Freeze-out Conditions from Lattice QCD

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Chemical freeze-out condition in HIC

\[ (T_f^f, \mu_B^f, \mu_Q^f, \mu_S^f) \]

by comparing hadron yields from HIC expt.

\[ \sqrt{s} \]

initial state

pre-equilibrium

QGP, hydro. expansion

hadronization

freeze-out

\[ T_c, \mu_c \]

\[ T_f^f, \mu_B^f \]

chemical freeze-out

no inelastic scattering, hadron abundances unchanged (freeze-out), chemical equilibration

statistical / thermal / hadron resonance gas (HRG) model

thermalized, non-interacting hadrons & resonances
Chemical freeze-out condition in HIC

Initial state

Pre-equilibrium

QGP, hydro. expansion

Hadronization

Freeze-out

Temperature

\( \sqrt{s} \)

\( (T^f, \mu_B^f, \mu_Q^f, \mu_S^f) \)

by comparing hadron yields from HIC expt.

Statistical / thermal / hadron resonance gas (HRG) model

Thermalized, non-interacting hadrons & resonances

No inelastic scattering, hadron abundances unchanged (freezes-out), chemical equilibration

Time

Model dependent connection, not from first principle QCD

\( T_c, \mu_c \)

\( T^f, \mu_B^f \)
Search for QCD critical point in RHIC BES

fluctuations of conserved charges

net-proton, net-charge, ...

characterize the chemical freeze-out condition

LQCD: $T_c(\mu_B)$

QCD critical point will be situated somewhere along this critical region
Freeze-out conditions from (L)QCD?

✓ how far is the critical region from the freeze-out line?
   ❖ if they are far enough, then locating QCD critical point in HIC experiments will be difficult

✓ to what extent the experimental observables are governed by non-critical QCD thermodynamics?
   1) fix the freeze-out conditions from QCD
   2) QCD calculations of observables at these freeze-out parameters
   3) comparison with experiments

✓ do thermal models work perfectly for the freeze-out conditions at LHC? (ALICE, 2012)

freeze-out parameters \( (T^f, \mu_B^f, \mu_Q^f, \mu_S^f) \) from LQCD
Strangeness and electric charge chemical potentials

1) strangeness neutrality: $\langle n_S \rangle = 0$

2) isospin asymmetry: $\langle n_Q \rangle = r \langle n_B \rangle$

Au-Au & Pb-Pb: $r \simeq 0.4$

fixed using HIC initial conditions

expand these two relations in powers of $\mu_B, \mu_Q, \mu_S$ around $\mu_B = \mu_Q = \mu_S = 0$

$\mu_Q (T, \mu_B) = q_1 (T) \mu_B + q_3 (T) \mu_B^3 + \cdots$

$\mu_S (T, \mu_B) = s_1 (T) \mu_B + s_3 (T) \mu_B^3 + \cdots$

$\langle N_Q \rangle = \frac{\langle N_p \rangle}{\langle N_B \rangle + \langle N_n \rangle} = r$

$\langle n_S \rangle = 0$

$\langle n_Q \rangle = r \langle n_B \rangle$

assume: homogeneous system

B: baryon, Q: charge, S: strangeness
p: proton, n: neutron

$N_x = N_x - N_x$, $n_x = N_x / V$
Strangeness and electric charge chemical potentials

\[ \mu_Q(T, \mu_B) = q_1(T) \mu_B + q_3(T) \mu_B^3 \]

\[ \mu_S(T, \mu_B) = s_1(T) \mu_B + s_3(T) \mu_B^3 \]

**LO:** continuum extrapolated

**NLO:** \( N_t = 6, 8 \), small cut-off dependence, *continuum estimate*

NLO corrections < 10%

\[ \frac{\mu_B}{T} \lesssim 1.3 \]
Strangeness and electric charge chemical potentials

\[ \mu_Q(T, \mu_B) \]

\[ \mu_S(T, \mu_B) \]

5-15% deviations from HRG

BNL-BI, arXiv:1208.1220
Temperature and baryon chemical potential

all observables can now be obtained as function of two independent parameters $T, \mu_B$

hadron yields are inaccessible in LQCD

comparison of 2 expt. measured ratios of cumulants of conserved charge fluctuations with LQCD calculations fixes 2 freeze-out parameters $T_f, \mu_B$

proton fluctuations (expt.) $\sim \sim$ baryon fluctuations (LQCD)

