Is a complete decomposition of nucleon spin possible ?

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1. **Introduction**
2. **Current status of nucleon spin decomposition problem**

   - 2 physically ineuivalent decompositions -
3. **What is "potential angular momentum" ?**

   - lessons from electrodynamics -
4. **Relations of the two decompositions with observables**
5. **Summary and outlook**

1. Introduction

current status and homework of nucleon spin problem

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^Q + \Delta g + \texttt{Orbital Angular Momenta ?}$$

(1) $\Delta\Sigma^Q$: fairly precisely determined ! ($\sim 1/3$)

(2) Δg : likely to be small , but large uncertainties

⇓

What carries the remaining 2 / 3 of nucleon spin ?

a fundamental question of QCD

⇓

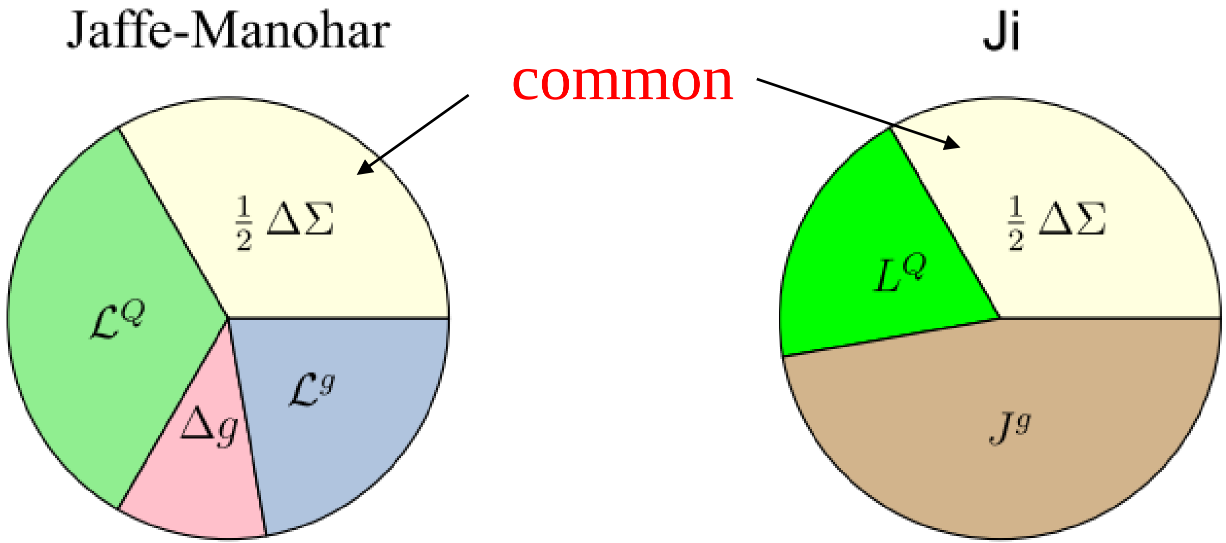
To answer this question unambiguously, we cannot avoid to clarify

- What is a **precise (QCD) definition** of each term of the decomposition ?
- How can we extract individual term by means of **direct measurements** ?

especially controversy are **orbital angular momenta** !

2. Current status of nucleon spin decomposition problem

two popular decompositions of the nucleon spin



Jaffe-Manohar:

$$\begin{aligned} \boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi \, d^3x \\ &+ \int \psi^\dagger \boldsymbol{x} \times \frac{1}{i} \nabla \psi \, d^3x \\ &+ \int \boldsymbol{E}^a \times \boldsymbol{A}^a \, d^3x \\ &+ \int E^{ai} \boldsymbol{x} \times \nabla A^{ai} \, d^3x \end{aligned}$$

Each term is not separately gauge-invariant !

Ji:

$$\begin{aligned} \boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi \, d^3x \\ &+ \int \psi^\dagger \boldsymbol{x} \times \frac{1}{i} \boldsymbol{D} \psi \, d^3x \\ &+ \int \boldsymbol{x} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a) \, d^3x \end{aligned}$$

J_g

No further decomposition !

First, pay attention to the **difference** of **quark OAM parts**

$$L_Q(\mathrm{JM}) \;\sim\; \psi^\dagger \boldsymbol{x} \times \boldsymbol{p}\,\psi \qquad\qquad L_Q(\mathrm{Ji}) \;\sim\; \psi^\dagger \boldsymbol{x} \times (\boldsymbol{p} - g\,\boldsymbol{A})\,\psi$$

canonical OAM | **dynamical OAM**

not gauge invariant ! | **gauge invariant** !

gauge principle

observables must be gauge-invariant !

- Observability of canonical OAM has been questioned ?
- On the other hand, it has been known that the dynamical quark OAM can be related to observables through GPDs. (X. Ji, 1997)

Chen-Wang-Goldman proposed a new gauge-invariant complete decomposition

X.-S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009) ; 100, 232002 (2008).

basic idea

$$A^\mu = A^\mu_{phys} + A^\mu_{pure}$$

which is a sort of generalization of the decomposition of photon field in QED into the transverse and longitudinal components :

$$\boldsymbol{A}_{phys} \Leftrightarrow \boldsymbol{A}_{\perp}, \qquad \boldsymbol{A}_{pure} \Leftrightarrow \boldsymbol{A}_{||}$$

Chen et al.'s decomposition (with "generalized Coulomb gauge" condition)

$$\begin{aligned} \boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi \, d^3x + \int \psi^\dagger \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}_{pure}) \, \psi \, d^3x \\ &+ \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x + \int E^{aj} \, (\boldsymbol{x} \times \nabla) \, \boldsymbol{A}^{aj}_{phys} \, d^3x \\ &= \boldsymbol{S}'^q + \boldsymbol{L}'^q + \boldsymbol{S}'^g + \boldsymbol{L}'^g \end{aligned}$$

- Each term is separately gauge-invariant !
- It reduces to gauge-variant **Jaffe-Manohar decomposition** in a particular gauge !

$$\boldsymbol{A}_{pure} = 0, \quad \boldsymbol{A} = \boldsymbol{A}_{phys}$$

Chen et al.'s papers created **quite a controversy** on the **feasibility** of complete decomposition of nucleon spin.

- X. Ji, Phys. Rev. Lett. 104 (2010) 039101 : 106 (2011) 259101.
- S. C. Tiwari, arXiv:0807.0699.
- X. S. Chen et al., arXiv:0807.3083 ; arXiv:0812.4336 ; arXiv:0911.0248.
- Y. M. Cho et al., arXiv:1010.1080 ; arXiv:1102.1130.
- X. S. Chen et al., Phys. Rev. D83 (2011) 071901.
- E. Leader, Phys. Rev. D83 (2011) 096012.
- Y. Hatta, Phys. Rev. D84 (2011) 041701R.
- P .M. Zhang and D. G. Pak, arXiv:1110.6516.
- H.-W. Lin and K.-F. Liu, arXiv:1111.0678.
- Y. Hatta, Phys. Lett. B708 (2012) 186.

………………………….

