Electric and baryonic charge fluctuations from lattice QCD

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Motivation I: The QCD critical point

Expected phase diagram of QCD:

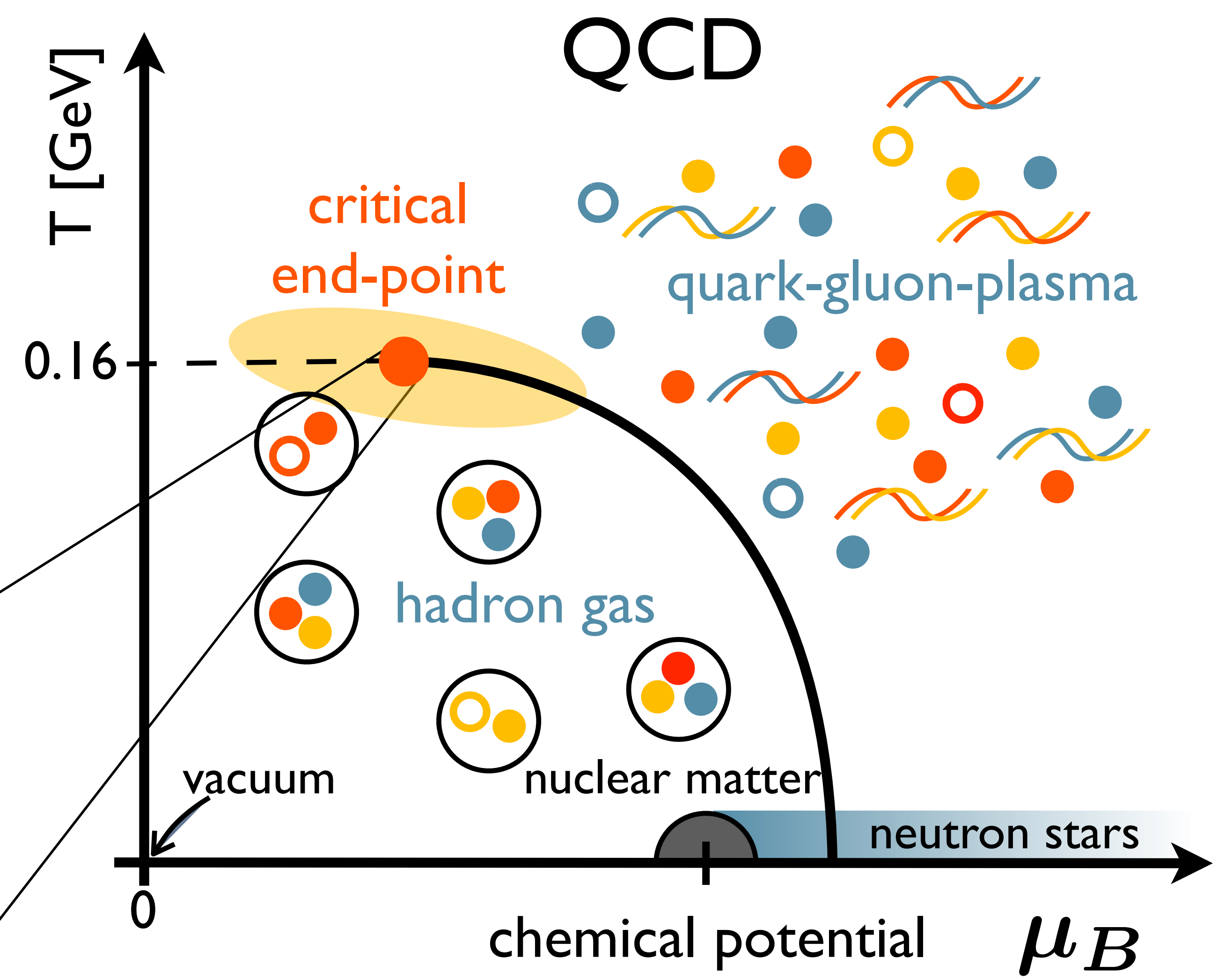
Phases:

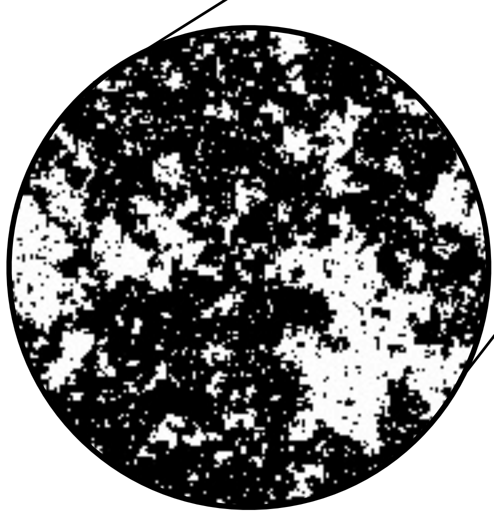
1) hadronic states at low T, low densities
2) quasi-free quarks and gluons hat high T, high densities

Mechanisms:

4) spontaneous chiral symmetry breaking
5) (de-)confinement

Critical end-point?





$\Rightarrow$ **Diverging correlation length and fluctuations. Universal behavior within a scaling region.**

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Overview

1) Introduction

- definition of generalized susceptibilities / charge fluctuations
- critical behavior of fluctuations

2) Status of lattice data

- 2nd, 4th and 6th order cumulants
- lattice cutoff effects for electric charge fluctuations

3) Summary

BNL-Bielefeld Collaboration:

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, S. Mukherjee, P. Petreczky, C. Schmidt, D. Smith, W. Soeldner, M. Wagner

HotQCD Collaboration:

A. Bazavov, T. Bhattacharya, M. Buchoff, M. Cheng, N. Christ, C. DeTar, H.-T. Ding, S. Gottlieb, R. Gupta, P. Hegde, U. Heller, C. Jung, F. Karsch, E. Laermann, L. Levkova, Z. Lin, R. Mawhinney, S. Mukherjee, P. Petreczky, D. Renfrew, C. Schmidt, C. Schroeder, W. Soeldner, R. Solz, R. Sugar, D. Toussaint, P. Vranas

1) Introduction: Definitions

Expansion of the pressure:

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk,0}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$X = B, Q, S$: conserved charges

Lattice

$$\chi_{n,0}^X = \frac{1}{VT} \frac{\partial^n \ln Z}{\partial(\mu_X/T)^n}\bigg|_{\mu_X=0}$$

generalized susceptibilities

$\Rightarrow$ only at $\mu_X = 0$!

Experiment

$$\begin{aligned} VT^3\,\chi_2^X &= \left\langle (\delta N_X)^2 \right\rangle \\ VT^3\,\chi_4^X &= \left\langle (\delta N_X)^4 \right\rangle - 3\left\langle (\delta N_X)^2 \right\rangle^2 \\ VT^3\,\chi_6^X &= \left\langle (\delta N_X)^4 \right\rangle \\ &\quad -15\left\langle (\delta N_X)^4 \right\rangle \left\langle (\delta N_X)^2 \right\rangle \\ &\quad +30\left\langle (\delta N_X)^2 \right\rangle^3 \end{aligned}$$

cumulants of net-charge fluctuations

$$\delta N_X \equiv N_X - \langle N_X \rangle$$

$\Rightarrow$ only at freeze-out $(\mu_f(\sqrt{s}), T_f(\sqrt{s}))$!

ratios of cumulants are volume independent
and can thus be compared between the two!

