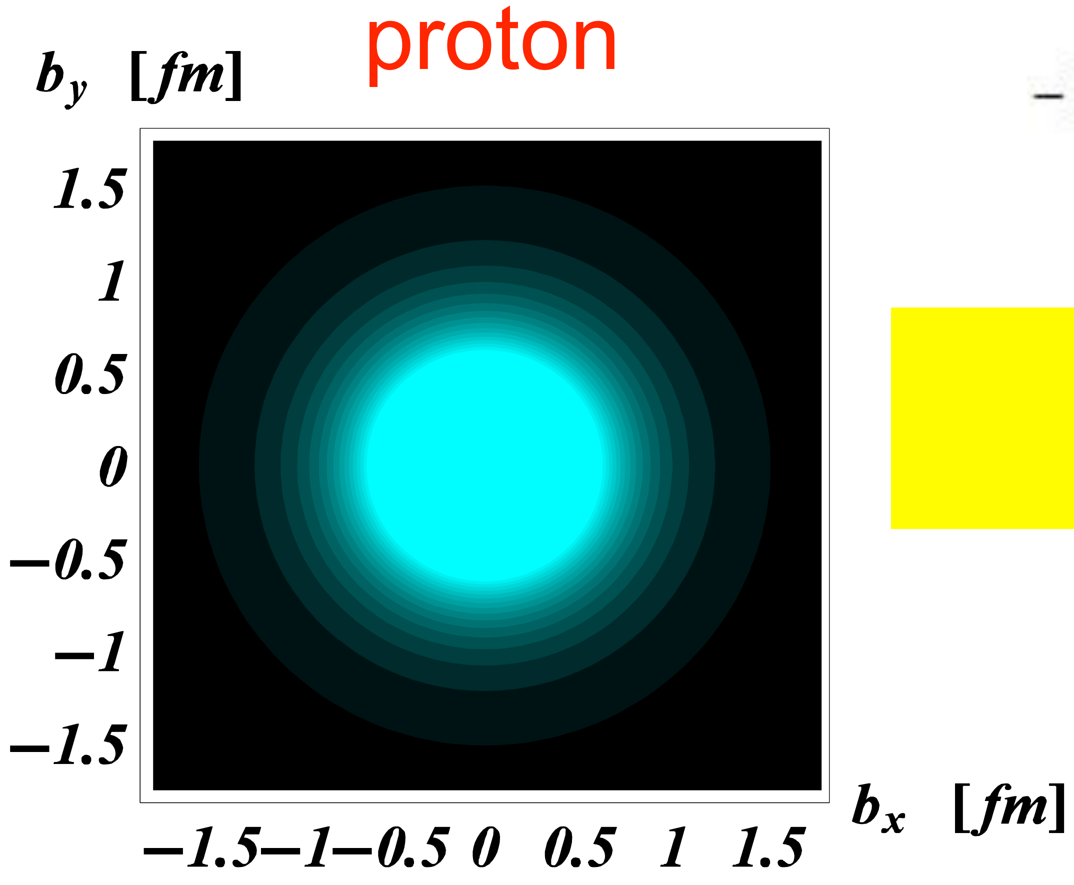


proton



charge distribution in the transverse plane

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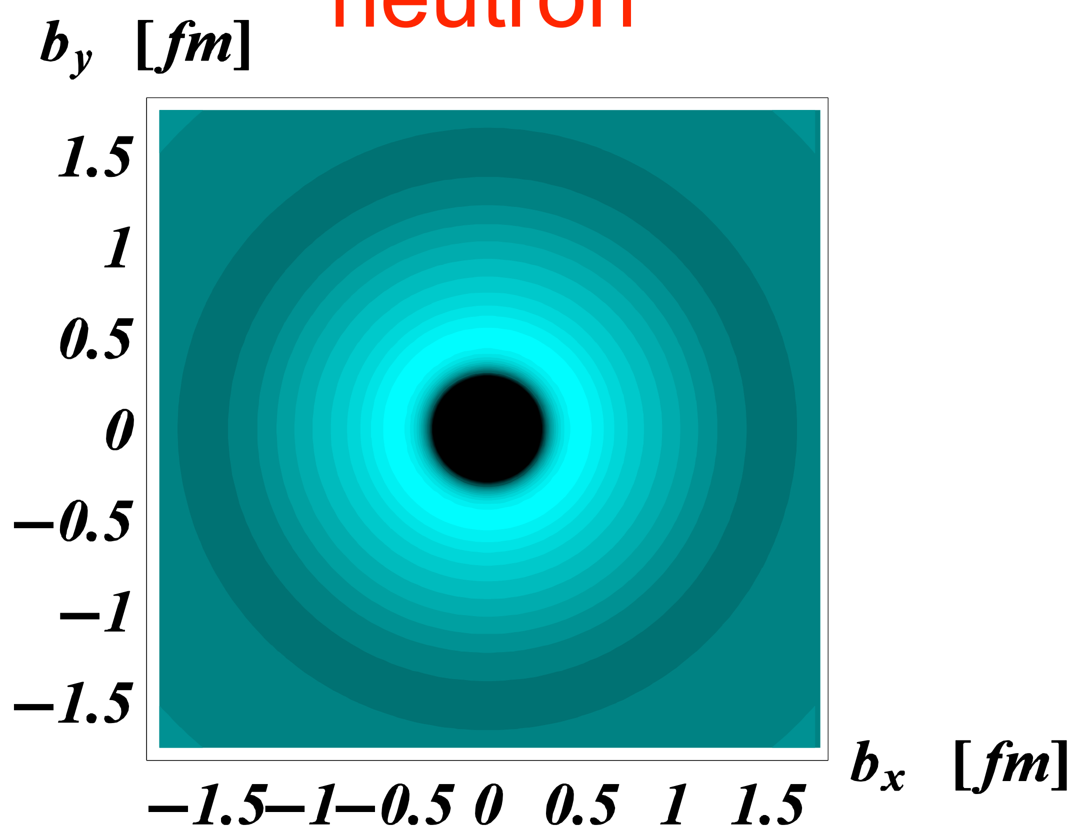
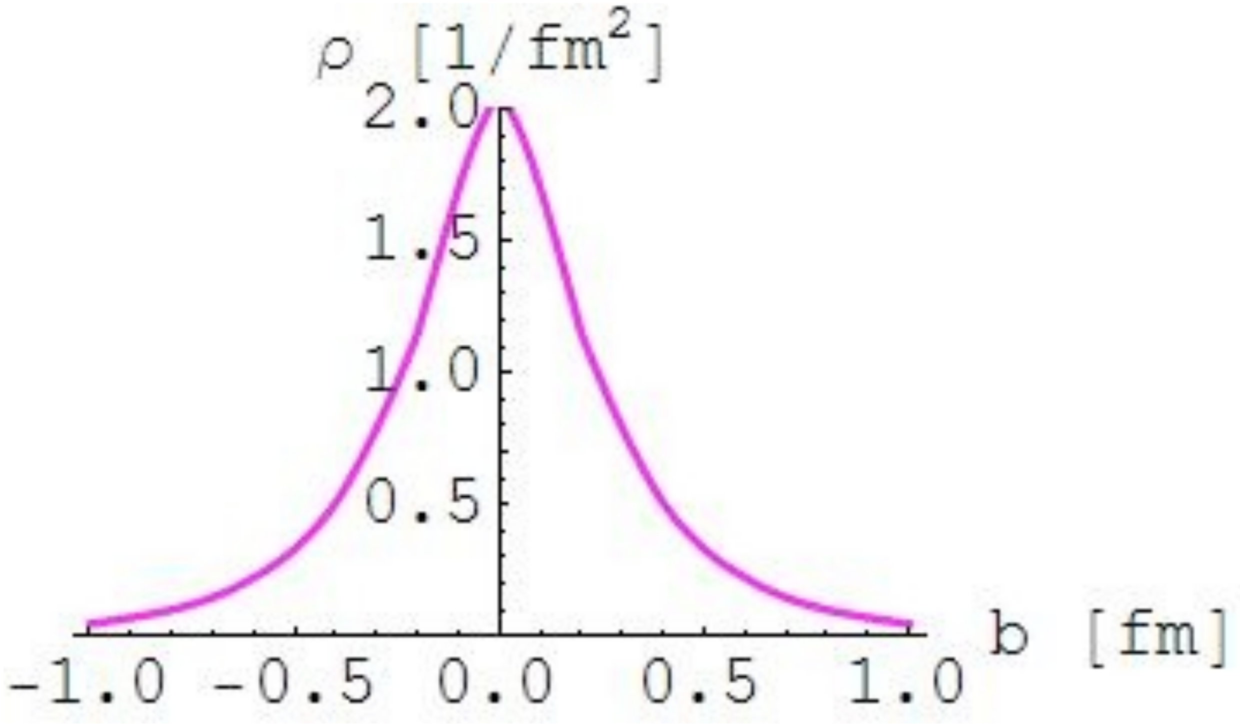
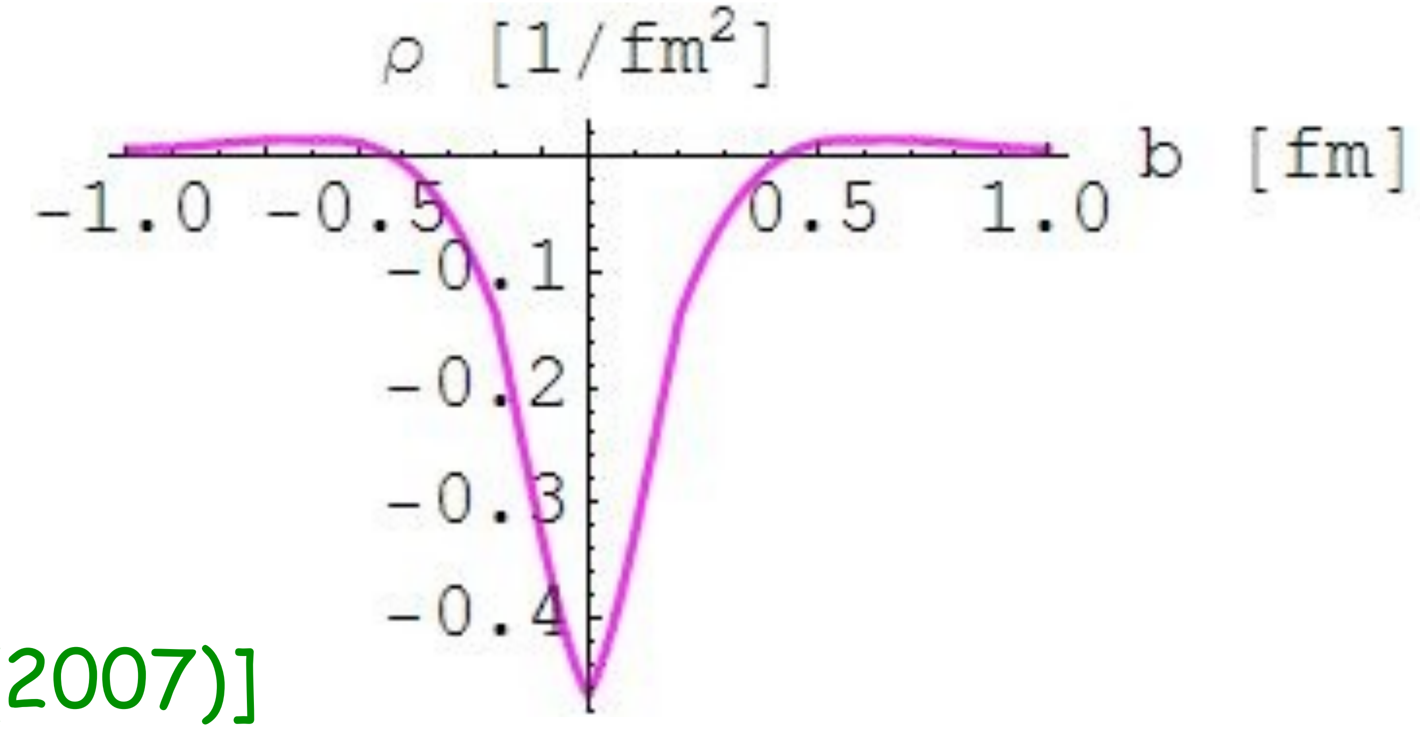