We believe that we have arrived at **one (satisfactory) solution** to the problem, step by step, through the following three papers :

(i) M. W., Phys. Rev. D81 (2010) 114010.
(ii) M. W., Phys. Rev. D83 (2011) 014012.
(iii) M. W., Phys. Rev. D84 (2011) 037501.

In the paper (i), we have shown that the way of gauge-invariant decomposition of nucleon spin is not necessarily unique, and proposed another G.I. decomposition :

$$\boldsymbol{J}_{QCD} = \boldsymbol{S}^q + \boldsymbol{L}^q + \boldsymbol{S}^g + \boldsymbol{L}^g$$

where

$$\boldsymbol{S}^q = \int \psi^\dagger \frac{1}{2} \Sigma \psi \, d^3x$$

$$\boldsymbol{L}^q = \int \psi \, \boldsymbol{x} \times (\boldsymbol{p} - g \boldsymbol{A}) \, \psi \, d^3x$$

$$\boldsymbol{S}^g = \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x$$

$$\boldsymbol{L}^g = \int E^{aj} \, (\boldsymbol{x} \times \nabla) \, A^{aj}_{phys} \, d^3x + \boxed{\int \rho^a \, (\boldsymbol{x} \times \boldsymbol{A}^a_{phys}) \, d^3x}$$

"**potential angular momentum**"

- The quark part of our decomposition is common with the **Ji decomposition**.
- The quark and gluon intrinsic spin parts are common with the **Chen decomp**.
- A crucial difference with the Chen decomp. appears in the orbital parts

$$\boldsymbol{L}^q + \boldsymbol{L}^g = \boldsymbol{L}'^q + \boldsymbol{L}'^g$$

$$\boldsymbol{L}^g - \boldsymbol{L}'^g = - (\boldsymbol{L}^q - \boldsymbol{L}'^q) = \int \rho^a \, (\boldsymbol{x} \times \boldsymbol{A}^a_{phys}) \, d^3x$$

The QED correspondent of this term is the orbital angular momentum carried by electromagnetic potential, appearing in the famous Feynman paradox.

An **arbitrariness** of the spin decomposition arises, because this potential angular momentum term is solely gauge-invariant !

$$\int \rho^a \, \boldsymbol{x} \times \boldsymbol{A}^a_{phys} \, d^3x \;=\; g \int \psi^\dagger(x) \, \boldsymbol{x} \times \boldsymbol{A}_{phys}(x) \, \psi(x) \, d^3x$$

$$\rightarrow \;\; \texttt{gauge invariant}$$

since

$$\boldsymbol{A}_{phys}(x) \;\rightarrow\; U^\dagger(x) \, \boldsymbol{A}_{phys}(x) \, U(x)$$

$$\psi^\dagger(x) \;\rightarrow\; \psi^\dagger(x) \, U^\dagger(x), \qquad \psi(x) \;\rightarrow\; U(x) \, \psi(x)$$

This means that one has a freedom to shift this potential OAM term to the quark OAM part in **our decomposition**, which leads to the **Chen decomposition**.

$$L^q \, (\text{Ours}) \;+\; \texttt{potential angular momentum}$$

$$= \int \psi^\dagger \, \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}) \, \psi \, d^3x \;+\; g \int \psi^\dagger \, \boldsymbol{x} \times \boldsymbol{A}_{phys} \, \psi \, d^3x$$

$$= \int \psi^\dagger \, \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}_{pure}) \, \psi \, d^3x \qquad = \qquad L'^q \, (\text{Chen})$$

Next, **in the paper (ii),** we found that we can make a covariant extension of the gauge-invariant decomposition of nucleon spin.

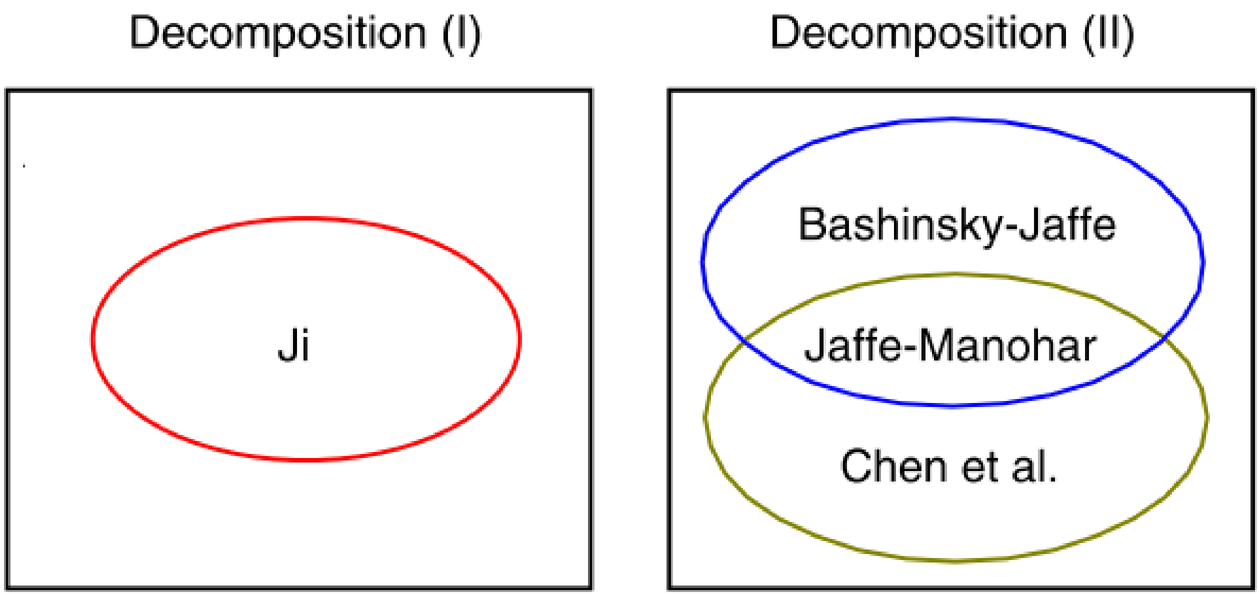
covariant generalization of the decomposition has several advantages.

(1) It is useful to find relations to high-energy DIS observables.

(2) It is vital to prove Lorentz frame-independence of the decomposition.

(3) It generalizes and unifies the nucleon spin decompositions in the market.

Basically, we find two physically nonequivalent decompositions (I) and (II) .



The basis of our treatment is the decomposition of gluon field, similar to Chen et al.

$$A^\mu = A^\mu_{phys} + A^\mu_{pure}$$

Different from theirs, we impose only the following general conditions :

$$F^{\mu\nu}_{pure} \equiv \partial^\mu A^\nu_{pure} - \partial^\nu A^\mu_{pure} - i\, g\, [A^\mu_{pure}, A^\nu_{pure}] = 0$$

and

$$A^\mu_{phys}(x) \rightarrow U(x)\, A^\mu_{phys}(x)\, U^{-1}(x)$$

$$A^\mu_{pure}(x) \rightarrow U(x) \left(A^\mu_{pure}(x) - \frac{i}{g}\, \partial^\mu \right) U^{-1}(x)$$

- Actually, these conditions are not enough to fix gauge uniquely !
- However, the **point of our analysis** is that we can postpone a concrete gauge-fixing until later stage, while accomplishing a gauge-invariant decomposition of $M^{\mu\nu\lambda}$ based on the **above general conditions** alone.

