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Nucleon tomography through exclusive and semi-inclusive processes

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and

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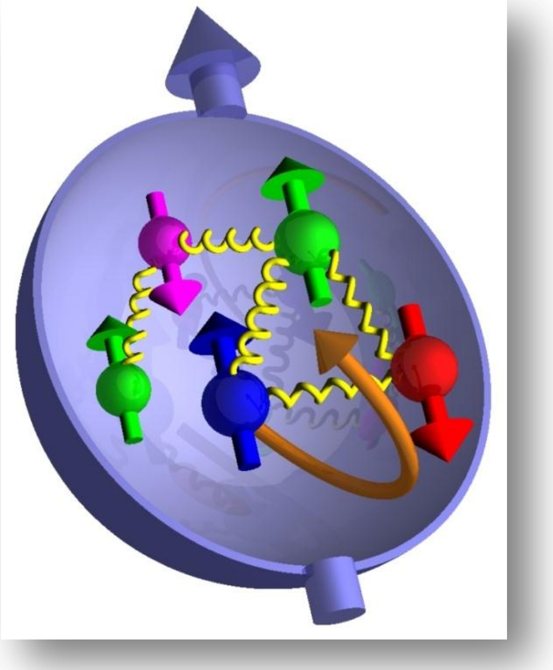
Outline

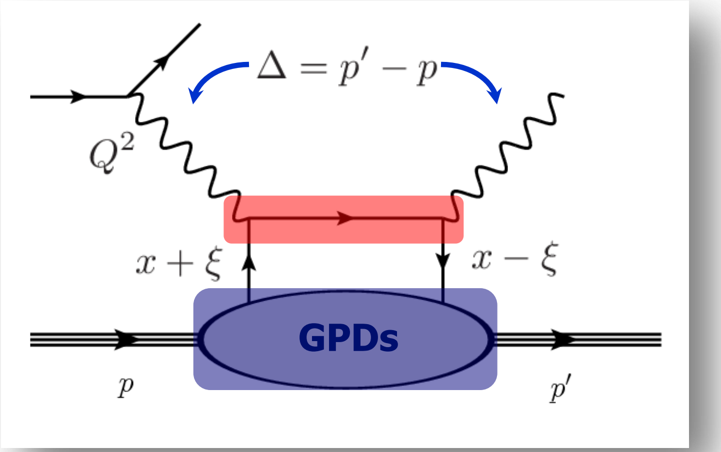
- Exclusive vs. Semi-inclusive
  - DVCS vs. SIDIS
  - GPDs vs. TMDs
- Partonic interpretation
  - 3D imaging
  - Twist-2 and Twist-3
- Hadron structure
  - Angular momentum decompositions
  - Quark spin and OAM
  - Interesting relations