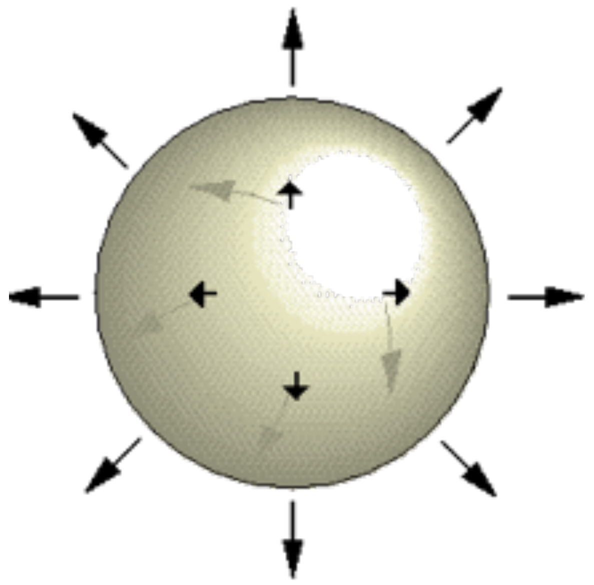


[Miller (2007); Burkardt (2007)]

Common assumptions :

- No gluons
- Independent quarks
- Spherical symmetry in the nucleon rest frame

spherical symmetry
in the rest frame



the quark distribution does not depend on
the direction of polarization

rest frame

$|\vec{0}, \sigma\rangle$

zero OAM

Light-cone boost

infinite-momentum frame

$|\vec{k}, \lambda\rangle_{LC}$

NON-zero OAM

LC polarizations of quark and nucleon are
NOT all independent

relations
among polarized TMDs

[Lorce',BP, 2011]

Wigner function
for transversely pol. quark in longitudinally pol. nucleon

$$\rho^{\Lambda,\,T}(\vec{b}^{\,2},\vec{k}_\perp^{\,2},\vec{b}\cdot\vec{k}_\perp) = \frac{1}{2}\left(F_{11} + \Lambda\,\vec{s}_T\cdot\vec{k}_\perp\frac{H_{17}}{M} + i\Lambda\,\vec{s}_T\cdot\vec{b}_\perp\frac{H'_{18}}{M}\right)$$