Again, we are left with two possibilities :

decomposition (I) & **decomposition (II)**

Gauge-invariant decomposition (II) : covariant generalization of Chen et al's

$$M^{\mu\nu\lambda}_{QCD} = M'^{\mu\nu\lambda}_{q-spin} + M'^{\mu\nu\lambda}_{q-OAM} + M'^{\mu\nu\lambda}_{g-spin} + M'^{\mu\nu\lambda}_{g-OAM} + \text{boost} + \text{total divergence}$$

with

pure-gauge covariant derivative

$$M'^{\mu\nu\lambda}_{q-spin} = \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\,\bar{\psi}\,\gamma_\sigma\,\gamma_5\,\psi$$

$$M'^{\mu\nu\lambda}_{q-OAM} = \bar{\psi}\,\gamma^\mu\,(\,x^\nu\,i\,D^\lambda_{pure} - x^\lambda\,i\,D^\nu_{pure}\,)\,\psi$$

$$M'^{\mu\nu\lambda}_{g-spin} = 2\,\mathrm{Tr}\,\{\,F^{\mu\lambda}\,A^\nu_{phys} - F^{\mu\nu}\,A^\lambda_{phys}\,\}$$

$$M'^{\mu\nu\lambda}_{g-OAM} = 2\,\mathrm{Tr}\,\{\,F^{\mu\alpha}\,(\,x^\nu\,D^\lambda_{pure} - x^\lambda\,D^\nu_{pure}\,)\,A^{phys}_\alpha\,\}$$

This decomposition reduces to any ones of Bashinsky-Jaffe, of Chen et al., and of Jaffe-Manohar, after an appropriate gauge-fixing in a suitable Lorentz frame, which reveals that **these 3 decompositions are all gauge-equivalent** !

These 3 must be **physically equivalent** decompositions !

Gauge-invariant decomposition (I) :

The difference with the decomposition (II) resides in **OAM parts** !

$$M^{\mu\nu\lambda} = M^{\mu\nu\lambda}_{q-spin} + M^{\mu\nu\lambda}_{q-OAM} + M^{\mu\nu\lambda}_{g-spin} + M^{\mu\nu\lambda}_{g-OAM}$$
$$+ \text{ boost } + \text{ total divergence}$$

with

full covariant derivative

$$M^{\mu\nu\lambda}_{q-spin} = M'^{\mu\nu\lambda}$$
$$M^{\mu\nu\lambda}_{q-OAM} = \bar{\psi}\,\gamma^\mu\,(\,x^\nu\,i\,D^\lambda - x^\lambda\,i\,D^\nu\,)\,\psi \neq M'^{\mu\nu\lambda}_{q-OAM}$$

$$M^{\mu\nu\lambda}_{g-spin} = M'^{\mu\nu\lambda}_{g-spin}$$
$$M^{\mu\nu\lambda}_{g-OAM} = M'^{\mu\nu\lambda}_{g-OAM} + 2\,\mathrm{Tr}\,[\,(\,D_\alpha\,F^{\alpha\mu}\,)\,(\,x^\nu\,A^\lambda_{phys} - x^\lambda\,A^\nu_{phys}\,)\,]$$

⇧

covariant generalization of potential OAM !

It was sometimes criticized that there are too many decompositions of nucleon spin.

$$\begin{aligned} \frac{1}{2} &= \frac{1}{2}\Sigma + L_Q + \Delta g + L_g \\ &= \frac{1}{2}\Sigma' + L'_Q + \Delta g' + L'_g \\ &= \frac{1}{2}\Sigma'' + L''_Q + \Delta g'' + L''_g \\ &\vdots \end{aligned}$$

In my viewpoint, however, this is not true any more. One should recognize now that there exist only two physically nonequivalent decompositions !

Decomposition (I)	Decomposition (II)
extension of **Ji's decomp.** including gluon part	gauge-invariant decomposition containing **Jaffe-Manohar's decomp.** as gauge-fixed form
dynamical (mechanical) OAMs	**"canonical" OAMs**

Since **both** decompositions are **gauge-invariant**, there arises a possibility that they both correspond to observables !

3. What is "potential angular momentum" ? - Lessons from QED -

We have shown that the **key quantity**, which distinguishes the two nucleon spin decompositions, is what-we-call the "**potential angular momentum**" term.

To understand its **physical meaning** more clearly, we find it instructive to study easier QED case, especially a system of **charged particles** and **photons**.

$$H = \sum_i \frac{1}{2} m_i \dot{\boldsymbol{r}}_i^2 + \frac{1}{2} \int d^3 r \, [\, \boldsymbol{E}^2 + \boldsymbol{B}^2 \,]$$

longitudinal-transverse decomposition : $\boldsymbol{A} = \boldsymbol{A}_{||} + \boldsymbol{A}_{\perp}$

$$\boldsymbol{E} = \boldsymbol{E}_{||} + \boldsymbol{E}_{\perp}, \qquad \boldsymbol{B} = \boldsymbol{B}_{\perp}$$

$$H = \sum_i \frac{1}{2} m_i \dot{\boldsymbol{r}}_i^2 + \frac{1}{2} \int d^3 r \, \boldsymbol{E}_{||}^2 + \frac{1}{2} \int d^3 r \, [\, \boldsymbol{E}_{\perp}^2 + \boldsymbol{B}_{\perp}^2 \,]$$

Here, by using the Gauss law $\nabla \cdot \boldsymbol{E}_{||} = \rho$

$$H = \sum_i \frac{1}{2} m_i \dot{\boldsymbol{r}}_i^2 + V_{Coul} + \frac{1}{2} \int d^3 r \, [\, \boldsymbol{E}_{\perp}^2 + \boldsymbol{B}_{\perp}^2 \,]$$

total momentum

$$\begin{aligned} \boldsymbol{P} &= \sum_i m_i \dot{\boldsymbol{r}}_i + \int d^3r \, (\boldsymbol{E}_{\|} + \boldsymbol{E}_{\perp}) \times \boldsymbol{B}_{\perp} \\ &= \sum_i m_i \dot{\boldsymbol{r}}_i + \boldsymbol{P}_{long} + \boldsymbol{P}_{trans} \\ &= \sum_i m_i \dot{\boldsymbol{r}}_i + \sum_i q_i \boldsymbol{A}_{\perp}(\boldsymbol{r}_i) + \boldsymbol{P}_{trans} \end{aligned}$$

↘ **potential momentum** : *a la* Konopinski

momentum associates with the longitudinal field of the particle i

Which of particle or photon should it be attributed to ?