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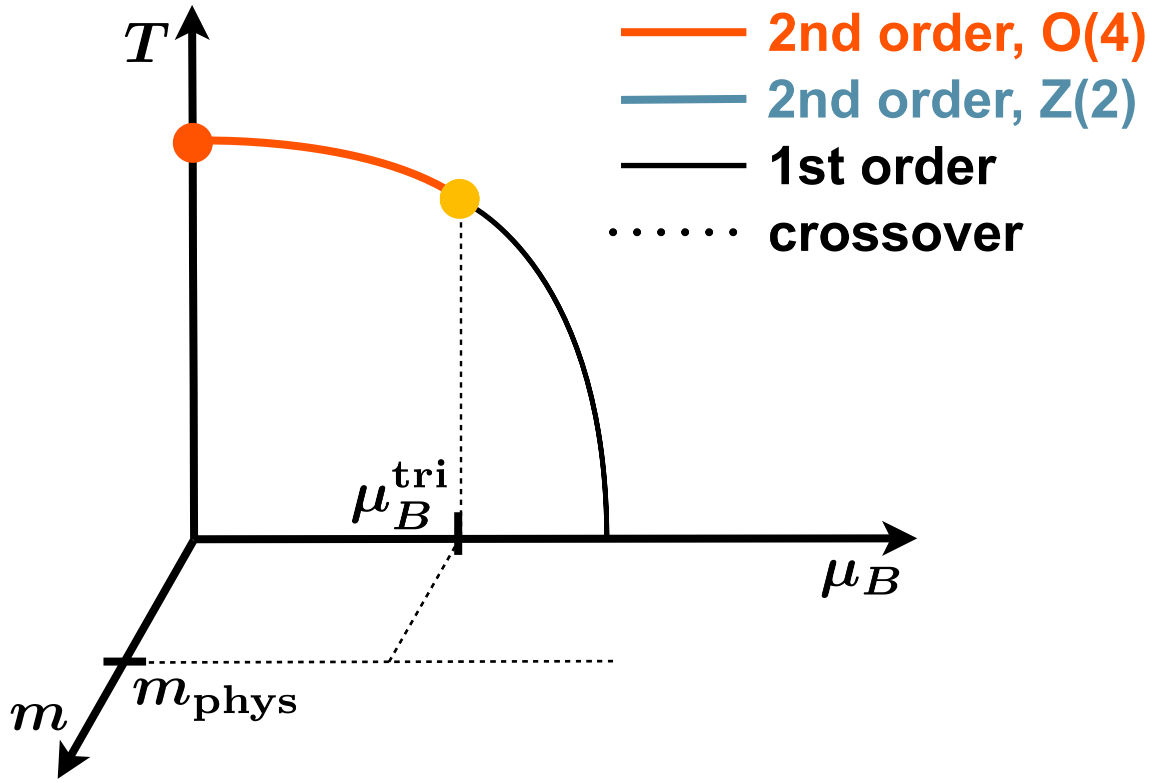
1) Introduction: Critical behavior



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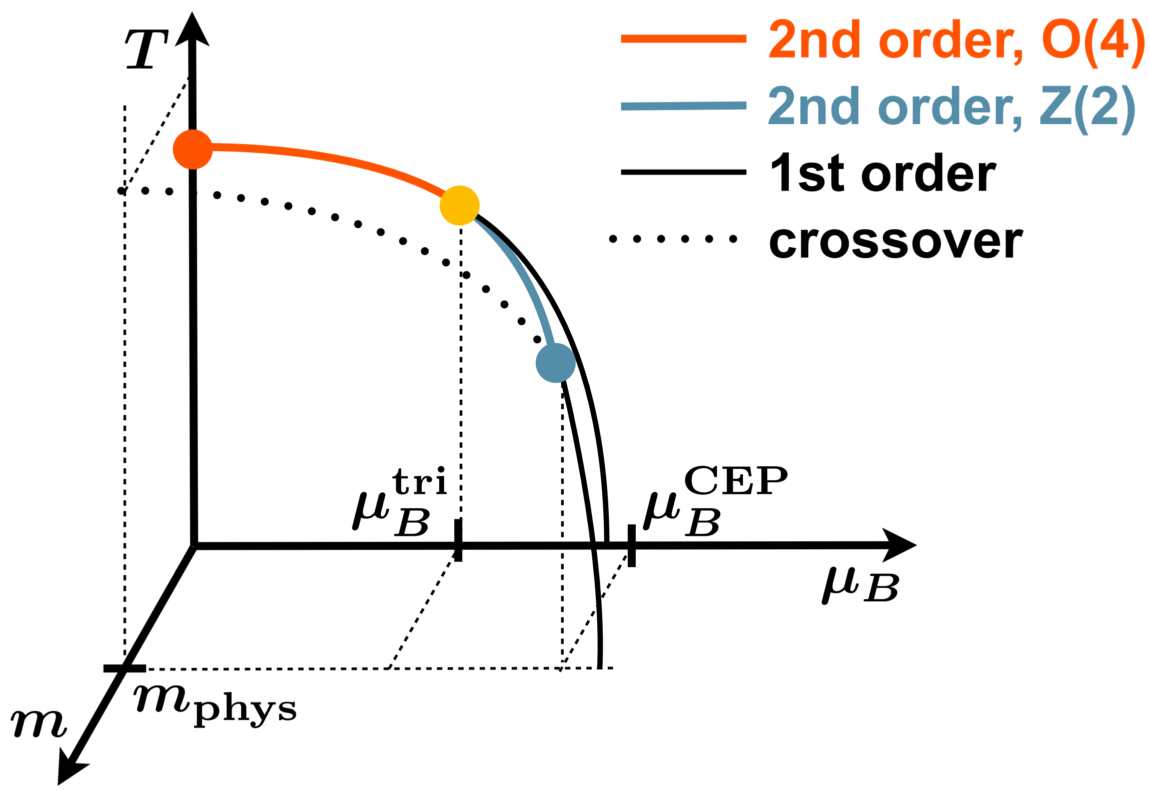

