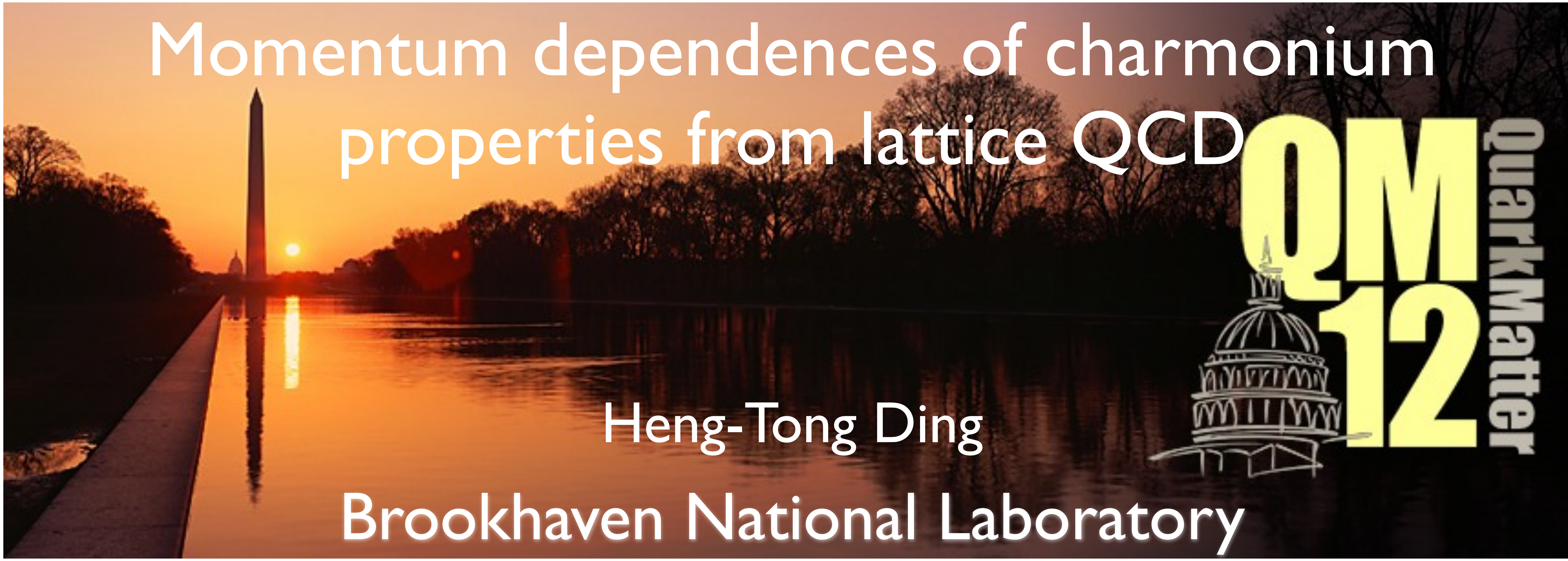

Momentum dependences of charmonium properties from lattice QCD

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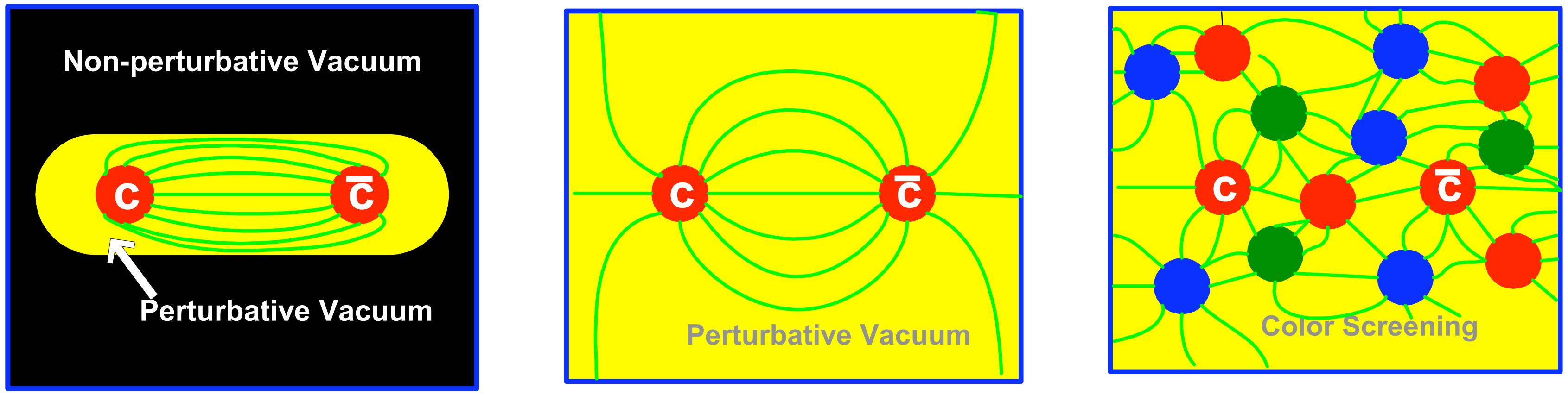
- Introduction & Motivation
- Fate of charmonium states at vanishing & nonzero momenta
- Summary & Conclusion

Charmonium at T>0

★ The suppression of the charmonium production can be a signature of the formation of the Quark Gluon Plasma [Matsui & Satz '86]

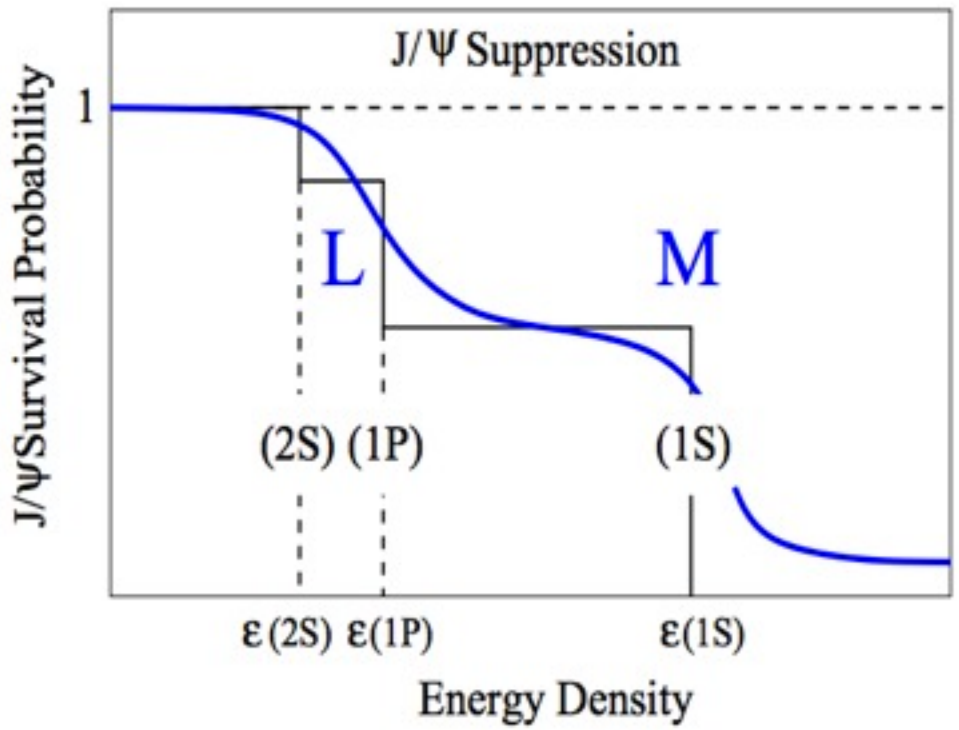
Debye screening picture



$$V(r) = -\frac{\alpha_{eff}}{r} + \sigma\, r$$

$$V(r,T) = -\frac{\alpha_{eff}}{r}\, e^{-r/\lambda_D(T)}$$

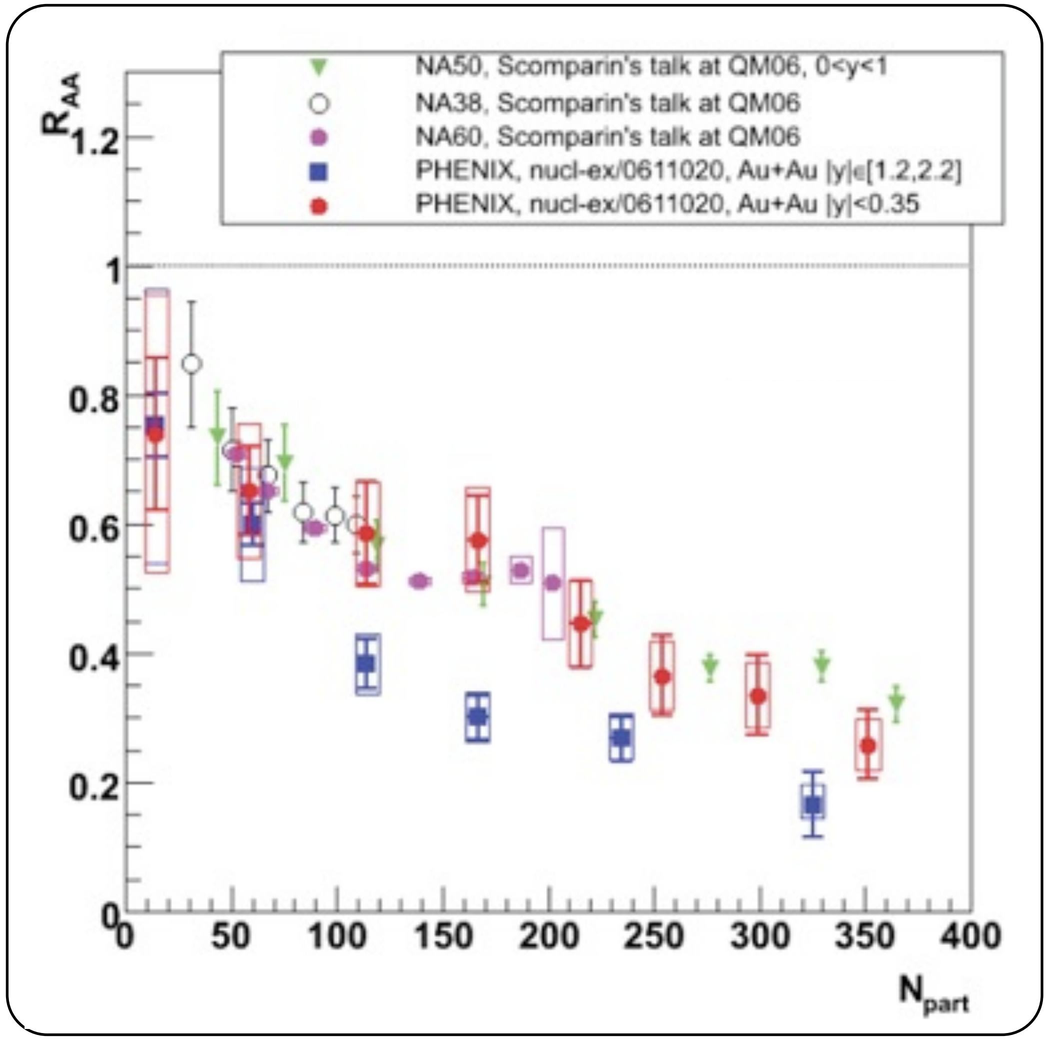
★ Sequential suppression of quarkonium states: QGP thermometer [F. Karsch, D. Kharzeev & H. Satz '06]