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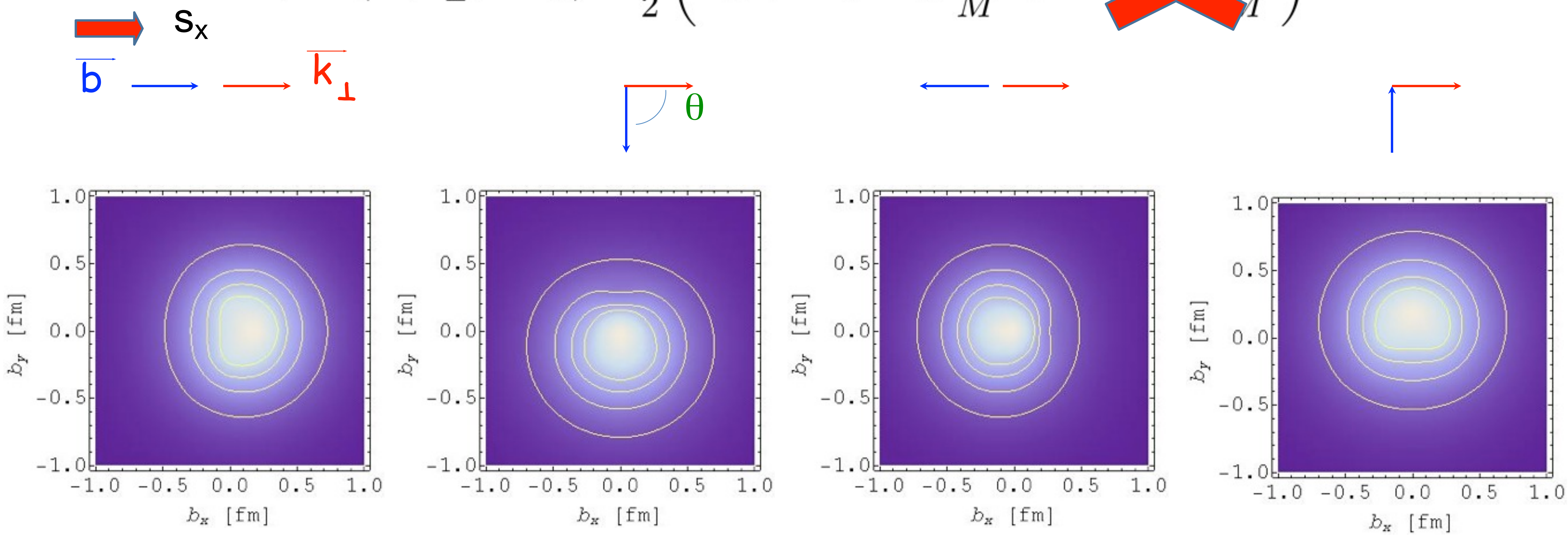

$|\vec{k}_\perp|$, μ fixed

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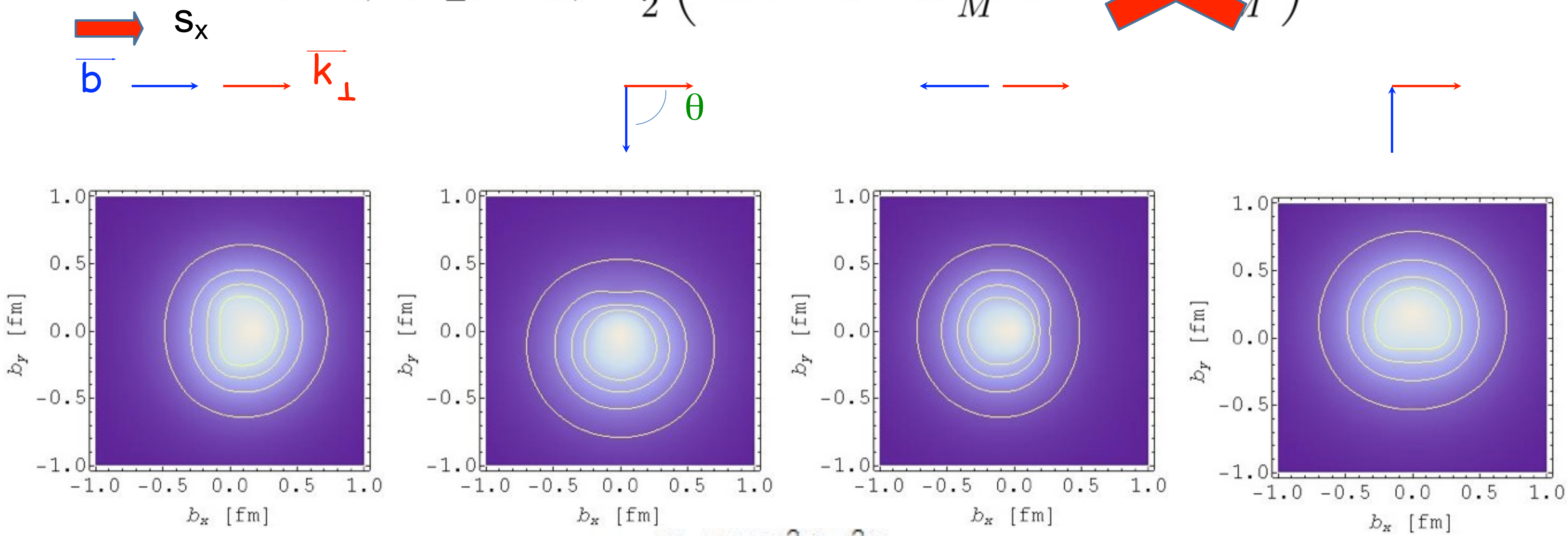
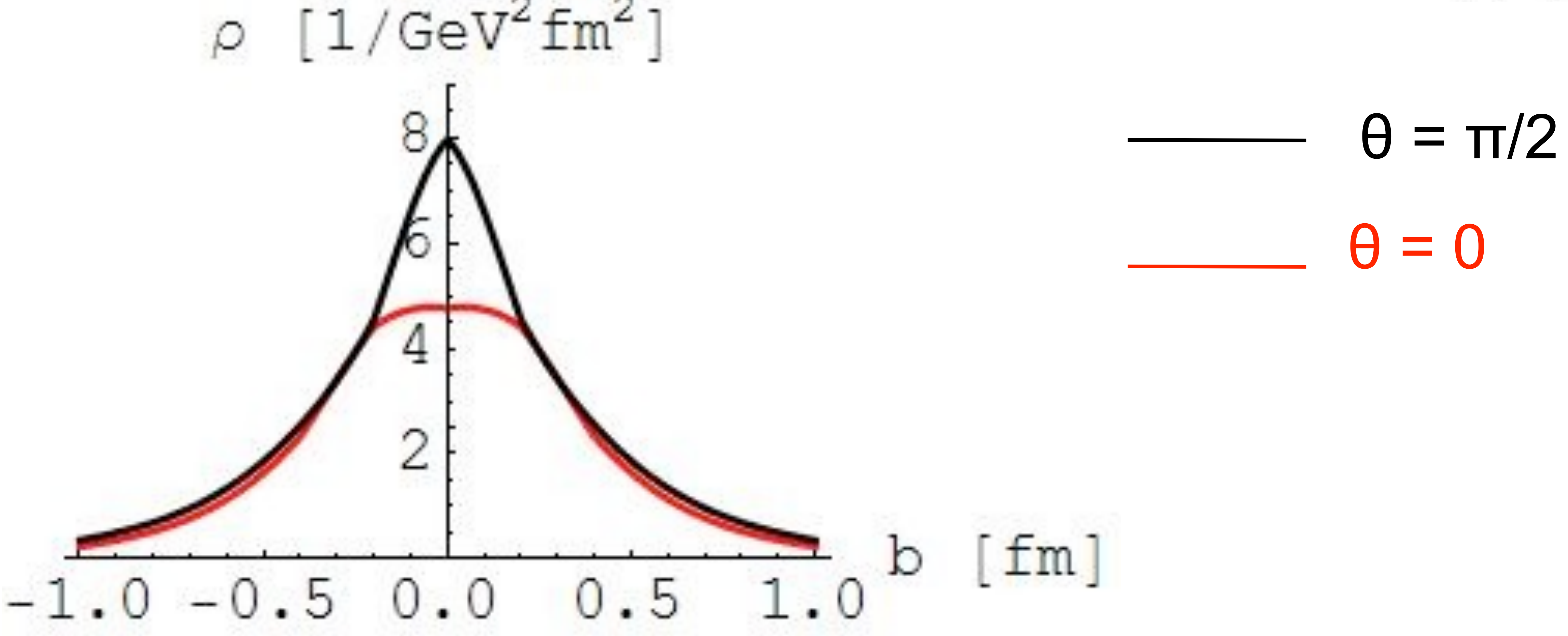