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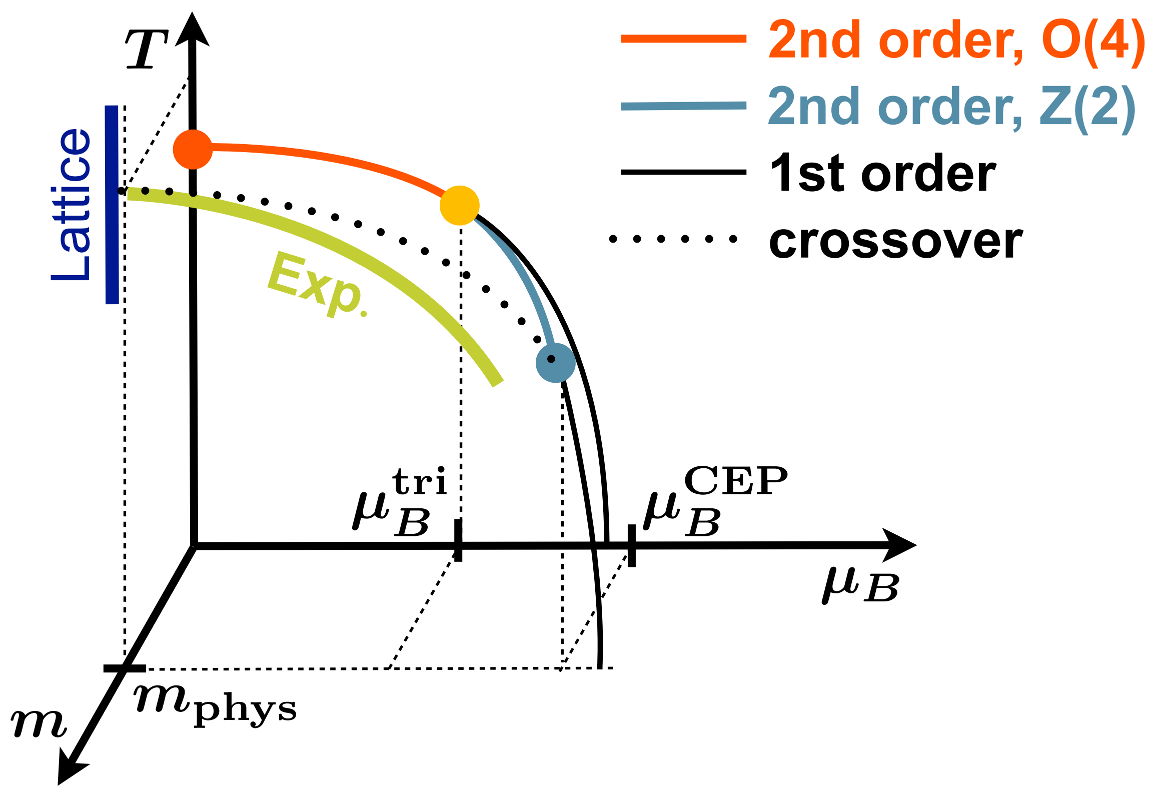

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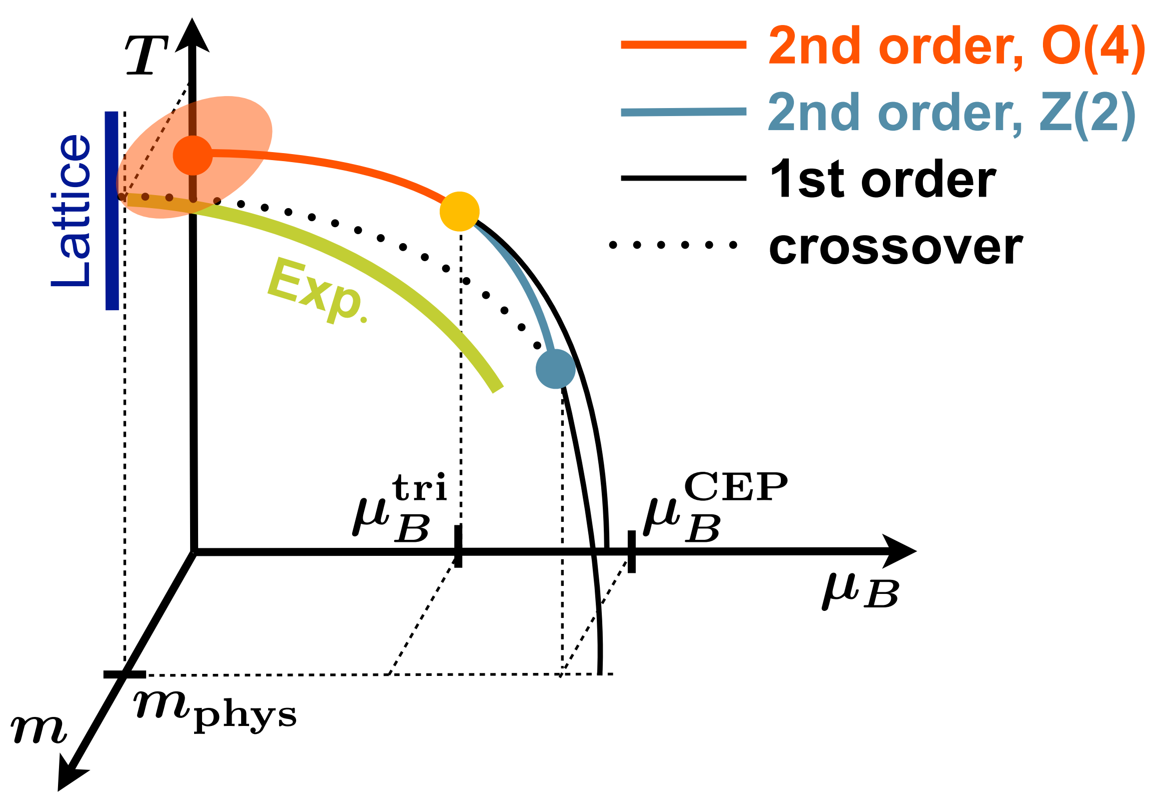

$\mu_B = 0,\ m = 0$

scaling field:

$$t = \frac{1}{t_0}\left(\left(\frac{T-T_c}{T_c}\right) + \kappa\left(\frac{\mu_B}{T}\right)^2\right)$$

derivatives: $\frac{\partial^2}{\partial \mu_B^2} \Leftrightarrow \frac{\partial}{\partial t}$

critical exponent: $-1 < \alpha < 0$

$$\Rightarrow \chi_B^{(2n)} \sim A_\pm^{(2n)} |t|^{2-\alpha-n}$$

$\left(\chi_B^{(4)} \to \text{kink},\ \chi_B^{(6)} \to \text{divergent}\right)$

assume scaling hypothesis for the free energy:

$$f = f_s(t,h) + \text{regular}$$

for t-derivatives:

$$f \sim A_\pm |t|^{2-\alpha} + \text{regular}$$

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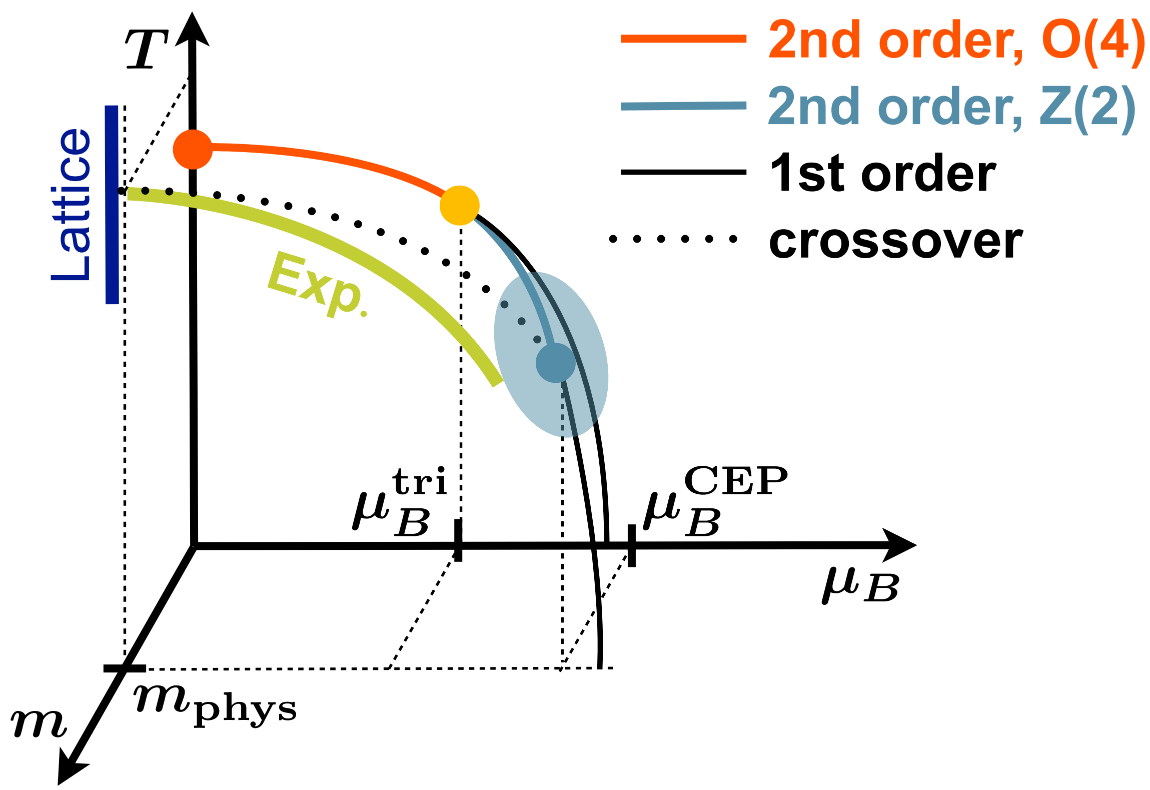

