Recent progress in hadron spectroscopy on the lattice

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The myths about lattice spectroscopy

Myths: lattice QCD can …

… only study hadronic **ground-states**

… not study states with **high spin**

… not study **isoscalar** meson with precision

… not deal with **resonances** or compute **scattering** properties

- Where do these myths come from?
- How close to solving these problems are we?
- New results: most of these myths need to be re-examined

Where do these myths come from?

Mostly restrictions with standard techniques used to perform **numerical simulations**, particularly those needed to study **quarks**

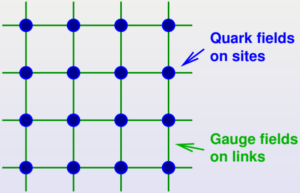
Are we close to solving these problems?

- New methods have enabled many of these challenges to be overcome.
- Can study excited and high spin states reliably
- New data on isoscalar mesons are almost as precise as isovector states
- Many collaborations publishing results on scattering and resonances.

Methods for lattice spectroscopy

Lattice regularisation

- Lattice provides a **non-perturbative, gauge-invariant** regulator for QCD
- Quarks live on sites
- Gluons live on links
- a - lattice spacing
- $a \sim 0.1$ fm



- The Nielson-Ninomiya theorem means chirally symmetric quarks are missing, but can discretise quarks by trading-off some symmetry
- In a finite volume $V = L^4$, finite number of degrees of freedom

Finite V: path-integral is an ordinary (but large) integral. Make predictions from the QCD lagrangian by **Monte Carlo**

Spectroscopy in lattice QCD

- Energies of colourless QCD states can be extracted from **two-point functions** in Euclidean time

$$C(t) = \langle 0|\, \Phi(t)\Phi^\dagger(0)\, |0\rangle$$

- Euclidean time: $\Phi(t) = e^{Ht}\Phi e^{-Ht}$ so $C(t) = \langle\Phi|e^{-Ht}|\Phi\rangle$. Insert a complete set of energy eigenstate and:

$$C(t) = \sum_{k=0}^{\infty} |\langle\Phi|k\rangle|^2 e^{-E_k t}$$

- $\lim_{t\to\infty} C(t) = Ze^{-E_0 t}$, so if observe large-t fall-off, then **energy of ground-state** is measured.

Euclidean metric very useful for spectroscopy; it provides a way of isolating and examining low-lying states

Excited states

- **Excited-state** energies can be measured by correlating between operators in a bigger set, $\{\Phi_1, \Phi_2, \ldots, \Phi_N\}$

$$C_{ij}(t) = \langle 0|\ \Phi_i(t)\Phi_j^\dagger(0)\ |0\rangle$$

- Solve generalised eigenvalue problem:

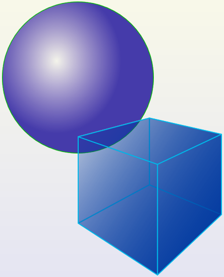
$$C(t_1)\ \underline{v} = \lambda\ C(t_0)\ \underline{v}$$

for different t_0 and t_1 [Lüscher & Wolff, C. Michael]

- Then $\lim_{(t_1-t_0)\to\infty} \quad \lambda_n = e^{-E_n(t_1-t_0)}$
- Method constructs optimal ground-state creation operator, then builds orthogonal states.

Excited states accessed if basis of creation operators is used and the matrix of correlators can be computed

Spin on the lattice



- Lattice breaks $O(3) \to O_h$
- Lattice states classified by quantum letter, $R \in \{A_1, A_2, E, T_1, T_2\}$.
- Continuum: subduce $O(3)$ irreps $\to O_h$
- Look for degeneracies. Problem: spin-4 has same pattern as $0 \oplus 1 \oplus 2$.
- Better spin assignment by constructing operators from lattice representation of convariant derivative.
- Start in continuum with operator of definite J, then subduce this into O_h and then replace derivatives with their lattice equivalent. Measure $\langle 0|\Phi|\ J^{PC}\rangle$ and look for remnants of continuum symmetries.