DVCS vs. SIDIS

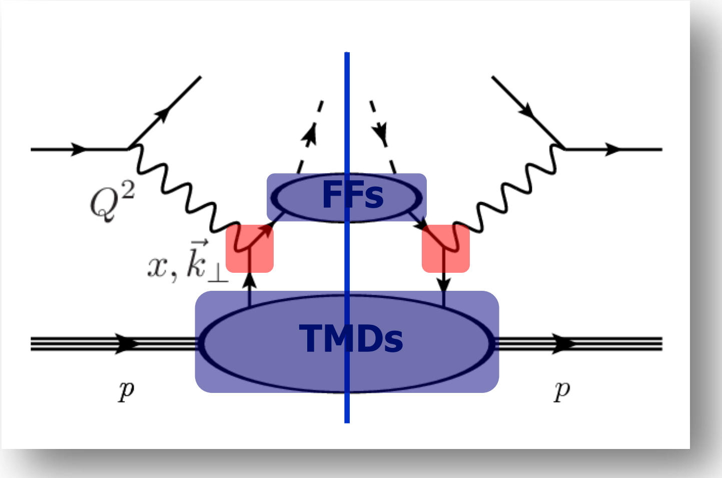
Incoherent scattering

DVCS



$$x = \frac{k^+}{P^+} \qquad -2\xi = \frac{\Delta^+}{P^+} \qquad t = \Delta^2$$

SIDIS



Factorization

Compton form factor
Cross section

$$= \quad \boxed{\text{hard}} \overset{\mu_F}{\otimes} \boxed{\text{soft}}$$

- **process dependent**
- **perturbative**

- **« universal »**
- **non-perturbative**

GPDs vs. TMDs

Γ **Dirac matrix**

$\mathcal{W}$ **Wilson line**

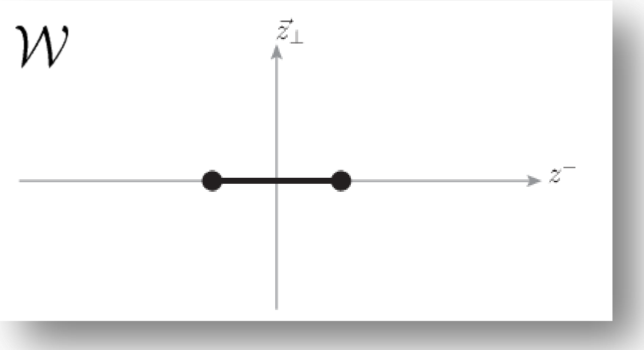
GPDs

Correlator

$$F^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ik^+z^-}$$

$$\langle p'^+, -\tfrac{\vec{\Delta}_\perp}{2}, \Lambda' | \overline{\psi}(-\tfrac{z^-}{2})\Gamma\mathcal{W}\psi(\tfrac{z^-}{2}) | p^+, \tfrac{\vec{\Delta}_\perp}{2}, \Lambda\rangle$$

Off-forward!



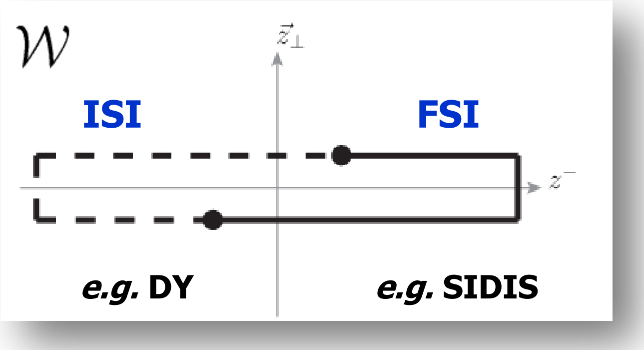
TMDs

Correlator

$$\Phi^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2k_\perp}{(2\pi)^3}\, e^{ik^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}$$

$$\langle p^+, \vec{0}_\perp, \Lambda' | \overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}\psi(\tfrac{z}{2}) | p^+, \vec{0}_\perp, \Lambda\rangle\Big|_{z^+=0}$$

Forward!



$$\boxed{f^{\mathrm{SIDIS}}_{\text{T-odd}} = -f^{\mathrm{DY}}_{\text{T-odd}}}$$

Partonic interpretation

Twist-2 ~ LO in P^+

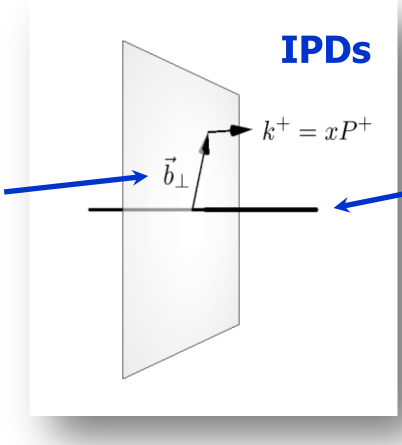
$$\boxed{\Gamma_{\text{twist-2}} = \gamma^+, \gamma^+\gamma_5, i\sigma^{j+}\gamma_5} \qquad j = 1, 2$$

U L T

GPDs

3D imaging $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

$$F^{[\Gamma]}|_{\xi=0} \sim \int \frac{\mathrm{d}^2 b_\perp}{(2\pi)^2}\, e^{i\vec{\Delta}_\perp \cdot \vec{b}_\perp}\, \rho^{[\Gamma]}(x, \vec{b}_\perp)$$



$\vec{b}_{i\perp} = \vec{r}_{i\perp} - \vec{R}_\perp$

Impact parameter

$\vec{R}_\perp = \sum_{i=1}^{N} x_i\, \vec{r}_{i\perp}$

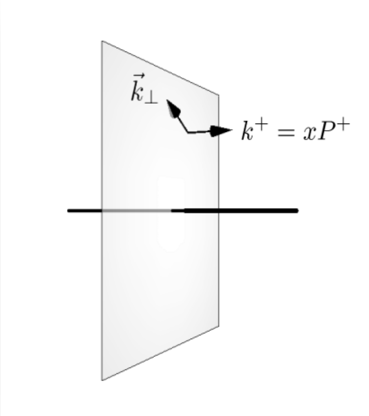
Transverse center of momentum

[Soper (1977)]
[Burkardt (2000,2003)]
[Diehl, Hägler (2005)]