If we combine it with the **mechanical momentum** $m_i \dot{\boldsymbol{r}}_i$

$$\boxed{m_i \dot{\boldsymbol{r}}_i + q_i \boldsymbol{A}_{\perp}(\boldsymbol{r}_i) = \boldsymbol{p}_i - q_i \boldsymbol{A}_{\|}(\boldsymbol{r}_i)}$$

with $\boldsymbol{p}_i \equiv m_i \dot{\boldsymbol{r}}_i - q_i \boldsymbol{A}(\boldsymbol{r}_i)$ being the usual **canonical momentum**, we get

$$\boldsymbol{P} = \sum_i (\boldsymbol{p}_i - q_i \boldsymbol{A}_{\|}(\boldsymbol{r}_i)) + \boldsymbol{P}_{trans} \quad : \texttt{ in general gauge}$$

or

$$\boldsymbol{P} = \sum_i \boldsymbol{p}_i + \boldsymbol{P}_{trans} \quad : \texttt{ in Coulomb gauge with } \boldsymbol{A}_{\|} = 0$$

total angular momentum

$$\boldsymbol{J} = \sum_i \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i + \int d^3r\, \boldsymbol{r} \times \left[(\boldsymbol{E}_{\parallel} + \boldsymbol{E}_{\perp}) \times \boldsymbol{B}_{\perp} \right]$$

$$= \sum_i \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i + \boldsymbol{J}_{long} + \boldsymbol{J}_{trans}$$

$$= \sum_i \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i + \sum_i \boldsymbol{r}_i \times q_i \boldsymbol{A}_{\perp}(\boldsymbol{r}_i) + \boldsymbol{J}_{trans}$$

what-we-call the "**potential angular momentum**"

angular momentum associates with the longitudinal field of the particle i

Again, combining this term with the **mechanical angular momentum**, we get

$$\boldsymbol{J} = \sum_i \boldsymbol{r}_i \times (\boldsymbol{p}_i - q_i \boldsymbol{A}_{\parallel}(\boldsymbol{r}_i)) + \boldsymbol{J}_{trans} \quad : \ \texttt{in general gauge}$$

or

$$\boldsymbol{J} = \boxed{\sum_i \boldsymbol{r}_i \times \boldsymbol{p}_i} + \boldsymbol{J}_{trans} \quad : \ \texttt{in Coulomb gauge with } \boldsymbol{A}_{\parallel} = 0$$

canonical OAM

We therefore find (in Coulomb gauge) the following simpler-looking relations.

$$\boldsymbol{P} = \sum_i \boldsymbol{p}_i + \boldsymbol{P}_{trans}$$

$$\boldsymbol{J} = \sum_i \boldsymbol{r}_i \times \boldsymbol{p}_i + \boldsymbol{J}_{trans}$$

compare !

At first sight, it appears to indicate physical superiority of canonical momentum and canonical angular momentum over the mechanical ones.

However, it is not true, as is clear from the following consideration for H .

total Hamiltonian

$$H = \sum_i \frac{1}{2} m_i \dot{\boldsymbol{r}}_i^2 + V_{coul} + H_{trans} \neq \sum_i \frac{\boldsymbol{p}_i^2}{2\,m} + H_{trans}$$

$\sum_i \frac{1}{2} m_i \dot{\boldsymbol{r}}_i^2$: mechanical kinetic energy

$\sum_i \frac{\boldsymbol{p}_i^2}{2\,m_i}$: "what" kinetic energy ?

~~canonical kinetic energy~~

Hydrogen atom (in Coulomb gauge)

$$H = \frac{1}{2} m \dot{\boldsymbol{r}}^2 + V_{Coul} + H_{trans} = H_0 + H_{trans} + H_{int}$$

$$H_0 = \frac{\boldsymbol{p}^2}{2m} + V_{Coul}(\boldsymbol{r})$$

$$H_{trans} = \sum_{\boldsymbol{k}} \sum_{\lambda=1,2} \hbar \omega_{\boldsymbol{k}} \, a^{\dagger}_{\boldsymbol{k},\lambda} a_{\boldsymbol{k},\lambda}$$

interaction term !

$$H_{int} = \frac{e}{2m} \left[\boldsymbol{p} \cdot \boldsymbol{A}_{\perp}(\boldsymbol{r}) + \boldsymbol{A}_{\perp}(\boldsymbol{r}) \cdot \boldsymbol{p} \right] + \frac{e^2}{2m} \boldsymbol{A}_{\perp}(\boldsymbol{r}) \cdot \boldsymbol{A}_{\perp}(\boldsymbol{r})$$

general form of eigen-states : $|\psi_n\rangle \otimes |\{n_{\boldsymbol{k},\lambda}\}\rangle$

$$H_0 |\psi_n\rangle = E_n |\psi_n\rangle$$

$$|\{n_{\boldsymbol{k},\lambda}\}\rangle = \prod_{\alpha} |n_{\boldsymbol{k}_\alpha,\lambda_\alpha}\rangle$$

In the usual description of hydrogen atom, we do not include

Fock components of transverse photons !

$$|\{n_{\boldsymbol{k},\lambda}\}\rangle \Rightarrow |0\rangle_{photon}$$

Electron alone saturates the spins of hydrogen atom (and any atoms) !

$$\boldsymbol{L}_{can} = \boldsymbol{r} \times \boldsymbol{p} \quad \Longleftrightarrow \quad \boldsymbol{L}_{mech} = \boldsymbol{r} \times (\boldsymbol{p} - e\, \boldsymbol{A}_{\perp})$$

No difference since $\langle \boldsymbol{A}_{\perp} \rangle = 0$, in such **restricted Fock space** !

Totally different from the nucleon spin problem of QCD.

Strongly-coupled gauge system of quarks and gluons !

We certainly need Fock component of transverse gluons . $(g(x) \neq 0 \;!)$

The meaning of what-we-call the "potential angular momentum" seems clear now !

- It represents angular momentum associates with the longitudinal part of color electric field of gluons generated by color-charged quarks !
- We attribute it to the nature of gluons, while Chen et al. to that of quarks.
- Since the choice is in a sense a matter of taste, any further claim on a superiority of one choice must be done in reference to relations with observables.

Important remark (1)

It is a wide-spread belief that, among the following two quantities :

$$\boldsymbol{L}_{can} = \boldsymbol{r} \times \boldsymbol{p} \qquad \Longleftrightarrow \qquad \boldsymbol{L}_{mech} = \boldsymbol{r} \times (\boldsymbol{p} - e\,\boldsymbol{A}_{\perp})$$

what is closer to physical image of orbital motion is the former, since the latter appears to contain an extra **interaction term** with the gauge field !

The fact is just opposite !

$$\begin{aligned} \boldsymbol{L}_{\text{“}can\text{”}} &= \boldsymbol{L}_{mech} + \sum_i \boldsymbol{r}_i \times q_i\,\boldsymbol{A}_{\perp}(\boldsymbol{r}_i) \\ &= \sum_i m_i\,\boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i + \int d^3r\,\boldsymbol{r} \times (\boldsymbol{E}_{\parallel} \times \boldsymbol{B}_{\perp}) \end{aligned}$$

orbital motion !