Remnants of continuum spin can be found on the lattice if we build operators more carefully and can measure their correlators

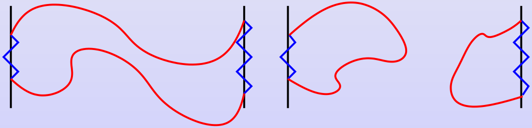
Isoscalar meson correlation functions

- Isovector mesons: Wick contraction gives

- Isoscalar meson correlator has extra diagram. Wick contraction:

$$\langle \psi_i \bar{\psi}_j \psi_k \bar{\psi}_l \rangle = M^{-1}_{ij} M^{-1}_{kl} - M^{-1}_{il} M^{-1}_{jk}$$

$$\begin{aligned} \langle 0|\Phi^{(I=0)}(t)\Phi^{\dagger(I=0)}(0)|0\rangle &= \\ \langle 0|\Phi^{(I=1)}(t)\Phi^{\dagger(I=1)}(0)|0\rangle &- \langle 0|\mathrm{Tr}\, M^{-1}\Gamma(t)\mathrm{Tr}\, M^{-1}\Gamma(0)|0\rangle \end{aligned}$$

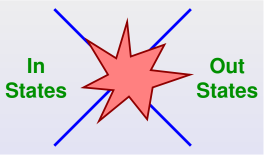


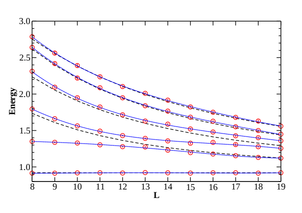
Measuring isoscalar meson correlation functions means also computing the disconnected Wick graphs by Monte Carlo.

Scattering

Scattering matrix elements not directly accessible from Euclidean QFT [Maiani-Testa theorem]

- Scattering matrix elements: asymptotic $|\mathrm{in}\rangle$, $|\mathrm{out}\rangle$ states. $\langle \mathrm{out}\,|e^{i\hat{H}t}|\,\mathrm{in}\rangle \to \langle \mathrm{out}\,|e^{-\hat{H}t}|\,\mathrm{in}\rangle$
- Euclidean metric: project onto ground-state





[D. McManus, P. Giudice & MP]

- **Lüscher's formalism:** information on elastic scattering inferred from **volume dependence** of spectrum
- Requires precise data, resolution of two-hadron and excited states.

Monte Carlo sampling the QCD lattice vacuum

Variance of Monte Carlo estimators is huge unless use importance sampling in Euclidean space-time

- In a **Euclidean** metric:

$$C(t_1, t_0) = \frac{\int DUD\bar{\psi}D\psi \quad \bar{\psi}_u(t_1)\Gamma\psi_d(t_1) \;\; \bar{\psi}_d(t_0)\Gamma\psi_u(t_0) \quad e^{-S_G - \bar{\psi}_u M\psi_u - \bar{\psi}_d M\psi_d}}{\int DUD\bar{\psi}D\psi \quad e^{-S_G - \bar{\psi}_u M\psi_u - \bar{\psi}_d M\psi_d}}$$

- Hard to deal with Grassmann algebra

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  ...so integrate out quark fields

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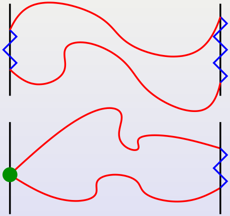
- Quenched approximation was to ignore $\det \ M^2$
- $N_f = 2$ **importance sampling measure**. Non-negative, thanks to Euclidean metric

The numerical tool-kit for quarks

- Physics focus of LQCD has been matrix elements, not spectroscopy.
- Traditionally, quark propagation computed starting with **point source**: $\eta(\underline{x}, t) = \delta_{t,0}\delta_{\underline{x},\underline{0}}$
- Solve $M\psi = \eta$, then ψ is one column of M^{-1}



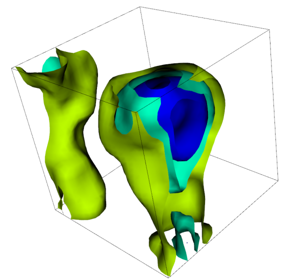
- QCD is translationally invariant
- With this trick, make simple mesons and baryons cheaply.
- Not so well suited to studying isoscalar mesons, higher-spin states, hybrids, large operator bases …