TMDs

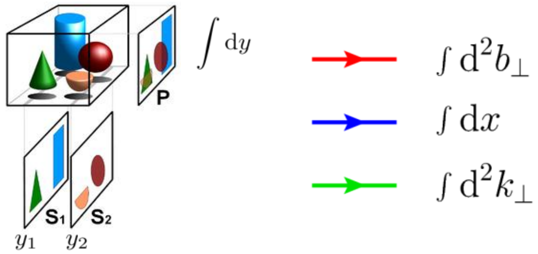
3D imaging $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$

$$\Phi^{[\Gamma]} \sim \rho^{[\Gamma]}(x, \vec{k}_\perp)$$

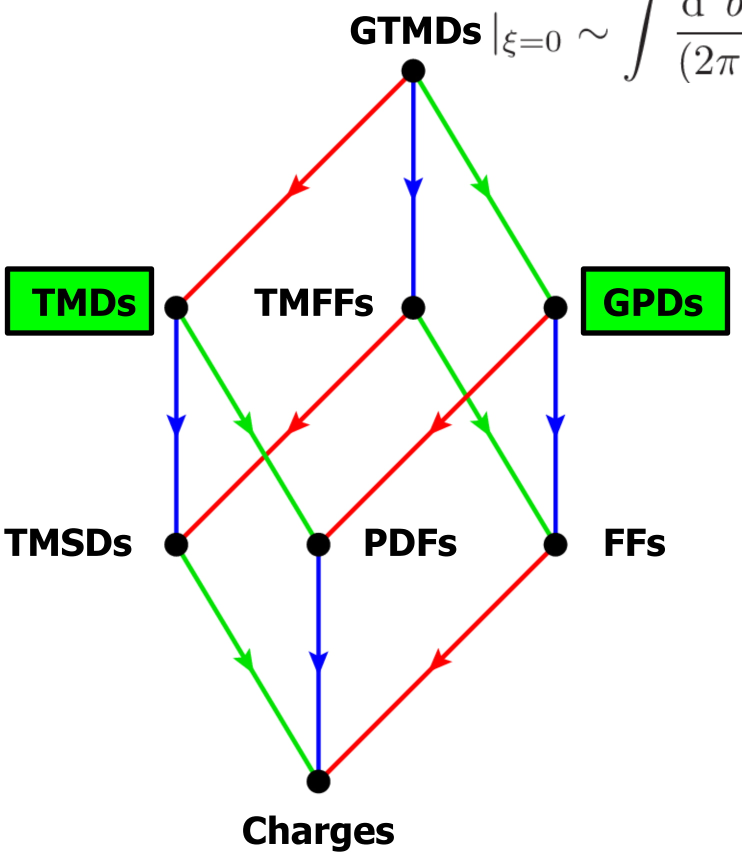


⚠ **Interpretation in $A^+ = 0$ gauge**

Complete picture



GTMDs and Wigner distributions

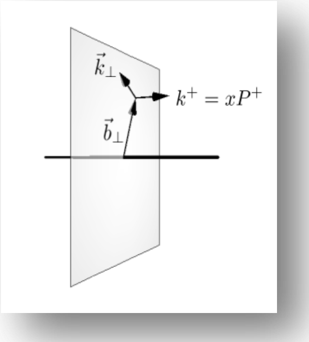


$$\textbf{GTMDs}\,|_{\xi=0} \sim \int \frac{\mathrm{d}^2 b_\perp}{(2\pi)^2}\, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp}$$

TMDs **TMFFs** **GPDs**

TMSDs **PDFs** **FFs**

Charges

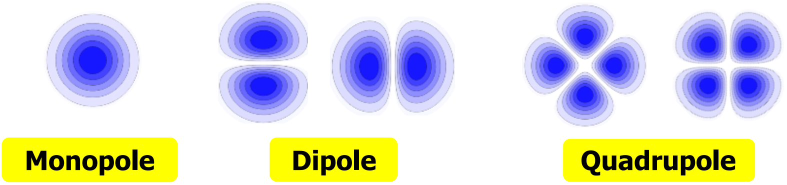


5D imaging

Wigner distribution

[C.L., Pasquini, Vanderhaeghen (2011)]
[C.L., Pasquini (2011)]

Twist-2 structure



GPDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}_T$	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}_T$	
T_x	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}$
T_y	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

