



Color Decoherence of Jets in HIC

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In collaboration with

C.A. Salgado and K. Tywoniuk

[PRL 106 \(2011\) 122002, arXiv:1009.2965 \[hep-ph\]](#)

[PLB 707 \(2011\) 156, arXiv:1102.4317 \[hep-ph\]](#)

[JHEP 1204 \(2012\) 064, arXiv:1112.5031 \[hep-ph\]](#)

[arXiv:1105.1346 \[hep-ph\]](#)

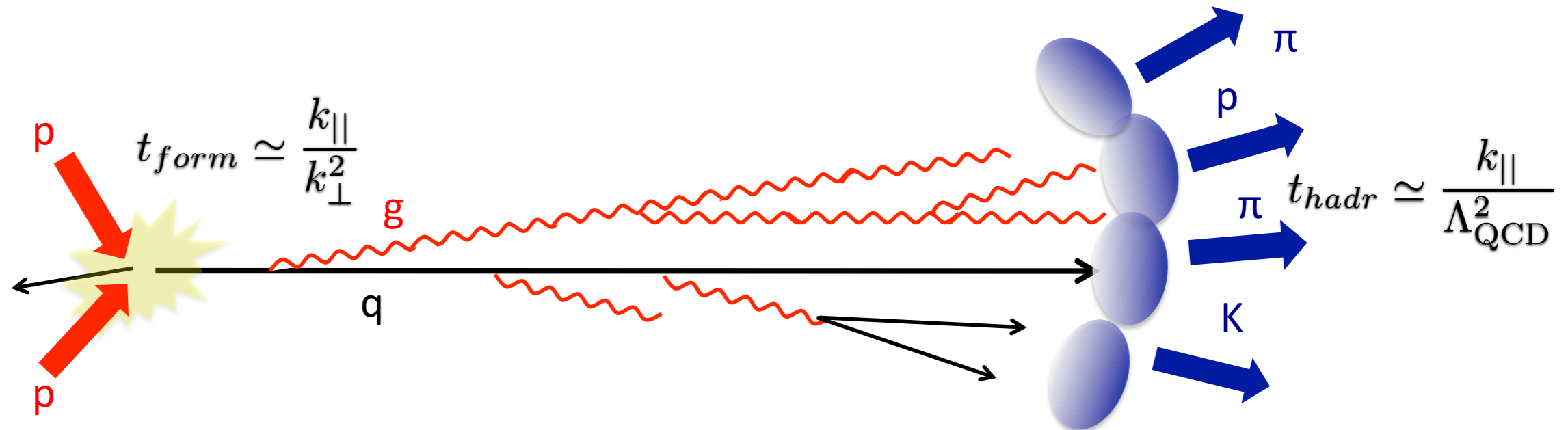
[arXiv:1205.5739 \[hep-ph\] \(To appear in JHEP\)](#)

August 16, 2012

Quark Matter 2012

Washington DC

JETS IN VACUUM

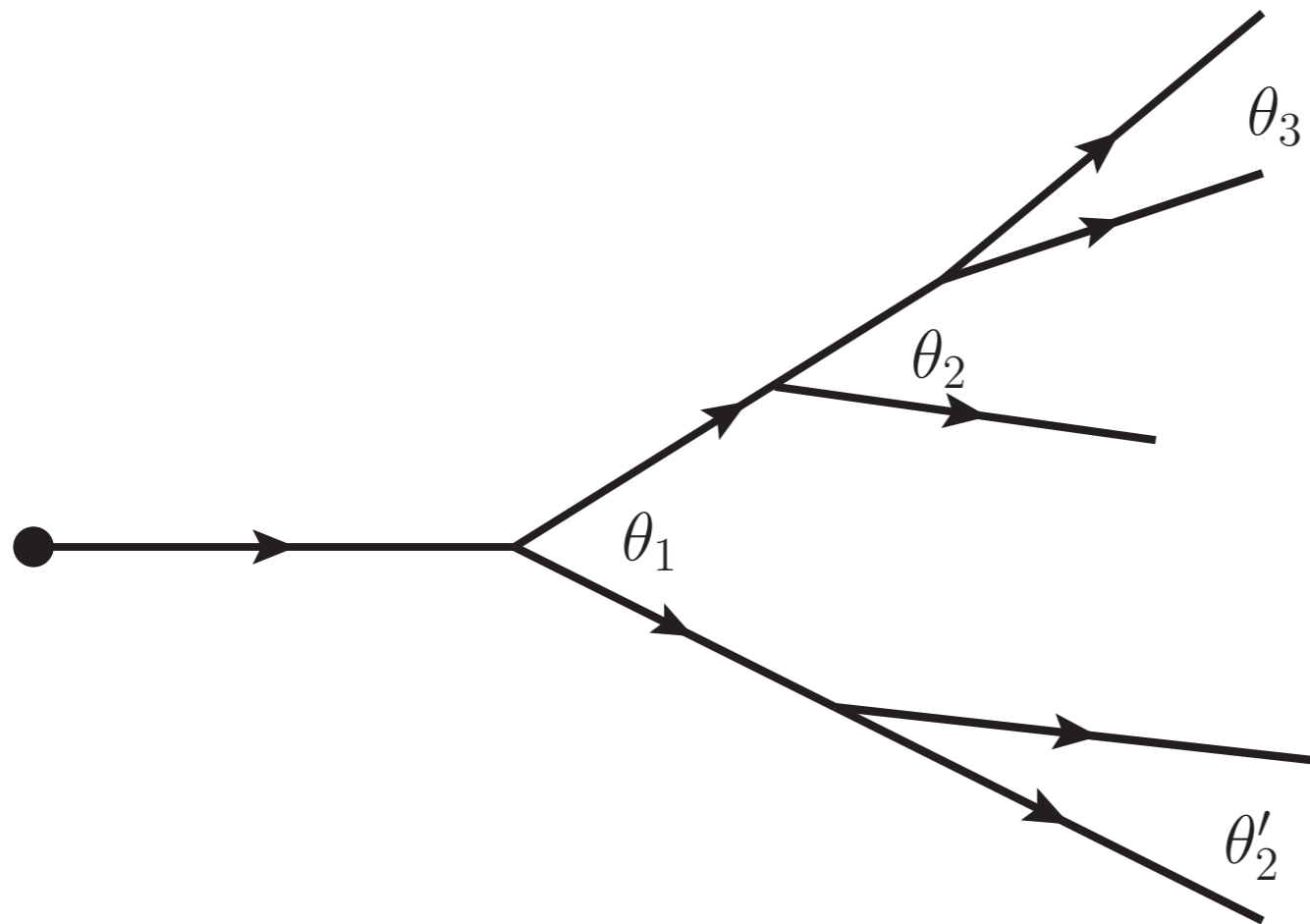


- Originally a **hard parton** (quark/gluon) which fragments into many partons with virtuality down to a non-perturbative scale where it **hadronizes**
- **LPHD**: Hadronization does not affect inclusive observables (jet shape, energy distribution etc..)

Large time domain for pQCD: $\frac{1}{\sqrt{s}} < t < \frac{\sqrt{s}}{\Lambda_{QCD}^2}$

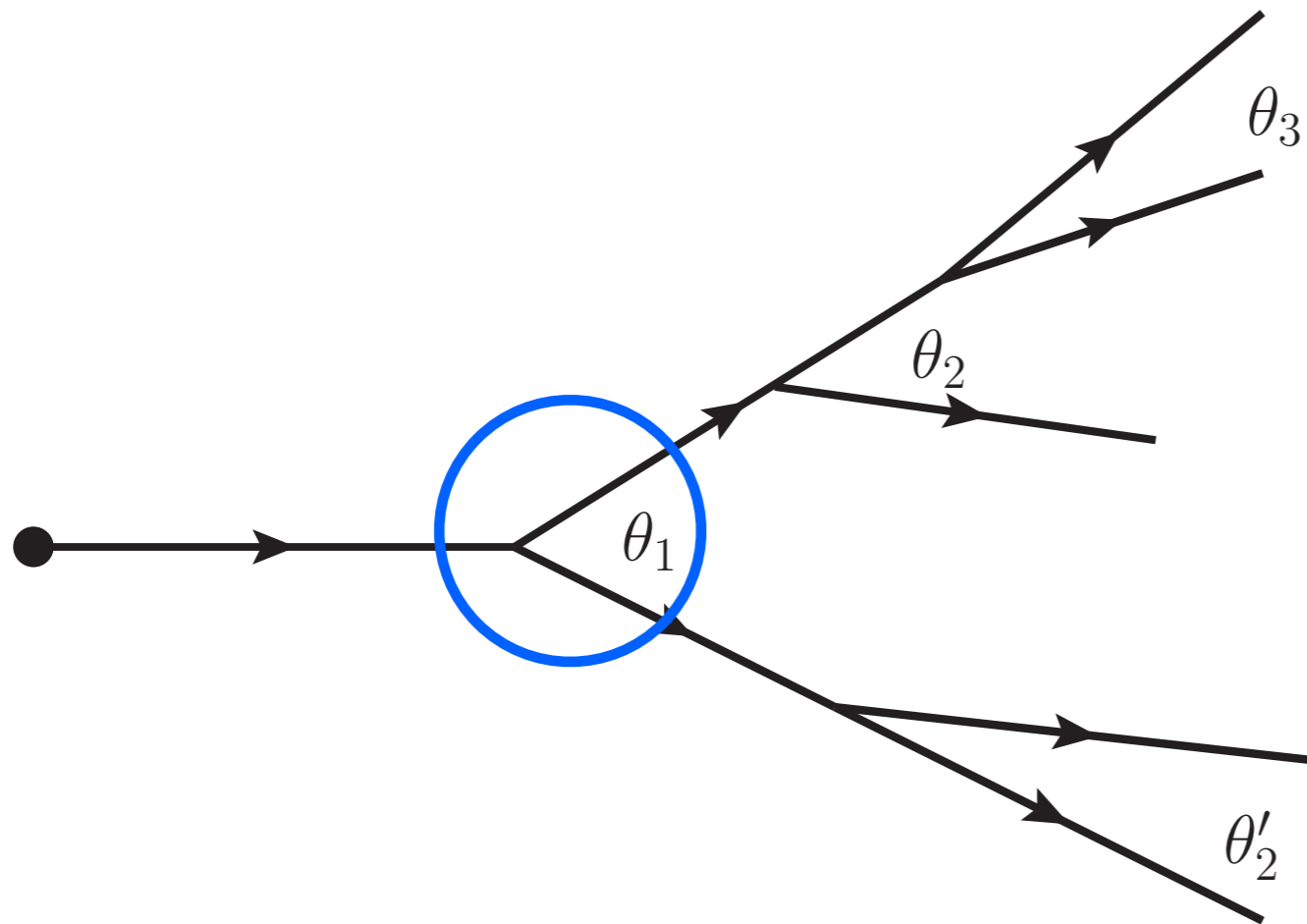
JETS IN VACUUM

[Bassetto, Mueller, Ciafaloni, Marchesini, Dokshitzer, Khoze, Troyan, Fadin, Lipatov, 80's]



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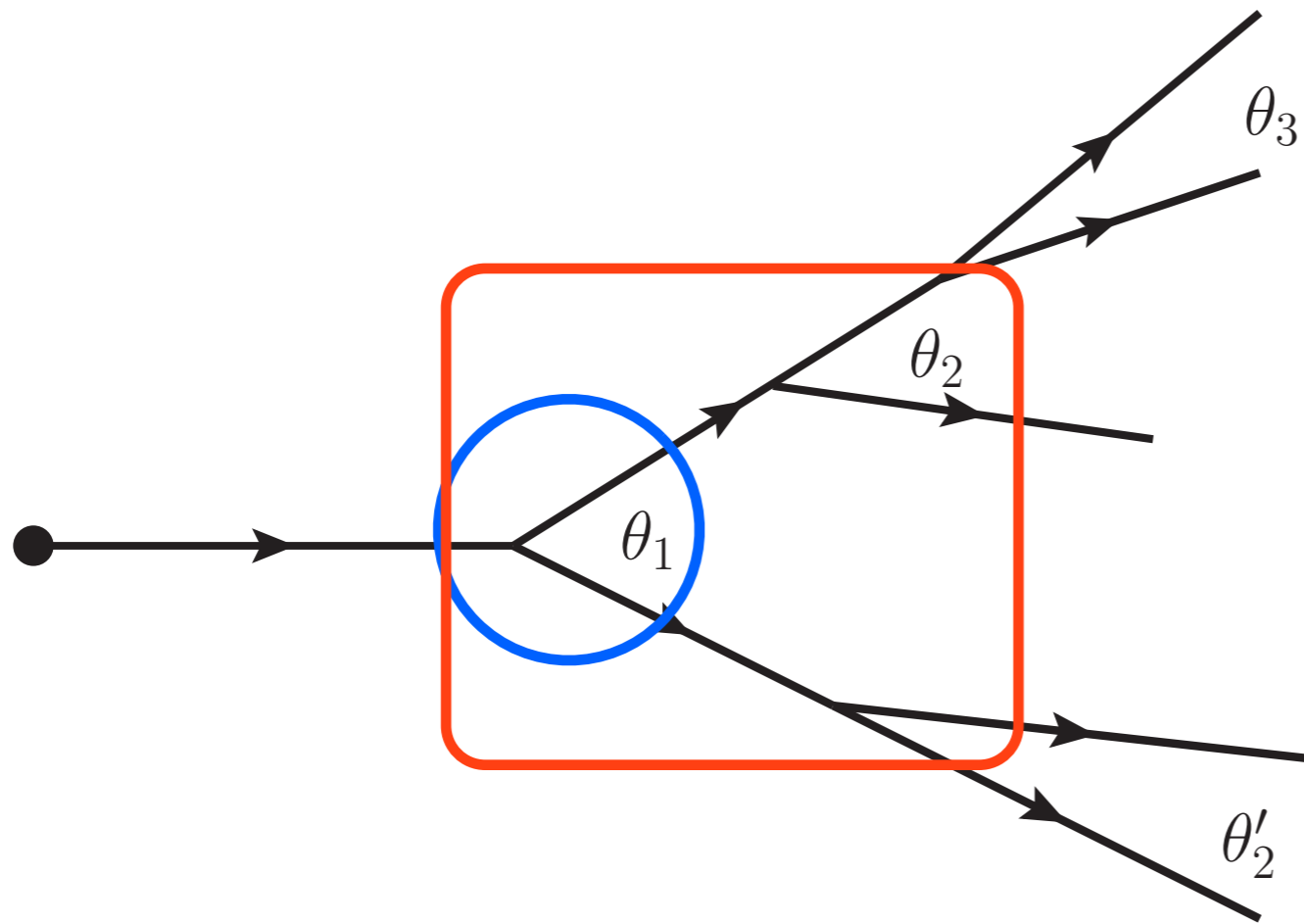


Leading Logarithms

$$dP \propto \frac{\alpha_s C_R}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\theta}$$