Due to various sizes and binding energies of charmonia, they may dissolve at different temperatures

The interpretation of experimental data

- Two different classes of effects on the modification of the charmonium production
  - ★ Cold nuclear effects: nuclear shadowing, comovers...
  - ★ Hot medium effects: color screening effects...
- Crucial to disentangle these two classes of effects to understand experimental pictures
  - ★ Dissociation temperatures from lattice QCD
  - ★ Examine the sequential dissociation picture
- Rapidity & pt dependences of R_{AA}



comparison produced by R. Granier de Cassagnac

★ What happens to charmonia when moving with respect to the hot thermal medium ? Will they see more energetic gluons and dissolve earlier ?

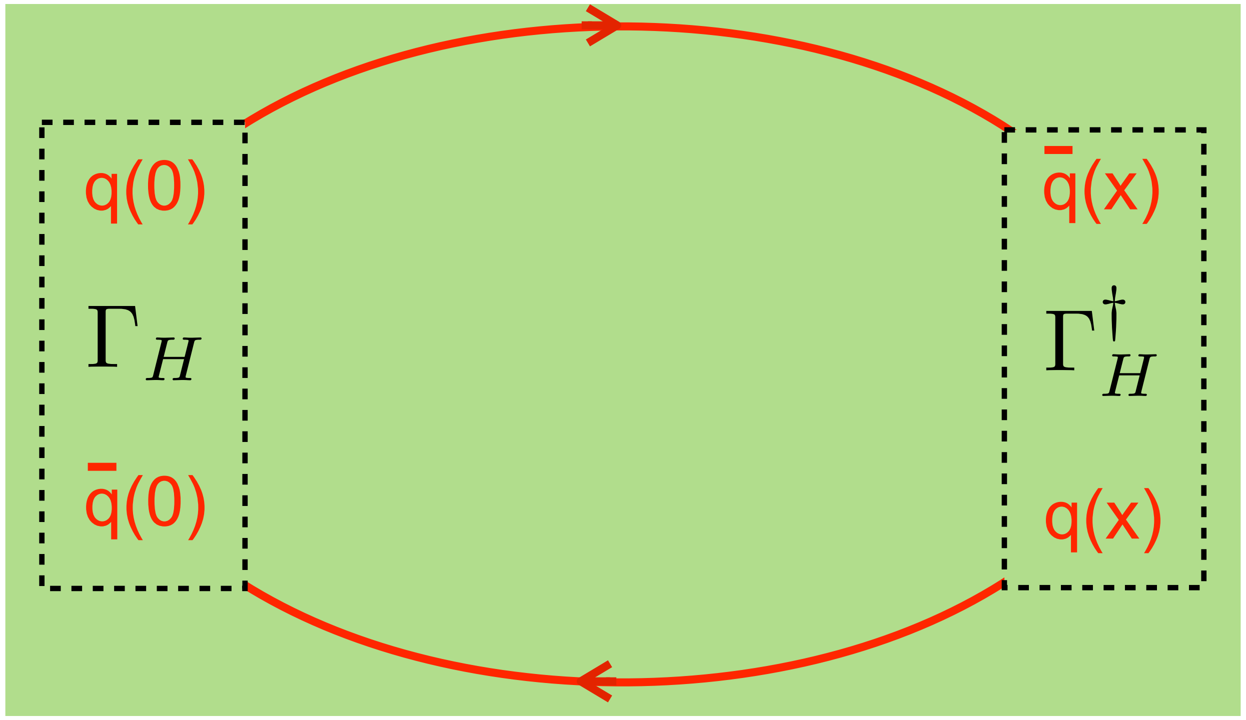
★ Model & Effective theory calculations indicate that width of J/ψ increases at finite p
[K. Haglin & C. Gale, PRC 63(2001)065201, M. Escobedo, M. Mannarelli & J. Soto, PRD84(2011)016008]

★ Lattice QCD: charmonium spectral functions, screening masses etc.

Temporal correlation and spectral functions

$$G(\tau,\vec{p},T)=\sum_{\vec{x}}\exp(-i\vec{p}\cdot\vec{x})\left\langle J_H(0,\vec{0})J_H^\dagger(\tau,\vec{x})\right\rangle$$

$$J_H(\tau,\vec{x})=\bar{q}(\tau,\vec{x})\,\Gamma_H\,q(\tau,\vec{x})$$



Channel	Γ_H	$^{2S+1}L_J$	J^{PC}	$c\bar{c}$	$M(c\bar{c})$[GeV]
PS	γ_5	$^1S_0$	0^{-+}	η_c	2.980(1)
VC	γ_μ	$^3S_1$	1^{--}	J/ψ	3.097(1)
SC	1	$^3P_0$	0^{++}	χ_{c0}	3.415(1)
AV	$\gamma_5\gamma_\mu$	$^3P_1$	1^{++}	χ_{c1}	3.510(1)

$$G(\tau,T)=D^+(-i\tau)\,,\qquad \rho(\omega)=2\mathrm{Im}D^R(\omega)=D^+(\omega)-D^-(\omega)$$

Spectral representation

$$G(\tau,\vec{p},T)=\int\frac{\mathrm{d}\omega}{2\pi}\,\rho(\omega,\vec{p},T)\,K(\tau,\omega,T);\quad K(\tau,\omega,T)=\frac{\cosh(\omega(\tau-\frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

Spatial correlation function and screening mass

- Spatial correlation functions: sum over transverse plane (x,y) and temporal direction τ

$$G(z,\vec{p}_\perp,p_\tau) = \sum_{\tau,x,y} \exp(-i\tilde{p}\cdot\tilde{x}) \left\langle J_H(0,\vec{0}) J_H^\dagger(\tau,\vec{x}) \right\rangle$$

- Relation to the spectral function

$$G(z,\vec{p}_\perp,p_\tau) = \int_{-\infty}^{\infty} \frac{\mathrm{d}p_z}{2\pi} \exp(ip_z\, z) \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{2\omega}{\omega^2+p_\tau^2} \rho(\omega,\vec{p}_\perp,p_z,p_\tau,T).$$

- Spatial correlator G(z) decays exponentially with E_{sc} at large distances

$$G(z) \sim \exp(-E_{sc}\, z), \quad E_{sc}^2 = \vec{p}^2 + M^2 + \Pi(\vec{p},T)$$

At p=0, E_{sc} gives the screening mass
At p=0 and T=0, E_{sc} gives the pole mass

- Temperature effects might be absorbed into M(T) and A(T)

$$E_{sc}^2 \simeq A^2(T)\vec{p}^2 + M^2(T)$$

Maximum Entropy Method [Asakawa, Hatsuda & Nakahara, '01]

- Difficult to extract spectral function (spf)

$$G(\tau,T) = \int_0^\infty \frac{d\omega}{2\pi}\, K(\tau,\omega,T)\, \rho(\omega,T)$$

Discretized $\mathcal{O}(10)$ — Continuous $\mathcal{O}(10^3)$

χ^2 fitting inconclusive

- Maximum Entropy Method (MEM) ⟵ Bayesian theorem
  - A method to obtain the most probable image from insufficient data
  - Ingredients of MEM: $P[\rho|GH] \propto P[G|\rho H]\ P[\rho|H]$

$P[G|\rho H] \propto \exp(-\chi^2/2)$: likelihood function

$P[\rho|H] \propto \exp(\alpha S)$: prior probability

ρ: spectral function
G: lattice data
H: prior information on ρ

Information entropy: $S = \int_0^\infty \frac{d\omega}{2\pi}\left[\rho(\omega) - m(\omega) - \rho(\omega)\ln\left(\frac{\rho(\omega)}{m(\omega)}\right)\right]$

Default Model (DM): m(ω), includes the prior information on ρ, e.g. ρ is positive-definite