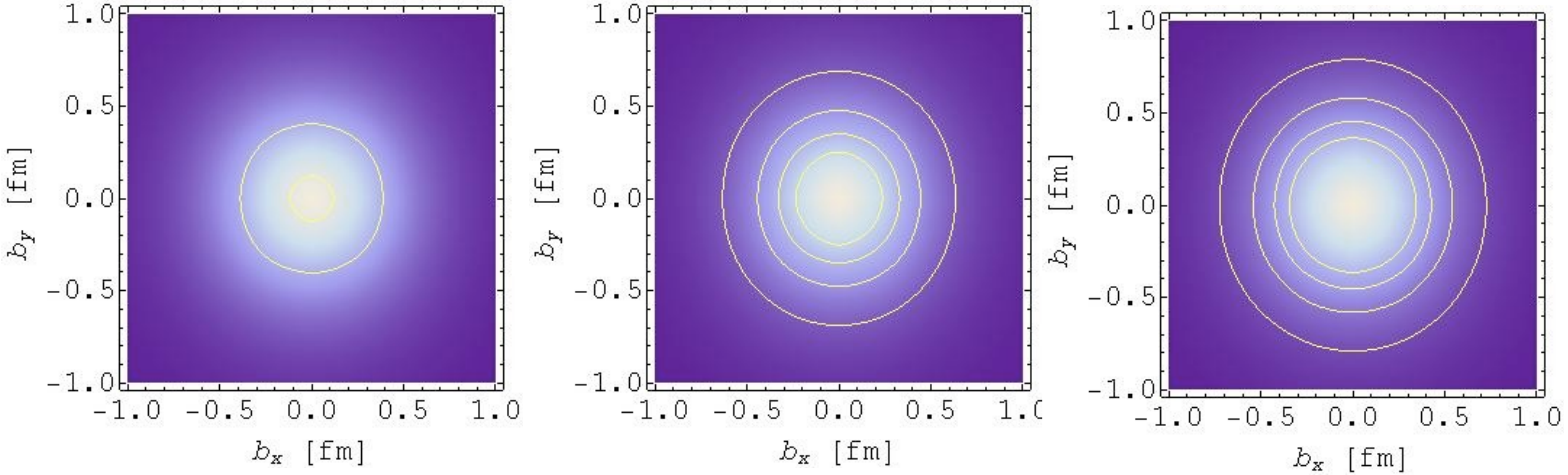
$|\vec{k}_\perp|$,μ fixed



Wigner function for transversely pol. quark in longitudinally pol. nucleon

$\rightarrow$ s_x

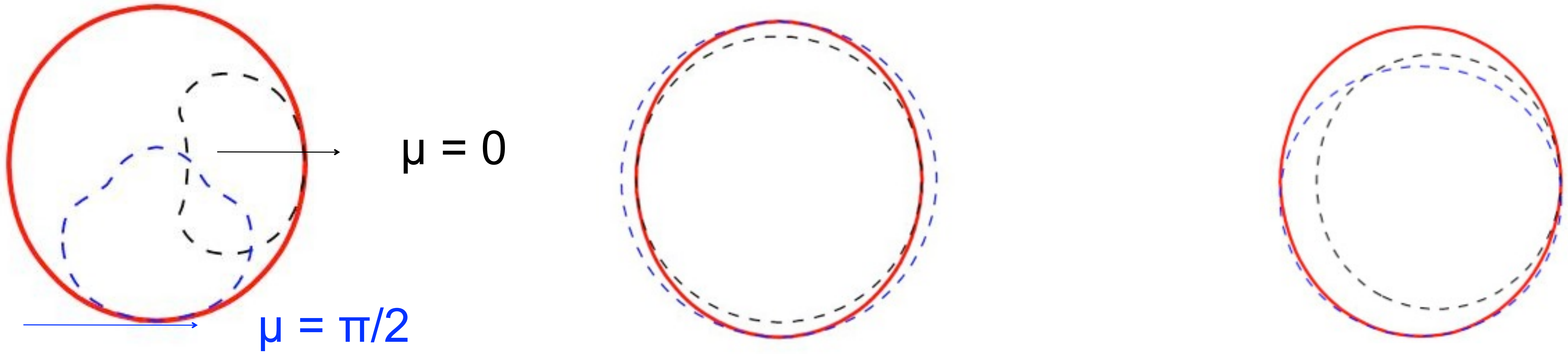
$\vec{k}_\perp \longrightarrow$ fixed



Dipole

Monopole

Monopole + Dipole



Light-Cone Constituent Quark Model

- momentum-space wf

[Schlumpf, Ph.D. Thesis, hep-ph/9211255]

$$\Psi(k_i) = \frac{N}{(M_0^2 + \beta^2)^\gamma} \qquad M_0 = \sum_i^3 \sqrt{m_i^2 + \vec{k}_i^2}$$

$\beta,\ \gamma$ parameters fitted to anomalous magnetic moments of the nucleon

N : normalization constant

- spin-structure:

$$q_\lambda^{LC}(k) = D_{\lambda s}^{(1/2)*}\, q_s^C(k) \qquad D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$

free quarks → $K_z = m + x\mathcal{M}_0 \quad \vec{K}_\perp = \vec{k}_\perp$ (Melosh rotation)

- SU(6) symmetry

Applications of the model to:

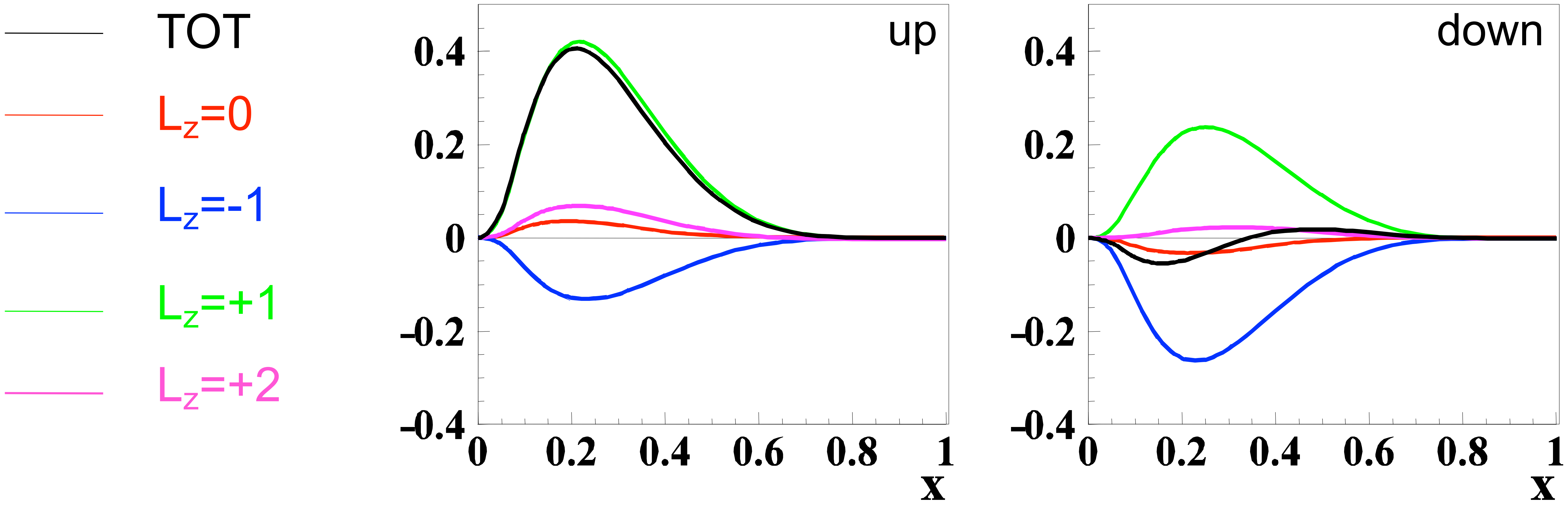
GPDs and Form Factors: BP, Boffi, Traini (2003)-(2005);
TMDs: BP, Cazzaniga, Boffi (2008); BP, Yuan (2010);
Azimuthal Asymmetries: Schweitzer, BP, Boffi, Efremov (2009)

Quark OAM: Partial-Wave Decomposition

$$\ell_z = \sum_{L_z} L_z \; {}^{L_z}\langle P,\uparrow | P,\uparrow\rangle^{L_z}$$