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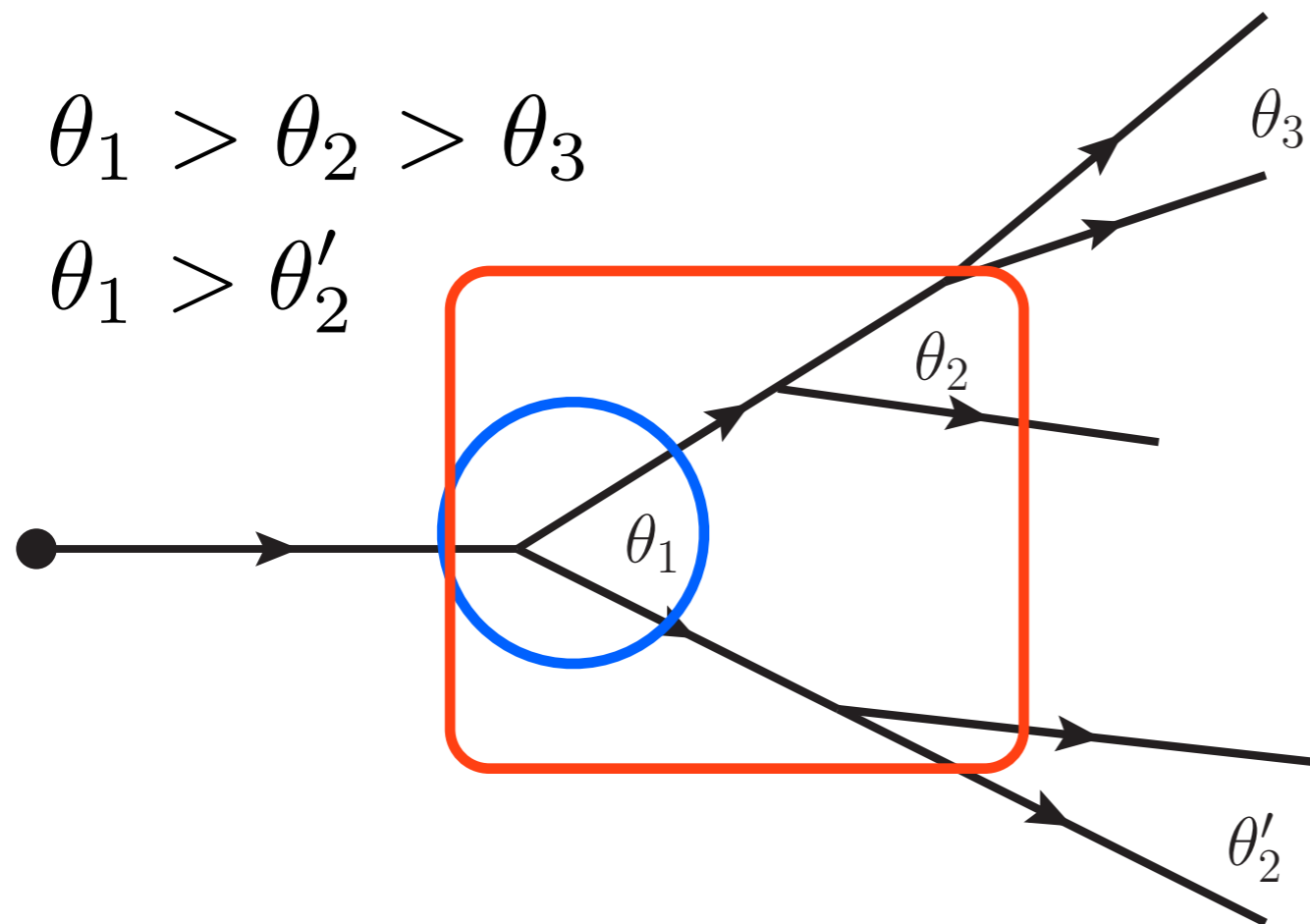
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Angular ordering

$$d^2 P \propto \Theta(\theta_1 - \theta_2) dP_1 dP_2$$

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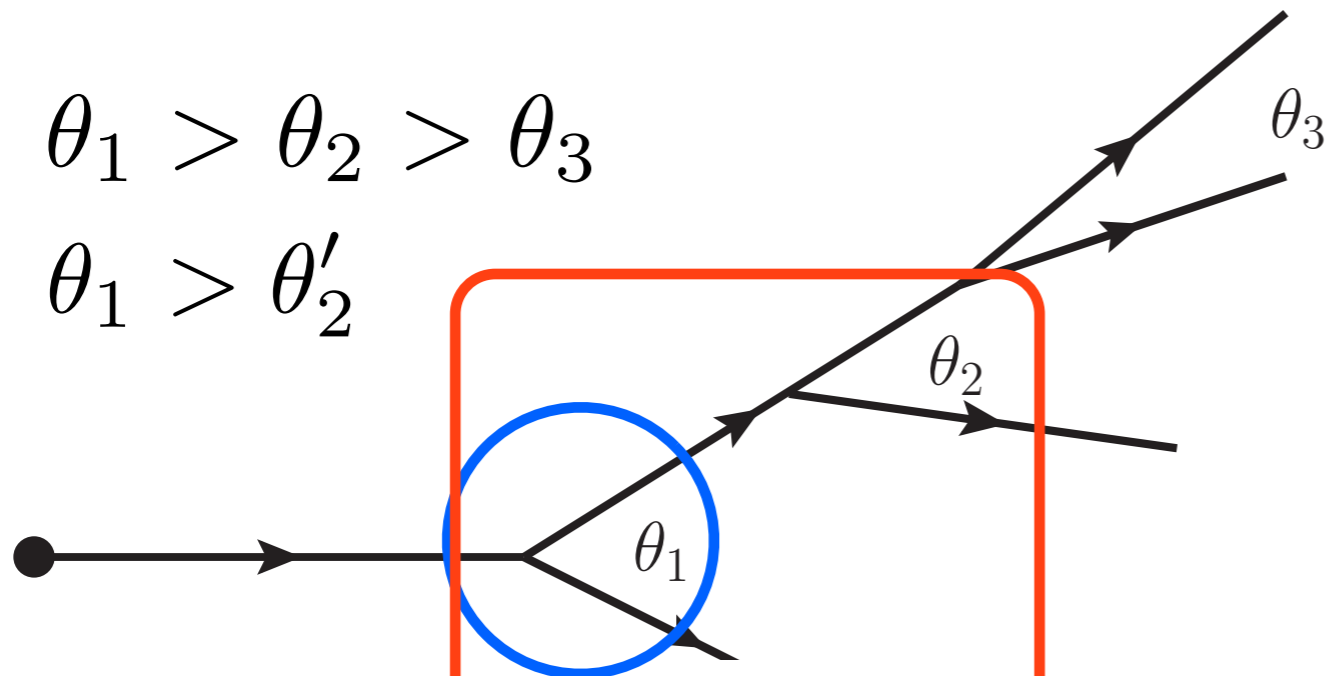
Markov process!

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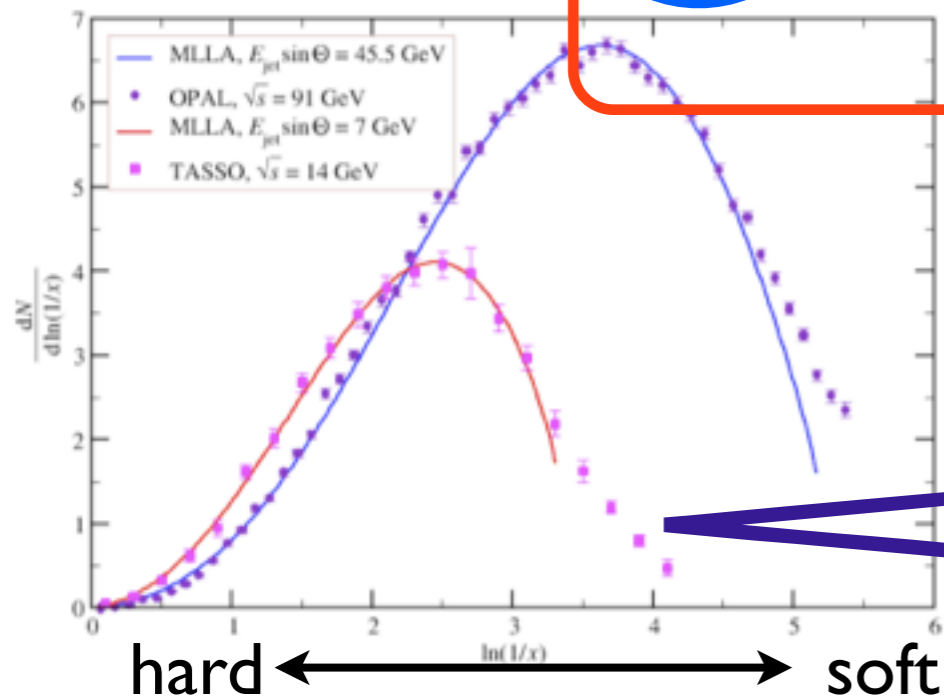
$$\theta_1 > \theta_2 > \theta_3$$

$$\theta_1 > \theta'_2$$



Leading Logarithms

$$dP \propto \frac{\alpha_s C_R}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\theta}$$



AO limits
phase space
for soft
emissions!

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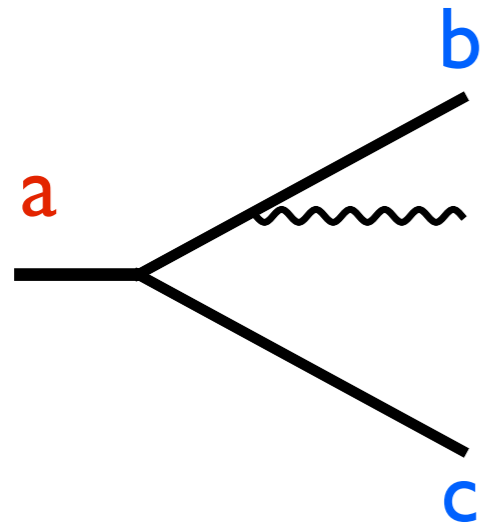
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TASSO Collaboration, Z. Phys. C 47 (1990)

OPAL Collaboration, Phys. Lett. B 247 (1990)

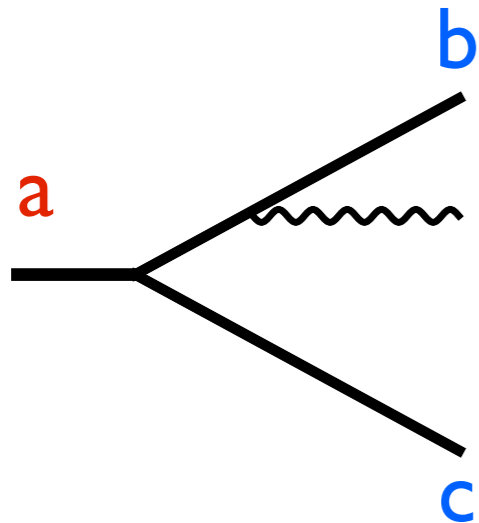
ANTENNA IN VACUUM (BUILDING BLOCK OF QCD EVOLUTION)

gluon radiation off a pair of color charges b and c which originates from a (highly virtual) charge a



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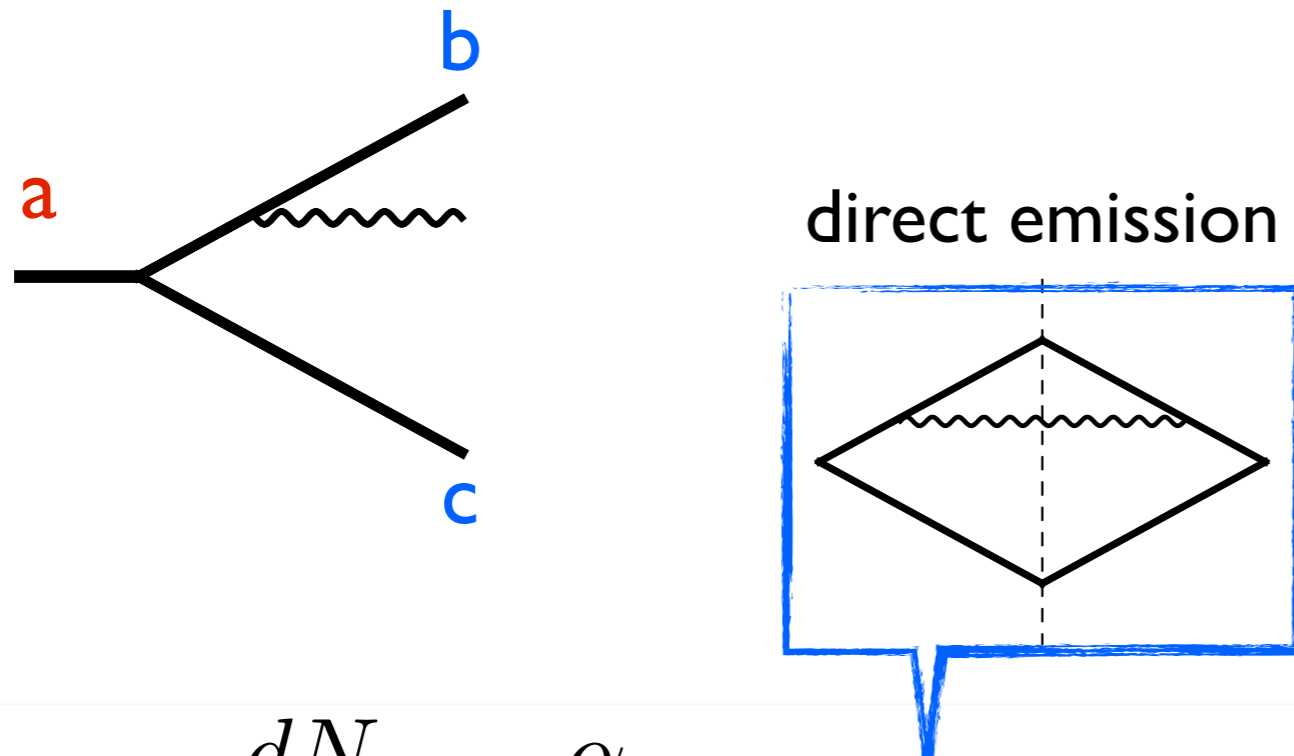
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$$(2\pi)^2 \omega \frac{dN_a}{d^3k} = \frac{\alpha_s}{\omega^2} [C_b (\mathcal{R}_b - \mathcal{J}) + (b \rightarrow c) + C_a \mathcal{J}]$$

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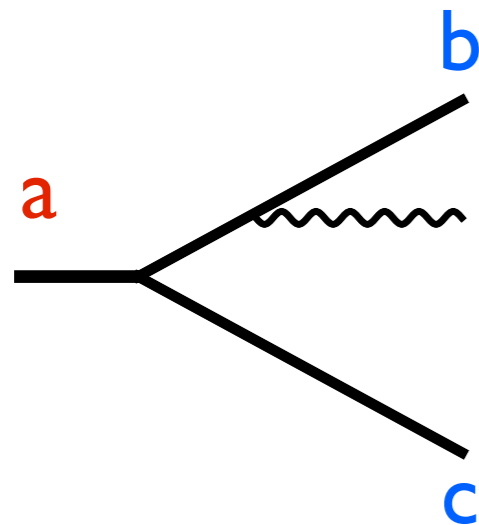
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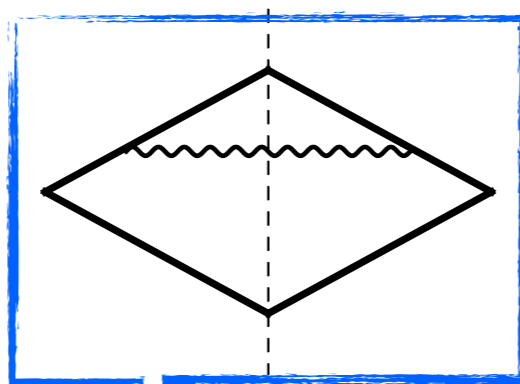
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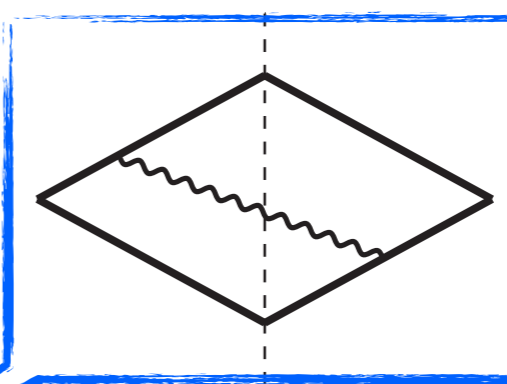
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direct emission



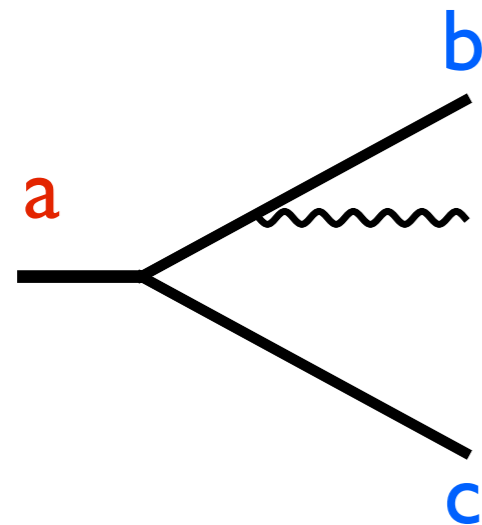
interference



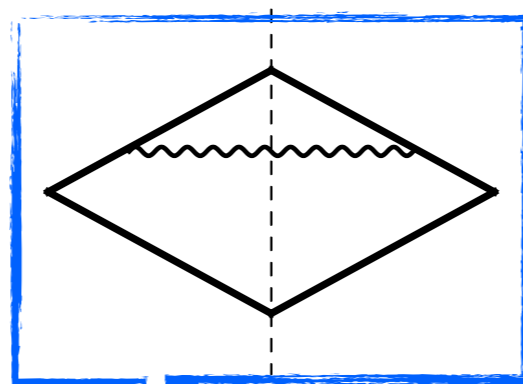
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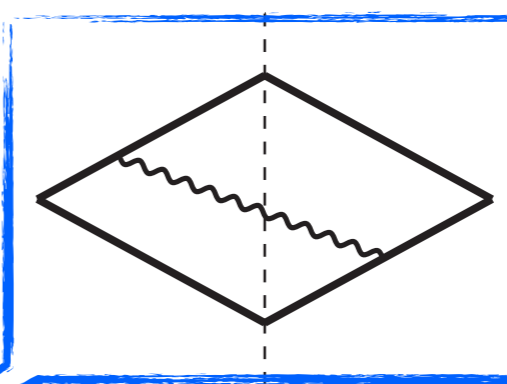
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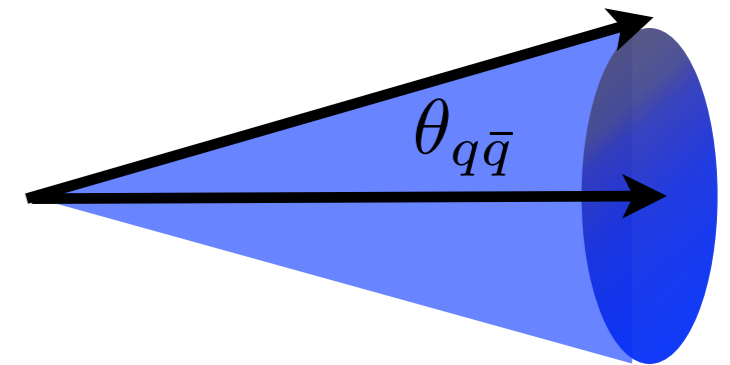
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For $g \rightarrow qq$ $C_b = C_c = C_F$ and $C_a = C_A$