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$\left(\chi_B^{(4)} \to \text{kink},\ \chi_B^{(6)} \to \text{divergent}\right)$

$\mu_B = \mu_B^{\mathrm{CEP}},\ m = m_{\mathrm{phys}}$

scaling field:

$$t = \frac{1}{t_0}\left(\left(\frac{T-T_c^{\mathrm{CEP}}}{T_c^{\mathrm{CEP}}}\right) + \kappa\left(\frac{\mu_B-\mu_B^{\mathrm{CEP}}}{\mu_B^{\mathrm{CEP}}}\right)\right)$$

derivatives: $\frac{\partial}{\partial \mu_B} \Leftrightarrow \frac{\partial}{\partial t}$

critical exponent: $0 < \alpha < 1$

$$\Rightarrow \chi_B^{(n)} \sim A_\pm^{(n)} |t|^{2-\alpha-n}$$

$\left(\chi_B^{(2)} \to \text{divergent}\right)$

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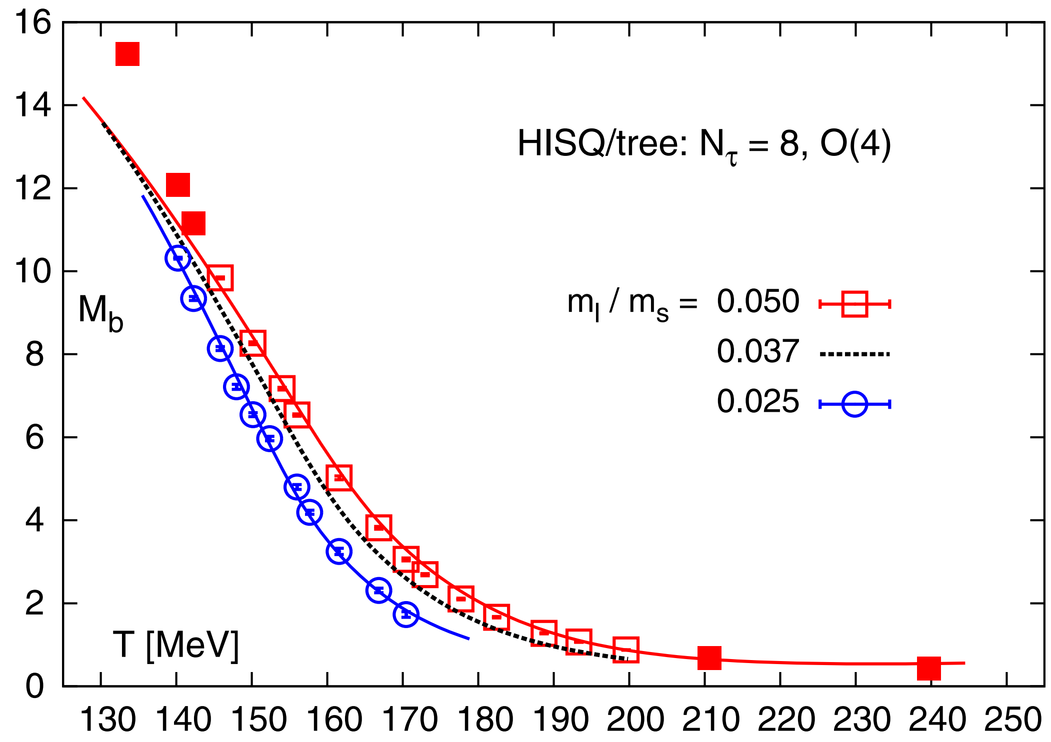
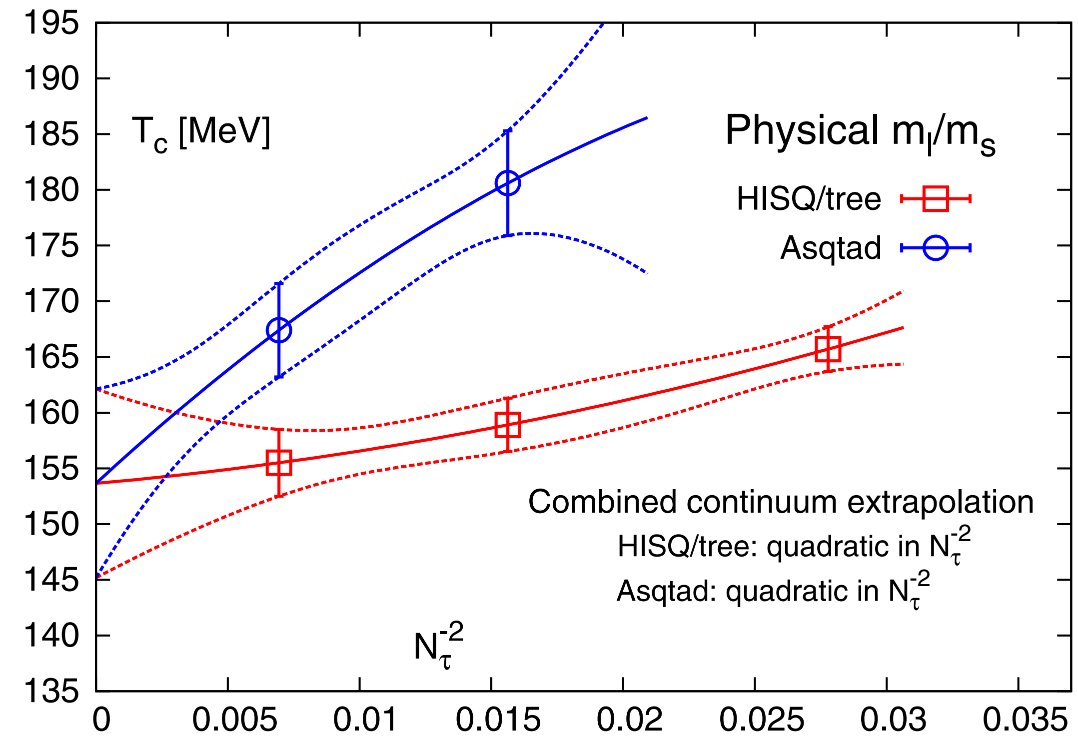
1) Introduction: Critical behavior

Important anchor point within the phase diagram:
Transition temperature at $\mu_B = 0$

$$T_c = 154(9)\mathrm{MeV}$$





HotQCD, PRD 85 (2012) 054503.