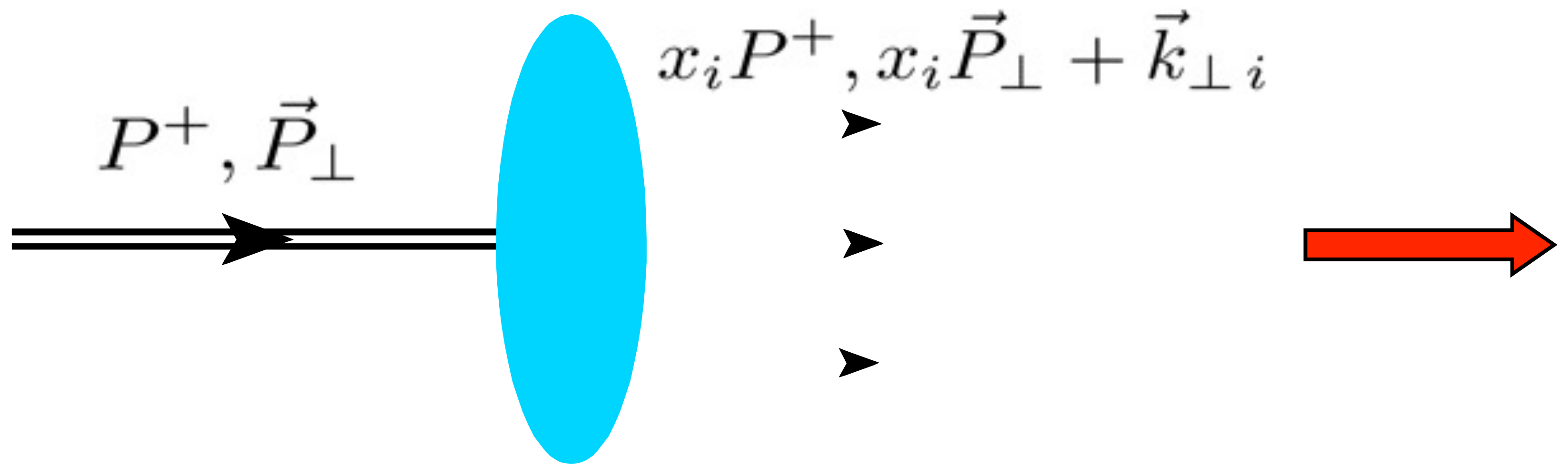
OAM	L_z=0	L_z=-1	L_z=+1	L_z=+2	TOT
UP	0.013	-0.046	0.139	0.025	**0.131**
DOWN	-0.013	-0.090	0.087	0.011	**-0.005**
UP+DOWN	0	-0.136	0.226	0.036	**0.126**
<P" |P">	0.62	0.136	0.226	0.018	**1**

distribution in x of OAM



Lorce,B.P., Xiong, Yuan, in preparation

LCWF Overlap Representation



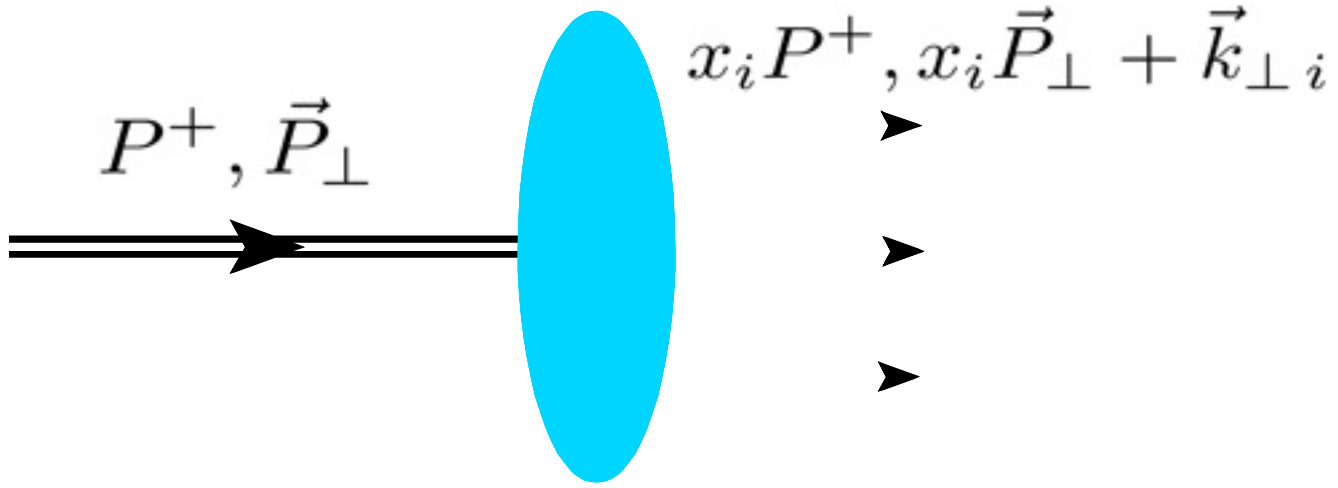
LCWF:$\Psi^{\Lambda;q_1 q_2 q_3}_{\lambda_1 \lambda_2 \lambda_3}(x_i, \vec{k}_{\perp,i})$

invariant under boost, independent of P$^\mu$

internal variables: $\sum_{i=1}^{3} x_i = 1, \sum_{i=1}^{3} \vec{k}_{\perp i} = \vec{0}_\perp$

[Brodsky, Pauli, Pinsky, '98]

LCWF Overlap Representation



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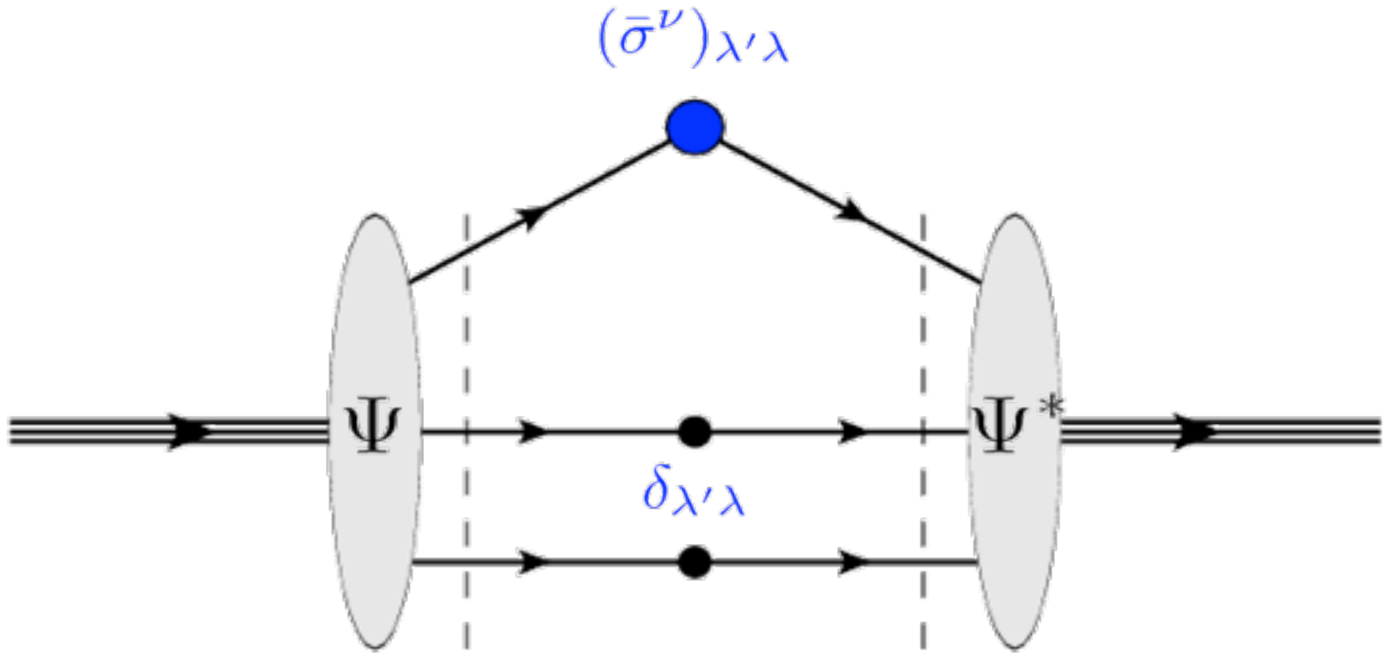
quark-quark correlator