For $\gamma \rightarrow qq$ $C_b = C_c = C_{F_4}$ and $C_a = 0$

ANTENNA IN VACUUM (BUILDING BLOCK OF QCD EVOLUTION)

$$dN_{q,\gamma^*}^{\text{vac}} = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{\sin \theta d\theta}{1 - \cos \theta} \Theta(\cos \theta - \cos \theta_{q\bar{q}}),$$

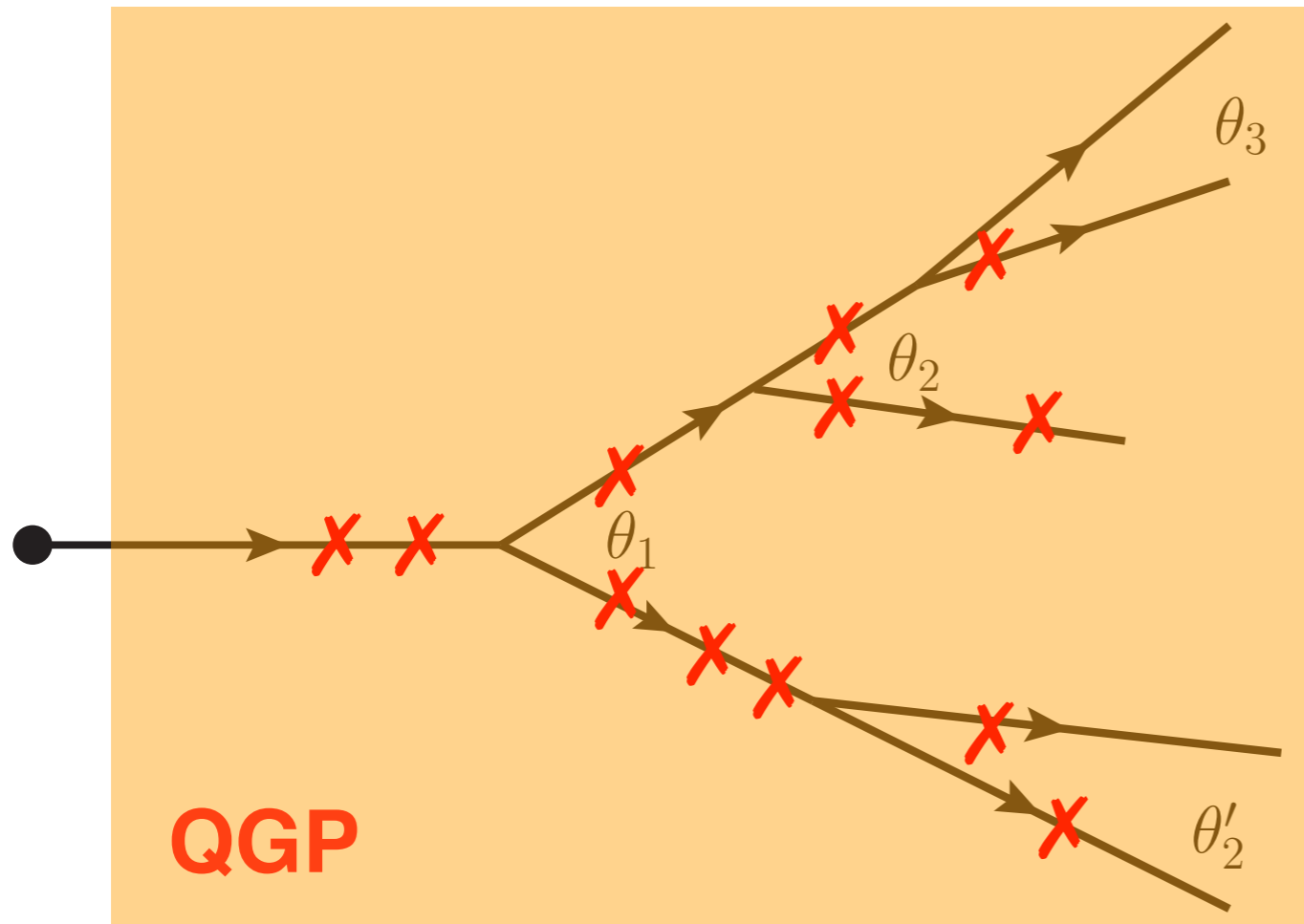


Angular ordering in vacuum

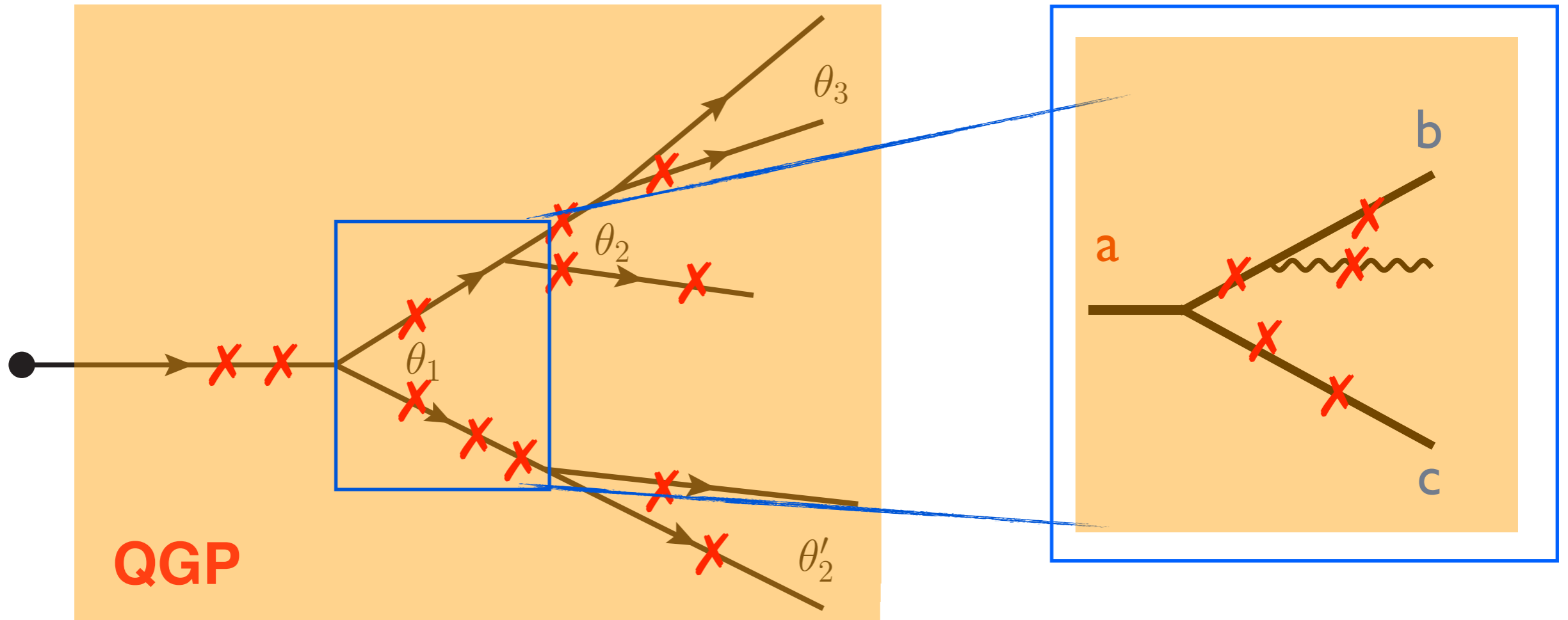
- Radiation confined inside the cone
- Why?

gluons emitted at angles larger than the pair opening angle cannot resolve its internal structure: Emission by the total charge (suppressed for a white antenna)

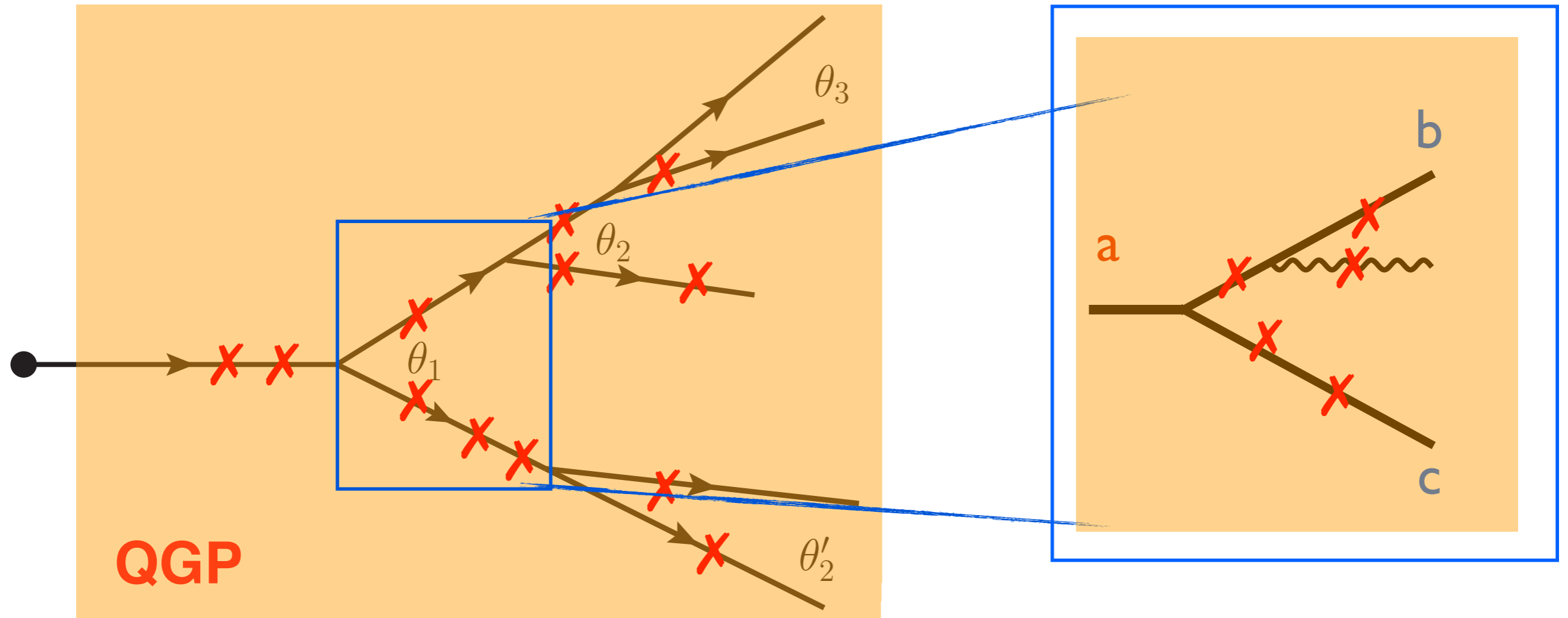
HOW IS THE JET MODIFIED?



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Q: how is the antenna radiation modified in a medium?

Building block of the in-medium cascade

SPACE-TIME STRUCTURE OF IN-MEDIUM INTERFERENCES

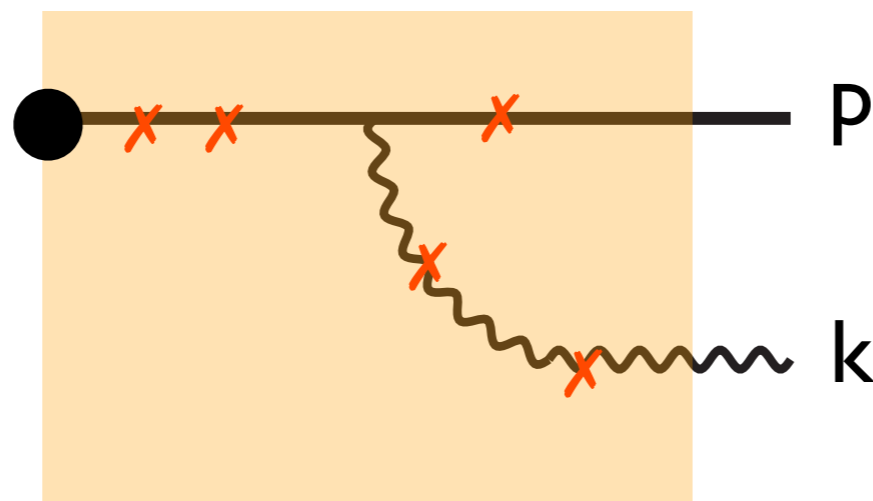
Consider an energetic charge q (eikonal current)

$$J_q(x) = g U_p(x^+, 0) \delta^{(3)}(\vec{x} - \frac{\vec{p}}{E}t) \Theta(t) Q_q$$

Color precession along the trajectory:

$$U_p(x^+, 0) = \mathcal{P}_+ \exp \left\{ ig \int_0^{x^+} dz^+ [T \cdot A_{\text{med}}^-(z^+, z^+ p_{\perp}/p^+)] \right\}$$

Setup: Solving the linearized Yang-Mills Eqs. for the gluon field



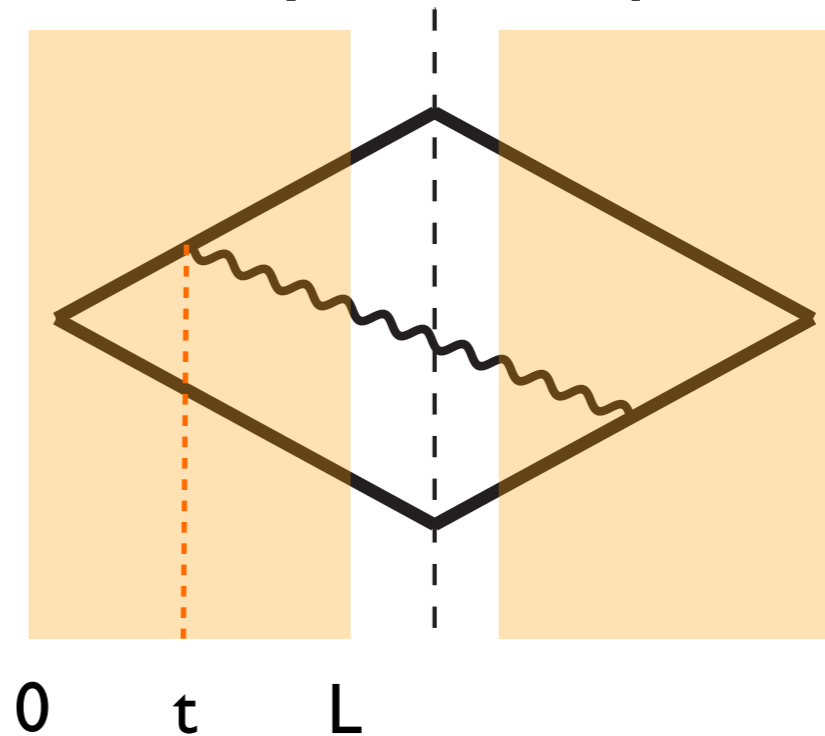
Non-eikonal gluon propagation₇

SPACE-TIME STRUCTURE OF IN-MEDIUM INTERFERENCES

\mathcal{R} : direct emission - BDMPS spectrum - anywhere up to L

\mathcal{J} : interferences \Rightarrow

interferences depend on the
decoherence parameter



$$\Delta_{\text{med}} = 1 - \frac{1}{N_c^2 - 1} \langle \text{Tr} U_p(t, 0) U_{\bar{p}}^\dagger(t, 0) \rangle \approx 1 - e^{-\frac{1}{12} \hat{q} \theta^2 t^3}$$

decoherence time

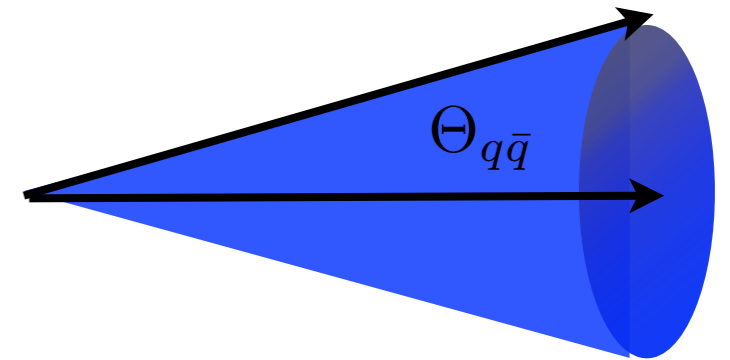
only gluons formed at $t < t_d \equiv (\hat{q} \theta_{q\bar{q}}^2)^{-1/3}$ interfere