- It is the "mechanical" angular momentum $\boldsymbol{L}_{mech}$ not the "canonical" angular momentum $\boldsymbol{L}_{\text{“}can\text{”}}$ that has a natural physical interpretation as orbital motion of particles !
- It may sound paradoxical, but what contains an extra interaction term is rather the "canonical" angular momentum than the "mechanical" angular momentum !

Important remark (2)

The reason of existence of two gauge-invariant decompositions is clear now !

$$\boldsymbol{J} \;=\; \boldsymbol{L}'_p + \boldsymbol{S}'_\gamma + \boldsymbol{L}'_\gamma \;=\; \boldsymbol{L}_p + \boldsymbol{S}_\gamma + \boldsymbol{L}_\gamma$$

decomposition (II) **decomposition (I)**

where

$$\boldsymbol{L}'_p = \sum_i \boldsymbol{r}_i \times (\boldsymbol{p}_i - q_i \boldsymbol{A}_{||}(\boldsymbol{r}_i)) \;\Rightarrow\; \sum_i \boldsymbol{r}_i \times \frac{1}{i} \boldsymbol{D}_{i,pure}$$

$$\boldsymbol{S}'_\gamma = \int d^3 r \, \boldsymbol{E}_\perp \times \boldsymbol{A}_\perp$$

$$\boldsymbol{L}'_\gamma = \int d^3 r \, E^k_\perp (\boldsymbol{r} \times \nabla) A^k_\perp$$

$$\boldsymbol{L}_p = \sum_i m_i \boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i \;\Rightarrow\; \boxed{\sum_i \boldsymbol{r}_i \times \frac{1}{i} \boldsymbol{D}_i}$$

$$\boldsymbol{S}_\gamma = \boldsymbol{S}'_\gamma$$

$$\boldsymbol{L}_\gamma = \int d^3 r \, E^k_\perp (\boldsymbol{r} \times \nabla) A^k_\perp + \int d^3 r \, \boldsymbol{r} \times (\boldsymbol{E}_{||} \times \boldsymbol{B}_\perp)$$

↘ **potential OAM term**

4. Relations of the two decompositions with observables

A clear relation with observables was first established for the **decomposition (I)**.

The basis of **nucleon spin sum rule**

$$\boxed{\langle Ps \,|\, W^\mu s_\mu \,|\, Ps \rangle \,/\, \langle Ps \,|\, Ps \rangle \;=\; 1/2}$$

where

$$W^\mu \;=\; -\,\varepsilon^{\mu\nu\alpha\beta} J_{\alpha\beta} P_\gamma \,/\, (2\sqrt{P^2}) \quad \texttt{with} \quad J^{\alpha\beta} \;=\; \int d^3x\, M^{0\alpha\beta}$$

W^μ → Pauli-Lubanski vector

Using the **key identities**, which hold in our decomposition (I) :

quark :

$$x^\nu T_q^{\mu\lambda} \;-\; x^\lambda T_q^{\mu\nu} \;=\; M_{q-spin}^{\mu\nu\lambda} \;+\; M_{q-OAM}^{\mu\nu\lambda} \;+\; \texttt{total divergence}$$

and

gluon :

$$x^\nu T_g^{\mu\lambda} \;-\; x^\lambda T_g^{\mu\nu} \;-\; \texttt{boost} \;=\; M_{g-spin}^{\mu\nu\lambda} \;+\; M_{g-OAM}^{\mu\nu\lambda} \;+\; \texttt{total divergence}$$

with

$$T_{QCD}^{\mu\nu} \;=\; T_q^{\mu\nu} \;+\; T_g^{\mu\nu} \quad : \quad \texttt{Belinfante symmetric form}$$

We can prove the following relations :

for the **quark part**

$$\begin{aligned} L_q &= \langle p \uparrow | M^{012}_{q-OAM} | p \uparrow \rangle \\ &= \frac{1}{2} \int_{-1}^{1} x \, [\, H^q(x,0,0) \; + E^q(x,0,0) \,] \, dx \; - \; \frac{1}{2} \int_{-1}^{1} \Delta q(x) \, dx \\ &= J_q \; - \; \frac{1}{2} \Delta q \end{aligned}$$

with

$$M^{012}_{q-OAM} \; = \; \bar{\psi} \left(\boldsymbol{x} \times \frac{1}{i} \boldsymbol{D} \right)^3 \psi \;\; \neq \;\; \begin{cases} \bar{\psi} \left(\boldsymbol{x} \times \frac{1}{i} \nabla \right)^3 \psi \\ \bar{\psi} \left(\boldsymbol{x} \times \frac{1}{i} \boldsymbol{D}_{pure} \right)^3 \psi \end{cases}$$

⇩

In other words

the quark OAM extracted from the combined analysis of GPD and polarized PDF is "dynamical OAM" (or "mechanical OAM") not "canonical OAM" !

This conclusion is nothing different from Ji's claim !

for the **gluon part** (this is a new observation)

$$\begin{aligned} L_g &= \langle p \uparrow | \, M^{012}_{g-OAM} \, | p \uparrow \rangle \\ &= \frac{1}{2} \int_{-1}^{1} x \, [\, H^g(x,0,0) \, + \, E^g(x,0,0) \,] \, dx \, - \, \int_{-1}^{1} \Delta g(x) \, dx \\ &= J_g \, - \, \Delta g \end{aligned}$$

with

$$M^{012}_{g-OAM} \; = \; 2 \, \mathrm{Tr} \, [\, E^j \, (\boldsymbol{x} \times \boldsymbol{D}_{pure})^3 \, A^{phys}_j \,] \quad : \quad \texttt{canonical OAM}$$

$$+ \; 2 \, \mathrm{Tr} \, [\, \rho \, (\boldsymbol{x} \times \boldsymbol{A}_{phys})^3 \,] \quad : \quad \texttt{potential OAM term}$$

The gluon OAM extracted from the combined analysis of GPD and polarized PDF contains "potential OAM" term, in addition to "canonical OAM" !

It is natural to call the whole part the gluon "**dynamical OAM**" .

We want to make several **important remarks** on our decomposition.

♣ Our decomposition is Lorentz-frame independent !

This should be clear from the fact that the (G)PDFs appearing in the r.h.s. of our sum rules are manifestly Lorentz-invariant quantities !

Goldman argued that the nucleon spin decomposition is frame-dependent !

- T. Goldman, arXiv:1110.2533.

This would be generally true. In fact, Leader recently proposed a sum rule for transverse angular momentum.

- E. Leader, arXiv:1109.1230.

$$\langle J_T(quark) \rangle \;=\; \frac{1}{2\,M} \left[P_0 \int_{-1}^{1} x\, E^q(x,0,0)\, dx \;+\; M \int_{-1}^{1} x\, H^q(x,0,0)\, dx \right]$$

It is clear that this sum rule **does not** have a frame-independent meaning !

Note that our interest here is the most fundamental **longitudinal spin sum rule**.

♣ The longitudinal spin decomposition is certainly frame-independent !

Underlying reason why the **longitudinal spin sum rule** (or **helicity sum rule**) is **Lorentz-frame independent** seems to be clear.