(» =0)



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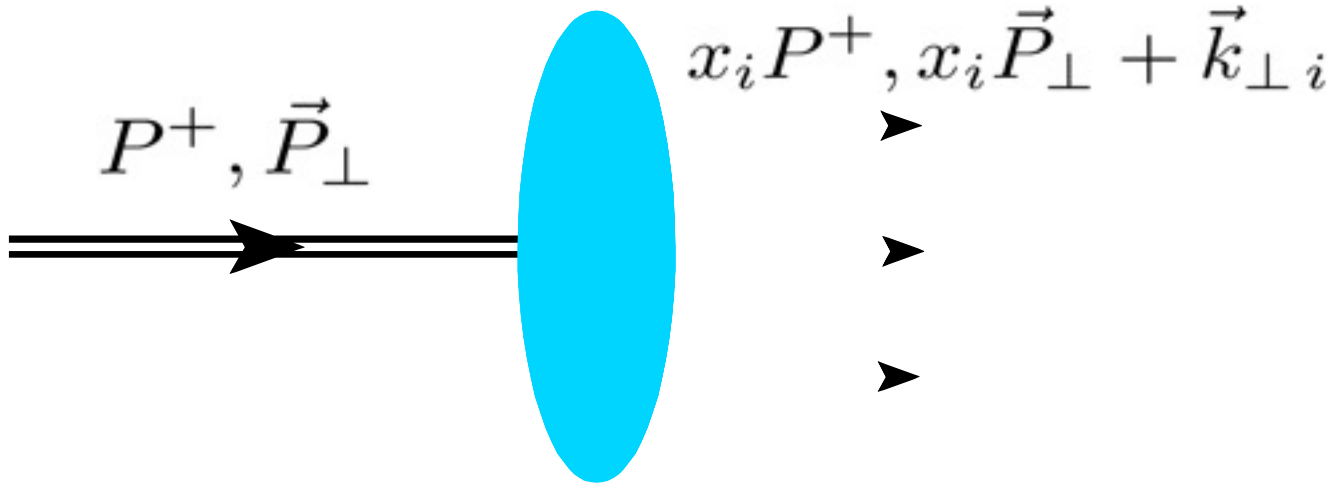

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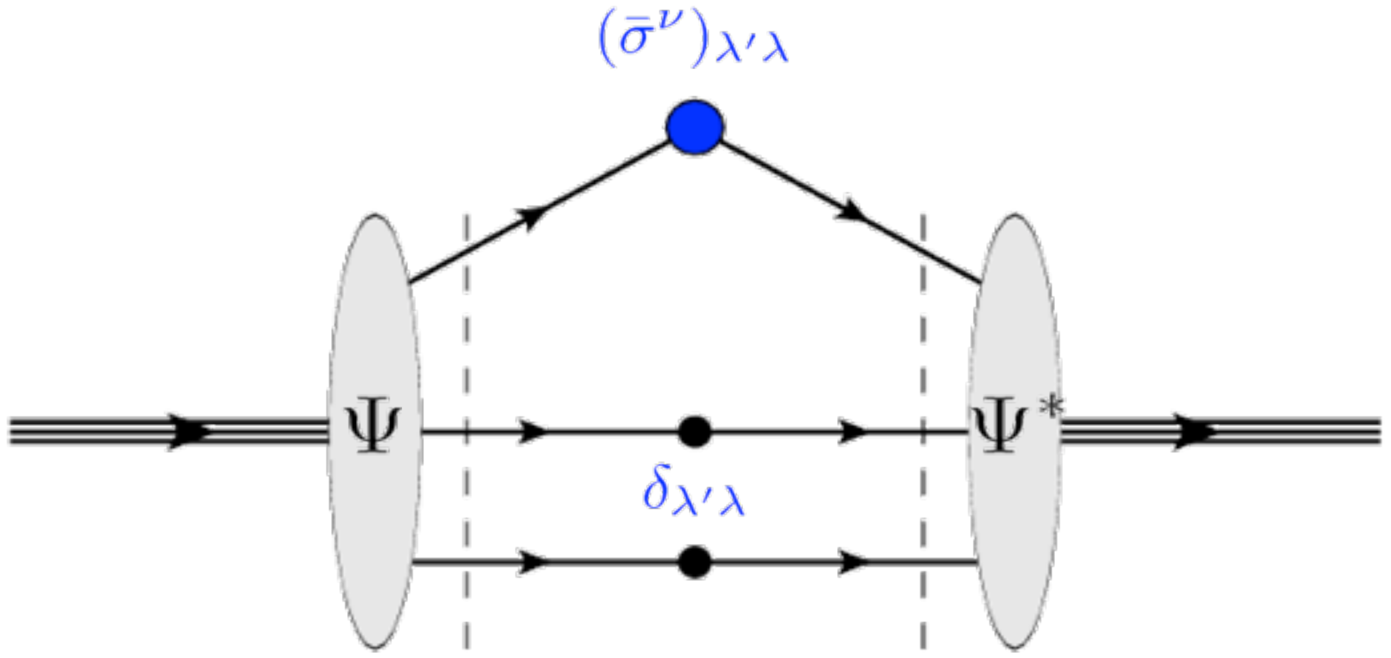


$$\Psi^{\Lambda;q_1q_2q_3}_{\lambda_1\lambda_2\lambda_3}(x_i, \vec{k}_{\perp,i}) = \sum_{s_i} \phi(x_i, \vec{k}_{\perp,i})\, \Phi^{\Lambda;q_1q_2q_3}_{s_1s_2s_3} \prod_i D^{1/2*}_{s_i\lambda_i}(R_{cf})$$

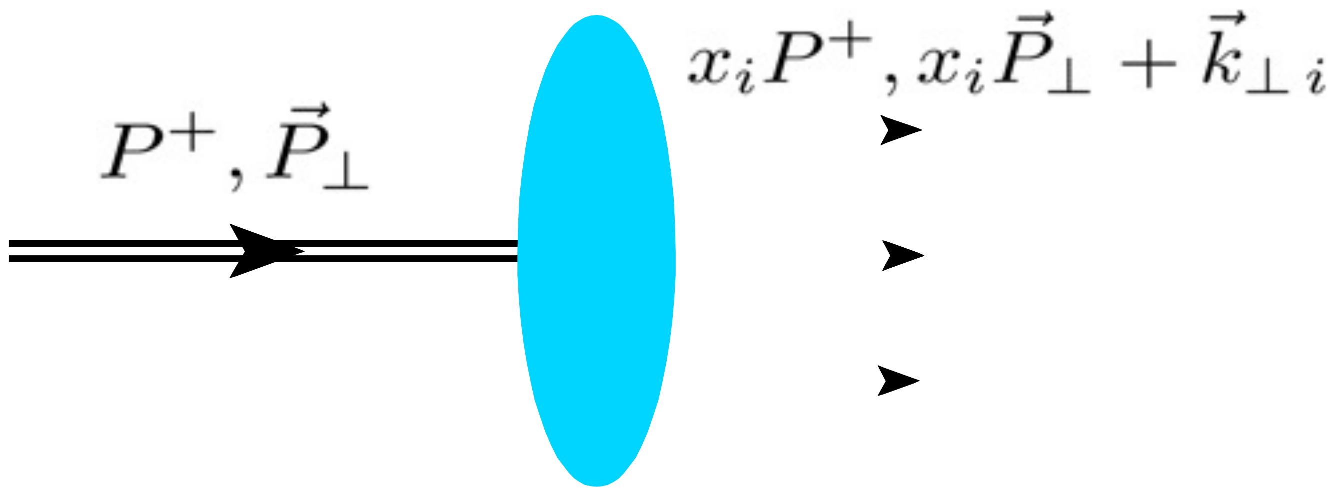
momentum wf

spin-flavor wf

rotation from canonical spin
to light-cone spin

[Lorce', BP, Vanderhaeghen (2011)]