DM is the only input parameter in the MEM analysis

- Important to check the dependence of output spf on DMs

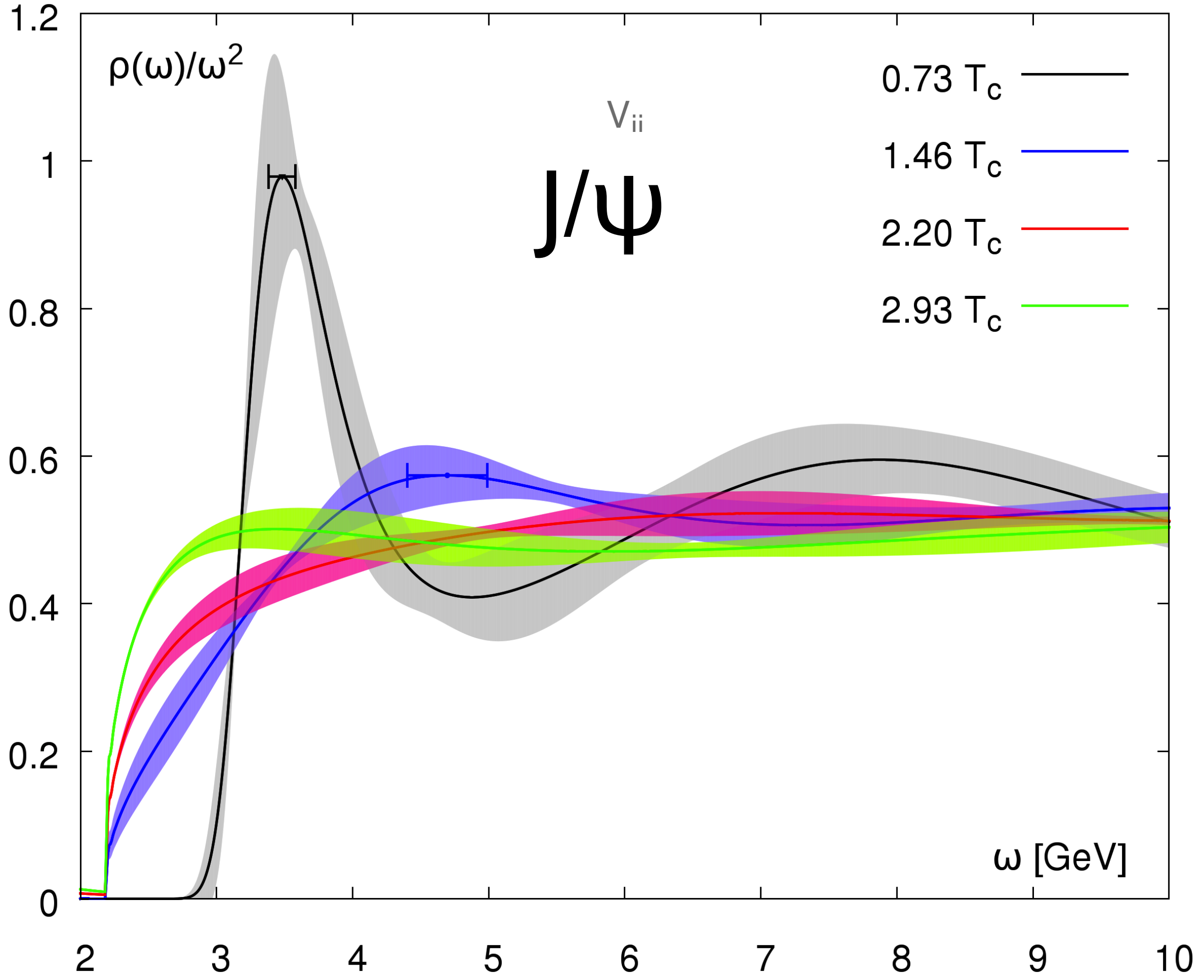
Lattice setup

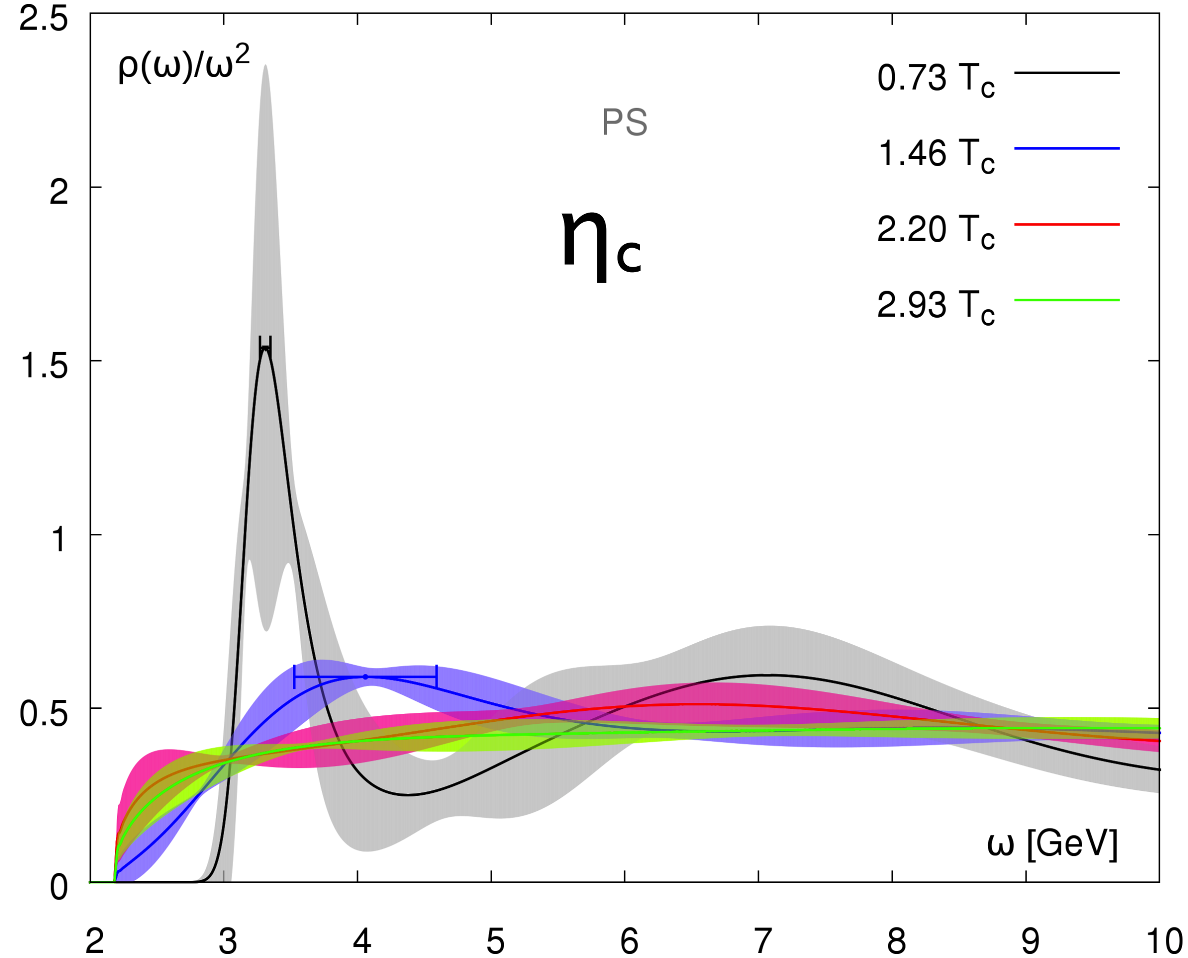
- ★ non-perturbatively clover improved Wilson fermions
- ★ isotropic quenched lattice
- ★ very fine lattice close to continuum
- ★ simulation parameters tuned to reproduce nearly physical J/ψ mass
- ★ large N_τ makes the extraction of spf more reliable

β	a [fm]	a^{-1}[GeV]	L_σ [fm]	c_{SW}	κ	$N_\sigma^3 \times N_\tau$	T/T_c	N_{conf}
7.793	0.010	18.974	1.33	1.310381	0.13200	$128^3 \times 96$	0.73	234
						$128^3 \times 48$	1.46	461
						$128^3 \times 32$	2.20	105
						$128^3 \times 24$	2.93	81

- ★ momentum of meson p on the lattice in units of $\tilde{p} = 2\pi/aN_\sigma = 0.93$ GeV
  $p = (0, 1, \sqrt{2}, \sqrt{5}, 2\sqrt{2}, 3, \ldots)\tilde{p}$

Charmonium spf at vanishing momentum



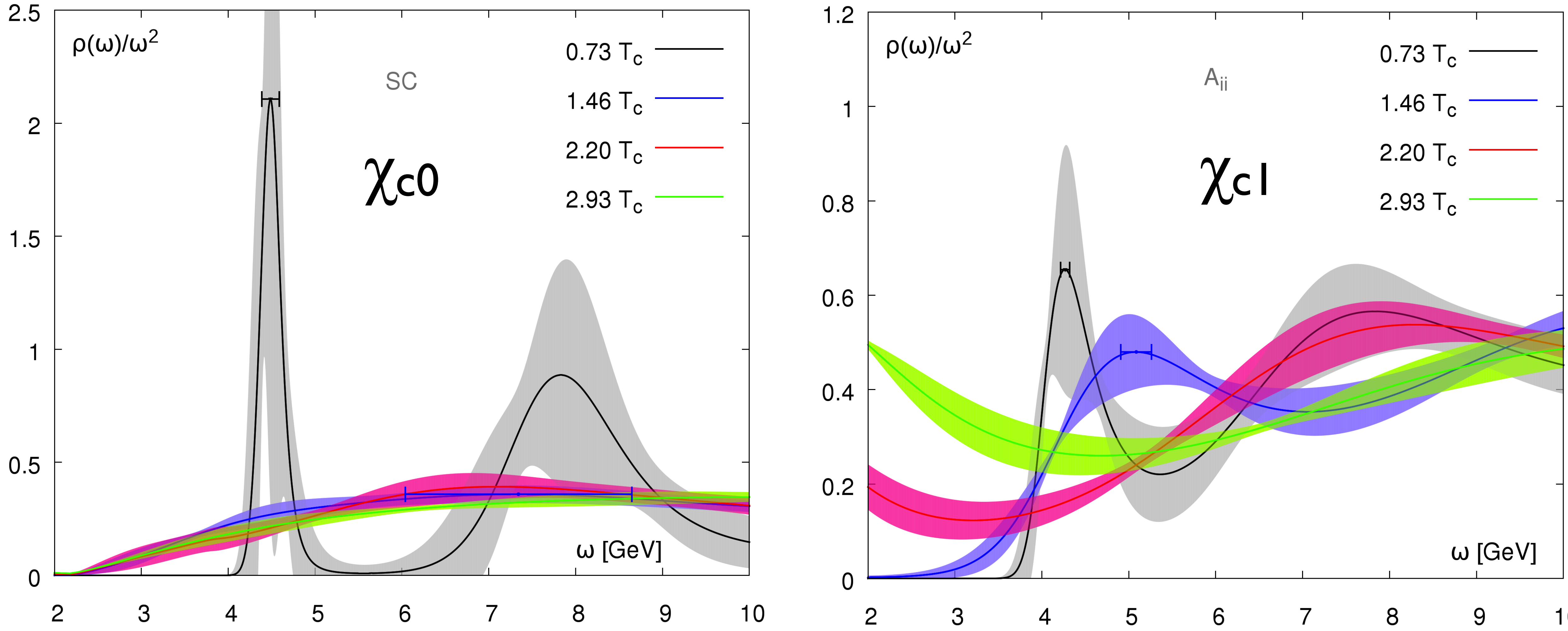


HTD, A. Francis, O. Kaczmarek, F. Karsch, H. Satz and W. Soeldner, PRD 86, 014509 (2012)