$\Rightarrow$ obtained by O(4) scaling fits to the chiral condensate and susceptibility

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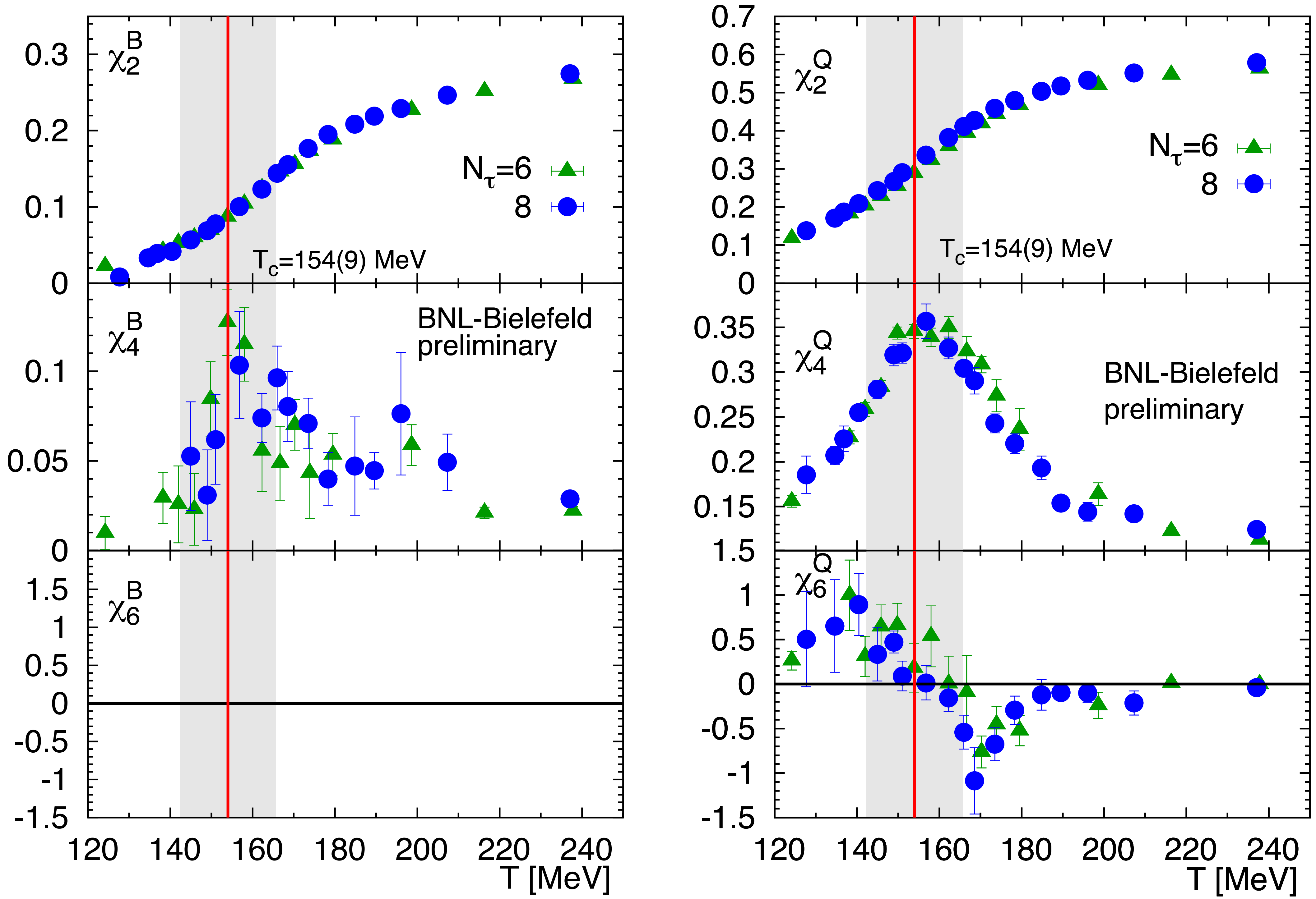
2) Status of the lattice data



$\Rightarrow$ structure consistent with O(4) critical behavior at $\mu_B = 0,\ m = 0$

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2) Status of the lattice data

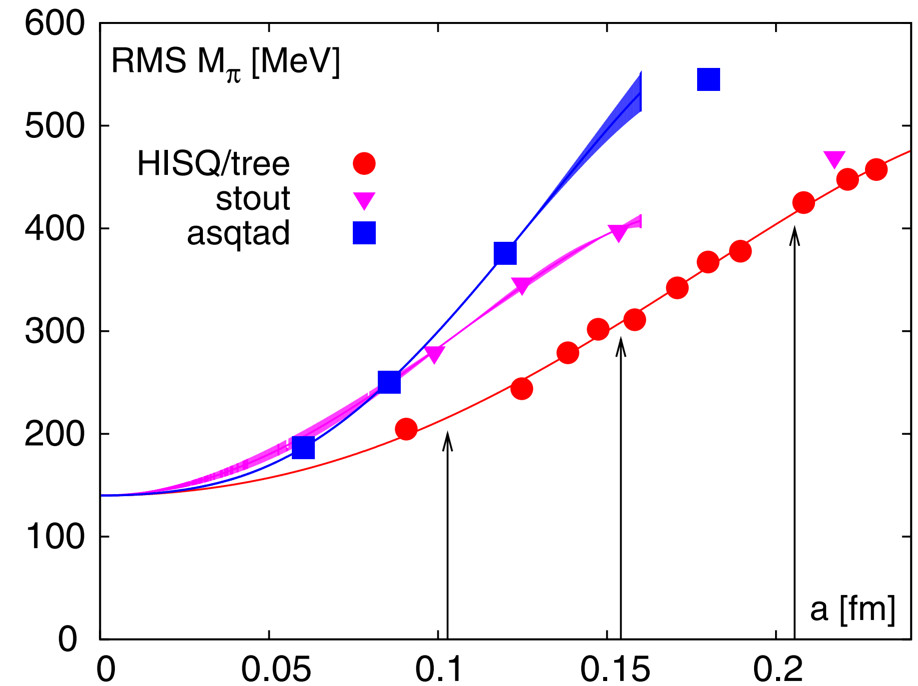
- **Goal:** compare measured electric charge fluctuations from LHC collisions to continuum extrapolated lattice data

  $\Rightarrow$ will allow to pin-down the relation between freeze-out and critical temperature (at $\mu_B = 0$)

- **Goal:** quantify deviations from the HRG model

  $\Rightarrow$ will allow to judge on the degree of criticality

- **Obstacle:** lattice cut-off effects modify the effective (RMS) pion mass. Electric charge fluctuations are extremely sensitive to the light pion sector



HotQCD, PRD 85 (2012) 054503.