🟩 **ξ-odd**

$$\mathcal{H} = \sqrt{1-\xi^2}\left(H - \frac{\xi^2}{1-\xi^2}\,E\right)$$

$$\mathcal{E} = \frac{E}{\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{H}} = \sqrt{1-\xi^2}\left(\tilde{H} - \frac{\xi^2}{1-\xi^2}\,\tilde{E}\right)$$

$$\tilde{\mathcal{E}} = \frac{\xi\tilde{E}}{\sqrt{1-\xi^2}}$$

$$\mathcal{H}_T = \sqrt{1-\xi^2}\left(H_T - \frac{\vec{\Delta}_\perp^2}{2M^2}\,\frac{\tilde{\mathcal{H}}_T}{\sqrt{1-\xi^2}} + \frac{\xi\,\tilde{\mathcal{E}}_T}{\sqrt{1-\xi^2}}\right)$$

$$\mathcal{E}_T = \frac{2\tilde{H}_T + E_T - \xi\,\tilde{E}_T}{\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{H}}_T = -\frac{\tilde{H}_T}{2\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{E}}_T = \frac{\tilde{E}_T - \xi\,E_T}{\sqrt{1-\xi^2}}$$

TMDs

Quark polarization

Nucleon polarization

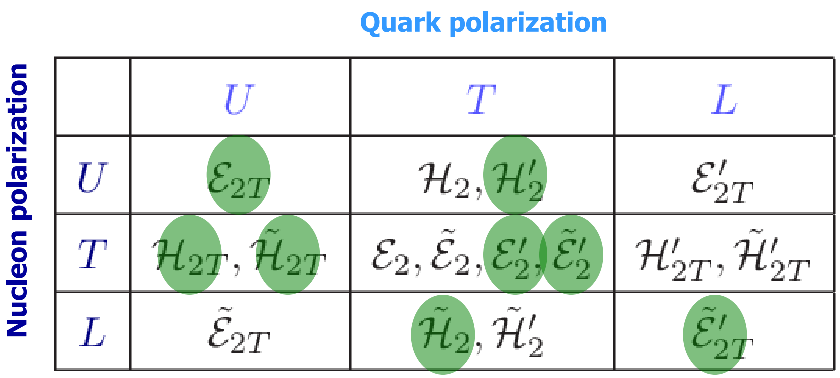
	U	T_x	T_y	L
U	f_1	$-i\,\frac{k_y}{M}\,h_1^\perp$	$i\,\frac{k_x}{M}\,h_1^\perp$	
T_x	$-i\,\frac{k_y}{M}\,f_{1T}$	$h_1 + \frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$\frac{k_x}{M}\,g_{1T}$
T_y	$i\,\frac{k_x}{M}\,f_{1T}$	$\frac{k_xk_y}{M^2}\,h_{1T}^\perp$	$h_1 - \frac{k_x^2-k_y^2}{2M^2}\,h_{1T}^\perp$	$\frac{k_y}{M}\,g_{1T}$
L		$\frac{k_x}{M}\,h_{1L}^\perp$	$\frac{k_y}{M}\,h_{1L}^\perp$	g_{1L}

🟥 **Naive T-odd**

Twist-3 structure

Parametrization **[Meißner, Metz, Schlegel (2009)]**

GPDs



Quark polarization

Nucleon polarization

	U	T	L
U	$\mathcal{E}_{2T}$	$\mathcal{H}_2, \mathcal{H}'_2$	$\mathcal{E}'_{2T}$
T	$\mathcal{H}_{2T}, \tilde{\mathcal{H}}_{2T}$	$\mathcal{E}_2, \tilde{\mathcal{E}}_2, \mathcal{E}'_2, \tilde{\mathcal{E}}'_2$	$\mathcal{H}'_{2T}, \tilde{\mathcal{H}}'_{2T}$
L	$\tilde{\mathcal{E}}_{2T}$	$\tilde{\mathcal{H}}_2, \tilde{\mathcal{H}}'_2$	$\tilde{\mathcal{E}}'_{2T}$

🟩 ξ **-odd**

$$\mathcal{H}_2^{(')} = \sqrt{1-\xi^2}\left(H_2^{(')} - \frac{\xi^2}{1-\xi^2}\,E_2^{(')}\right)$$

$$\mathcal{E}_2^{(')} = \frac{E_2^{(')}}{\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{H}}_2^{(')} = \sqrt{1-\xi^2}\left(\tilde{H}_2^{(')} + \frac{\xi}{1-\xi^2}\,\tilde{E}_2^{(')}\right)$$