S wave states (J/ψ and η_c) are dissociated at $T \geq 1.46\, T_c$

- the location of the first peak at $0.73T_c$ corresponds to meson mass in the vacuum, and width of the peak at $0.73T_c$ is too large to be physical
- "peak location" shifted to large w region by ~1 GeV from 0.73 to $1.46\, T_c$
- different from previous lattice calculations: S wave states survive up to $T \gtrsim 2T_c$
- isotropic lattices with larger N_T used in the current study

Charmonium spf at vanishing momentum

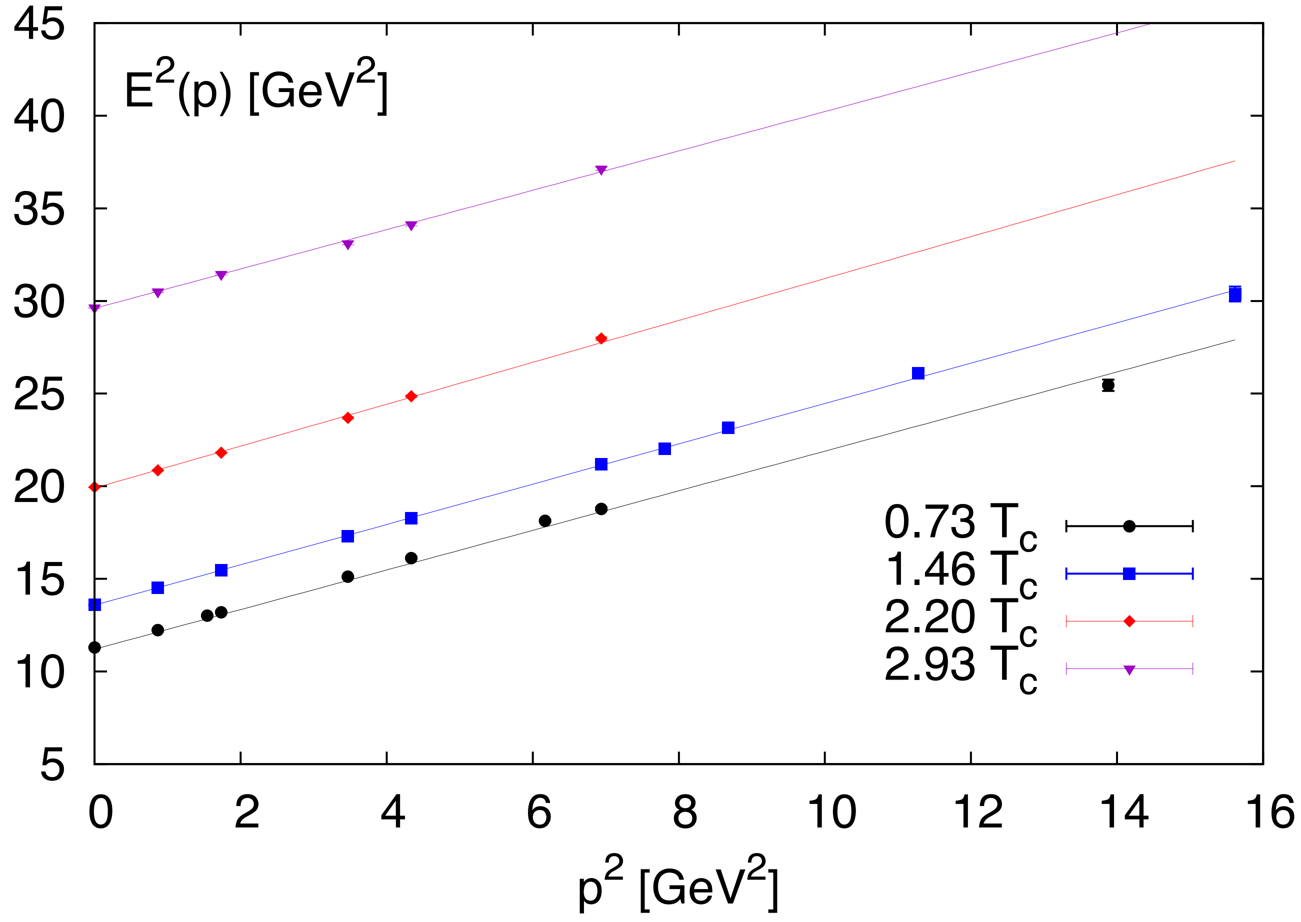


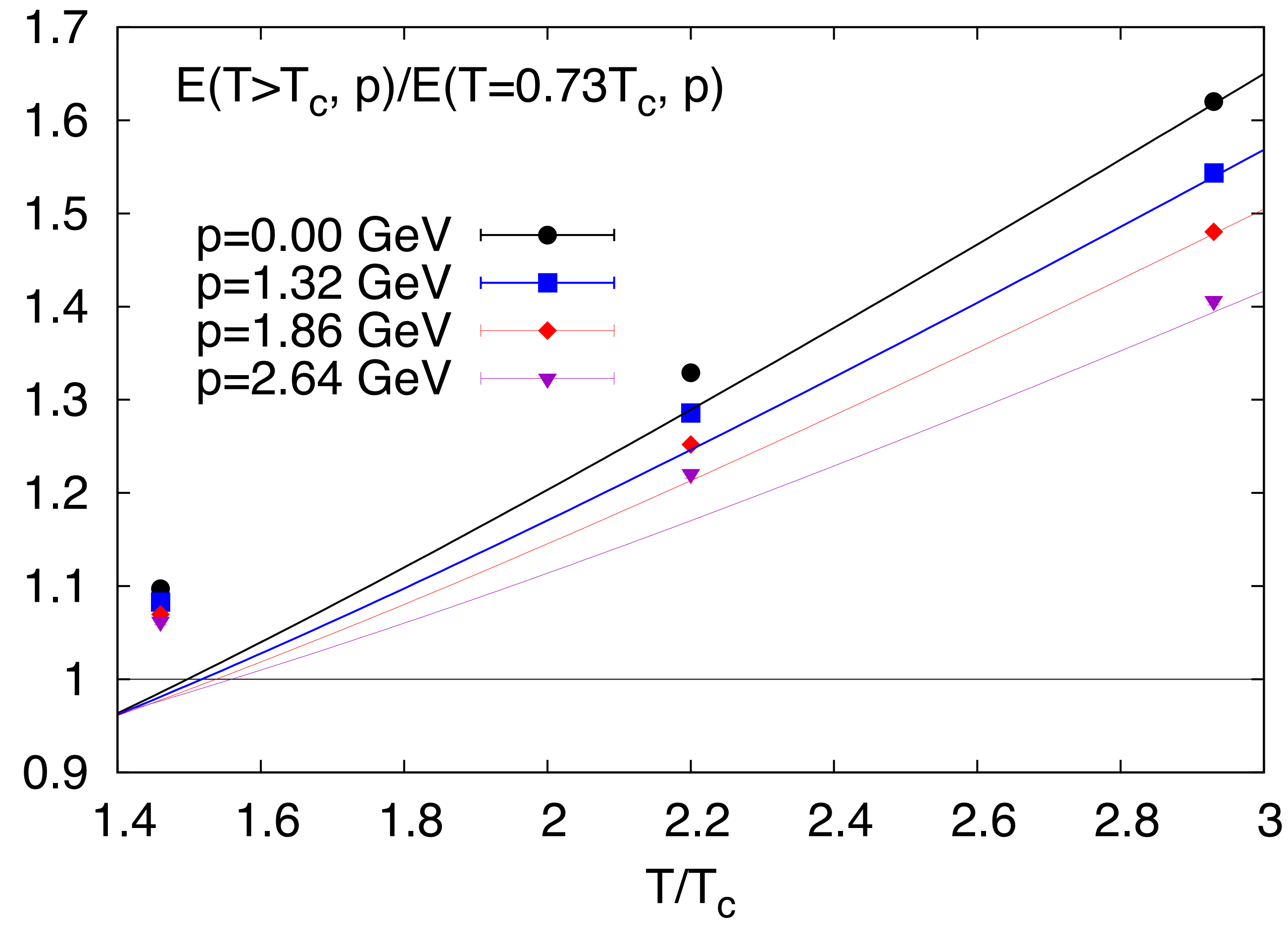
HTD, A. Francis, O. Kaczmarek, F. Karsch, H. Satz and W. Soeldner, PRD 86, 014509 (2012)

P wave states (χ_{c0} and χ_{c1}) are dissociated at $T \geq 1.46\, T_c$

- consistent with previous lattice calculations

Dispersion relation in the PS channel





- The dispersion relation $E^2=A^2(T)\ p^2 + M^2(T)$ changes only in the intercept i.e. M(T)
- The slope of dispersion relation, A(T), has minor dependence on T
- Ratio of E at $T>T_c$ to that at $T<T_c$ approach to the free field theory as expected