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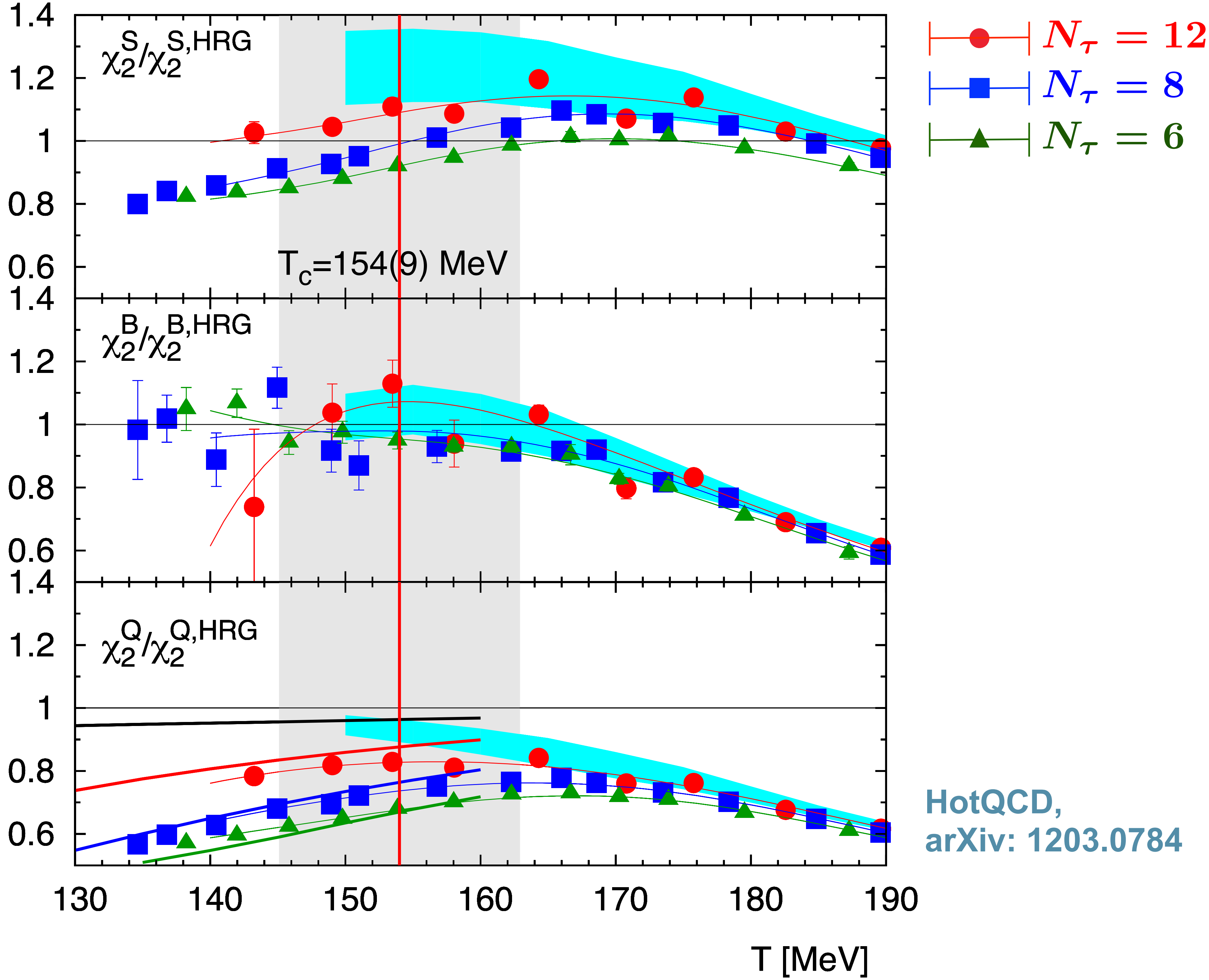
2) Status of the lattice data: $\chi_{2,0}^{X}$



$\Rightarrow$ quadratic fluctuations agree with HRG for $T \lesssim 150$ MeV

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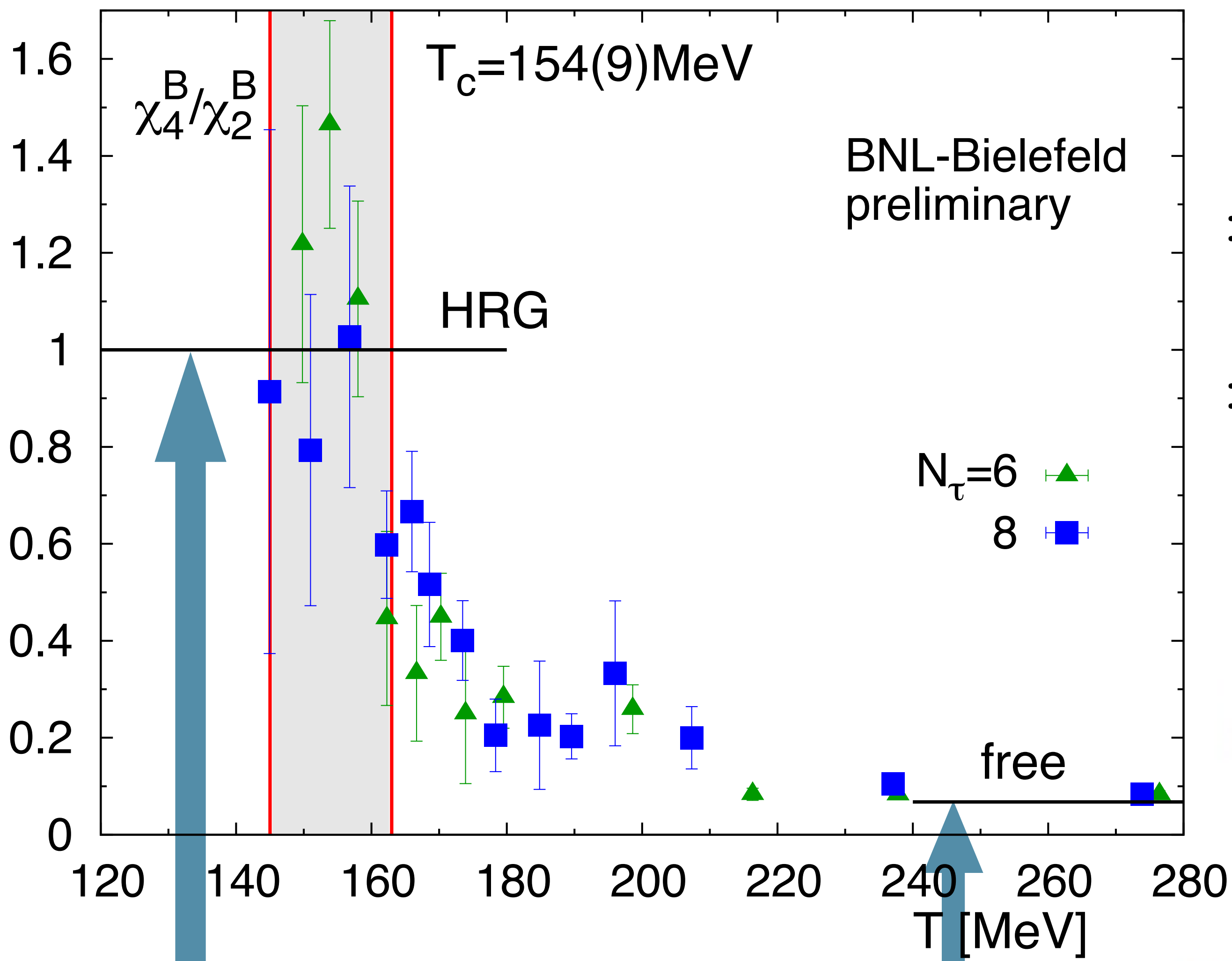
2) Status of the lattice data: $\chi^B_{4,0}/\chi^B_{2,0}$