$$\tilde{\mathcal{E}}_2^{(')} = -\frac{\tilde{E}_2^{(')}}{\sqrt{1-\xi^2}}$$

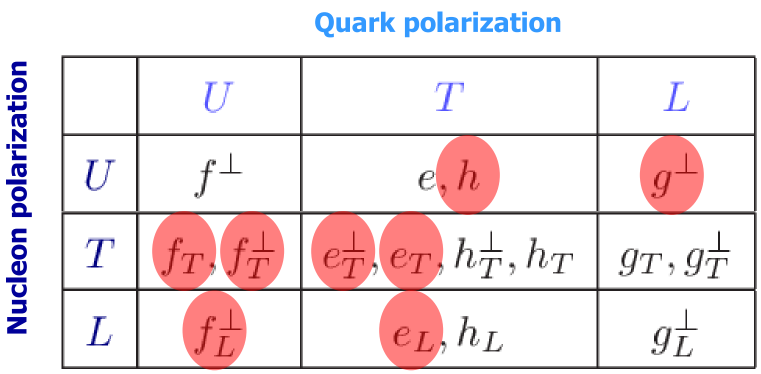
$$\mathcal{H}_{2T}^{(')} = \sqrt{1-\xi^2}\left(H_{2T}^{(')} - \frac{\vec{\Delta}_\perp^2}{2M^2}\,\frac{\tilde{\mathcal{H}}_{2T}^{(')}}{\sqrt{1-\xi^2}} + \frac{\xi\,\tilde{\mathcal{E}}_{2T}^{(')}}{\sqrt{1-\xi^2}}\right)$$

$$\mathcal{E}_{2T}^{(')} = \frac{2\tilde{H}_{2T}^{(')} + E_{2T}^{(')} - \xi\,\tilde{E}_{2T}^{(')}}{\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{H}}_{2T}^{(')} = -\frac{\tilde{H}_{2T}^{(')}}{2\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{E}}_{2T}^{(')} = \frac{\tilde{E}_{2T}^{(')} - \xi\,E_{2T}^{(')}}{\sqrt{1-\xi^2}}$$

TMDs



Quark polarization

Nucleon polarization

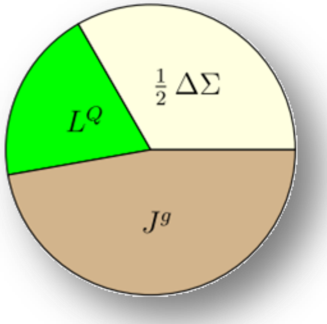
	U	T	L
U	$f^\perp$	e, h	$g^\perp$
T	$f_T, f_T^\perp$	$e_T^\perp, e_T, h_T^\perp, h_T$	$g_T, g_T^\perp$
L	$f_L^\perp$	e_L, h_L	$g_L^\perp$

🟥 **Naive T-odd**

Angular momentum decompositions

Ji

[Ji (1997)]



$$\vec{J}_{\text{QCD}} = \frac{1}{2}\int \mathrm{d}^3r\,\psi^\dagger\vec{\Sigma}\psi + \int \mathrm{d}^3r\,\psi^\dagger\vec{r}\times(-i\vec{D})\psi + \int \mathrm{d}^3r\,\vec{r}\times(\vec{E}^a\times\vec{B}^a)$$

Kinematic

Pros:
- Gauge-invariant decomposition
- Accessible in DIS and DVCS

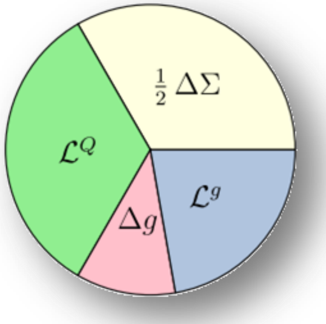
Cons:
- Does not satisfy canonical relations
- Incomplete decomposition

Improvements:
- Complete decomposition

[Wakamatsu (2009,2010)]

Jaffe-Manohar

[Jaffe, Manohar (1990)]



$$\vec{J}_{\text{QCD}} = \frac{1}{2}\int \mathrm{d}^3r\,\psi^\dagger\vec{\Sigma}\psi + \int \mathrm{d}^3r\,\psi^\dagger\vec{r}\times(-i\vec{\nabla})\psi + \int \mathrm{d}^3r\,\vec{E}^a\times\vec{A}^a + \int \mathrm{d}^3r\,E^{ai}\vec{r}\times\vec{\nabla}A^{ai}$$

Canonical

Pros:
- Satisfies canonical relations
- Complete decomposition

Cons:
- Gauge-variant decomposition
- Missing observables for the OAM

Improvements:
- Gauge-invariant extension

[Chen *et al.* (2008)]
- OAM accessible *via* Wigner distributions

[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan(2011)]
[Hatta (2011)]