Ratio of G/G$_{rec}$ in the pseudo scalar channel

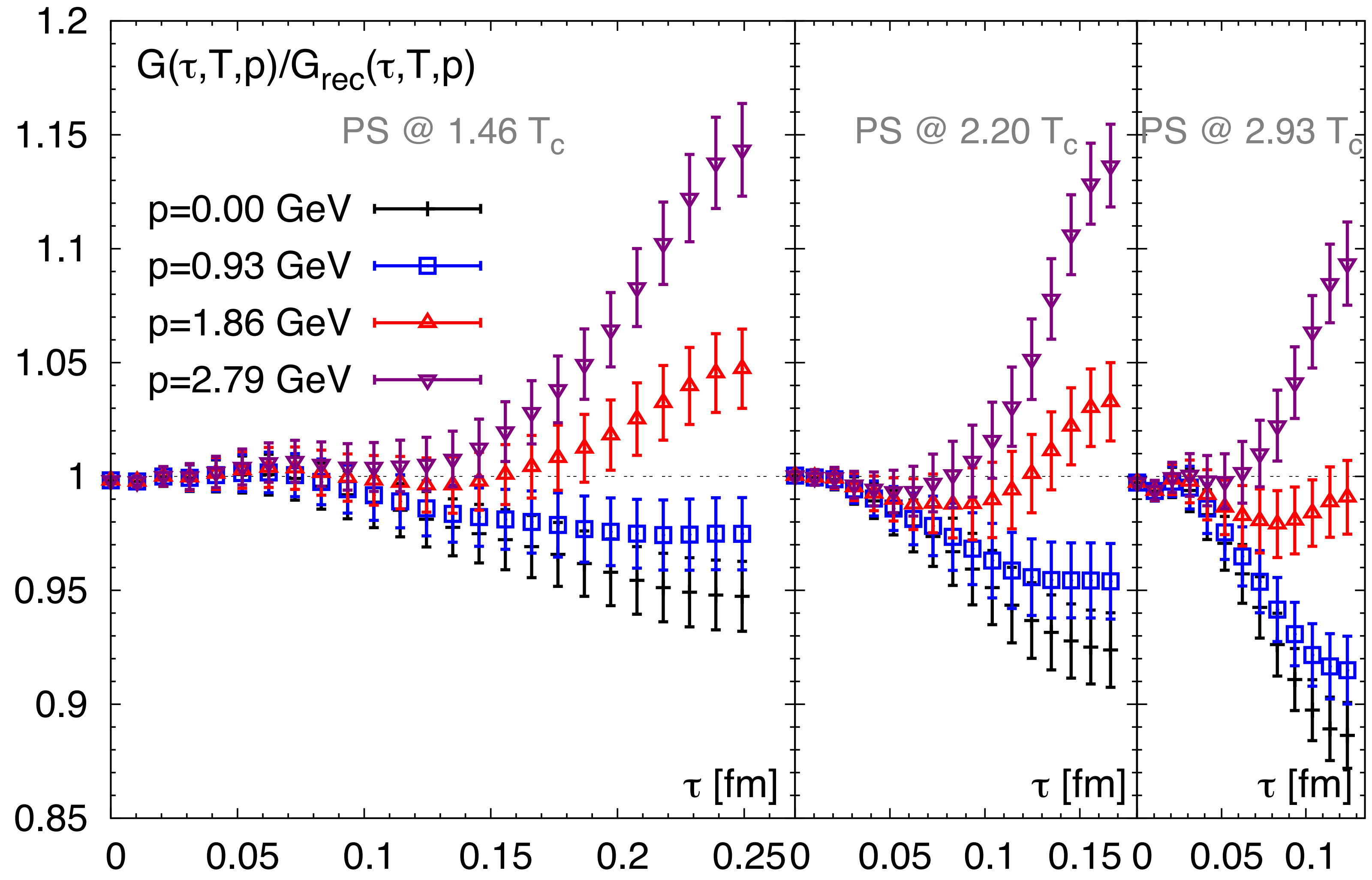
$$\frac{G(\tau,T,p)}{G_{rec}(\tau,T,p)} = \frac{\int \frac{d\omega}{2\pi}\, \rho(\omega,p,T)\, K(\omega,\tau,T)}{\int \frac{d\omega}{2\pi}\, \rho(\omega,p,T')\, K(\omega,\tau,T)}$$

Deviation of the ratio from unity indicates thermal modifications

- Evaluate $G(\tau,T,p)/G_{rec}(\tau,T,p)$ directly from correlator data without knowing $\rho(\omega,p,T')$

[HTD et al., '12]

η_c



- The temperature effects bring the ratio down while the momentum effects bring the ratio up at the large distance

Ratio of G/G_{rec} in the vector channel

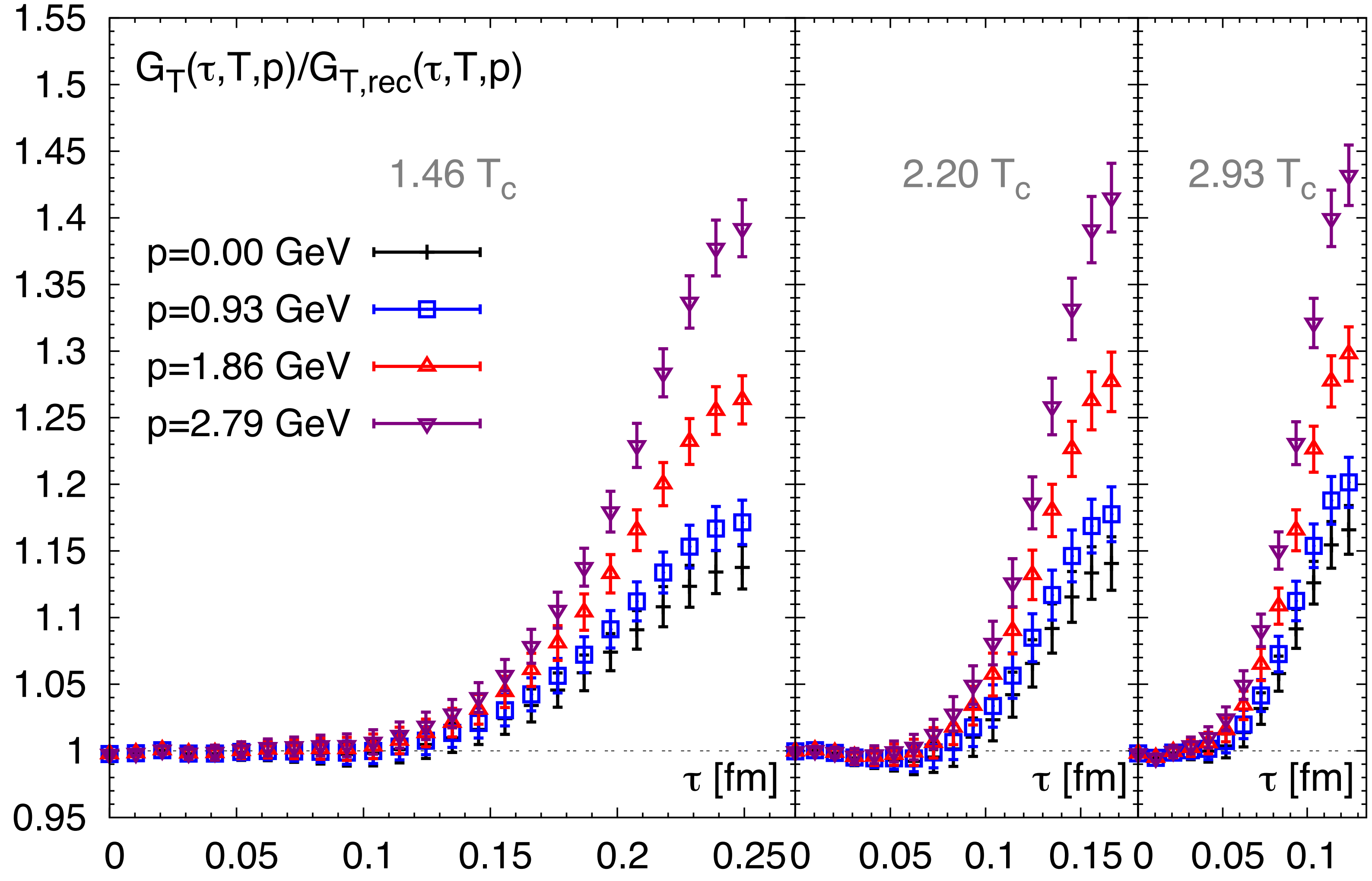
$$\frac{G(\tau,T,p)}{G_{rec}(\tau,T,p)} = \frac{\int \frac{d\omega}{2\pi}\, \rho(\omega,p,T)\, K(\omega,\tau,T)}{\int \frac{d\omega}{2\pi}\, \rho(\omega,p,T')\, K(\omega,\tau,T)}$$

Deviation of the ratio from unity indicates thermal modifications

- Evaluate $G(\tau,T,p)/G_{rec}(\tau,T,p)$ directly from correlator data without knowing $\rho(\omega,p,T')$

[HTD et al., '12]