\Rightarrow **Suppression of interferences**

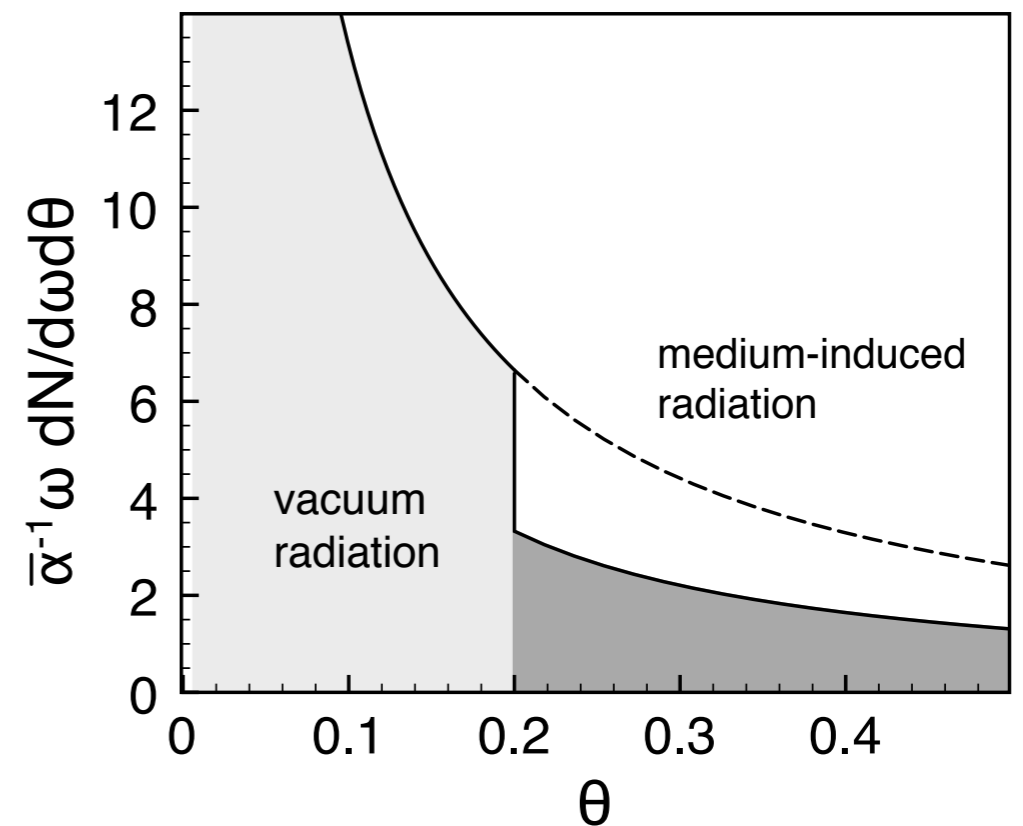
ONSET OF DECOHERENCE - SOFT GLUONS

$$\Delta_{\text{med}} \equiv \Delta_{\text{med}}(L)$$

$$\Delta_{\text{med}} \rightarrow 0 \quad \text{Coherence}$$



$$dN_{q,\gamma^*}^{\text{tot}} = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{\sin \theta d\theta}{1 - \cos \theta} [\Theta(\cos \theta - \cos \theta_{q\bar{q}}) + \Delta_{\text{med}} \Theta(\cos \theta_{q\bar{q}} - \cos \theta)] .$$



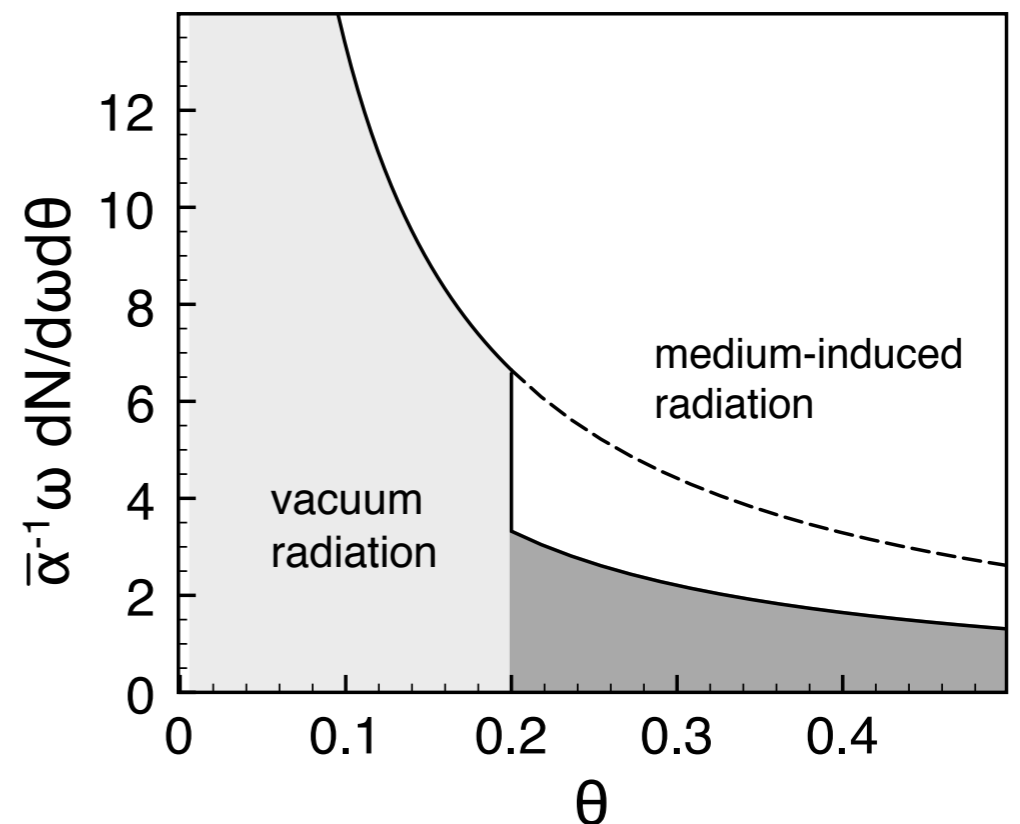
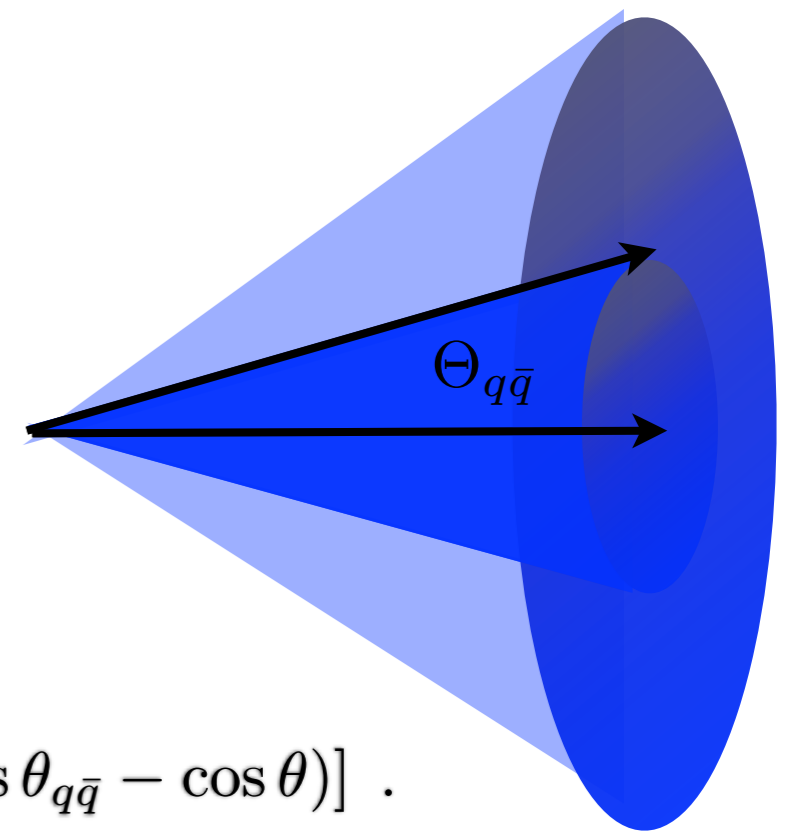
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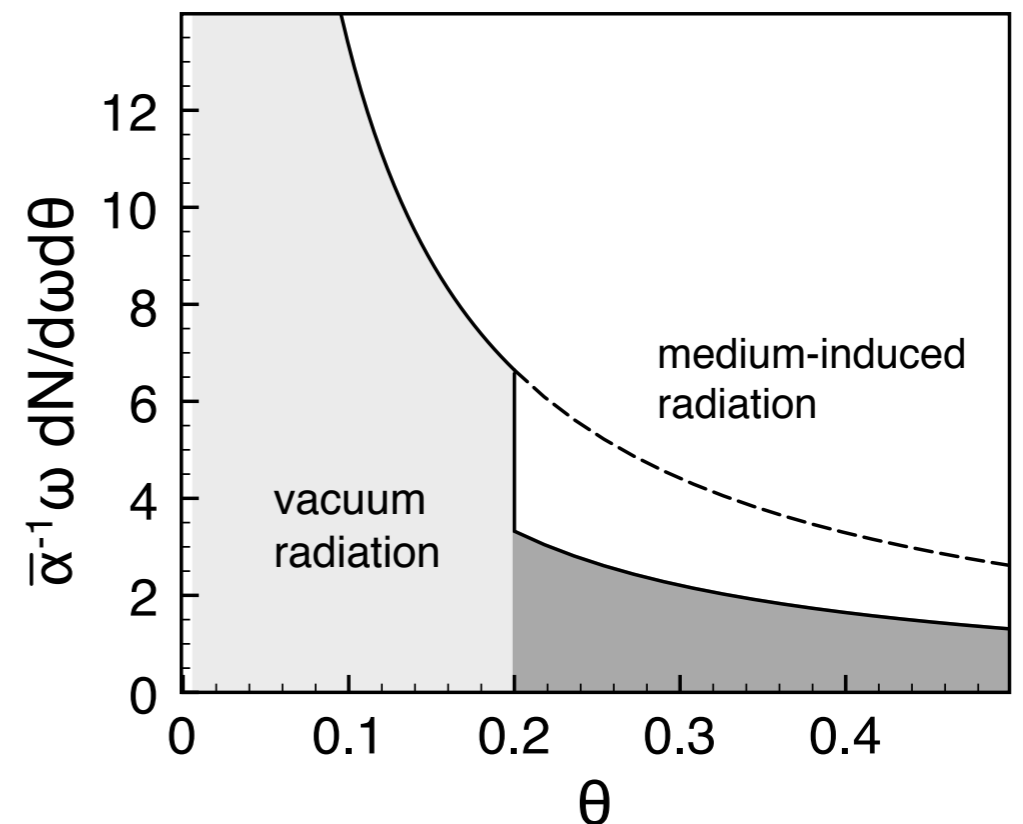
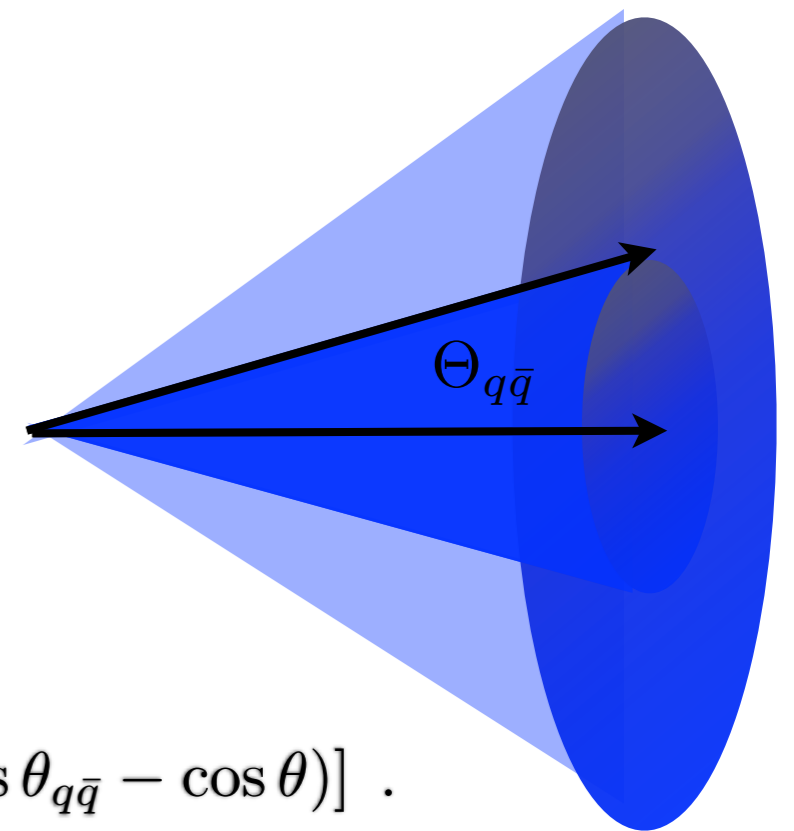
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\Rightarrow geometrical separation!



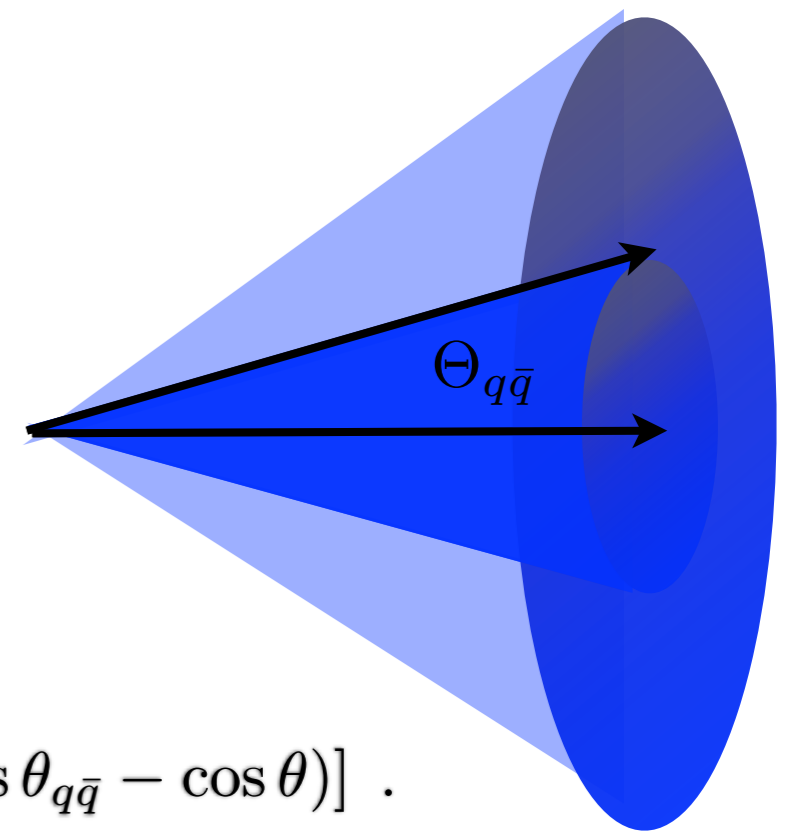
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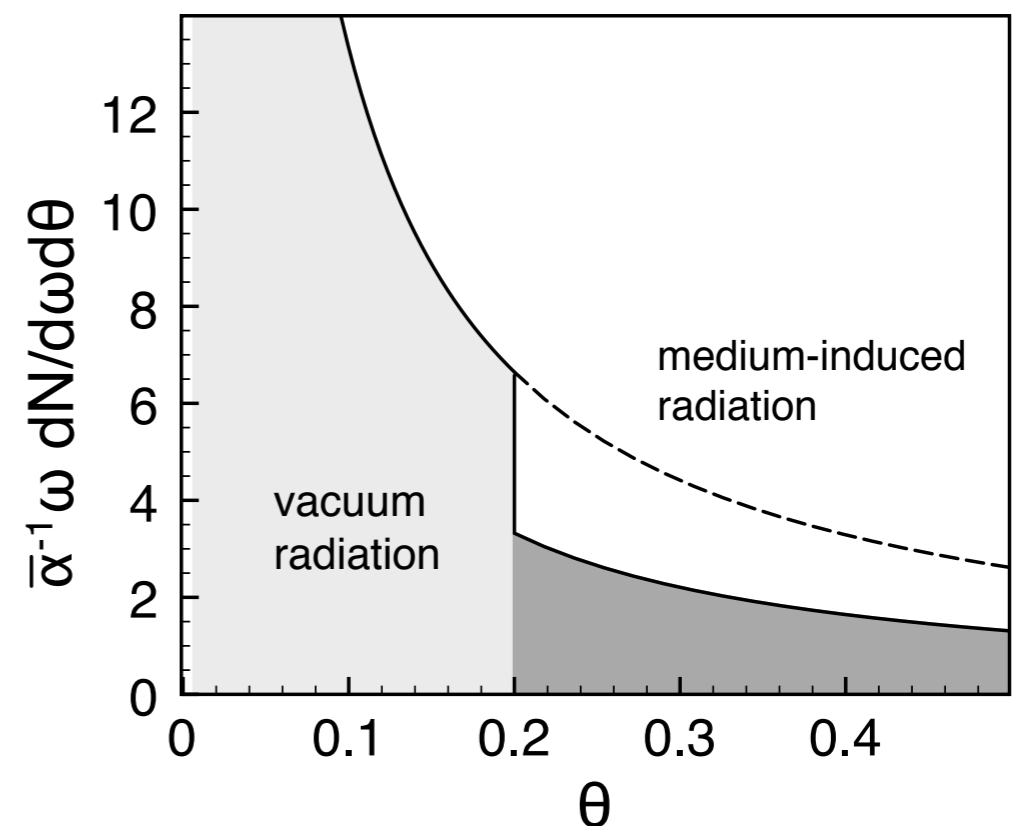
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$$dN_{q,\gamma^*}^{\text{tot}} \Big|_{\text{opaque}} = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{\sin \theta d\theta}{1 - \cos \theta} .$$