Quark spin and OAM

GPDs

Quark spin

$$S^q_z = \frac{1}{2}\int \mathrm{d}x\,\tilde H^q(x,0,0)$$

Ji sum rule [Ji (1997)]

$$L^q_z + S^q_z = \frac{1}{2}\int \mathrm{d}x\,x\,[H^q(x,0,0) + E^q(x,0,0)]$$

Twist-3

$$L^q_z + 2S^q_z = -\int \mathrm{d}x\,x\,\tilde E^q_{2T}(x,0,0)$$

$$\tilde E^q_{2T} = -(G^q_2 + H^q + E^q) + 2(G^q_4 + \xi\,G^q_3)$$

$$\int \mathrm{d}x\,x\,G^q_{3,4} = 0$$

PPSS sum rule

$$L^q_z = -\int \mathrm{d}x\,x\,G^q_2(x,0,0)$$

← **Pure twist-3!**

[Penttinen, Polyakov, Shuvaev, Strikman (2000)]

Genuine sum rule

$$\boxed{\int \mathrm{d}x\left[x\left(H^q + E^q + 2\tilde E^q_{2T}\right) + \tilde H^q\right] = 0}$$

TMDs

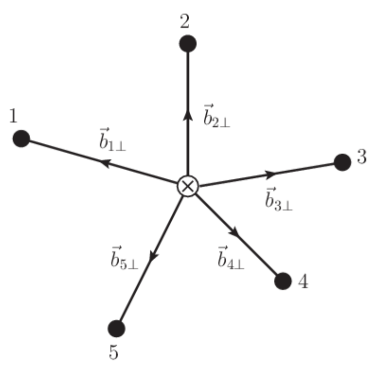
Quark spin

$$S^q_z = \frac{1}{2}\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,g^q_{1L}(x,\vec k_\perp)$$

Pretzelosity

$$\mathcal{L}^q_z(x,\vec k_\perp) = -\frac{\vec k^2_\perp}{2M^2}\,h^{\perp q}_{1T}(x,\vec k^2_\perp)$$

[Burkardt (2007)]
[Efremov, Schweitzer, Teryaev, Zavada (2008,2010)]
[She, Zhu, Ma (2009)]
[Avakian, Efremov, Schweitzer, Yuan (2010)]
[C.L., Pasquini (2011)]



- **Model-dependent**
- **Not intrinsic OAM**

TMDs $\mathcal{L}_{iz} = \vec r_{i\perp} \times \vec k_{i\perp}$

GTMDs $\ell^{\text{int}}_{iz} = \vec b_{i\perp} \times \vec k_{i\perp}$

Interesting relations

***=SU(6)**

Model relations

	Linear relations	**Quadratic relation**
Flavor-dependent $D^u = \frac{2}{3},\ D^d = -\frac{1}{3}$	$D^q f_1^q + g_{1L}^q = 2h_1^q$ * * *	
Flavor-independent	$g_{1T}^q = -h_{1L}^{\perp q}$ * * * ● ● ● $g_{1L}^q - h_1^q = \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp q}$ * * * ● ● ●	$2h_1^q\, h_{1T}^{\perp q} = -(g_{1T}^q)^2$ * * * ● ● ●

Bag	**[Jaffe, Ji (1991), Signal (1997), Barone & *al.* (2002), Avakian & *al.* (2008-2010)]**
LFχQSM	**[C.L., Pasquini, Vanderhaeghen (2011)]**
LFCQM	**[Pasquini & *al.* (2005-2008)]**
S Diquark	**[Ma & *al.* (1996-2009), Jakob & *al.* (1997), Bacchetta & *al.* (2008)]**
AV Diquark	**[Ma & *al.* (1996-2009), Jakob & *al.* (1997)] [Bacchetta & *al.* (2008)]**
Cov. Parton	**[Efremov & *al.* (2009)]**
Quark Target	**[Meißner & *al.* (2007)]**

Geometrical explanation

[C.L., Pasquini (2011)]

Preliminaries

Conditions:

- ✓ **Quasi-free quarks**
- ✓ **Spherical symmetry**

Wigner rotation

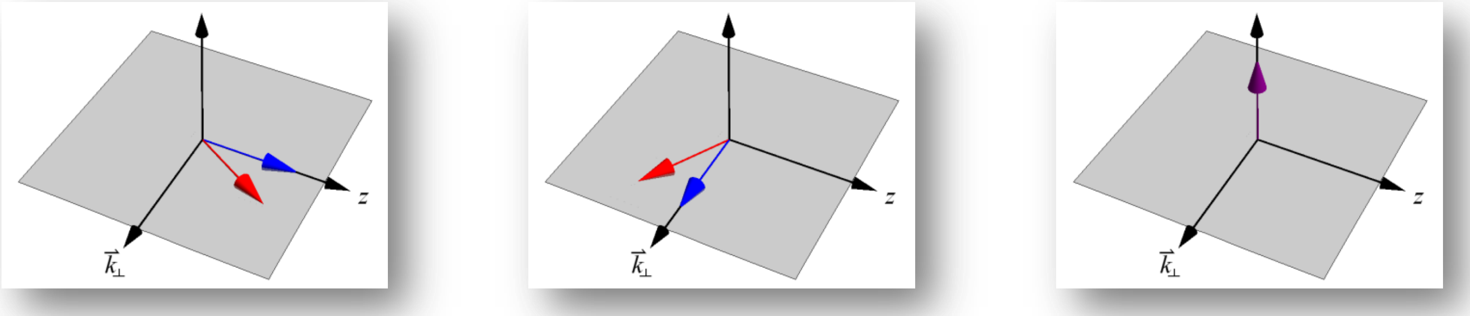
$$q_\lambda^{LC}(k) = \sum_s D^{(1/2)*}_{\lambda s}(k)\, q_s^{C}(k)$$

$$D^{(1/2)*}_{\lambda s}(k) = \begin{pmatrix} \cos\frac{\theta}{2} & \hat{k}_L \sin\frac{\theta}{2} \\ -\hat{k}_R \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

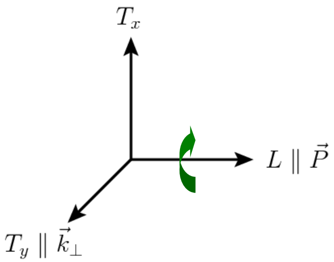
Light-front helicity

Canonical spin

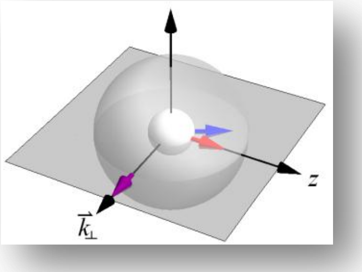
(reduces to Melosh rotation in case of FREE quarks)



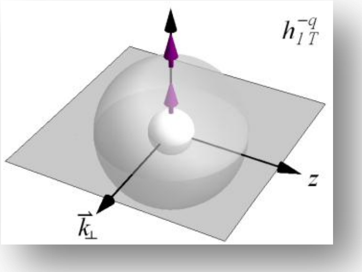
Geometrical explanation



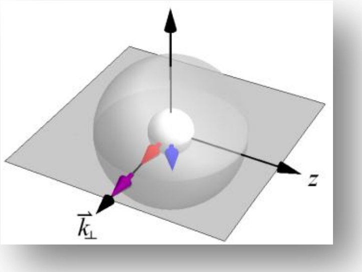
Axial symmetry about z



$= 0$

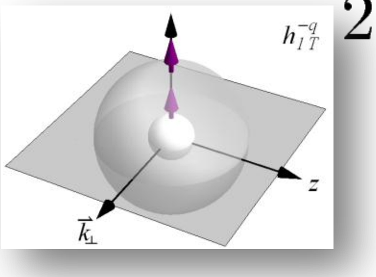


$=$

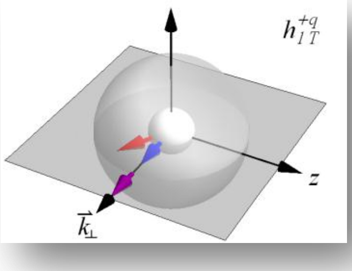


$$\boxed{h_{1T}^{\pm q} \equiv h_1^q \pm \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q}}$$

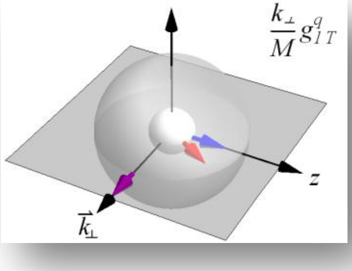
$$\boxed{2h_1^q\, h_{1T}^{\perp q} = -(g_{1T}^q)^2}$$