The OAM component along the longitudinal direction comes from the motion in the perpendicular plane to this axis, and such transverse motion is not affected by the Lorentz boost along this axis.

$$
\begin{aligned}
M^{+12}_{q-OAM} &= \frac{1}{2}\,\bar{\psi}\,\gamma^{+}\,(\,x^{1}\,i\,\partial^{2} \;+\; x^{2}\,i\,\partial^{1}\,)\,\psi \\
&\quad + \; g\,\bar{\psi}\,\gamma^{+}\,(\,x^{1}\,A^{2}_{\perp} \;-\; x^{2}\,A^{1}_{\perp}\,)\,\psi
\end{aligned}
$$

$$
\begin{cases}
x_0 \;\rightarrow\; x'_0 \;=\; \gamma\left(x_0 \;-\; \frac{v}{c}\,x_3\right) \\
x_1 \;\rightarrow\; x'_1 \;=\; x_1 \\
x_2 \;\rightarrow\; x'_2 \;=\; x_2 \\
x_3 \;\rightarrow\; x'_3 \;=\; \gamma\left(x_3 \;-\; \frac{v}{c}\,x_0\right)
\end{cases}
\qquad
\begin{cases}
A_0 \;\rightarrow\; A'_0 \;=\; \gamma\left(A_0 \;-\; \frac{v}{c}\,A_{||}\right) \\
A_1 \;\rightarrow\; A'_1 \;=\; A_1 \\
A_2 \;\rightarrow\; A'_2 \;=\; A_2 \\
A_{||} \;\rightarrow\; A'_{||} \;=\; \gamma\left(A_{||} \;-\; \frac{v}{c}\,A_0\right)
\end{cases}
$$

with

$$
\gamma \;=\; \frac{1}{\sqrt{1-(v\,/\,c)^{2}}}
$$

Although our decomposition seems satisfactory in many respects, one subtle question remained. It is a role of **quantum-loop effects.**

[remaining important question]

Is ΔG gauge-invariant even at quantum level ? ⇨ **delicate question**

In fact, it was often claimed that ΔG has its meaning only in the LC gauge and in the infinite-momentum frame (for instance, by X. Ji and P. Hoodbhoy).

More specifically, in

- P. Hoodbhoy, X. Ji, and W. Lu, Phys. Rev. D59 (1999) 074010.

they claim that ΔG evolves differently in the LC gauge and the Feynman gauge.

However, the gluon spin operator used in their Feynman gauge calculation is

$$M^{+12}_{g-spin} = 2\,\mathrm{Tr}\,[\,F^{+1}\,A^2 \;-\; F^{+2}\,A^1\,]$$

which is delicately different from our gauge-invariant gluon spin operator

$$M^{+12}_{g-spin} = 2\,\mathrm{Tr}\,[\,F^{+1}\,A^2_{phys} \;-\; F^{+2}\,A^1_{phys}\,]$$

The problem is how to take account of this difference in the Feynman rule of evaluating 1-loop anomalous dimension of the quark and gluon spin operator.

This problem was attacked and solved in our 3rd paper

(iii) M. W., Phys. Rev. D84 (2011) 037501.

♣ We find that the calculation in the Feynman gauge (as well as in any covariant gauge including the Landau gauge) reproduces the answer obtained in the LC gauge, which is also the answer obtained by the Altarelli-Parisi method.

♣ So far, a direct check of the answer of Altarelli-Pasiri method for the evolution equation of ΔG within the operator-product-expansion (OPE) framework was limited to the LC gauge calculation, just because it was believed that there is **no gauge-invariant definition of gluon spin** in the **OPE framework**.

♣ This is the reason why the question of gauge-invariance of ΔG has been left in unclear status for a long time !

After establishing satisfactory natures of the decomposition (I), now we come to discussing another decomposition (II).

According to Chen-Wang-Goldman, the greatest advantage of the decomposition (II) is that their quark OAM operator $\boldsymbol{L}'_q \equiv -i\,\boldsymbol{x} \times (\nabla - i\,g\,\boldsymbol{A}_{pure})$ satisfies

$$\boldsymbol{L}'_q \times \boldsymbol{L}'_q = i\,\boldsymbol{L}'_q \quad \text{due to} \quad \nabla \times \boldsymbol{A}_{pure} = 0$$

It was claimed that this is crucial for its physical interpretation as an OAM.

However, this is not necessarily true, as discussed in

"**Commutation rules and eigenvalues of spin and orbital angular momentum of radiation fields**", S.J. Van Enk, G. Nienhuis, J. of Modern Optics, 41 (1994)963.

$$\boldsymbol{J}_\gamma \times \boldsymbol{J}_\gamma = i\,\boldsymbol{J}_\gamma \quad \text{but} \quad \boldsymbol{L}_\gamma \times \boldsymbol{L}_\gamma \neq i\,\boldsymbol{L}_\gamma, \quad \boldsymbol{S}_\gamma \times \boldsymbol{S}_\gamma \neq i\,\boldsymbol{S}_\gamma$$

Then, the claimed superiority of decomposition (II) over (I) is not actually present.

Nevertheless, since the decomposition (II) is also gauge-invariant, there still remains a possibility that it can be related to observables.

Recently, Hatta made important step toward this direction.

- Y. Hatta, Phys. Lett. B708 (2012) 186.

based on his formal **decomposition formula**

- Y. Hatta, P. R. D84, 041701 (R) (2011).

$$A^\mu(x) = A^\mu_{phys}(x) + A^\mu_{pure}(x)$$

$$A^\mu_{phys}(x) = -\int dy^- \, \mathcal{K}(y-x) \, \mathcal{W}^-_{xy} \, F^{+\mu}(y^-, \boldsymbol{x}) \, \mathcal{W}^-_{yx}$$

$$A^\mu_{pure}(x) = -\frac{i}{g} \mathcal{W}^-_{x,\pm\infty} \mathcal{W}_{\pm\infty} \left(\mathcal{W}^-_{x,\pm\infty} \mathcal{W}_{\pm\infty} \right)^\dagger$$

where

$$\mathcal{W}^-_{xy} \equiv \mathcal{P} \exp \left(-i\, g \int_{y^-}^{x^-} A^+(y'^-, \boldsymbol{x}) \, dy'^- \right)$$

$\mathcal{W}_{\pm\infty}$: Wilson line in the spatial ($\boldsymbol{x}$) direction at $x^- = \pm\infty$)

$\mathcal{K}(y^-)$ is either of the followings depending on the choice of LC gauge

$$\mathcal{K}(y^-) = \frac{1}{2}\epsilon(y^-), \quad \text{or} \quad \theta(y^-), \quad \text{or} \quad -\theta(-y^-)$$