- 1) Independent emissions!
- 2) “Memory loss” effect



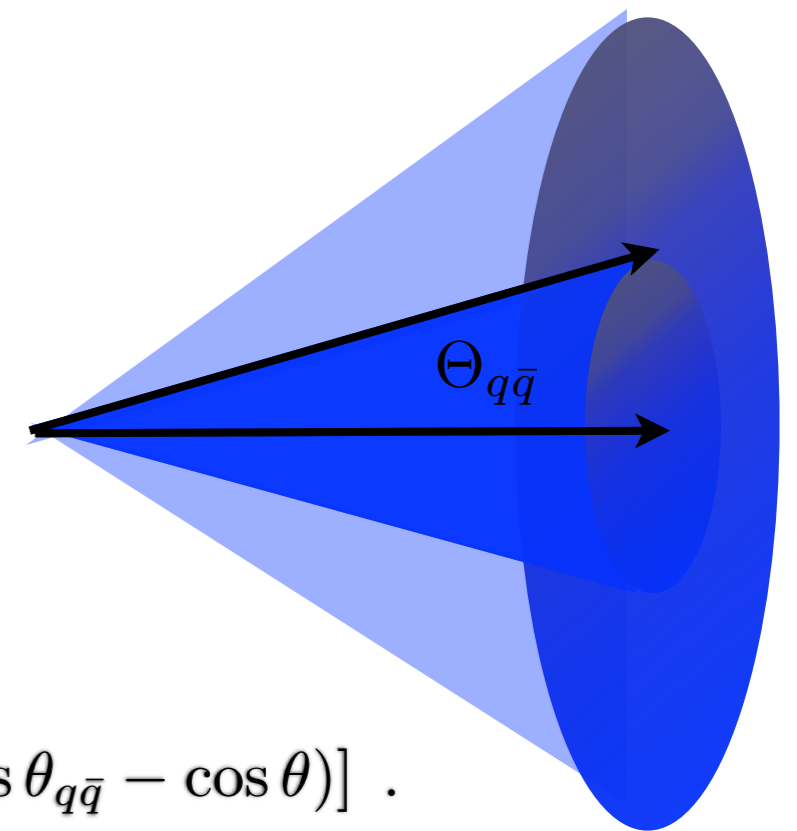
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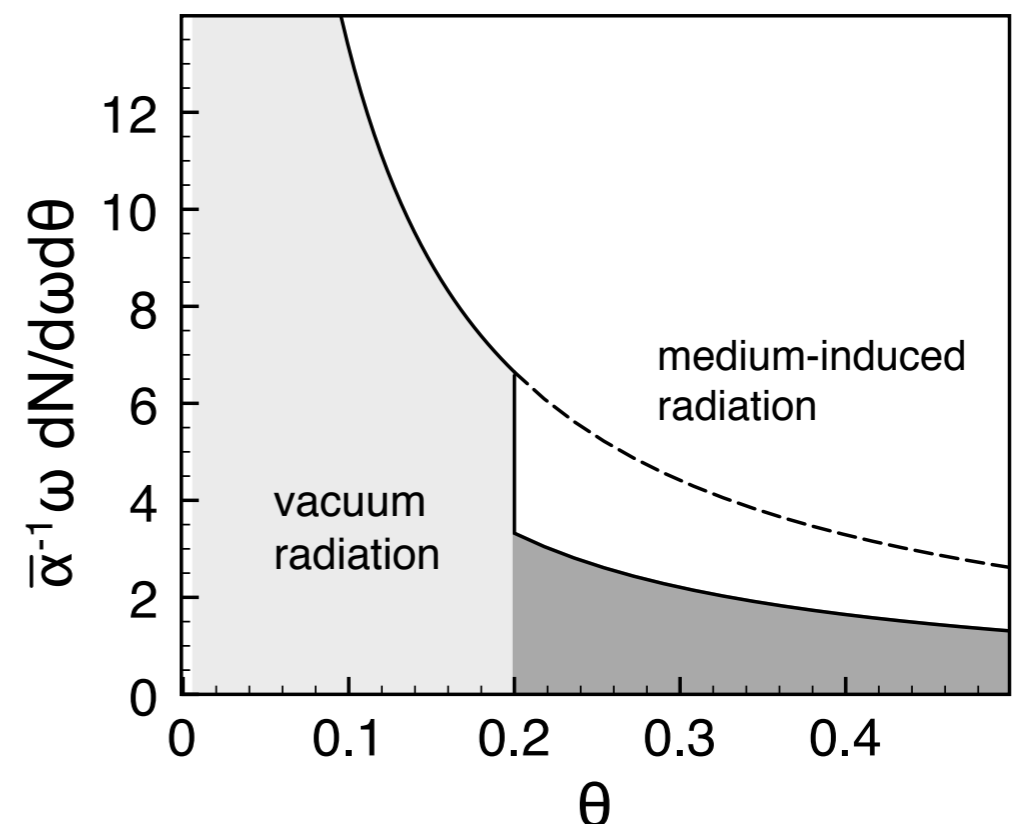


⇒ geometrical separation!

$$dN_{q,\gamma^*}^{\text{tot}} \Big|_{\text{opaque}} = \frac{\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{\sin \theta}{1 - \cos \theta} .$$

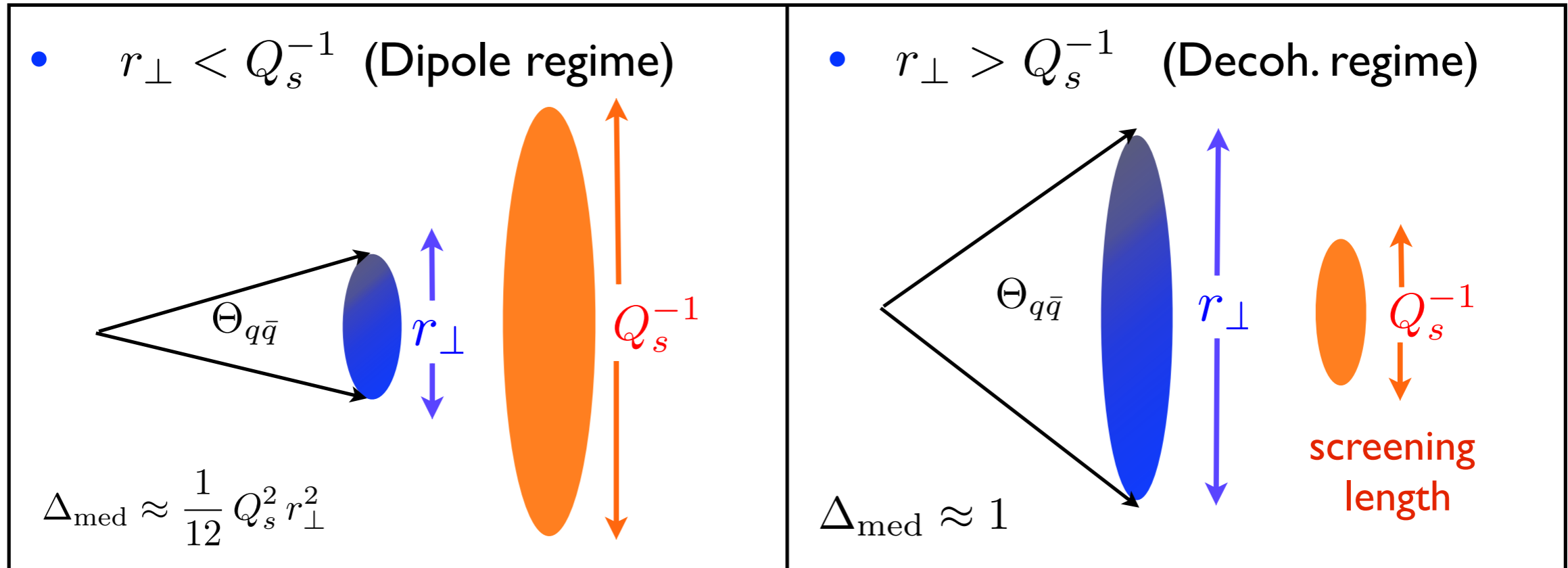
and gluon!

- 1) Independent emissions!
- 2) “Memory loss” effect



HARD SCALES IN THE PROBLEM

$$Q_s^2 = \hat{q} L \quad r_{\perp} = \theta_{q\bar{q}} L \quad \text{- a two scale problem!}$$



$$\tau_d = (\hat{q} \theta_{q\bar{q}}^2)^{-1/3}$$

$$\Delta_{\text{med}} \approx 1 - \exp\left[-\frac{1}{12} Q_s^2 r_{\perp}^2\right]$$

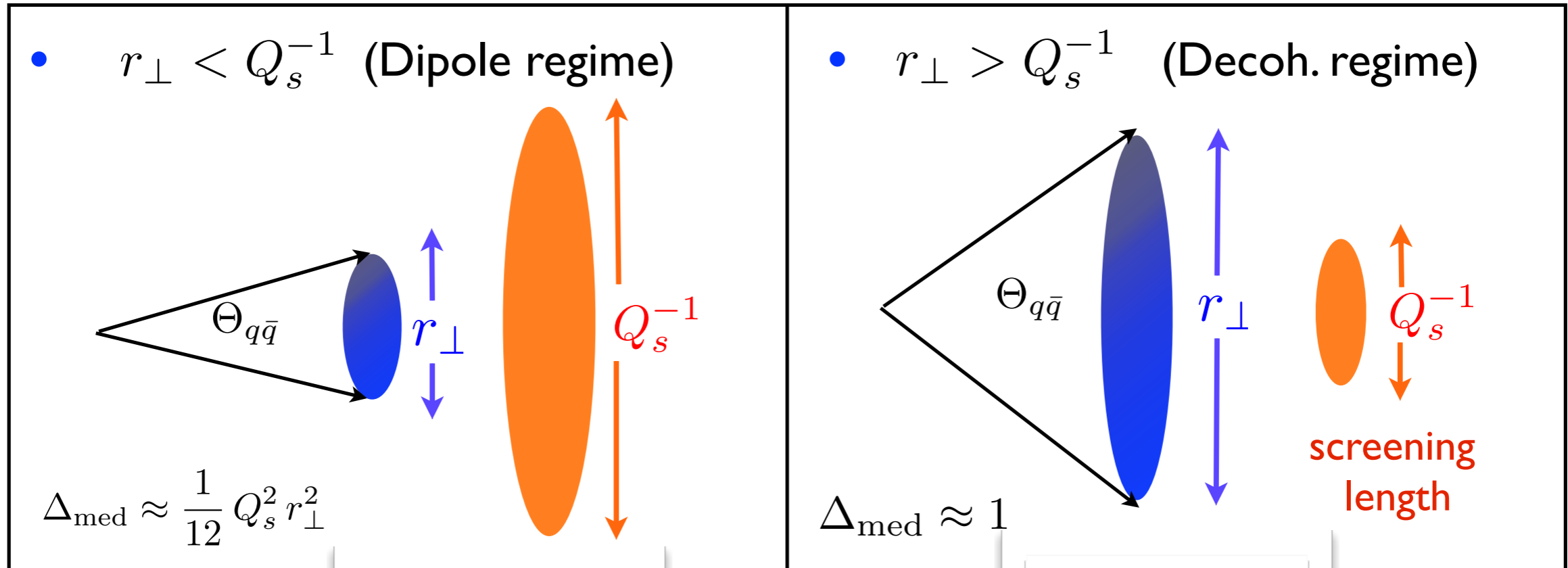
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Q_s : characteristic momentum scale of the medium

HARD SCALES IN THE PROBLEM

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- a two scale problem!



$$\tau_d = (\hat{q} \theta_{q\bar{q}}^2)^{-1/3}$$

$$\tau_d \gg L$$

$$\tau_d \ll L$$

$$\Delta_{\text{med}} \approx 1 - \exp\left[-\frac{1}{12} Q_s^2 r_{\perp}^2\right]$$

Q_s : characteristic momentum scale of the medium

HARD SCALES IN THE PROBLEM

Glueon spectrum characterized by the hardest scale in the problem

$$Q_{\text{hard}} = \max(r_{\perp}^{-1}, Q_s)$$

$$Q_s^2 = \hat{q} L \qquad r_{\perp} = \theta_{q\bar{q}} L$$

medium-induced color randomization destroys the coherence of the antenna and opens up phase space for gluon radiation up to

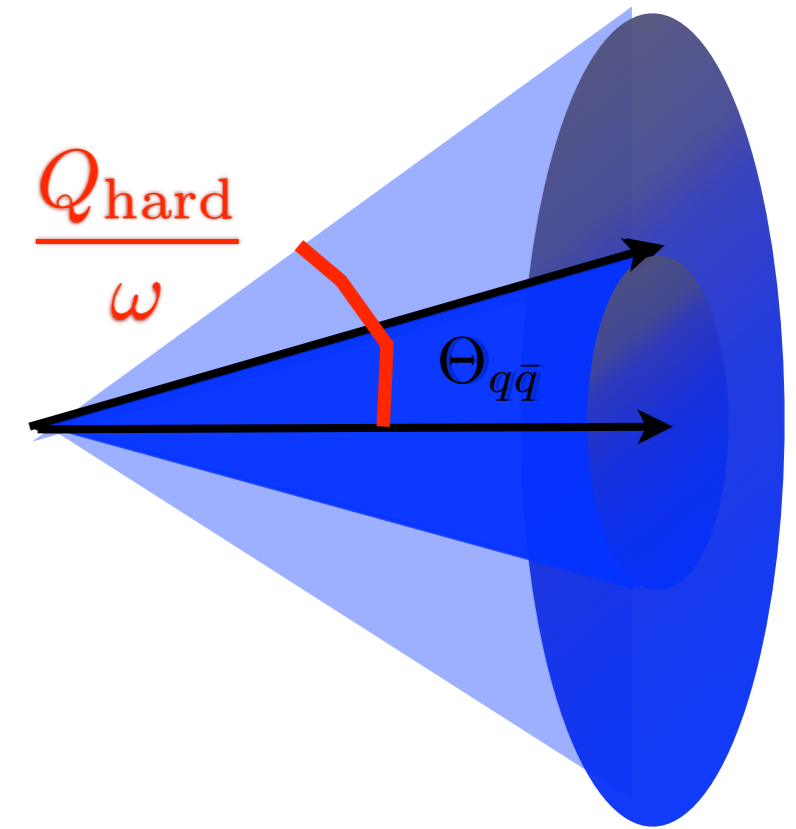
for gluon momentum $k_{\perp} > Q_{\text{hard}}$ the spectrum is suppressed and coherence is restored

✓ The system cannot induced radiations harder than the intrinsic scales of the problem

ONSET OF DECOHERENCE - FINITE GLUON ENERGIES

$$Q_{\text{hard}} = \max(r_{\perp}^{-1}, Q_s)$$

In terms of angular variables:



Vacuum: Independent emissions for $\theta < \theta_{q\bar{q}}$
Emission off the total charge otherwise (coherence)

Full decoherence: Independent emissions for $\theta < \theta_{\text{med}} = \frac{Q_s}{\omega}$
Emission off the total charge otherwise (coherence)

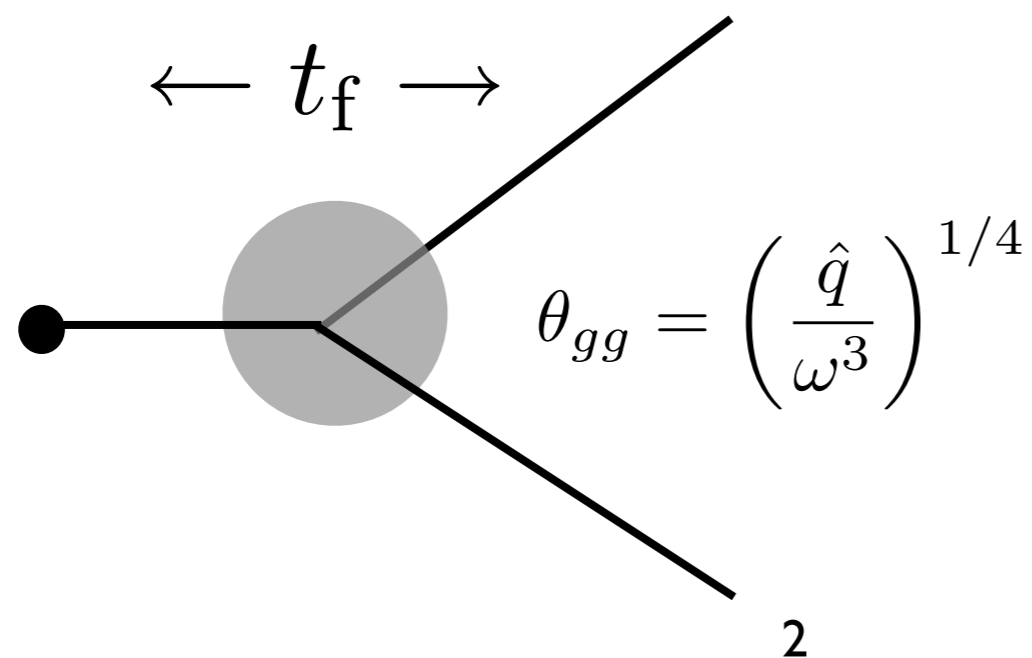
⇒ In the opaque limit, simple shift of the angular constraint!