LCWF Overlap Representation



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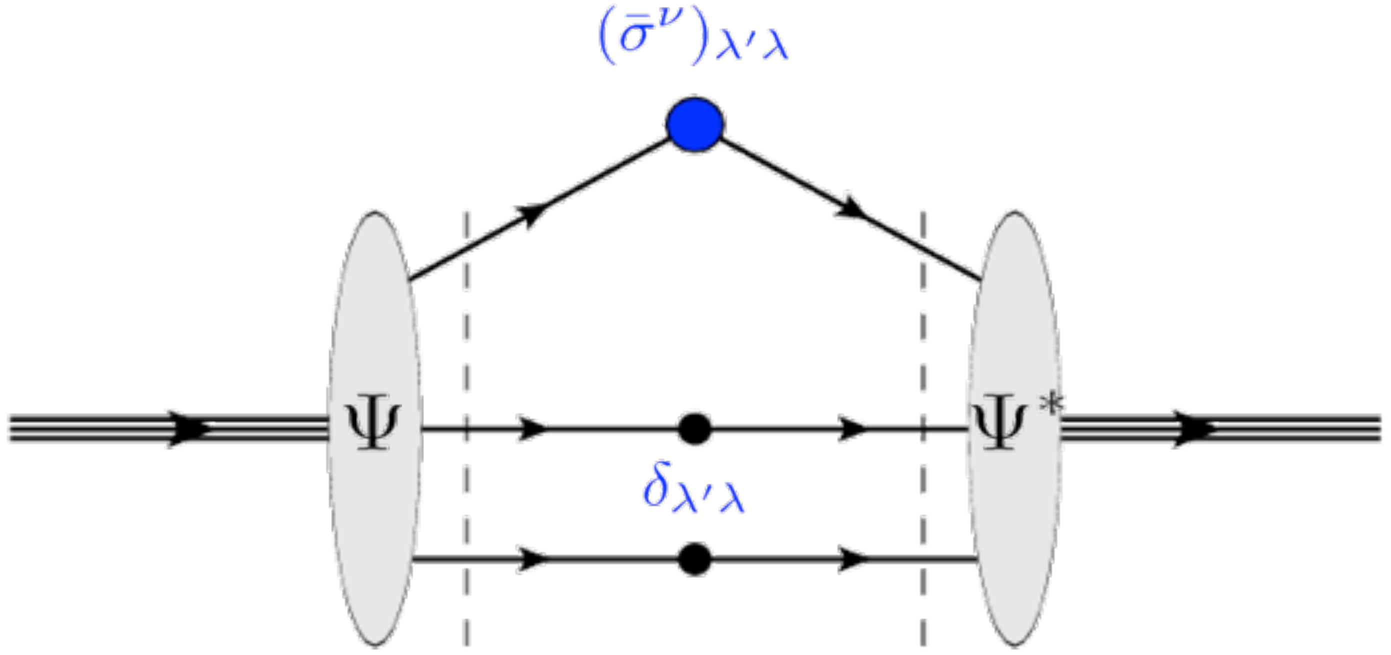

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momentum wf

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rotation from canonical spin
to light-cone spin

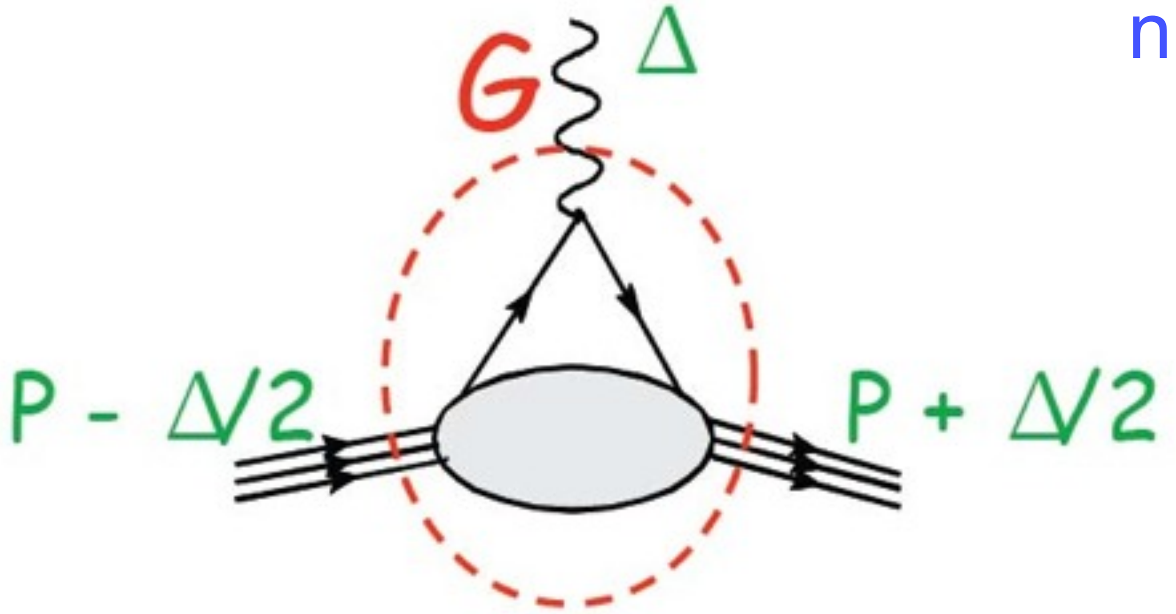
Bag Model, ÂQSM, LCCQM, Quark-Diquark and Covariant Parton Models

Common assumptions :

- ➢ No gluons
- ➢ Independent quarks

[Lorce', BP, Vanderhaeghen (2011)]

GPDs and Form Factors of Energy-Momentum Tensor



nucleon in an external classical gravitational field

→ G couples to energy-momentum tensor

$$\langle p' \,|\hat{T}^{q,g}_{\mu\nu}(0)|\, p\rangle = \bar{u}(p')[\, M_2^{q,g}(t)\frac{P_\mu P_\nu}{M} + J^{q,g}(t)\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + d_1^{q,g}(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M} \pm \bar{c}(t)\, g_{\mu\nu}\,]u(p)$$

↓

GPDs open the possibility to study these form factors on hard exclusive processes

[Ji, 1997;Polyakov, 2008]

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x,\xi,t) = A^q_{2,0}(t) + 4C^q_{2,0}(t)\xi^2 = M_2^q(t) + \frac{4}{5}d_1^q(t)\xi^2$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, E^q(x,\xi,t) = B^q_{2,0}(t) - 4C^q_{2,0}(t)\xi^2 = 2J^q(t) - M_2^q(t) - \frac{4}{5}d_1^q(t)\xi^2$$