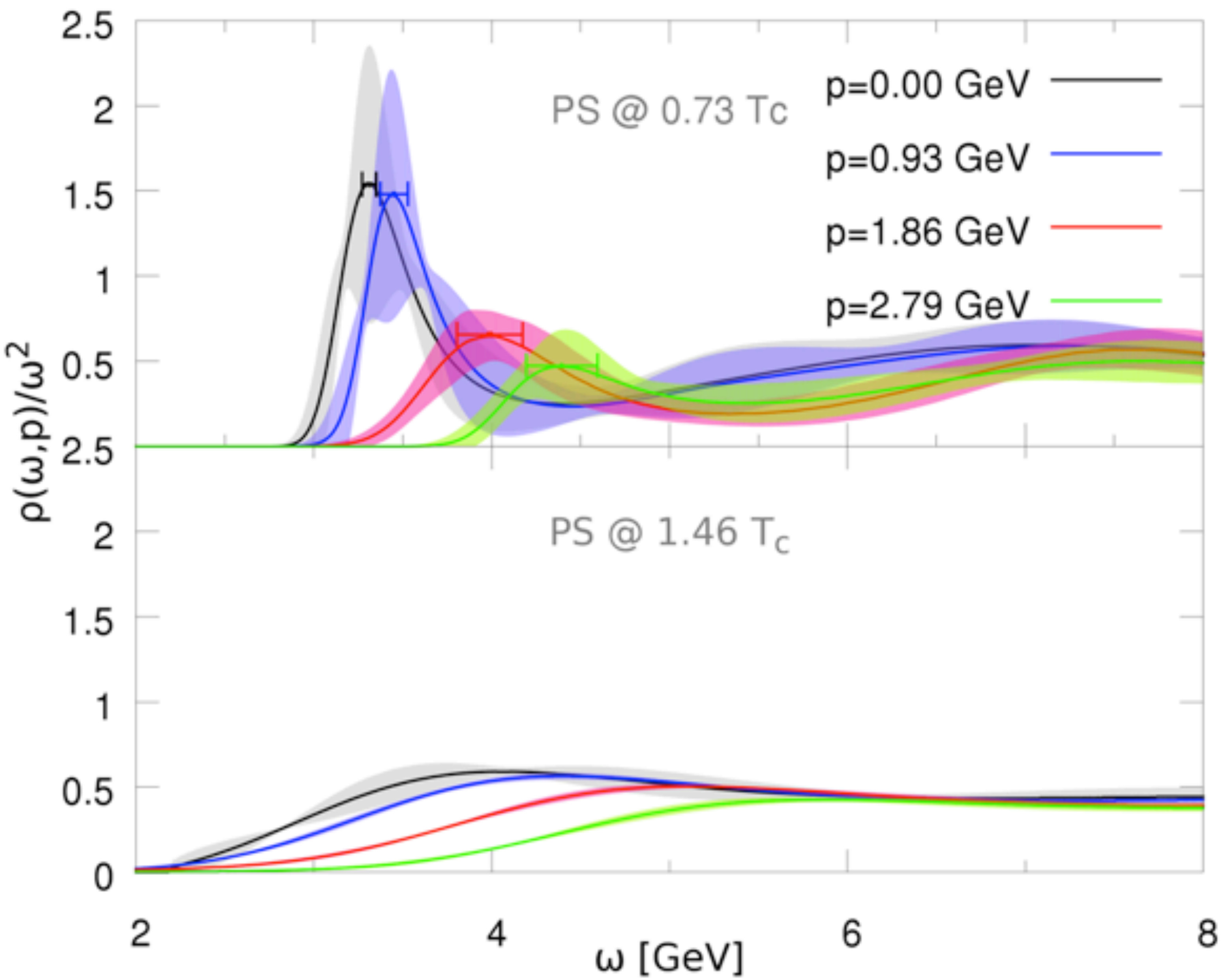
J/ψ_T



- G/G_{rec} grows with momentum at large distance

Charmonium spf at nonzero momenta

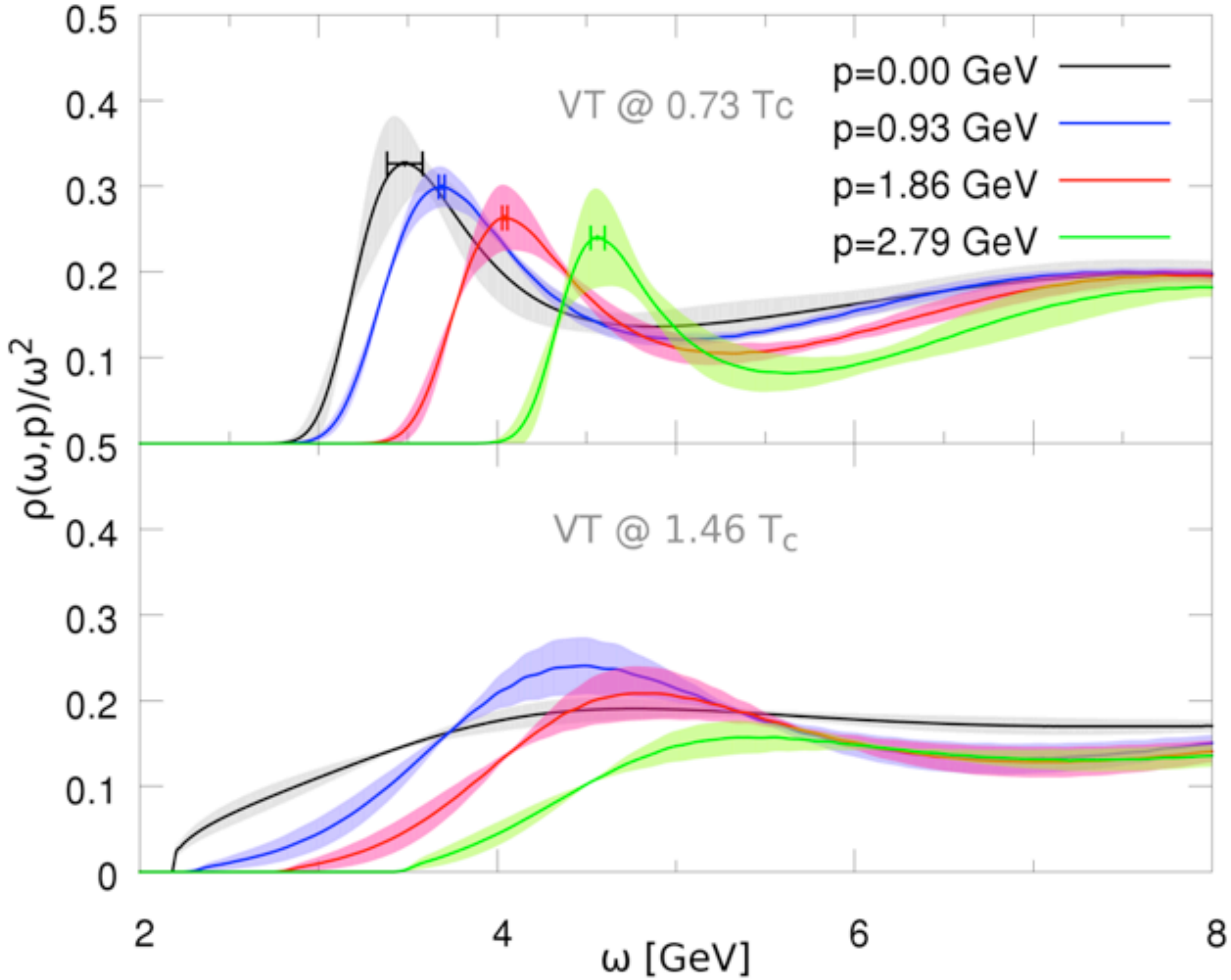
η_c



- At T=0.73T_c:The shape of η_c peak persists at finite momenta
- The peak locations at T<T_c are consistent with the dispersion relation obtained from spatial correlators
- At T=1.46 T_c spf gets closer to the free theory with larger p

Charmonium spf at nonzero momenta

J/ψ_T



- Similar as the PS channel

Conclusion

- We have calculated charmonium correlation functions on large and fine isotropic lattices: $128^3 \times N_T$, N_T=96,48,32,24 for 0.73, 1.46, 2.20 and 2.93 T_c

- Through MEM analysis, all the charmonium ground states at vanishing momenta are dissociated at $T \geq 1.46\, T_c$

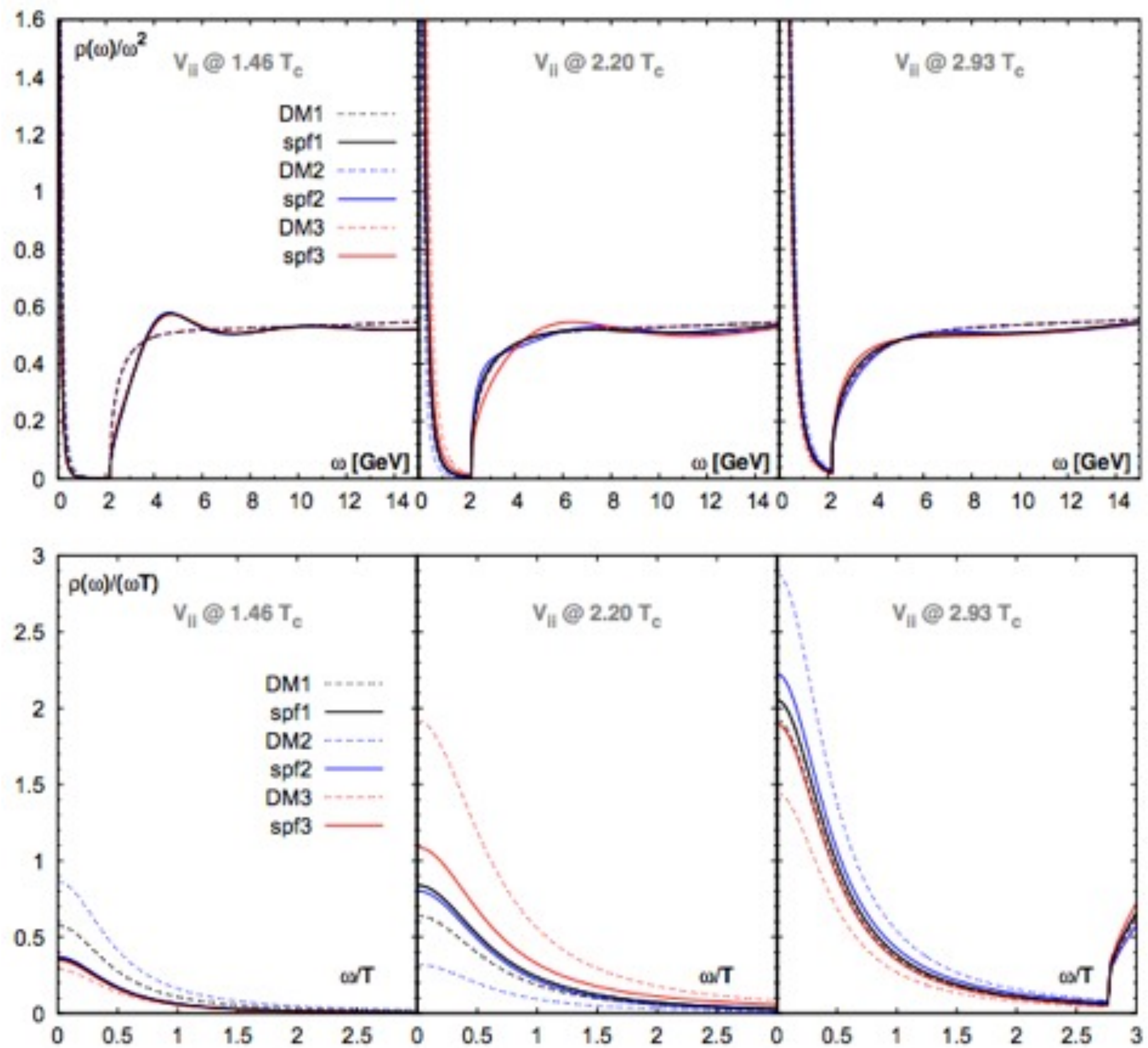
- Simulations at temperature region $0.73\, T_c < T < 1.46\, T_c$ are crucially needed to examine the sequential dissociation picture of charmonia. Calculations are on the way

- The dispersion relation in the PS channel changes only in the thermal mass term