DECOHERENCE OF MULTI-GLUON EMISSIONS

Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)

$$\omega \frac{dN}{d\omega} = \frac{C_F \alpha_s}{\pi} \sqrt{\frac{\hat{q} L^2}{\omega}} \propto \alpha_s \frac{L}{t_f}$$

The antenna is formed in the medium



formation time

$$t_f = \sqrt{\frac{\omega}{\hat{q}}}$$

Emission time

$$t \sim L \gg t_f$$

DECOHERENCE OF MULTI-GLUON EMISSIONS

decoherence time

$$t_d = (\hat{q} \theta_{gg}^2)^{-1/3}$$

formation time

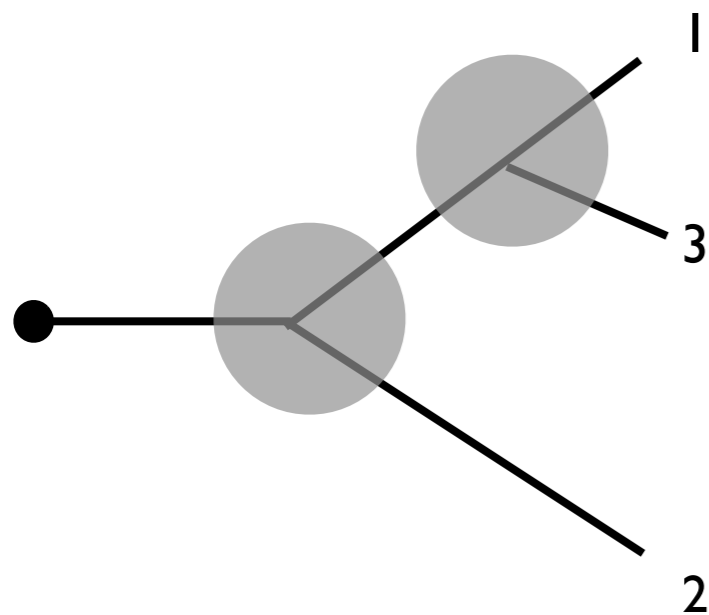
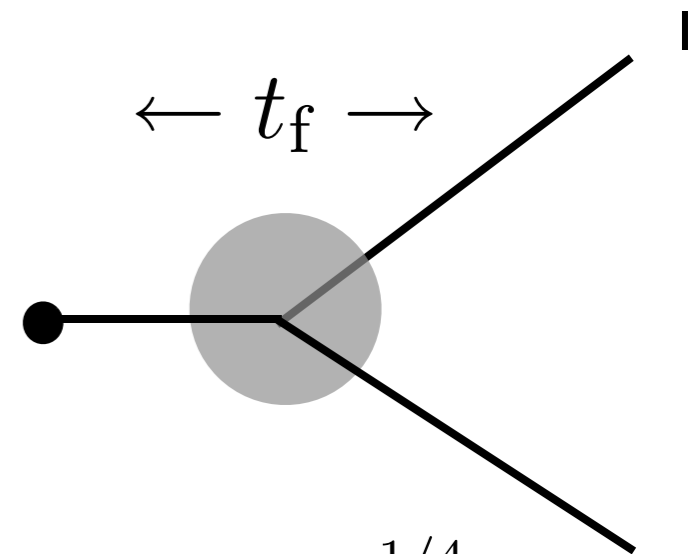
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typical angle for medium-induced gluon

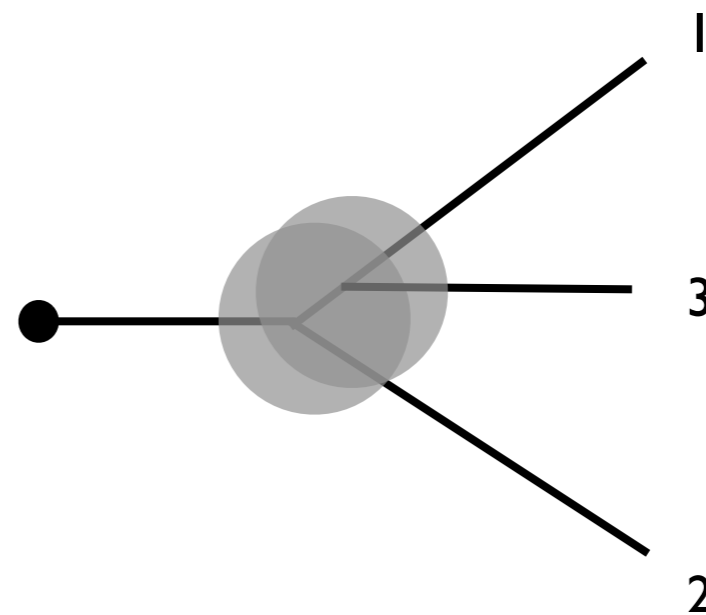
$$\theta_{gg} = \left(\frac{\hat{q}}{\omega^3} \right)^{1/4}$$

$$t_d \equiv t_f$$

(see F. Dominguez 's talk)



incoherent emissions



coherent emissions

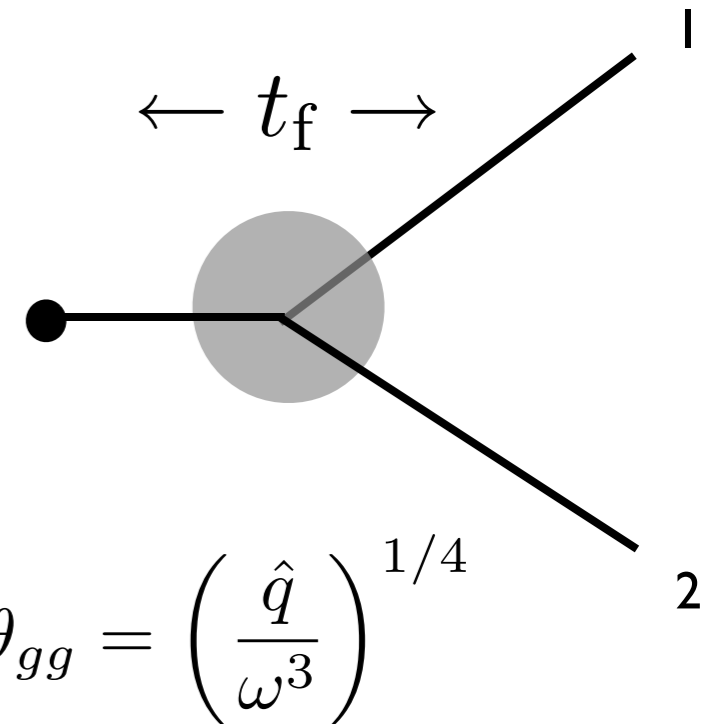
DECOHERENCE OF MULTI-GLUON EMISSIONS

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formation time

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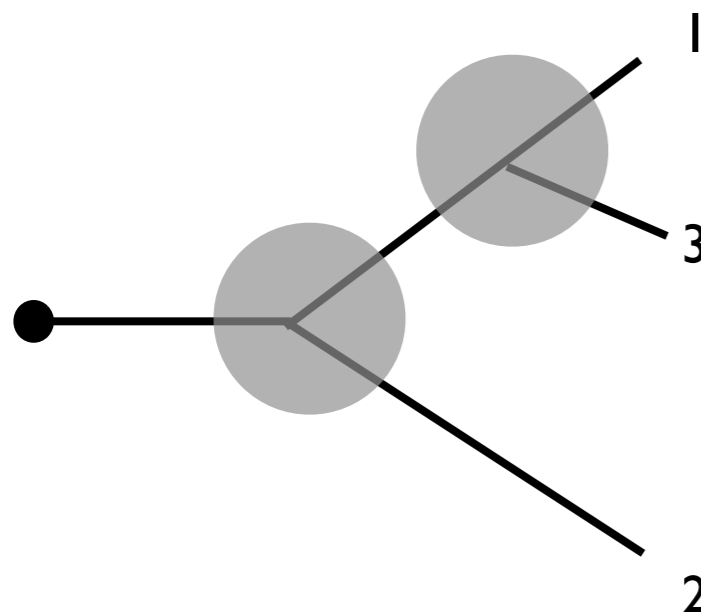


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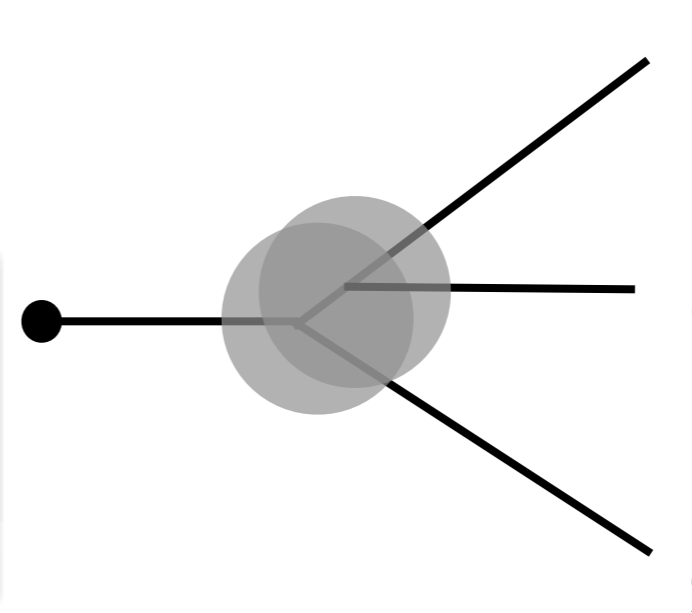
$$t_d \equiv t_f$$

(see F. Dominguez 's talk)



$$\propto \left(\alpha_s \frac{L}{t_f}\right)^2$$

incoherent emissions



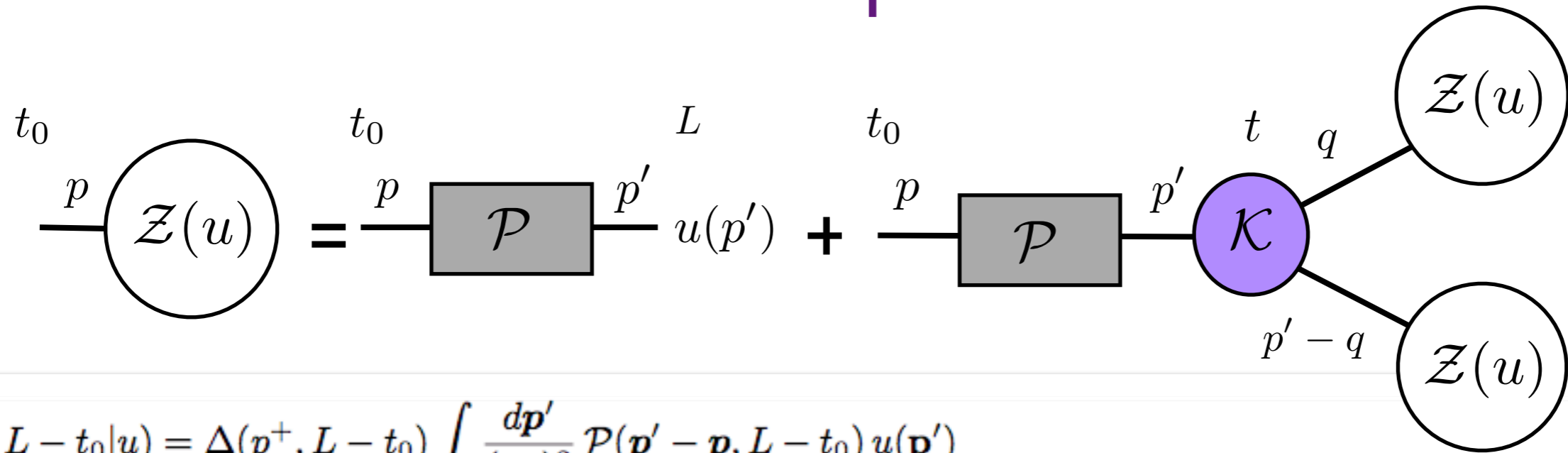
$$\propto \alpha_s \left(\alpha_s \frac{L}{t_t}\right)$$

coherent emissions

SUMMARY

- ✓ In the limit of a dense medium, parton branchings **decohere** due to rapid color randomization
- ✓ A two scale problem: the hardest scale sets the transition from incoherent emissions to coherent emissions
- ✦ Interplay **decoherence** ($k_{\perp} < Q_{\text{hard}}$) vs **coherence** ($k_{\perp} > Q_{\text{hard}}$)
- ✓ Building block of jet calculus? Probabilistic picture?

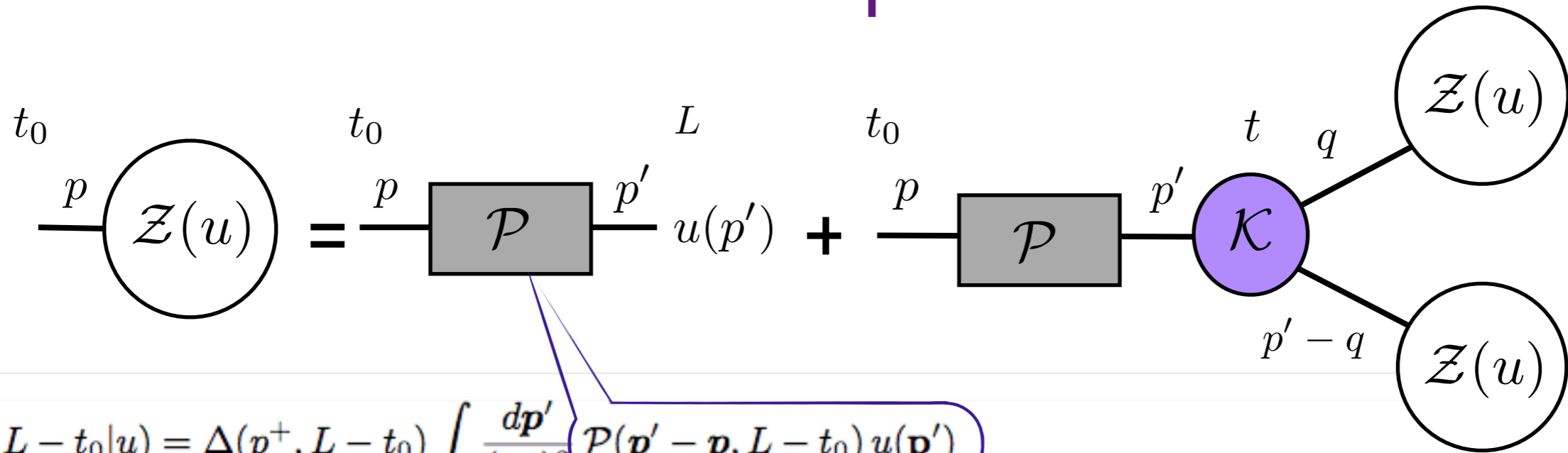
Master Equation



$$\mathcal{Z}(\mathbf{p}, L - t_0 | u) = \Delta(p^+, L - t_0) \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) u(\mathbf{p}')$$

$$+ \alpha_s \int_{t_0}^L dt \Delta(p^+, t - t_0) \int_0^1 \frac{dz}{z} \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) \int \frac{d\mathbf{q}}{(2\pi)^2} \mathcal{K}(\mathbf{q} - z\mathbf{p}' | z) \mathcal{Z}(\mathbf{q}, L - t | u) \mathcal{Z}(\mathbf{p}' - \mathbf{q}, L - t | u)$$

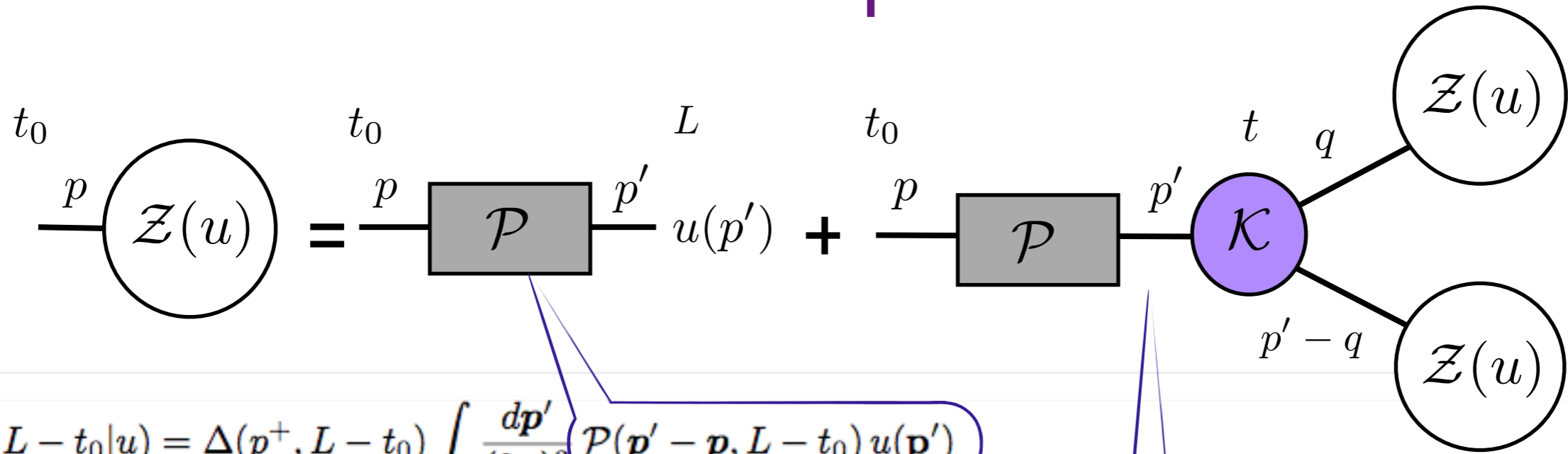
Master Equation



$$\mathcal{Z}(\mathbf{p}, L - t_0 | u) = \Delta(p^+, L - t_0) \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) u(\mathbf{p}')$$