The “point-to-all” propagator has limited the scope of physics lattice QCD has addressed. Better calculations need “all-to-all”

New methods: distillation

- We can ameliorate the problems coming from measuring quark propagation by looking more carefully at how hadrons are constructed most efficiently
- **Smeared fields:** determine $\tilde{\psi}$ from the “raw” field in the path-integral, ψ:

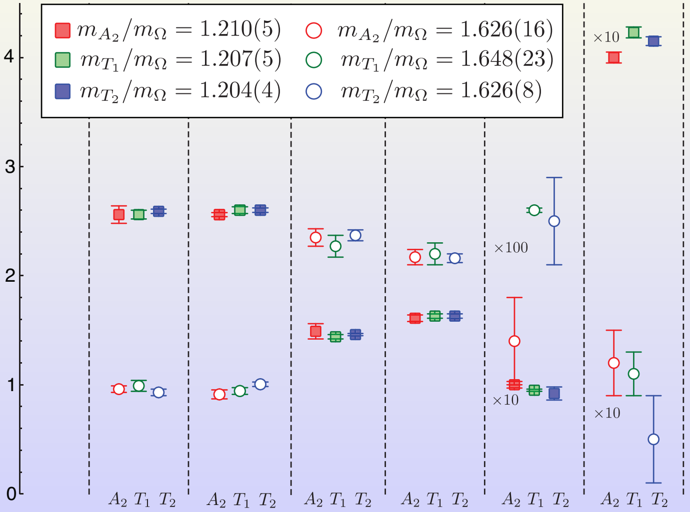
$$\tilde{\psi}(t) = \Box[U(t)]\psi(t)$$



- Extract confinement-scale degrees of freedom while preserving symmetries
- Build creation operators on smeared fields
- Re-define smearing to be a projection operator into a small vector space smooth fields: **distillation**

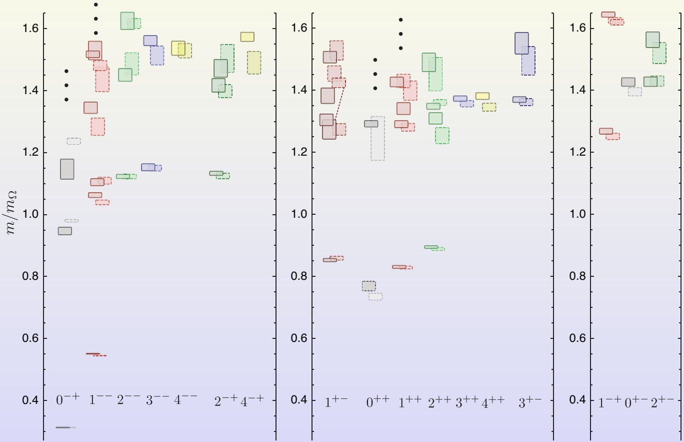
Results

Spin identification — $J = 3$ example



[Dudek et.al Phys.Rev.D82:034508,2010]

Isovector meson spectroscopy



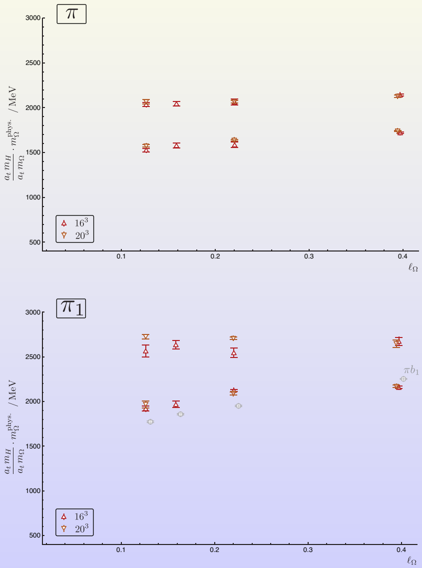
[Dudek et.al. Phys.Rev.D82:034508,2010]

- $m_\pi = 400$ MeV
- No 2-meson operators

Should be a dense spectrum of two-meson states:

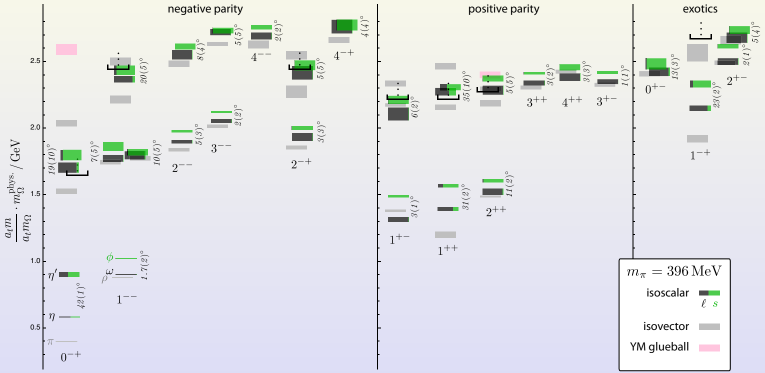
— Not seen at all

Light quark mass dependence



[Dudek et.al. Phys.Rev.D82:034508,2010]

Isoscalar mesons



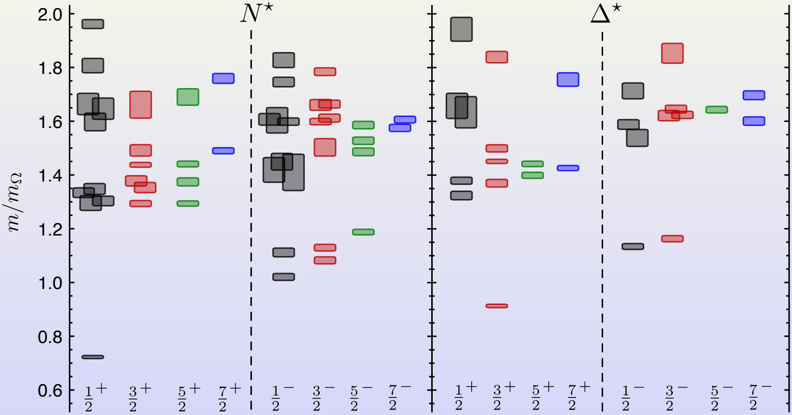
[Dudek et.al. Phys.Rev.D83:111502,2011]

- $m_\pi = 400$ MeV, finite a
- No 0^{++} data presented
- No glueball or two-meson operators

Statistical precision:

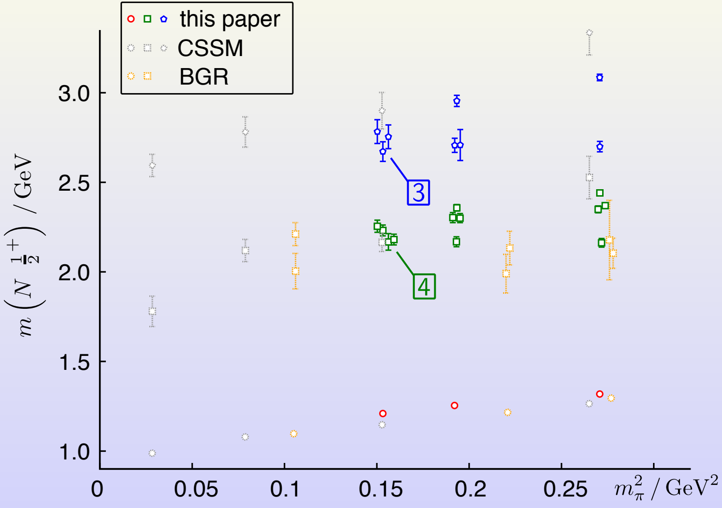
η	0.5 %
η'	1.9 %

N and Δ spectroscopy



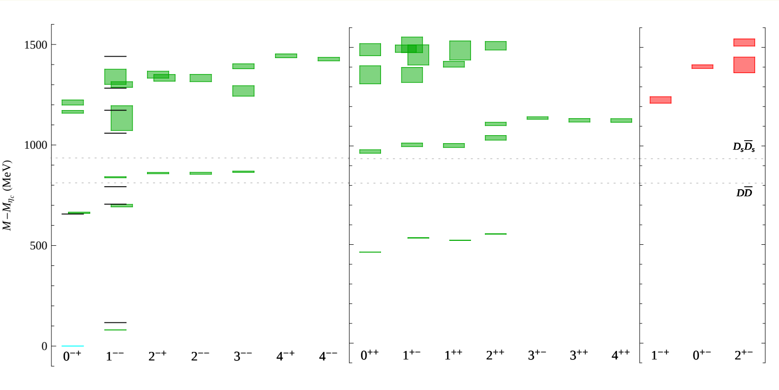
[Edwards et.al. Phys.Rev.D84:074508,2011]

Light quark mass dependence — the Roper?