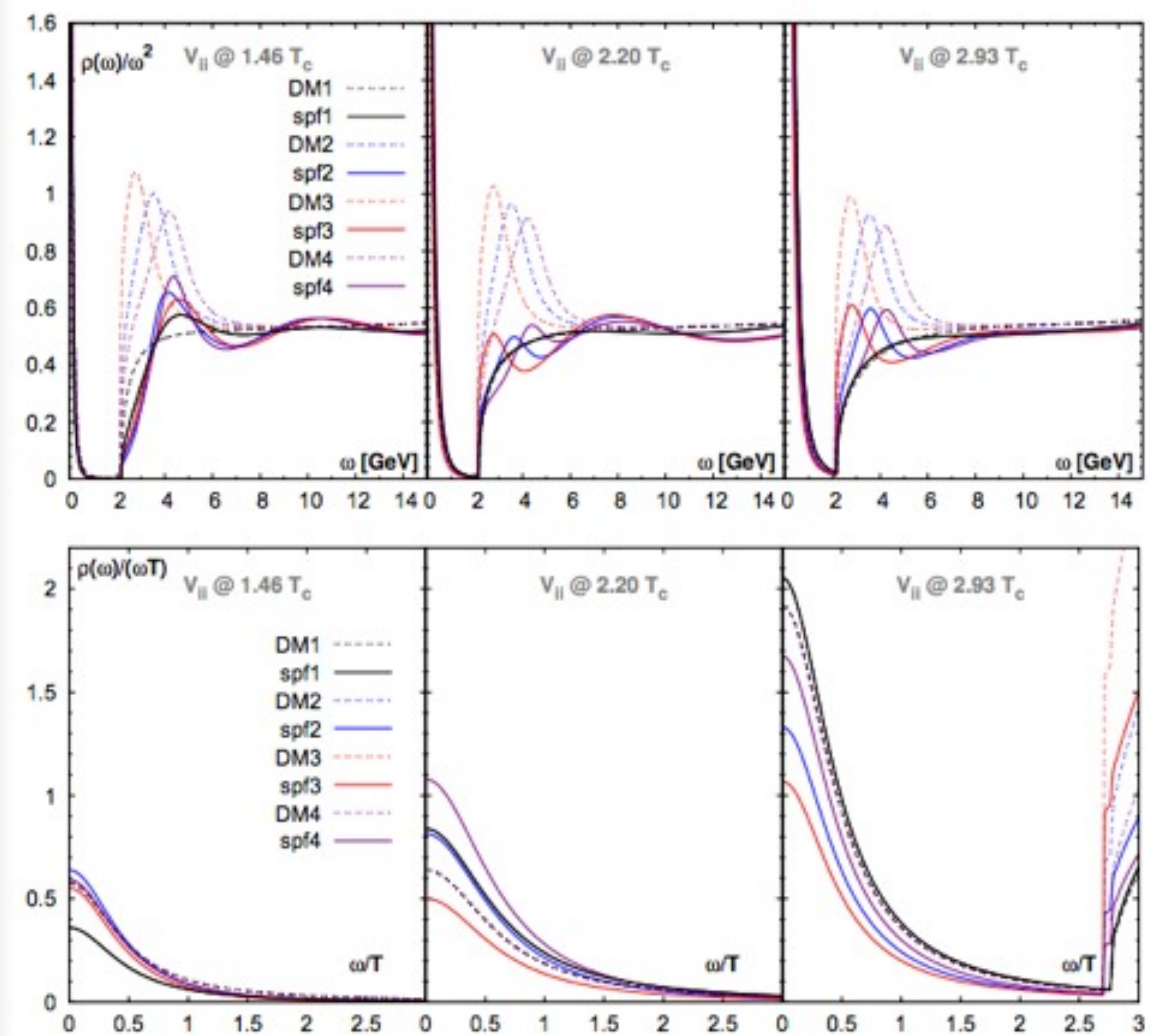
- The spectral functions of S wave states persist at finite momenta at $T<T_c$ and suffer more modifications at $T>T_c$

default model dependences of charmonium spf

variations on transport peaks



variations on resonance peaks



HTD, A. Francis, O. Kaczmarek, F. Karsch, H. Satz and W. Soeldner, PRD 86, 014509 (2012)

Previous lattices studies on charmonium spf

	$N_\sigma^3 \times N_\tau$	T/T_c	#conf	$a_\sigma[10^{-3}\text{fm}]$	a_σ/a_τ
Umeda02 EPJC 37S1(04)9	$20^3 \times 32$	0.88	1000		
	$20^3 \times 26$	1.08	1000	96	4
Asakawa03 PRL 92(04)012001	$32^3 \times 96$	0.78	194		
	$32^3 \times 54$	1.38	150	39	4
	$32^3 \times 46$	1.62	182		
	$32^3 \times 40$	1.87	181		
	$32^3 \times 32$	2.33	141		
Datta03 PRD 69(04)094507	$40^3 \times 40$	0.9	85		
	$64^3 \times 24$	1.5	80	20	1
	$48^3 \times 16$	2.25	100		
	$48^3 \times 12$	3	90		
Jakovác06 PRD 75(2007)014506	$24^3 \times 44$	1.09	110		
	$24^3 \times 40$	1.20	1680	56	4
	$24 \cdot 32 \times 32$	1.50	1000		
	$24^3 \times 24$	1.99	300		
	$24^3 \times 20$	2.39	640		
	$24^3 \times 16$	2.99	310		
Aarts07 PRD 76(07)094513	$12^3 \times 80$	0.42	250		
	$8^3 \times 32$	1.05	1000	162	6
	$8^3 \times 24$	1.40	1000		
	$8^3 \times 16$	2.09	1000		

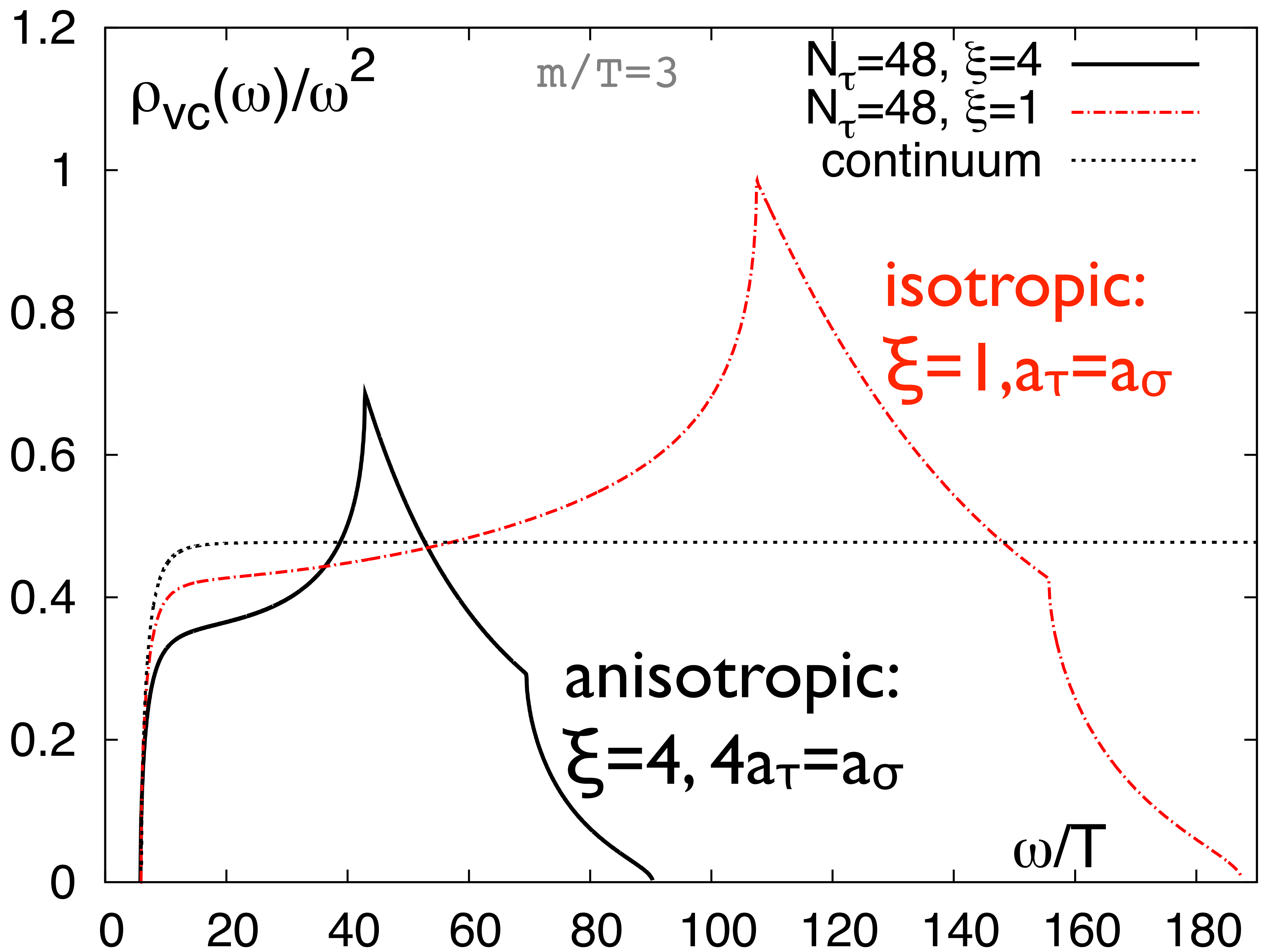
a_σ: lattice spacing in the spatial direction
a_τ: lattice spacing in the temporal direction

- Redefinition of the kernel to cure the divergence at w~0 is needed
- Detailed default model dep. analysis as well as systematic uncertainty study are important

- Lattice cutoff effects are much larger on anisotropic lattices
- N_τ used in the current study on isotropic lattices is doubled compared to Datta03, and comparable to the study on anisotropic lattices (Asakawa03), and at least 1.5 times larger than other studies on anisotropic lattices (Umeda02, Jakovac06, Aarts07) at similar temperatures

HTD, A. Francis, O. Kaczmarek, F. Karsch, H. Satz and W. Soeldner, PRD 86, 014509 (2012)