M_2 (t)
quark momentum distribution

↓ t=0

momentum sum rule of PDF

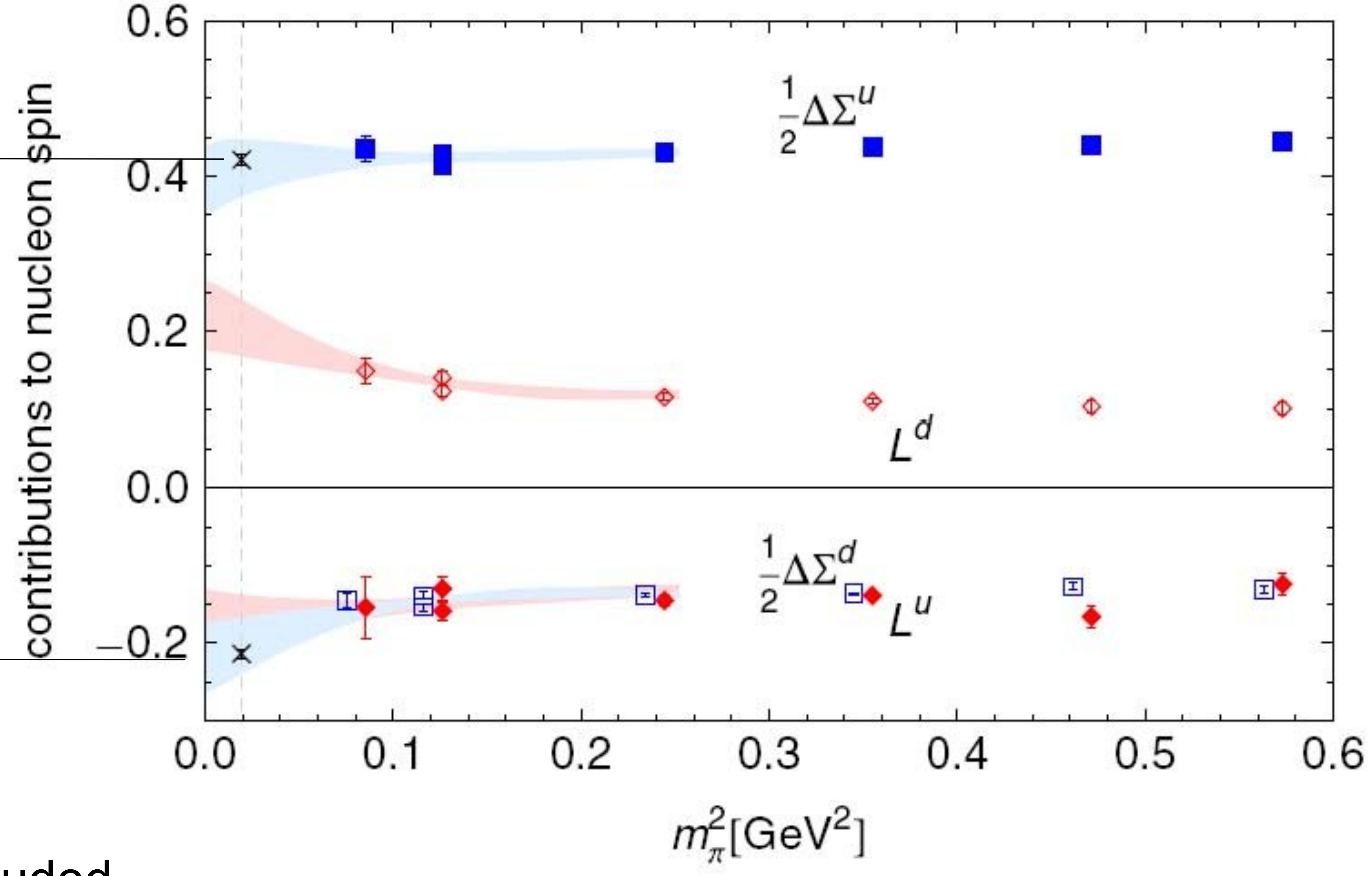
J (t)
quark angular-momentum distr.

↓ t=0

spin sum rule

d_1 (t)
distribution of “shear forces”

Lattice Results



[HERMES,2007]

[HERMES,2007]

- “disconnected diagrams” not included
- Error bands: chiral extrapolation in m¼; and extrapolation to t=0

Lattice results (μ = 2 GeV)

J^u=0.236(6) J^d= -0.0018(37) L^{u+d} =0.056(11)

⬇

$J^g \simeq$ 70%

- General trend from lattice QCD and models adjusted to data (μ = 2 GeV)

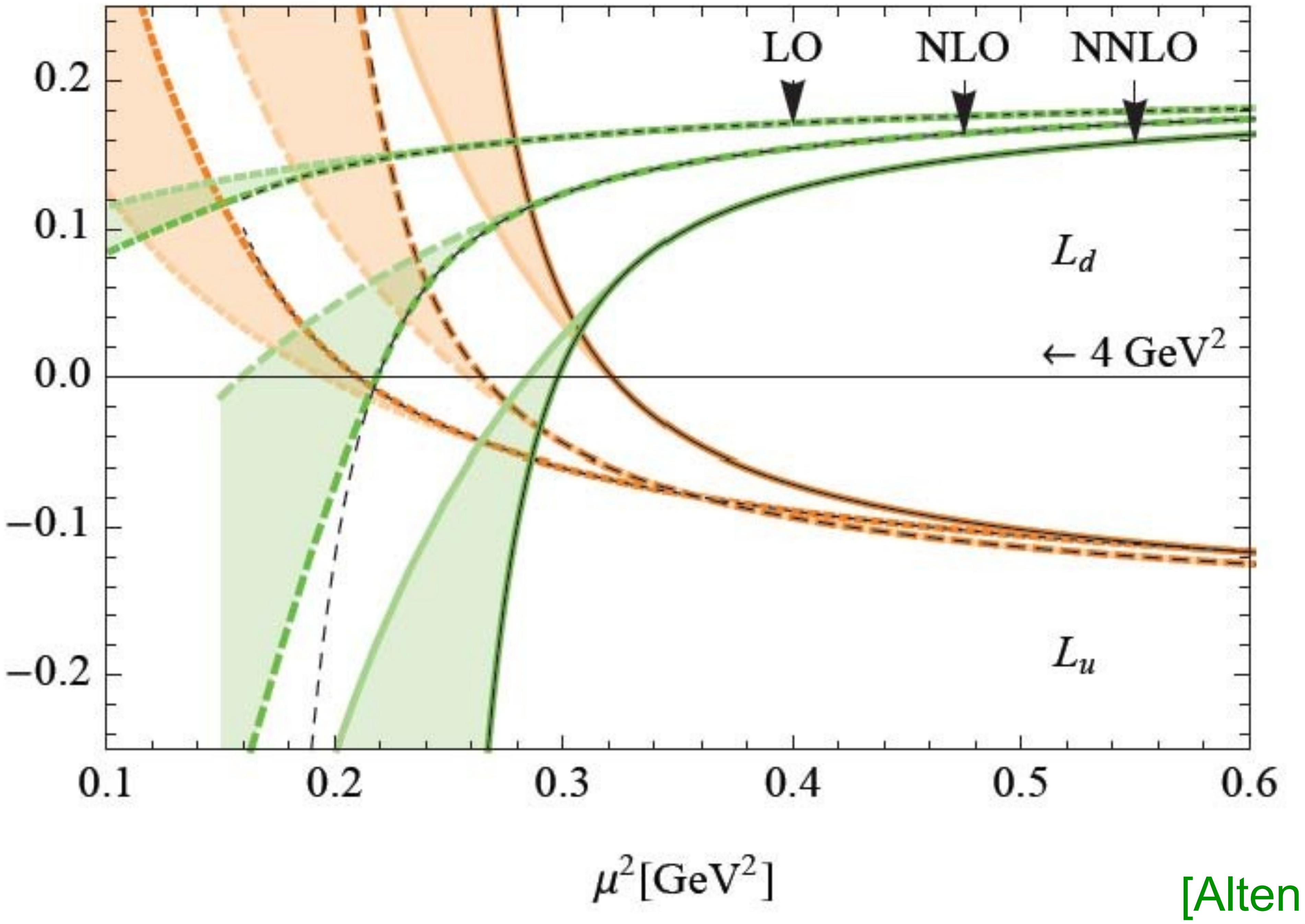
$J^u>0 \quad J^d \approx 0 \quad L^u<0 \quad L^d>0$

- Quark models at lower scales tend to predict

$L^u > 0 \quad L^d < 0$

Caution in comparing results at different scales

evolution to larger scales can flip the sign



[Altenbuchinger, Haegler, Weise, Henley, 2011]