Asakawa-Kitazawa; Bzdak-Skokov

safe to work with net electric charge fluctuations measured both in expt. and LQCD

ratio of cumulants: cancels the unknown volume of the fireball
Temperature and baryon chemical potential

\[ \frac{M_Q(\sqrt{s})}{\sigma_Q(\sqrt{s})} = \frac{\langle N_Q \rangle}{\langle (\delta N_Q)^2 \rangle} = \frac{\chi_2^Q(T, \mu_B)}{\chi_1^Q(T, \mu_B)} = R_{12}^Q(T) \mu_B + R_{12}^{Q,3}(T) \mu_B^3 + \cdots = R_{12}^Q(T, \mu_B) \]

\[ \frac{S_Q(\sqrt{s}) \sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\langle (\delta N_Q)^3 \rangle}{\langle N_Q \rangle} = \frac{\chi_3^Q(T, \mu_B)}{\chi_1^Q(T, \mu_B)} = R_{31}^{Q,0}(T) + R_{31}^{Q,2}(T) \mu_B^2 + \cdots = R_{31}^Q(T, \mu_B) \]

**HIC**
- Mean: \( M_Q \)
- Variance: \( \sigma_Q^2 \)
- Skewness: \( S_Q \)
- \( \delta N_Q = N_Q - \langle N_Q \rangle \)

**LQCD**
- Generalized charge susceptibilities:
  \[ \chi_n^Q(T, \vec{\mu}) = \frac{1}{VT^3} \frac{\partial^n \ln Z(T, \vec{\mu})}{\partial (\mu_Q/T)^n} \]
Temperature and baryon chemical potential

\[ R_{12}^Q = \frac{M_Q}{\sigma_Q^2} \]

\[ R_{12}^Q(T, \mu_B) = R_{12}^{Q,1}(T)\mu_B + R_{12}^{Q,3}(T)\mu_B^3 \]

LO: continuum extrapolated

NLO: \( N_\tau = 6,8 \)

NLO corrections < 10%

\( \mu_B/T \lesssim 1.3 \)
Temperature and baryon chemical potential

$$R_{31}^Q = S_Q \sigma^3_Q / M_Q$$

LO: $N_\tau = 6, 8$

NLO corrections: $\leq 10\%$, $\mu_B / T \leq 1.3$

$$R_{31}^Q(T, \mu_B) = R_{31}^{Q,0}(T) + R_{31}^{Q,2}(T) \mu_B^2$$

large deviation from HRG for $T > 155$ MeV
Temperature and baryon chemical potential

\[ R_{12}^Q = \frac{M_Q}{\sigma_Q^2} \]

\[ R_{31}^Q = \frac{S_Q \sigma_Q^3}{M_Q} \]

<table>
<thead>
<tr>
<th>( S_Q \sigma_Q^3 / M_Q )</th>
<th>( T^f [\text{MeV}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lesssim 2 )</td>
<td>( \lesssim 155 )</td>
</tr>
<tr>
<td>( \sim 1.5 )</td>
<td>( \sim 160 )</td>
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<tr>
<td>( \lesssim 1 )</td>
<td>( \gtrsim 170 )</td>
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</tbody>
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<table>
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<tr>
<th>( M_Q / \sigma_Q^2 )</th>
<th>( \mu_B^f / T^f )</th>
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</thead>
<tbody>
<tr>
<td>0.01–0.02</td>
<td>0.1–0.2</td>
</tr>
<tr>
<td>0.03–0.04</td>
<td>0.3–0.4</td>
</tr>
<tr>
<td>0.05–0.08</td>
<td>0.5–0.7</td>
</tr>
</tbody>
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for: \( T^f \sim 160 \text{ MeV} \)

BNL-BI, arXiv:1208.1220
Thermodynamic consistency

\[ \frac{R_{12}^Q}{R_{12}^B} = \frac{M_Q/\sigma_Q^2}{M_B/\sigma_B^2} \]

all other cumulant ratios should be reproduced w/o fixing any further parameter, provided the system is in thermal & chemical equilibrium
Systematic: estimates using HRG

series truncation: $\sim 5\%$

$m_u \neq m_d: \sim 5\%$

continuum limit: $\sim 5\%$
chemical freeze-out conditions can be determined from first principle (lattice) QCD calculations by comparing with cumulants of net charge fluctuations currently being measured by STAR, PHENIX.

controlled NLO Taylor expansion up to

$$\mu_B \sim 200 \text{ MeV}, \sqrt{s} \sim 19.6 \text{ GeV}$$

general agreement with thermal models within 15%