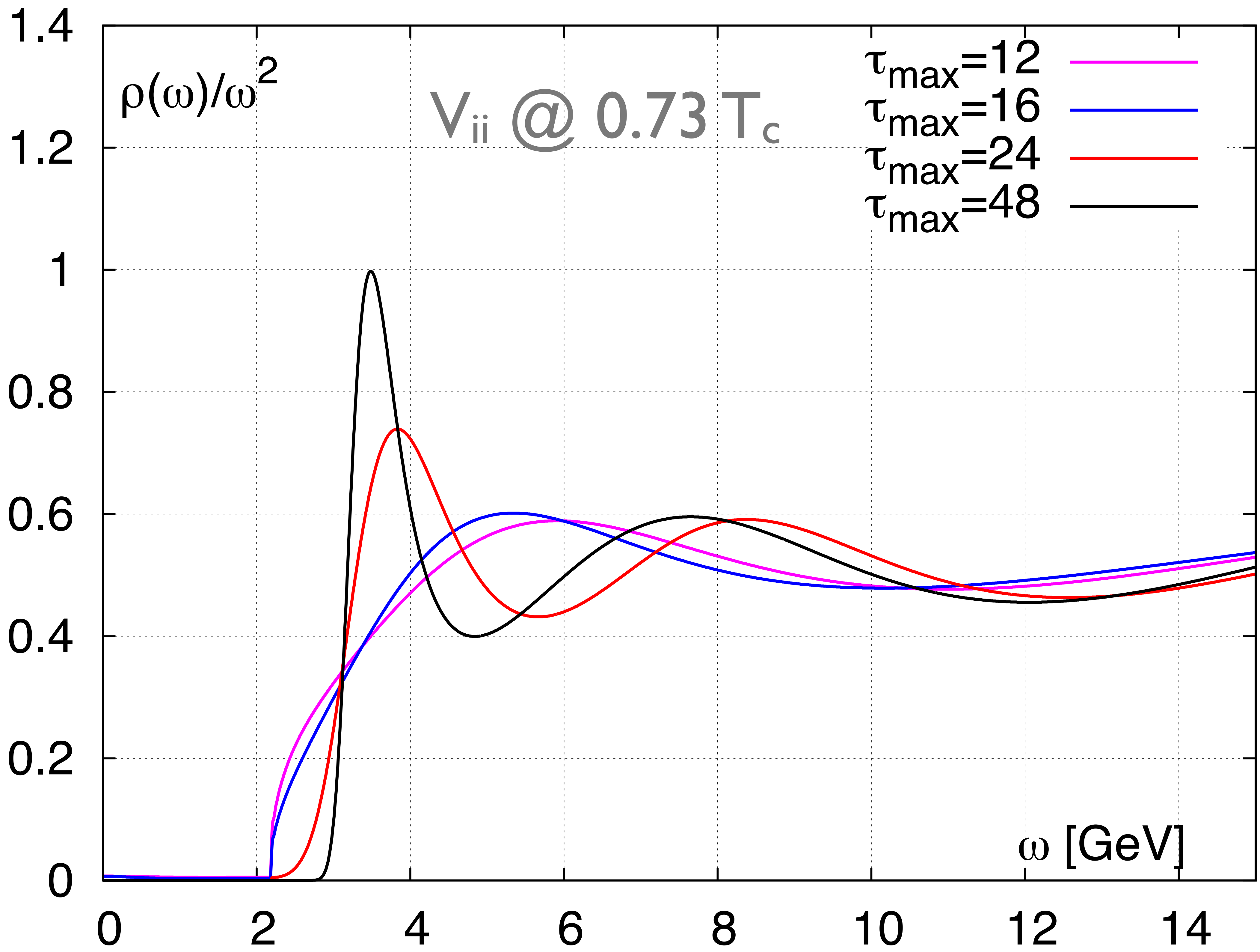
isotropic v.s. anisotropic lattices



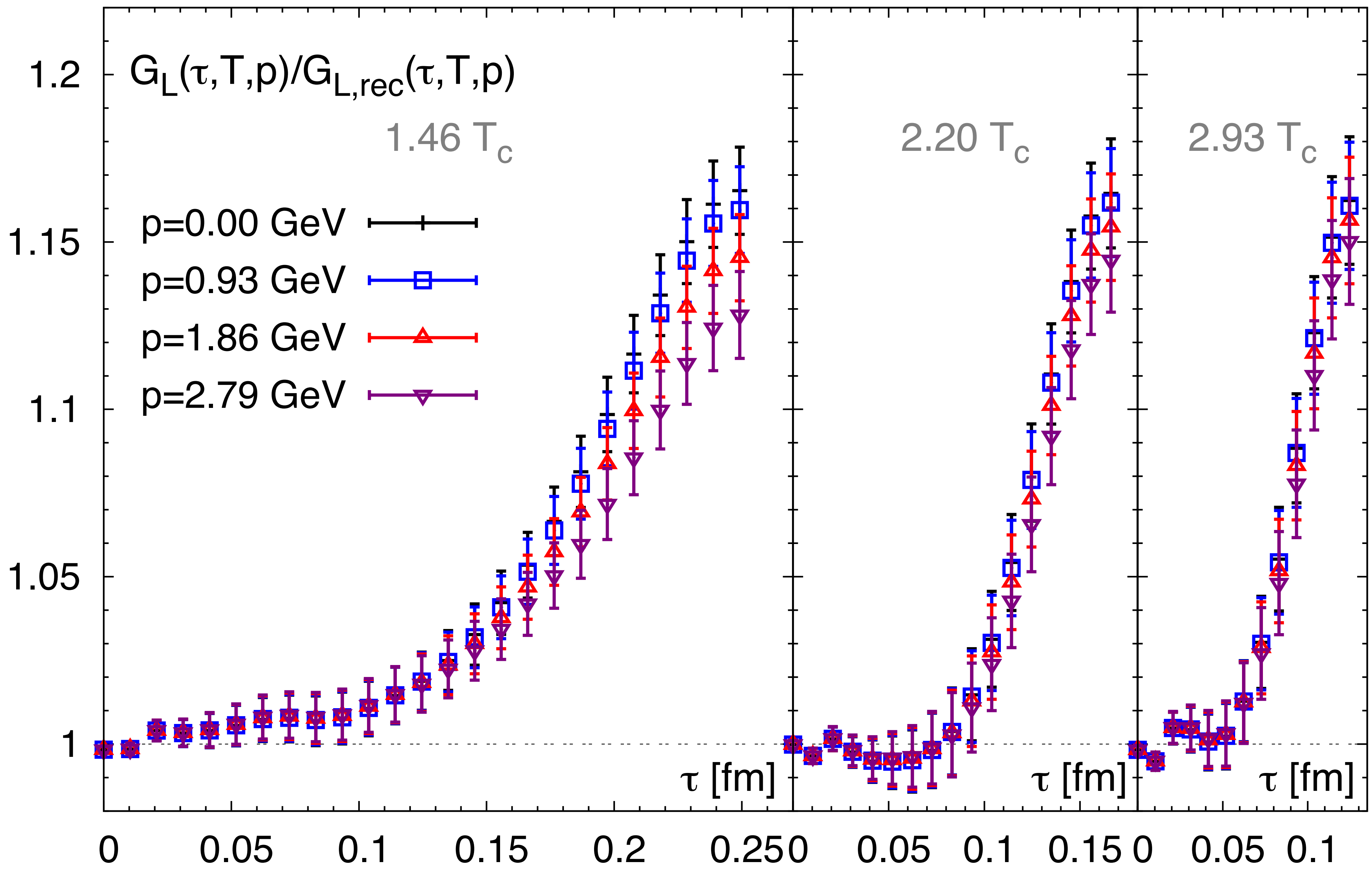
- Lattice cutoff effects on ξ=4 lattices with N_τ are even more sever than lattice cutoff effects on isotropic lattices with $N_\tau/2$

HTD, A. Francis, O. Kaczmarek, F. Karsch, H. Satz and W. Soeldner, PRD 86, 014509 (2012)

dependences on the temporal extent

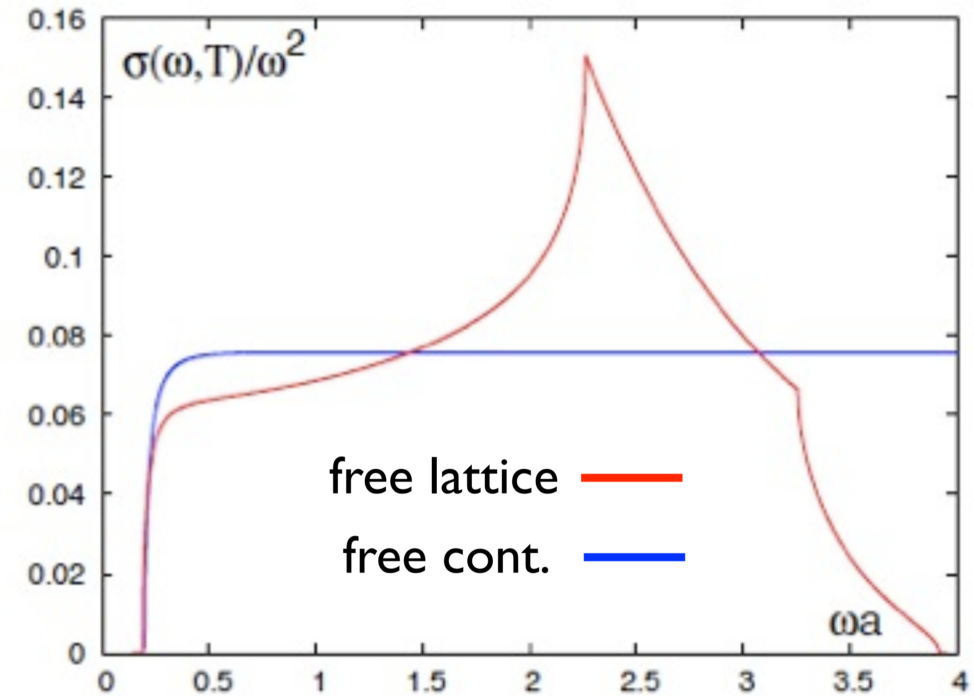


ratio of G/G_{rec} for the longitudinal vector corr.



Prior information in the default model

- high frequency behavior of spf should resemble the spf in the free case
  - ★ free lattice spf rather than free continuum spf [HTD et al.,'09]



- low frequency behavior of spf

I: Non-interacting case: [Karsch et al., 03, Aarts et al., '05]

$$\sigma_H(\omega,0) = N_c\left[\left(a_H^{(1)}+a_H^{(3)}\right)I_1+\left(a_H^{(2)}-a_H^{(3)}\right)I_2\right]\omega\,\delta(\omega)$$

- ωδ(ω) term corresponds to a τ independent constant in the correlator, i.e. zero mode contribution [Umeda, '07]
- No zero mode contribution in the PS channel
- Zero mode contribution exists in the Vector, Scalar and Axial Vector channels

$$D \equiv \frac{\pi}{3\chi_{00}}\lim_{\omega\to 0}\sum_{i=1}^{3}\frac{\sigma_{ii}(\omega,0)}{\omega} = \infty$$

II: Interacting case: [Aarts & Martinez-Resco '02, Petreczky & Teaney '06]

$$\delta(\omega) \to \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2} \quad \longrightarrow \quad D \propto 1/\eta$$

δ(ω) is smeared into a transport peak

spf in vector channel should be linear in ω at ω~0