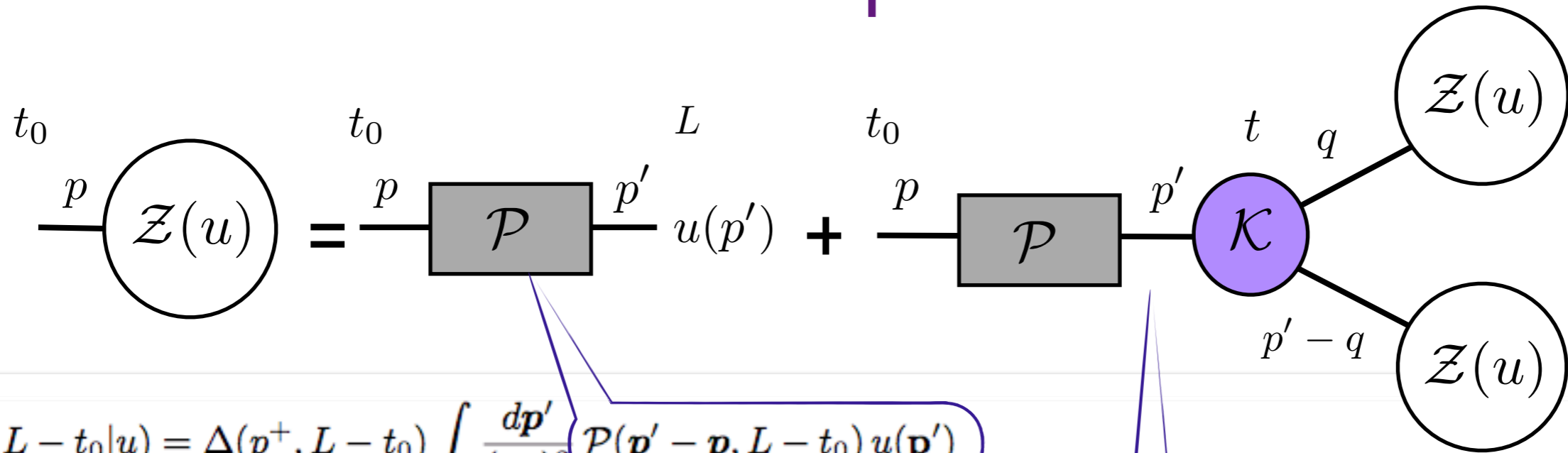
$$+ \alpha_s \int_{t_0}^L dt \Delta(p^+, t - t_0) \int_0^1 \frac{dz}{z} \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) \int \frac{d\mathbf{q}}{(2\pi)^2} \mathcal{K}(\mathbf{q} - z\mathbf{p}' | z) \mathcal{Z}(\mathbf{q}, L - t | u) \mathcal{Z}(\mathbf{p}' - \mathbf{q}, L - t | u)$$

Master Equation



$$\begin{aligned}
 \mathcal{Z}(\mathbf{p}, L - t_0 | u) &= \Delta(p^+, L - t_0) \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) u(\mathbf{p}') \\
 &+ \alpha_s \int_{t_0}^L dt \Delta(p^+, t - t_0) \int_0^1 \frac{dz}{z} \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) \int \frac{d\mathbf{q}}{(2\pi)^2} \mathcal{K}(\mathbf{q} - z\mathbf{p}' | z) \mathcal{Z}(\mathbf{q}, L - t | u) \mathcal{Z}(\mathbf{p}' - \mathbf{q}, L - t | u)
 \end{aligned}$$

Master Equation



$$\mathcal{Z}(\mathbf{p}, L - t_0 | u) = \Delta(p^+, L - t_0) \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) u(\mathbf{p}')$$

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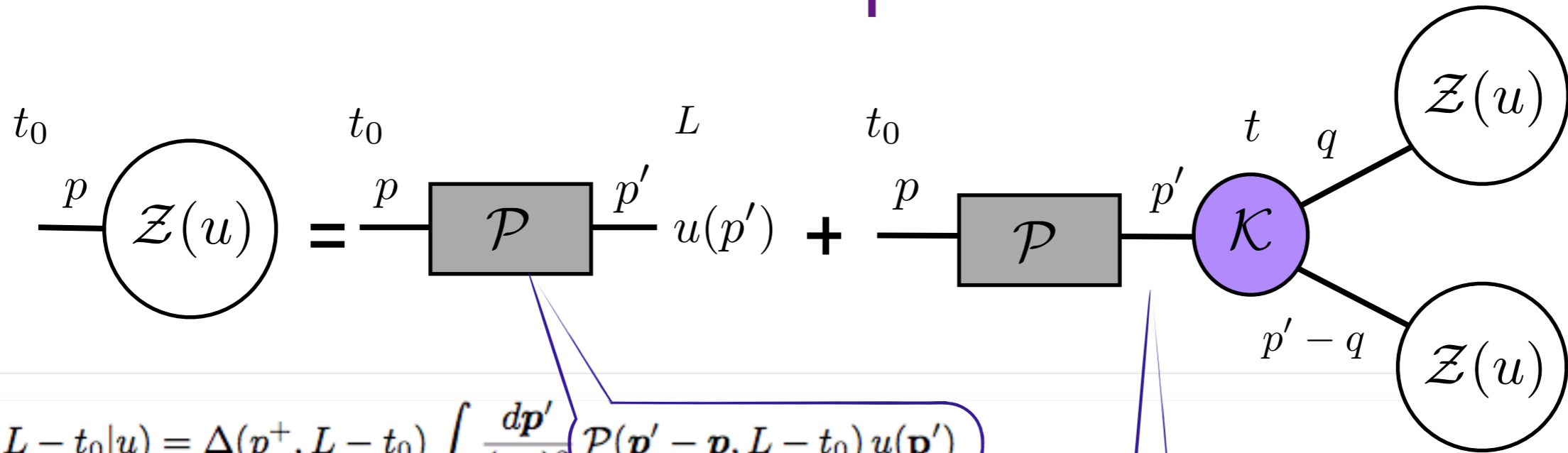
- In-medium splitting function

$$\mathcal{K}_{BC}^A(\mathbf{q} - z\mathbf{p}, z) = \frac{2}{p^+} P_{AB}(z) \sin \left[\frac{(\mathbf{q} - z\mathbf{p})^2}{2k_{\text{br}}^2} \right] \exp \left[-\frac{(\mathbf{q} - z\mathbf{p})^2}{2k_{\text{br}}^2} \right]$$

- Relative pT at branching time

$$k_{\text{br}}^2 = \sqrt{z(1-z)p^+ \hat{q}_{\text{eff}}}$$

Master Equation



$$\mathcal{Z}(\mathbf{p}, L - t_0 | u) = \Delta(p^+, L - t_0) \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) u(\mathbf{p}')$$

$$+ \alpha_s \int_{t_0}^L dt \Delta(p^+, t - t_0) \int_0^1 \frac{dz}{z} \int \frac{d\mathbf{p}'}{(2\pi)^2} \mathcal{P}(\mathbf{p}' - \mathbf{p}, L - t_0) \int \frac{d\mathbf{q}}{(2\pi)^2} \mathcal{K}(\mathbf{q} - z\mathbf{p}' | z) \mathcal{Z}(\mathbf{q}, L - t | u) \mathcal{Z}(\mathbf{p}' - \mathbf{q}, L - t | u)$$

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- Sudakov form factor:
Prob. not to emit
(Unitarity)

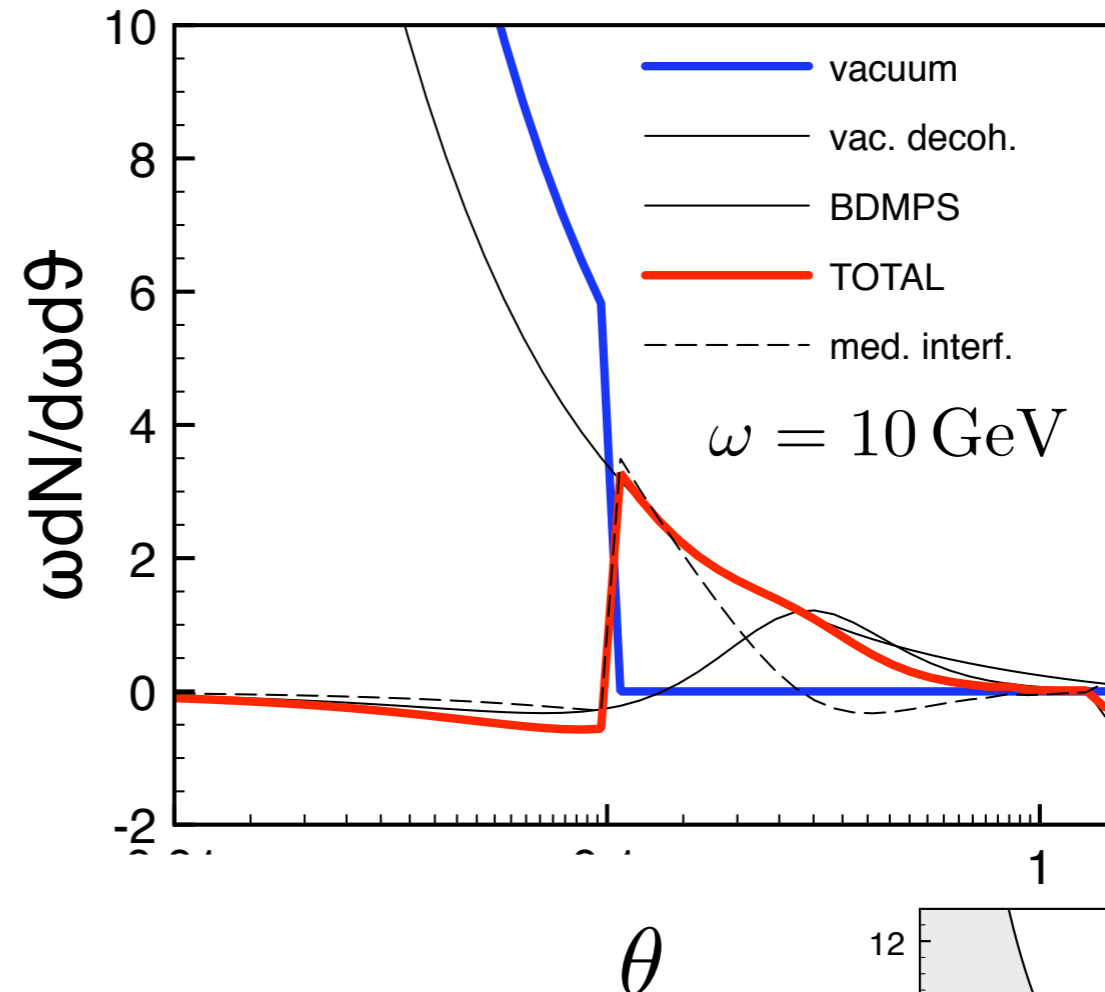
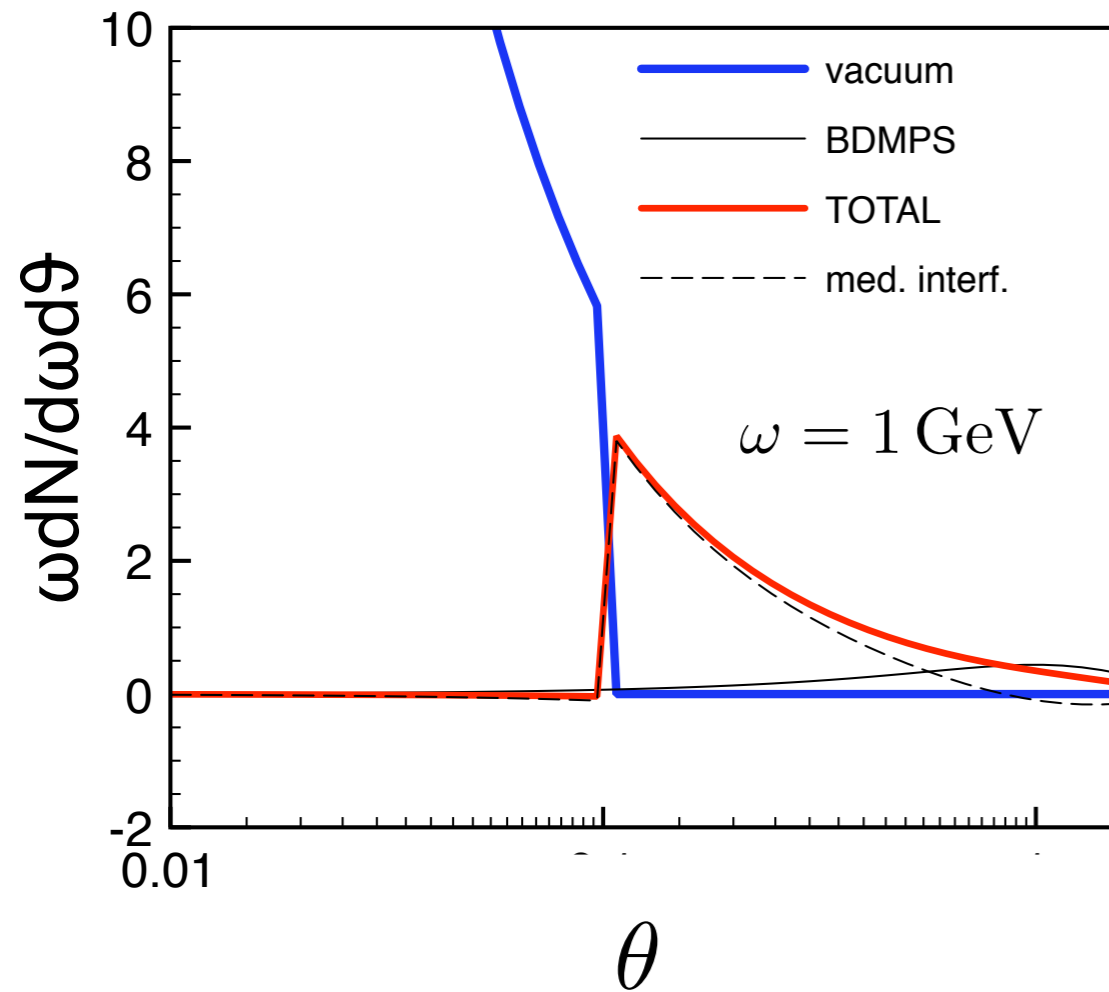
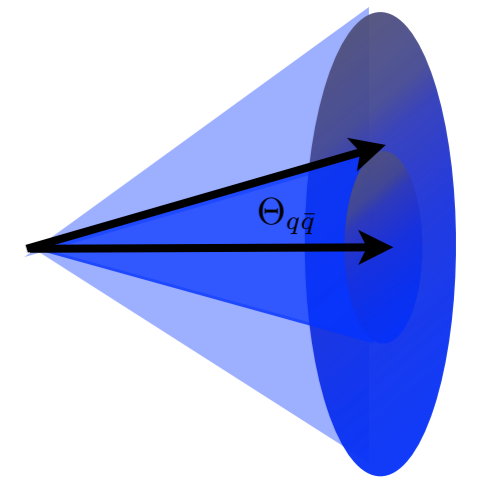
$$\Delta(p^+, L - t_0) = \exp \left[-\alpha_s (L - t_0) \int_0^1 \frac{dz}{z} \mathcal{K}(z) \right]$$

- Relative pT at branching time

Angular distribution ($r_{\perp} < Q_s^{-1}$)

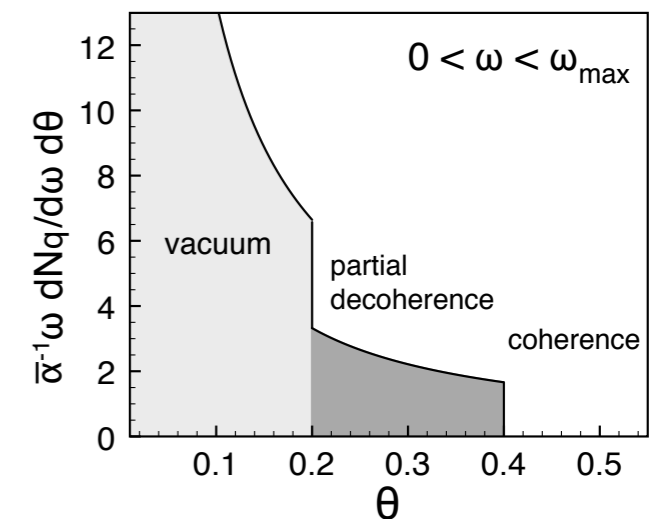
- Dipole regime

$$\hat{q} = 1 \text{ GeV}^2/\text{fm} \quad \theta_{q\bar{q}} = 0.1 \quad L = 5 \text{ fm}$$



$$dN_q^{\text{ind}} \propto \mathcal{R}_q^{\text{ind}} - \mathcal{J}^{\text{ind}}$$