[Edwards et.al. Phys.Rev.D84:074508,2011]

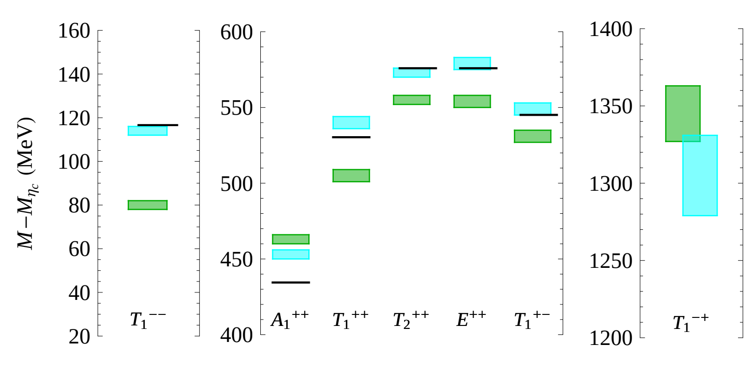
Charmonium spectrum [PRELIMINARY]



Hadron Spectrum: in preparation

- Resolve states up to $J = 4$; most fit into quark model
- 1S, 1P, 2S, 1D, 2P, 1F, 2D all seen
- Not all fit quark model: spin-exotic (and non-exotic) hybrids seen

Charmonium hyperfine structure



Hadron Spectrum Collab. in preparation

- Hyperfine structure is sensitive to finite-lattice-spacing artefacts
- Change lattice action to investigate their influence
- Spin-exotic 1^{-+} moves only about 50MeV

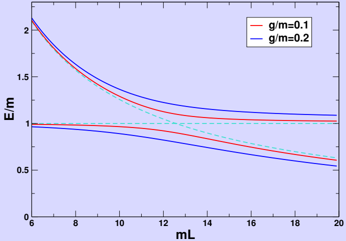
Hadrons in a finite box: scattering

- On a finite lattice with periodic b.c., hadrons have quantised momenta; $\underline{p} = \frac{2\pi}{L}\left\{n_x, n_y, n_z\right\}$
- Two hadrons with total $P = 0$ have a discrete spectrum
- These states can have same quantum numbers as those created by $\bar{q}\Gamma q$ operators and QCD can mix these

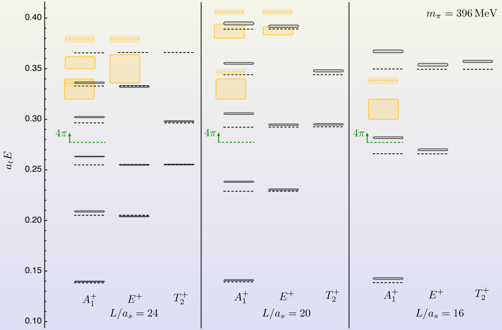
- This leads to shifts in the spectrum in finite volume
- This is the same physics that makes resonances in an experiment
- Lüscher's method - relate elastic scattering to energy shifts