4th moment of baryon number fluctuations



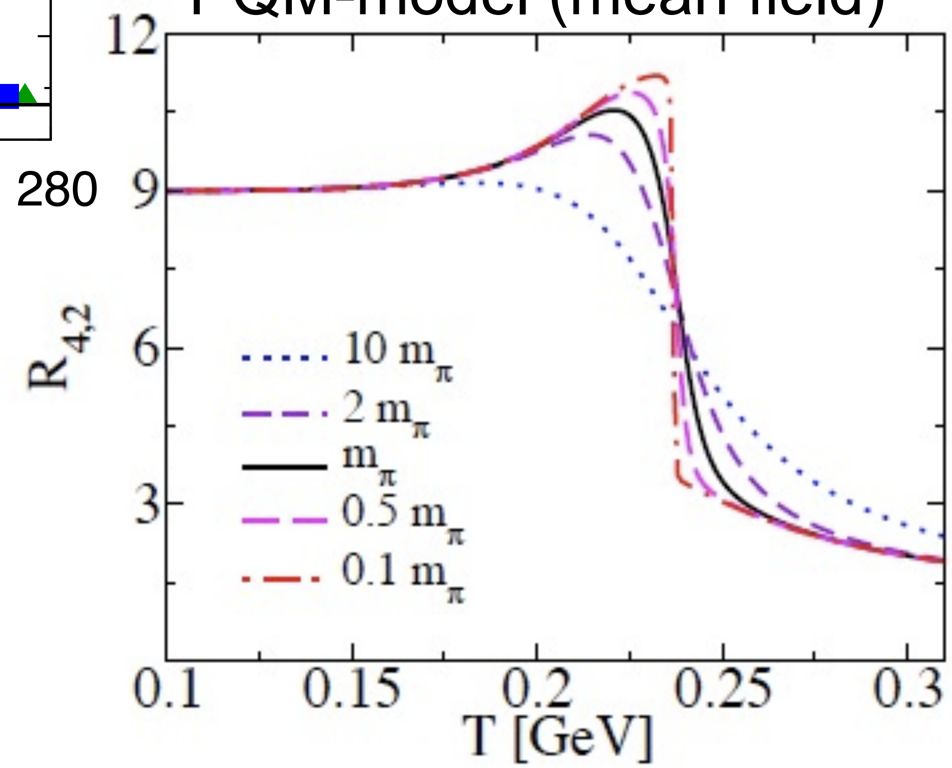
baryon number carried by **hadrons**

baryon number carried by **quarks**

⇒ interpolation between HRG and free quarks

⇒ no or very little sign of critical behavior

for comparison:
PQM-model (mean field)



Skokov *et al.*, PRD 82 (2010) 034029.

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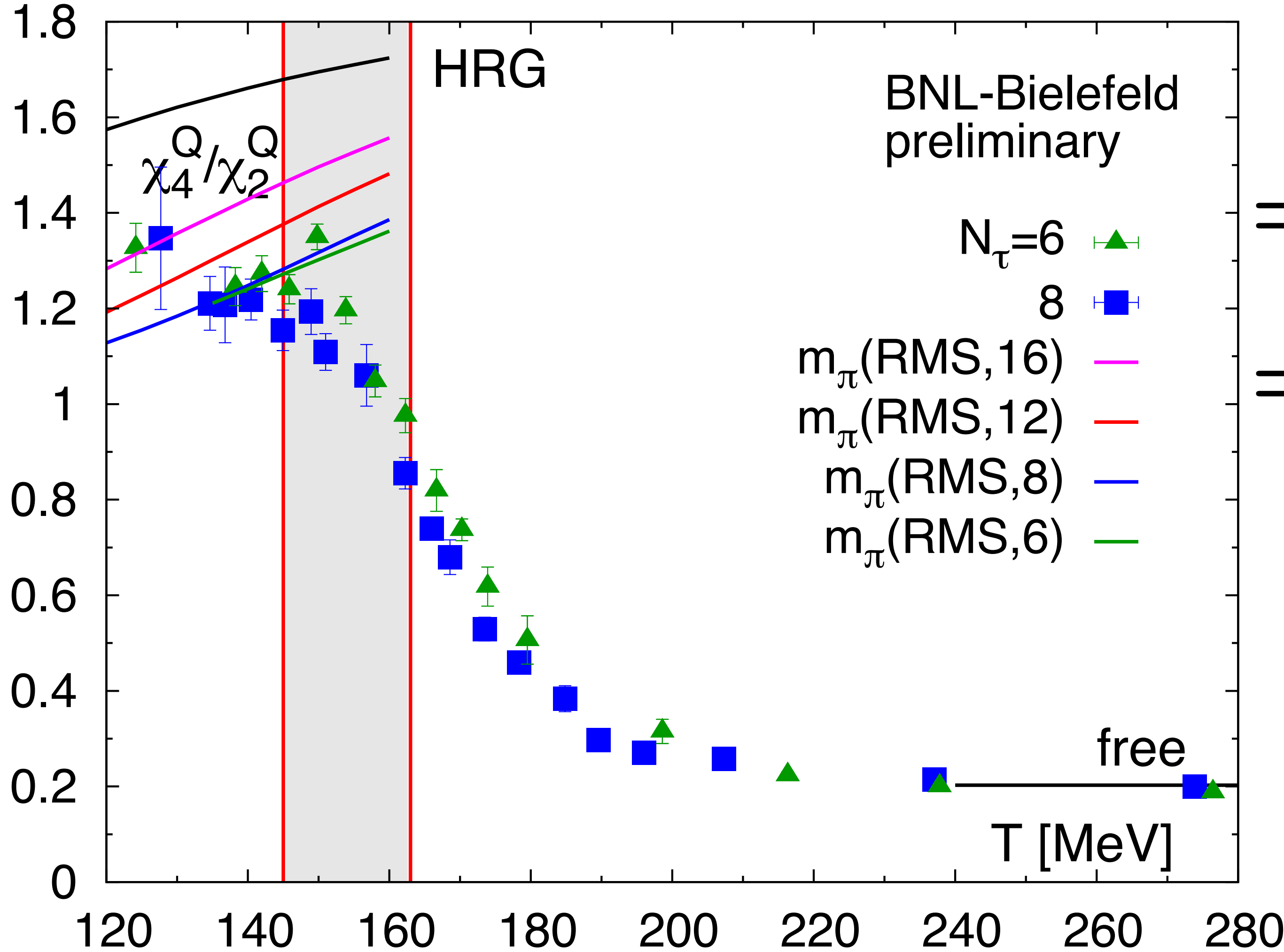
2) Status of the lattice data: $\chi^Q_{4,0}/\chi^Q_{2,0}$

4th moment of electric charge fluctuations



⇒ interpolation between HRG and free quarks

⇒ no or very little sign of critical behavior

⇒ electric charge fluctuations are sensitive to the light pion sector (which is distorted on the lattice). For higher order fluctuations this problem becomes more severe