$^2 =$

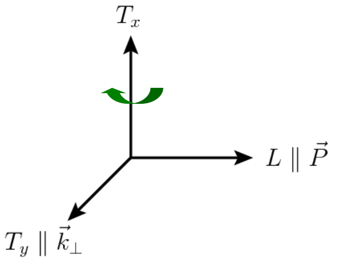


$^2 +$

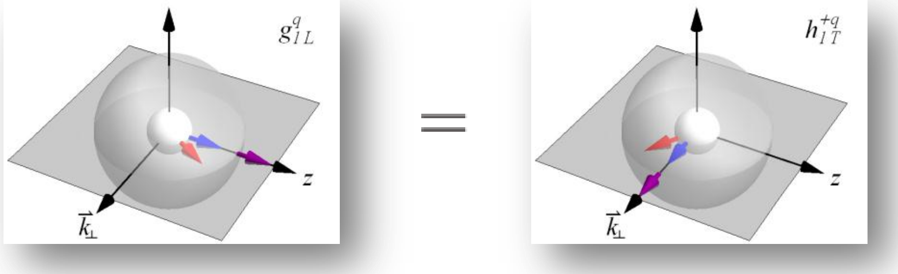


^{2}

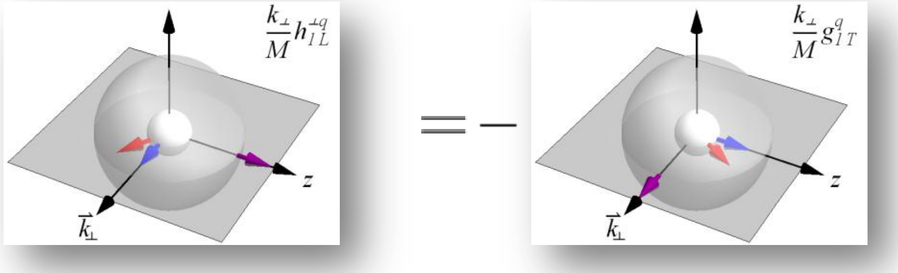
Geometrical explanation



Axial symmetry about $\vec{k}_\perp$



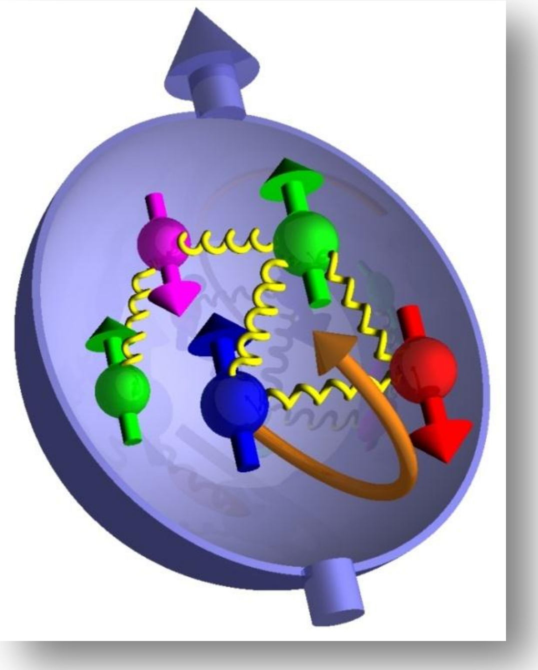
$$g_{1L}^{q} - h_{1}^{q} = \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp q}$$



$$g_{1T}^{q} = -h_{1L}^{\perp q}$$

Summary

- Exclusive vs. Semi-inclusive
  - DVCS vs. SIDIS
    - Factorization approach
  - GPDs vs. TMDs
    - Quark-quark correlator
- Partonic interpretation
  - 3D imaging
    - Probabilistic interpretation
  - Twist-2 and Twist-3
    - Helicity structure
- Hadron structure
  - Angular momentum decompositions
    - Short overview
  - Quark spin and OAM
    - Relation with observables
  - Interesting relations
    - Spherical symmetry in quark models



LC helicity and canonical spin

[C.L., Pasquini (2011)]

$$\boxed{h_{1T}^{\pm q} \equiv h_1^q \pm \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q}}$$

LC helicity

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	f_1^q	$\frac{k_\perp}{M} h_1^{\perp q}$		
T_x	$\frac{k_\perp}{M} f_{1T}^{\perp q}$	h_{1T}^{-q}		
T_y			h_{1T}^{+q}	$\frac{k_\perp}{M} g_{1T}^q$
L			$\frac{k_\perp}{M} h_{1L}^{\perp q}$	g_{1L}^q

Canonical spin

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	f_1^q	$\frac{k_\perp}{M} h_1^{\perp q}$		
T_x	$\frac{k_\perp}{M} f_{1T}^{\perp q}$	h_{1T}^{-q}		
T_y			$\sin\theta\,\frac{k_\perp}{M}\,g_{1T}^q + \cos\theta\,h_{1T}^{+q}$	$\cos\theta\,\frac{k_\perp}{M}\,g_{1T}^q - \sin\theta\,h_{1T}^{+q}$
L			$\sin\theta\,g_{1L}^q + \cos\theta\,\frac{k_\perp}{M}\,h_{1L}^{\perp q}$	$\cos\theta\,g_{1L}^q - \sin\theta\,\frac{k_\perp}{M}\,h_{1L}^{\perp q}$