Toy model

$$H = \begin{pmatrix} m & g \\ g & \frac{4\pi}{L} \end{pmatrix}$$



$I = 2$ $\pi - \pi$ phase shift

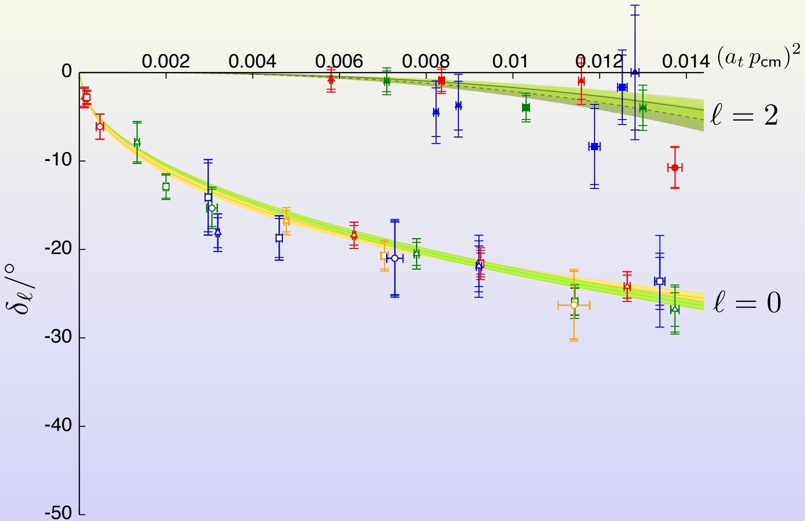


- Lüscher's method: first determine energy shifts as volume changes
- Data for $L = 16a_s, 20a_s, 24a_s$
- Small energy shifts are resolved

- Measured δ_0 and δ_2 (δ_4 is very small)
- $I = 2$ a useful first test - simplest Wick contractions

Dudek et.al. [Phys.Rev.D83:071504,2011, arXiv:1203.6041]

I = 2 $\pi - \pi$ phase shift

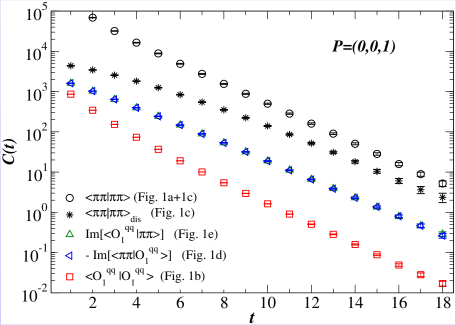


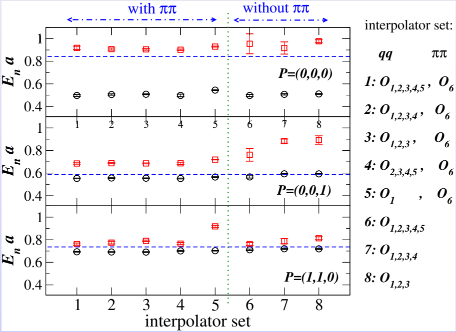
Dudek et.al. [Phys.Rev.D83:071504,2011, arXiv:1203.6041]

I = 1 scattering using distillation

[C.Lang et.al. arXiv:1105.5636]

- Number of groups have measured Γ_ρ on the lattice.
- Need non-zero relative momentum of pions in final state (P-wave decay)
- New calculation using distillation

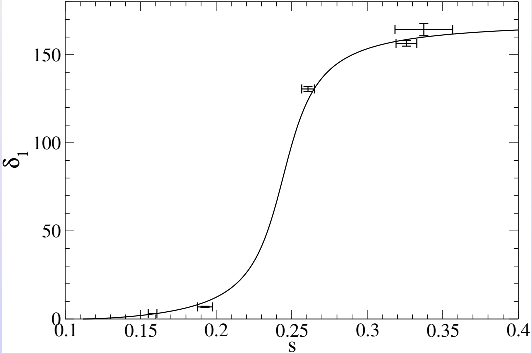




$I = 1$ $\pi\pi$ phase shift

[C.Lang et.al. arXiv:1105.5636]

- $m_\pi \approx 266$ MeV
- Better resolution by studying moving ρ as well
- ρ resonance resolved clearly, with $m_\rho = 792(7)(8)$ MeV
- $g_{\rho\pi\pi} = 5.13(20)$



Conclusions

- Precision spectroscopy from the lattice is improving rapidly:
  - Variational methods have enabled **excited states** to be studied
  - More sophisticated operator construction allows us to disentangle **higher spins** in lattice data
  - **Isoscalar mesons** are determined at similar precision
  - **Lüscher's method** links the spectrum in finite volume to scattering properties.
- These developments have been enabled by **extending the toolkit** for measuring quark propagation on the lattice
- Look out for better data on scattering soon …
- …but inelastic thresholds remain a challenge