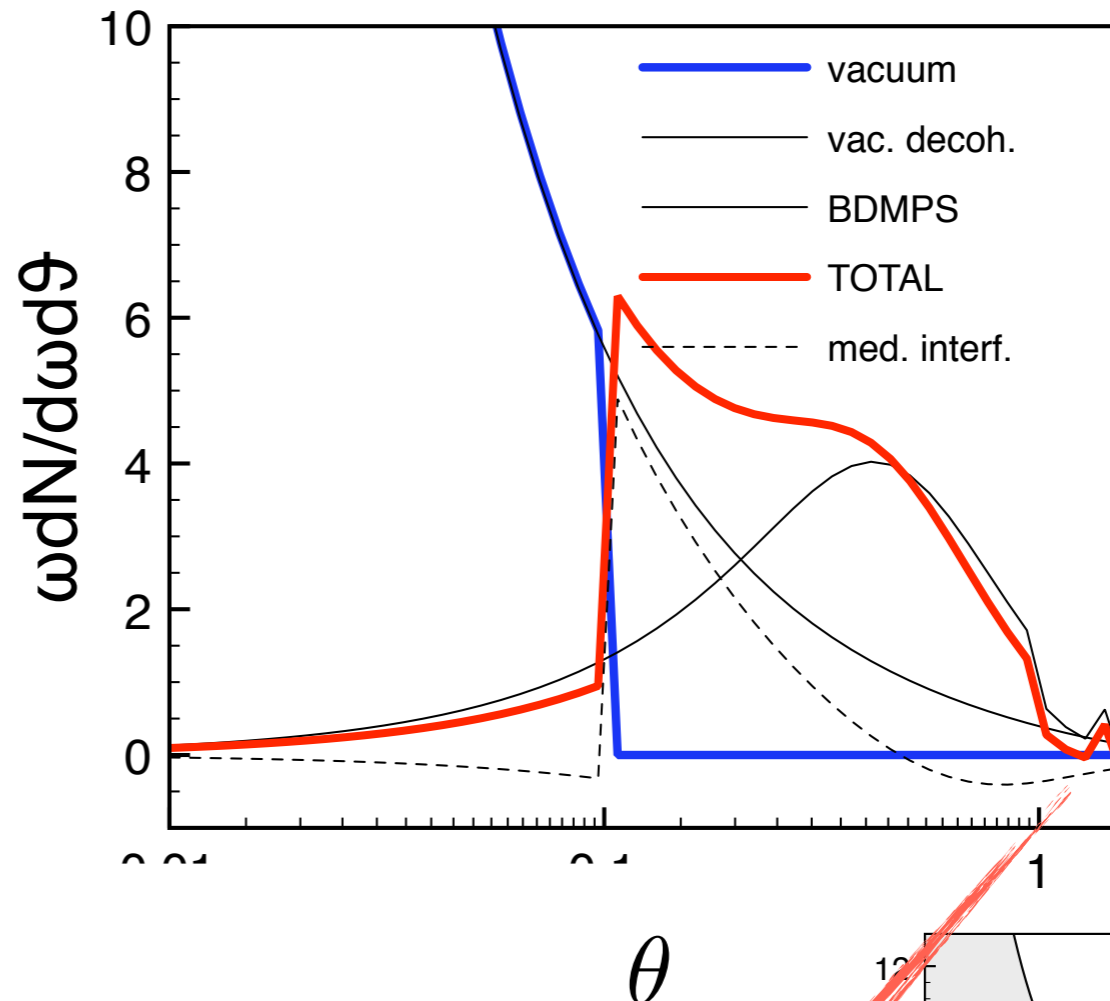
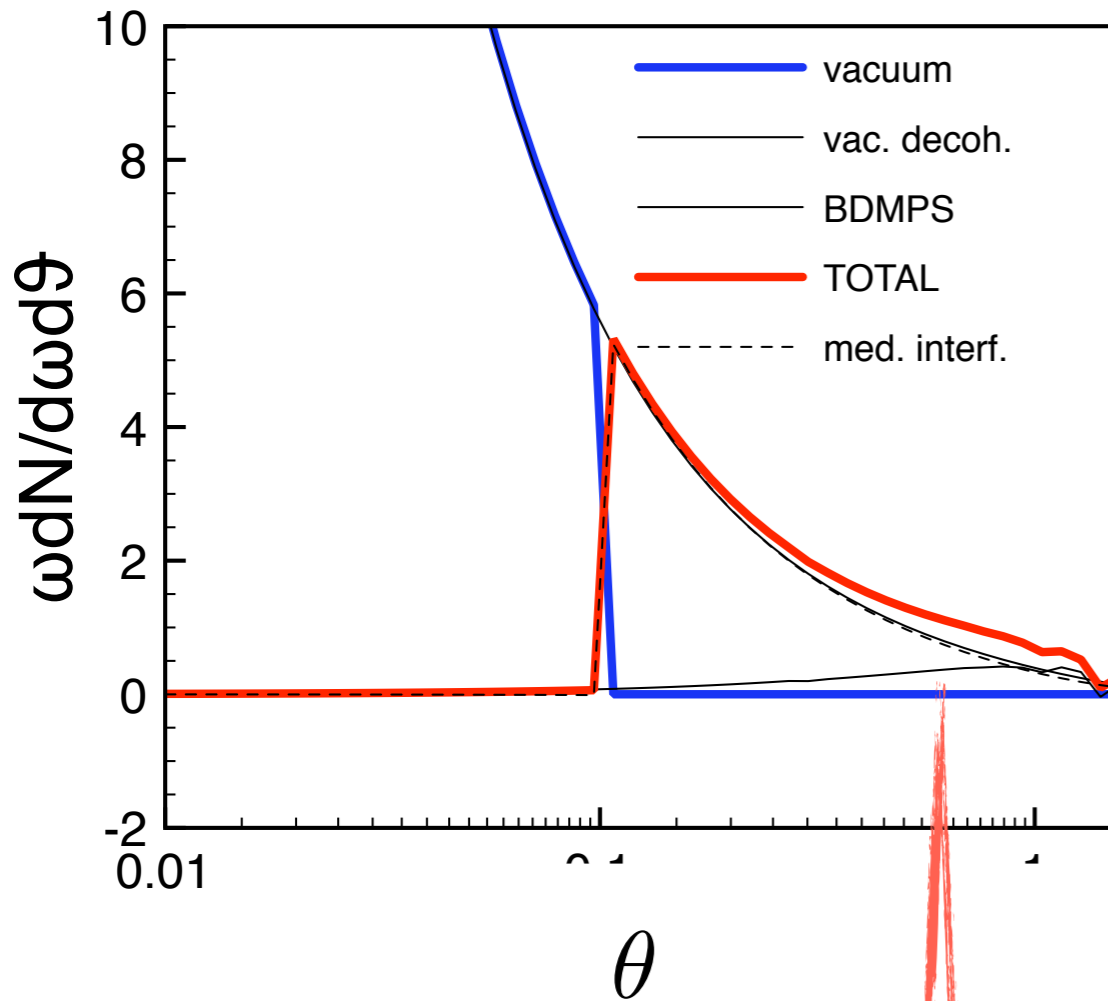
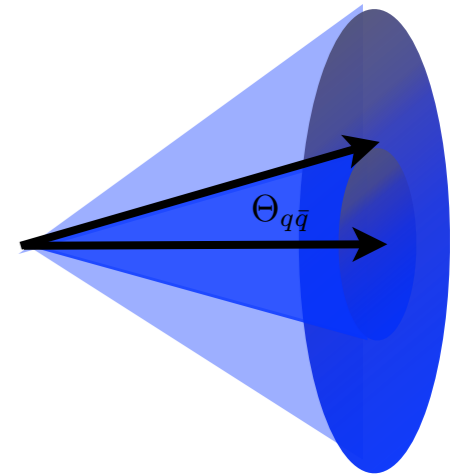
integrating over the azimuth



Angular distribution ($r_{\perp} > Q_s^{-1}$)

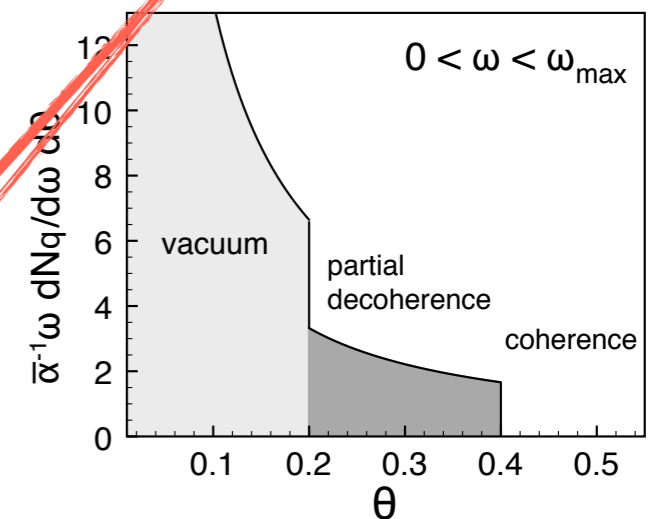
- Decoherence regime

$\hat{q} = 5 \text{ GeV}^2/\text{fm}$ $\theta_{q\bar{q}} = 0.1$ $L = 10 \text{ fm}$



Decoherence

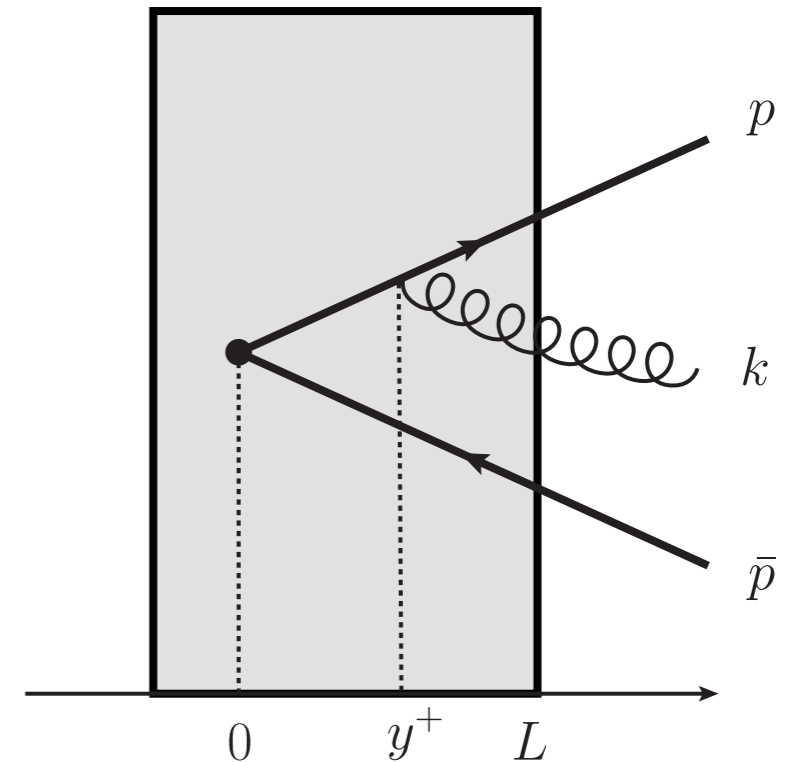
Coherence: $k_{\perp} > Q_s$



Classical Yang Mills

Glueon propagator (Brownian motion in the transverse plane)

$$\mathcal{G}(x^+, \mathbf{x}; y^+, \mathbf{y} | k^+) = \int \mathcal{D}[\mathbf{r}] \exp \left[i \frac{k^+}{2} \int_{y^+}^{x^+} d\xi \dot{\mathbf{r}}^2(\xi) \right] U(x^+, y^+; [\mathbf{r}])$$



The amplitude for gluon radiation off the quark

$$\mathcal{M}_{\lambda, q}^a(\vec{k}) = \frac{g}{k^+} \int_{x^+ = -\infty} d^2 \mathbf{x} e^{i k^- x^+ - i \mathbf{k} \cdot \mathbf{x}} \int_0^{+\infty} dy^+ e^{i \frac{k^+ p^-}{p^+} y^+} \times \epsilon_\lambda \cdot (i \partial_y + k^+ \mathbf{n}) \mathcal{G}^{ab}(x^+, \mathbf{x}; y^+, \mathbf{y} | k^+) \Big|_{\mathbf{y} = \mathbf{n} y^+} U_p^{bc}(y^+, 0) Q_q^c,$$

$$\mathbf{n} = \mathbf{p} / p^+$$

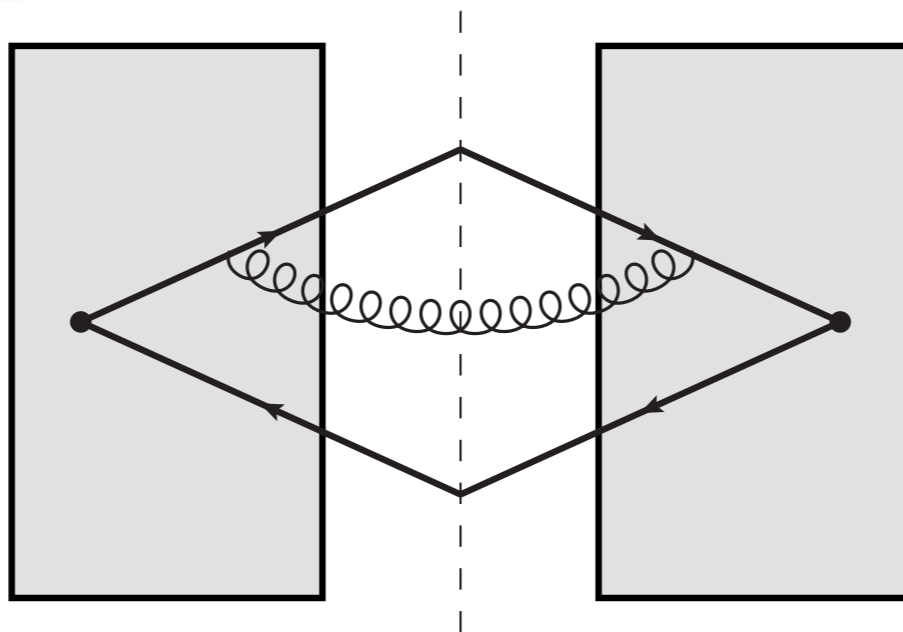
Classical Yang Mills

Independent emission off the quark (BDMPS-Z spectrum)

$$\mathcal{R}_q = 2 \operatorname{Re} \left\{ \int_0^\infty dy'^+ \int_0^{y'^+} dy^+ \int d^2 z \exp \left[-i \boldsymbol{\kappa} \cdot \mathbf{z} - \frac{1}{2} \int_{y^+}^\infty d\xi n(\xi) \sigma(\mathbf{z}) \right] \right. \\ \left. \times \boldsymbol{\partial}_y \cdot \boldsymbol{\partial}_z \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y} = \mathbf{0} | k^+) \right\},$$

$$\boldsymbol{\kappa} = \mathbf{k} - x \mathbf{p}$$

$$x = k^+ / p^+$$



Classical Yang Mills

Interferences

Y. M.-T. and K. Tywoniuk 1105.1346 [hep-ph]
 J. Casalderrey-Solana and E. Iancu arXiv:1105.1760 (JHEP 2011)

$$\begin{aligned} \mathcal{J} = & \text{Re} \left\{ \int_0^\infty dy'^+ \int_0^{y'^+} dy^+ (1 - \Delta_{\text{med}}(y^+, 0)) \right. \\ & \times \int d^2 \mathbf{z} \exp \left[-i \bar{\boldsymbol{\kappa}} \cdot \mathbf{z} - \frac{1}{2} \int_{y'^+}^\infty d\xi n(\xi) \sigma(\mathbf{z}) + i \frac{k^+}{2} \delta \mathbf{n}^2 y^+ \right] \\ & \left. \times (\boldsymbol{\partial}_y - i k^+ \delta \mathbf{n}) \cdot \boldsymbol{\partial}_z \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y} | k^+) \Big|_{\mathbf{y} = \delta \mathbf{n} y^+} \right\} + \text{sym.} \end{aligned}$$

$$\boldsymbol{\kappa} = \mathbf{k} - x \mathbf{p}$$

$$x = k^+ / p^+$$

$$|\delta \mathbf{n}| \equiv \sin \theta_{q\bar{q}} \sim \theta_{q\bar{q}}$$

Decoherence parameter

$$\frac{1}{N_c^2 - 1} \langle \text{Tr} U_p(y^+, 0) U_{\bar{p}}^\dagger(y^+, 0) \rangle = 1 - \Delta_{\text{med}}(y^+, 0)$$