⇒ replace $m_\pi \longrightarrow m_\pi^{\mathrm{RMS}}$ within HRG model

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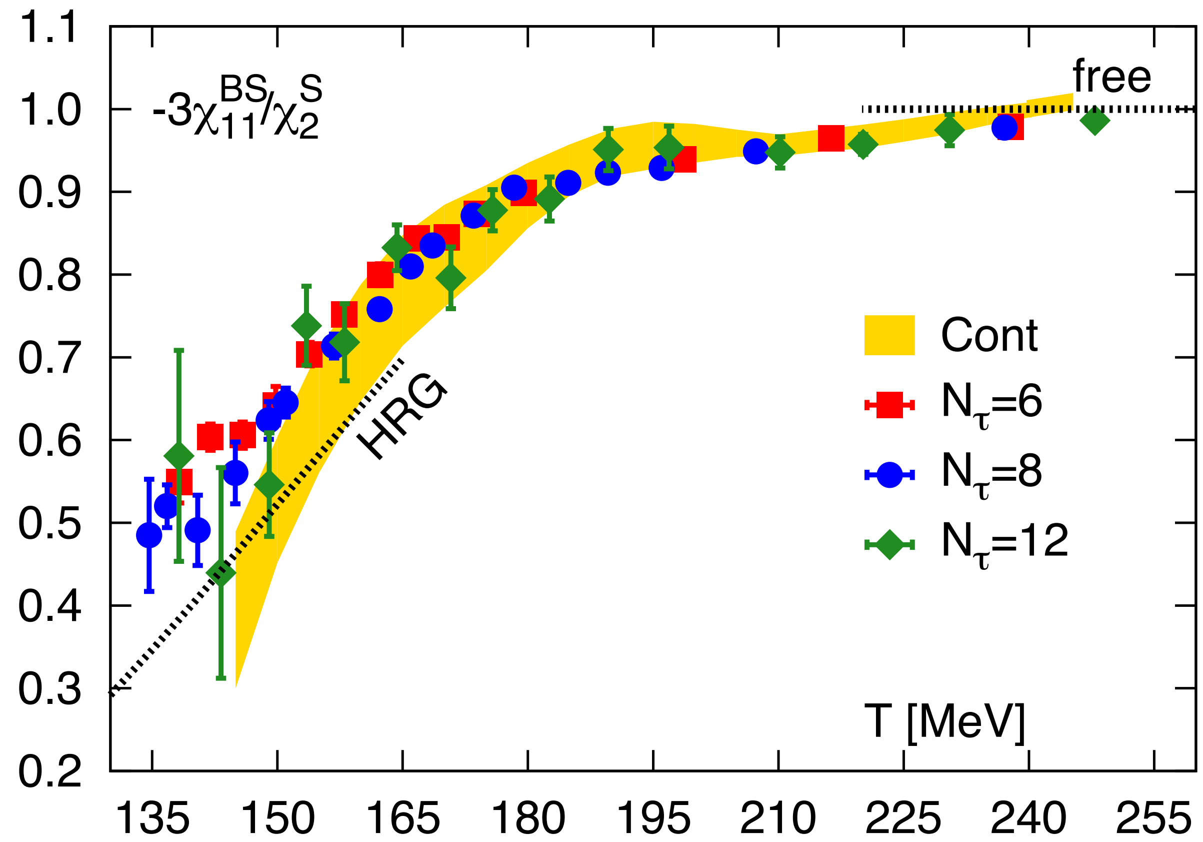
2) Status of the lattice data

- **Goal:** calculate LO and NLO correction to the fluctuation observables at finite chemical potential and compare to experimental data from RHIC

$\Rightarrow$ see talk by S. Mukherjee

$\Rightarrow$ we find NLO corrections typically of the order of 10% for $\mu_B/T < 1$

- **Example:** baryon number and strangeness correlation



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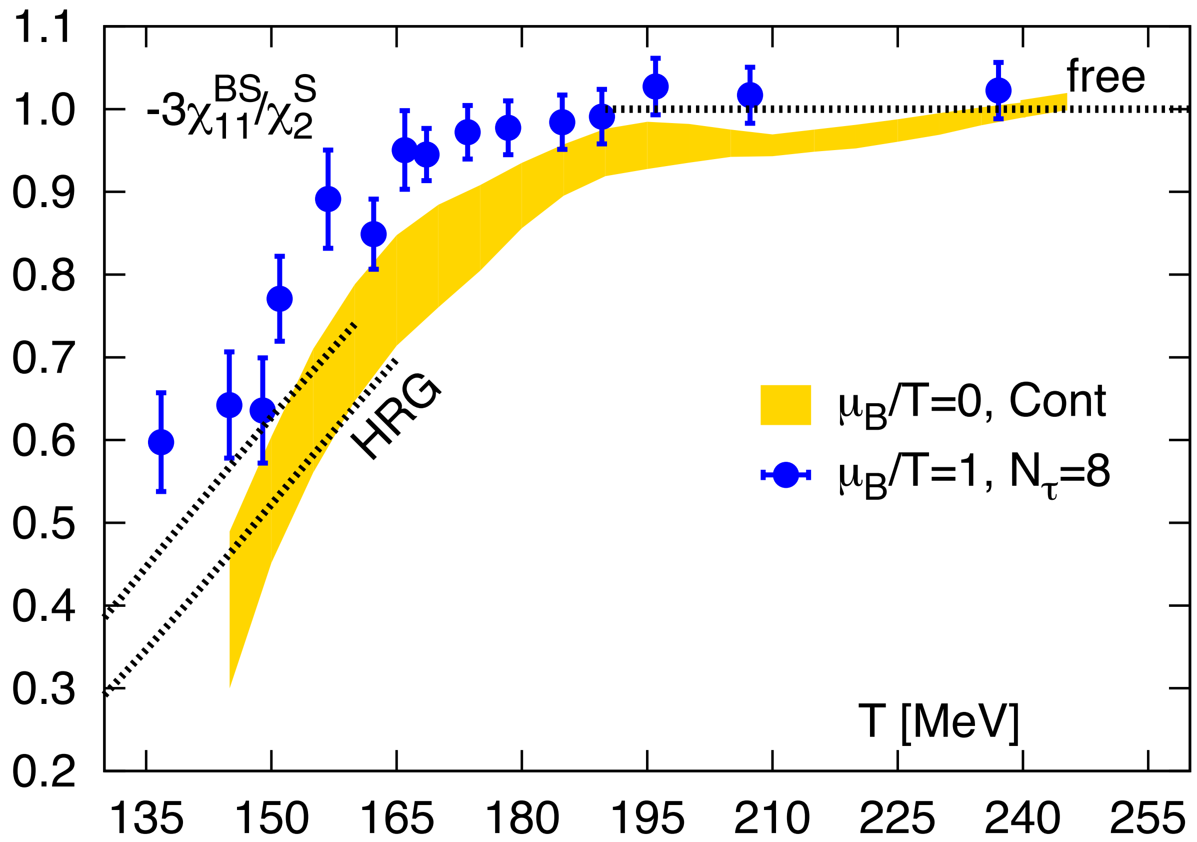

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3) Summary

- Higher moments of charge fluctuations are increasingly more more sensitive to critical behavior (even at $\mu_B = 0$)
  $\Longrightarrow$ **So far, we do not see a sign of large 'critical' fluctuations**

- Cutoff effects are under control for quadratic fluctuations. Higher order fluctuations are work in progress.

- Electric charge fluctuations suffer from the distortion of the light pion sector on the lattice for $T \lesssim 160\ \mathrm{MeV}$.

- Experimental results on moments of Q fluctuations can be used to determine freeze-out conditions from QCD.

- Comparing this analysis to calculations of the QCD transition temperature will help to quantify the relation between freeze-out and transition temperature.
  $\Longrightarrow$ **Establishing this at $\mu_B \approx 0$ provides an anchor point for the analysis of the entire phase diagram.**

- Finite chemical potential corrections are small and typically of the order of 10% for $\mu_B/T < 1$
  $\Longrightarrow$ **This covers $200\ \mathrm{GeV} > \sqrt{s} > 25\ \mathrm{GeV}$ in the BES at RHIC**

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