Starting from a gauge-invariant expression of the **Wigner distribution** (or **generalized transverse-momentum-dependent PDF**, i.e. **GTMD**) as follows :

$$\boxed{f_L(x, q_T, \Delta)} \equiv \int \frac{dz^- \, d^2 z_T}{(2\pi)^3} \, e^{i\,x\,\bar{P}^+ z^- - i\,q_T\,z_T}$$

$$\times \langle P' S' \,|\, \bar{\psi}\left(-\frac{z^-}{2}, -\frac{z_T}{2}\right) \gamma^+ \, \mathcal{W}^-_{-\frac{z}{2}, \pm\infty} \, \mathcal{W}^T_{-\frac{z_T}{2}, \frac{z_T}{2}} \, \mathcal{W}^-_{\pm\infty, \frac{z}{2}} \, \psi\left(\frac{z^-}{2}, \frac{z_T}{2}\right) |\, PS \rangle$$

$\mathcal{W}^T$: Wilson line in the transverse direction at $x^- = - \pm \infty$

he showed the **relation**

$$\epsilon^{ij} \, L_{\text{"}can\text{"}} = \frac{1}{2P^+} \lim_{\Delta \to 0} \frac{\partial}{\partial \, i \, \Delta^i} \langle P' S' \,|\, \bar{\psi}(0) \, \gamma^+ \left(i \overrightarrow{D}^j_{pure} - i \overleftarrow{D}^j_{pure} \right) \psi(0) \,|\, PS \rangle$$

$$= \lim_{\Delta \to 0} \frac{\partial}{\partial \, i \, \Delta^i} \int dx \, d^2 q_T \; q^i_T \; f(x, q_T, \Delta)$$

$$= \epsilon^{ij} \frac{S^+}{P^+} \frac{1}{2} \int dx \, d^2 q_T \; q^2_T \; \tilde{f}(x, q^2_T, \xi = 0, \Delta_T \cdot q_T = 0)$$

"canonical" OAM $\Longleftrightarrow$ M.E. of a manifestly gauge invariant op.

♣ The GTMD $\tilde{f}$ does not appear in the usual classification of TMDs !

GTMDs (S. Meissner, A. Metz, and M. Schlegel, JHEP08(2009)056)

$$W^{[\gamma^+]}(x,\xi,\boldsymbol{q}_T^2,\boldsymbol{q}_T\cdot\boldsymbol{\Delta}_T,\boldsymbol{\Delta}_T^2;\eta)$$
$$= \frac{1}{2}\int\frac{dz^-\,d^2z_T}{(2\pi)^2}\,e^{k\cdot z}\,\langle p',\lambda'|\,\bar\psi\left(-\frac{z}{2}\right)\gamma^+\,\mathcal{W}\left(-\frac{z}{2},\frac{z}{2}\,|\,n\right)\psi\left(\frac{z}{2}\right)|p,\lambda\rangle_{z^+=0}$$
$$= \frac{1}{2M}\,\bar u(p',\lambda')\left[F_{1,1}+\frac{i\,\sigma^{i+}\,q_T^i}{P^+}F_{1,2}+\frac{i\,\sigma^{i+}\,\Delta_T^i}{P^+}F_{1,3}+\frac{i\,\sigma^{ij}\,q_T^i\,\Delta_T^j}{M^2}\boxed{F_{1,4}}\right]u(p,\lambda)$$

forward limit $(\Delta_T\to 0)$

$$F_{1,1}^e(x,0,\boldsymbol{q}_T^2,0,0)\;\Rightarrow\;f_1(x,\boldsymbol{q}_T^2),\qquad -F_{1,2}^o(x,0,\boldsymbol{q}_T^2,0,0;\eta),\;\Rightarrow\;f_{1T}^{\perp}(x,\boldsymbol{q}_T^2;\eta)$$

$F_{1,3}$, $F_{1,4}$ term : vanish !

Within the framework of light-cone quark model (non-gauge theory)

- C. Lorce and B. Pasquini, P.R. D84, 014015 (2011).

$$L_{can} = -\int dx\,d^2q_T\,\frac{\boldsymbol{q}_T^2}{M^2}\boxed{F_{1,4}^q(x,0,q_T^2,0,0)}$$

This is just the sum rule, to which Hatta gave gauge-invariant meaning.

really observable ?

5. Summary and outlook

- ♣ We have established the existence of **two** physically inequivalent decompositions of the nucleon spin, the **decompositions (I) and (II)**, with particular emphasis upon the existence of two types of OAM, i.e.

  "canonical" OAM & dynamical OAM

- ♣ It was shown that the dynamical OAMs of quarks and gluons appearing in the decomposition (I) can in principle be extracted model-independently from combined analysis of GPD and polarized DIS measurements.

- ♣ It is important to recognize that this longitudinal spin decomposition, has a Lorentz-frame independent meaning !

- ♣ Besides, the sum rule persists even at quantum level !

- ♣ This means that we now have at least one satisfactory solution to the nucleon spin decomposition problem.

- ♣ On the other hand, Hatta's recent work opened up a possibility that the OAM appearing in the decomposition (II) may also be related to observables.

- ♣ Since the relation between the OAM appearing in the decomposition (I) and the observables is already known, this means that we may be able to isolate the correspondent of "potential angular momentum" term appearing in Feynman's paradox of electodynamics.

$$L_{pot} = L_{mech} - L_{\text{“}can\text{”}}$$

- ♣ However, one must be careful about the presence of very delicate problem hidden in the sum rules containing generalized (and/or ordinary) TMDs.

- ♣ Once quantum loop effects is included, the very existence of TMDs satisfying gauge-invariance and factorization (universality or process independence) at the same time is being questioned !

$L_{\text{“}can\text{”}}$ ⇨ Is process-independent extraction possible ?

Still a challenging open question !

[Backup Slide] Nuclear spin decomposition problem

It is not a well-defined problem, because of the ambiguities of nuclear force.

$$H \;=\; \sum_i \frac{\boldsymbol{p}_i^2}{2\,m} \;+\; \sum_{i<j} V_{NN}(\boldsymbol{r}_i - \boldsymbol{r}_j)$$

To explain it, let us consider the deuteron, the simplest nucleus.

$$H\,\psi_d(\boldsymbol{r}) \;=\; E\,\psi_d(\boldsymbol{r})$$

$$\psi_d(\boldsymbol{r}) \;=\; \left[u(r) \;+\; \frac{S_{12}(\hat{\boldsymbol{r}})}{\sqrt{8}}\, w(r) \right] \frac{\chi}{\sqrt{4\,\pi}}$$

deuteron w.f. and S- and D-state probabilities

$$P_S \;=\; \int_0^\infty u^2(r)\, r^2\, dr, \quad P_D \;=\; \int_0^\infty w^2(r)\, r^2\, dr$$

angular momentum decomposition of deuteron spin

$$1 \;=\; \langle J_3 \rangle \;=\; \langle L_3 \rangle \;+\; \langle S_3 \rangle \;=\; \frac{3}{2} P_D \;+\; \left(P_S - \frac{1}{2} P_D \right)$$

We however know the fact that the D-state probability is not a direct observable !

- R.D. Amado, Phys. Rev. C20 (1979) 1473.
- J.L. Friar, Phys. Rev. C20 (1979) 325.

♣ 2-body unitary transformation arising in the theory of meson-exchange currents can change the D-state probability, while keeping the deuteron observables intact.

♣ The ultimate origin is non-uniqueness of short range part of NN potential.

infinitely many phase-equivalent potential !

♣ The D-state probability, for instance, depends on the cutoff Λ of short range physics in an effective theory of 2-nucleon system.

- S.K. Bogner et al., Nucl. Phys. A784 (2007) 79.

⇩

See the figure in the next page !

Deuteron **D-state probability** in an effective theory (Bogner et al., 2007)

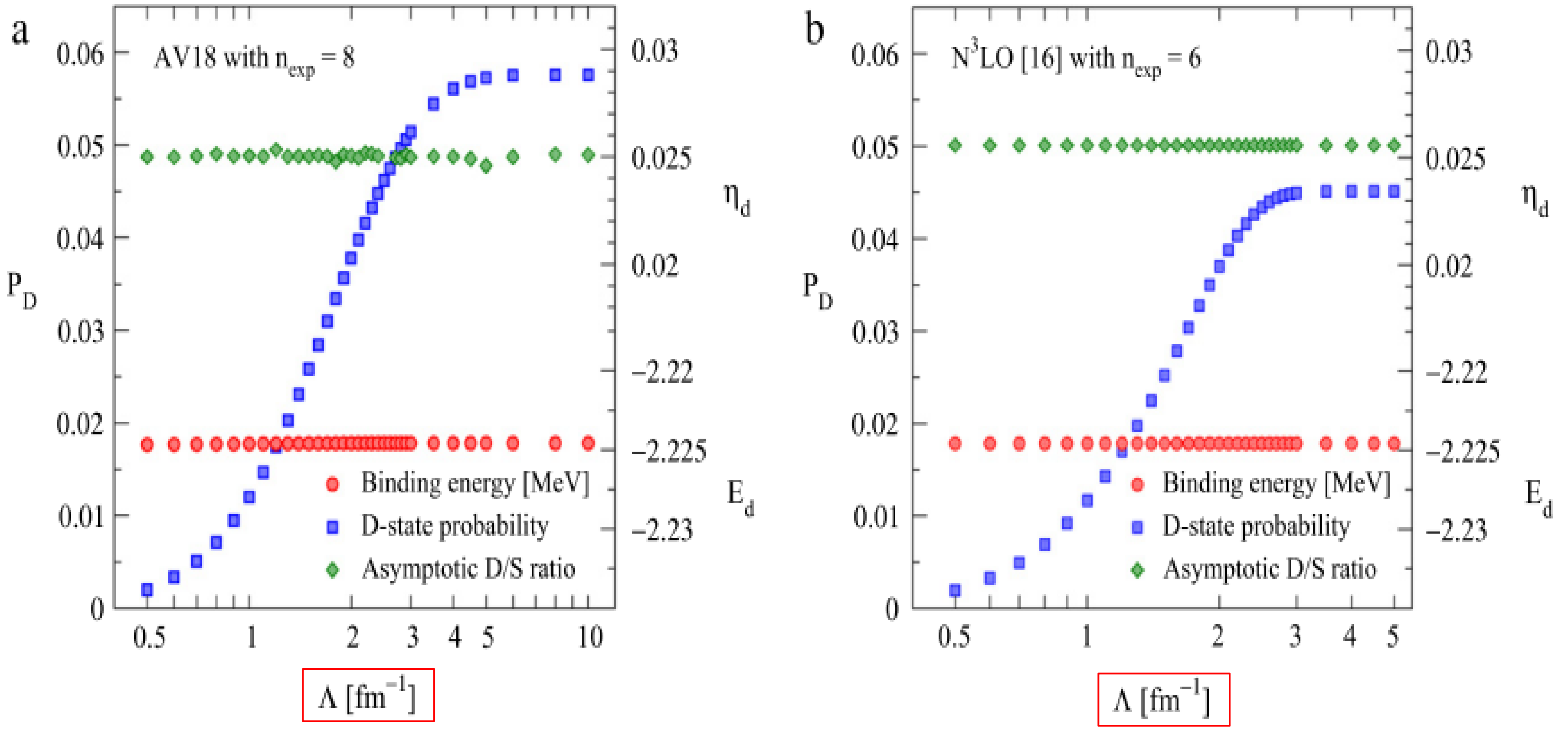


Fig. 57. D-state probability P_D (left axis), binding energy E_d (lower right axis), and asymptotic D/S-state ratio η_d (upper right axis) of the deuteron as a function of the cutoff [6], starting from (a) the Argonne v_{18} [18] and (b) the N^3LO NN potential of Ref. [20] using different smooth $V_{low\,k}$ regulators. Similar results are found with SRG evolution.

Note that the asymptotic D/S ratio corresponds to observables, although the D-state probability not !

[**A natural question**] **Why can we observe "dynamical OAM"** ?

- motion of a charged particle in static electric and magnetic fields

(See the textbook of J.J. Sakurai, for instance.)

$$\boldsymbol{E} \;=\; -\nabla\phi, \quad \boldsymbol{B} \;=\; \nabla \times \boldsymbol{A}$$

Hamiltonian

$$H \;=\; \frac{1}{2\,m}\,(\boldsymbol{p} - e\,\boldsymbol{A})^2 \;+\; e\,\phi$$

Heisenberg equation

$$\frac{d x_i}{dt} \;=\; \frac{1}{i\,\hbar}\,[\,x_i, H\,] \;=\; \frac{p_i - e\,A_i}{m}$$

One finds

$$\boldsymbol{\Pi} \;\stackrel{def}{\equiv}\; m\,\frac{d\boldsymbol{x}}{dt} \;=\; \boldsymbol{p} - e\,\boldsymbol{A} \;\neq\; \boldsymbol{p}$$

"dynamical momentum" "canonical momentum"

Equation of motion

$$m\,\frac{d^2\boldsymbol{x}}{dt^2} \;=\; \frac{d\boldsymbol{\Pi}}{dt} \;=\; e\left[\,\boldsymbol{E} \;+\; \frac{1}{2}\left(\frac{d\boldsymbol{x}}{dt}\times\boldsymbol{B} \;-\; \boldsymbol{B}\times\frac{d\boldsymbol{x}}{dt}\right)\right]$$

Equation of motion

$$m\frac{d^2\boldsymbol{x}}{dt^2} = \frac{d\boldsymbol{\Pi}}{dt} = e\left[\boldsymbol{E} + \frac{1}{2}\left(\frac{d\boldsymbol{x}}{dt}\times\boldsymbol{B} - \boldsymbol{B}\times\frac{d\boldsymbol{x}}{dt}\right)\right]$$

- What appears in Newton-Lorentz equation is dynamical momentum $\boldsymbol{\Pi}$ not canonical one $\boldsymbol{p}$.

- "Equivalence principle" of Einstein dictates that the "flow of inertia mass" can in principle be detected by using gravitational force as a probe.

- Naturally, the gravitational force is too weak to be used as a probe of mass flow in microscopic system.

- However, remember the fact that the 2nd moments of unpolarized GPDs are also called the gravito-electric and gravito-magnetic form factors.

- The fact that the dynamical OAM as well as dynamical linear momentum can be extracted from GPD analyses is therefore not a mere accident !