Thermal Photon Production at NLO

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$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3k} = \text{Photon emission rate per phase-space}$$

The photon emission rate at weak coupling:

- The rate is function of the coupling coupling constant and k/T:

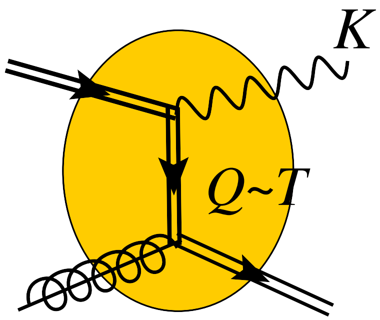
$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma}{\mathrm{d}^3k} \propto e^2T^2 \Big[\underbrace{O(g^2 \log) + O(g^2)}_{\text{LO AMY}} + \underbrace{O(g^3 \log) + O(g^3)}_{\text{From soft } gT \text{ gluons, } n_B \simeq \frac{T}{\omega} \simeq \frac{1}{g}} + \ldots$$

$O(g^3)$ is closely related to open issues in energy loss:

- At NLO must include drag, collisions, bremsstrhalung, and kinematic limits

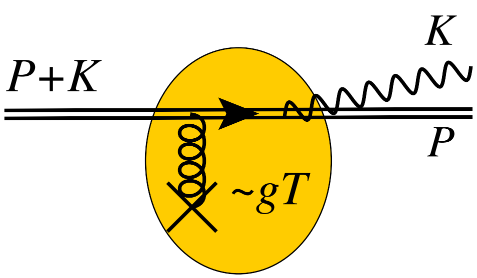
Three rates for photon production at Leading Order — Baier, Kapusta, AMY

1. Hard Collisions – a $2 \leftrightarrow 2$ processes



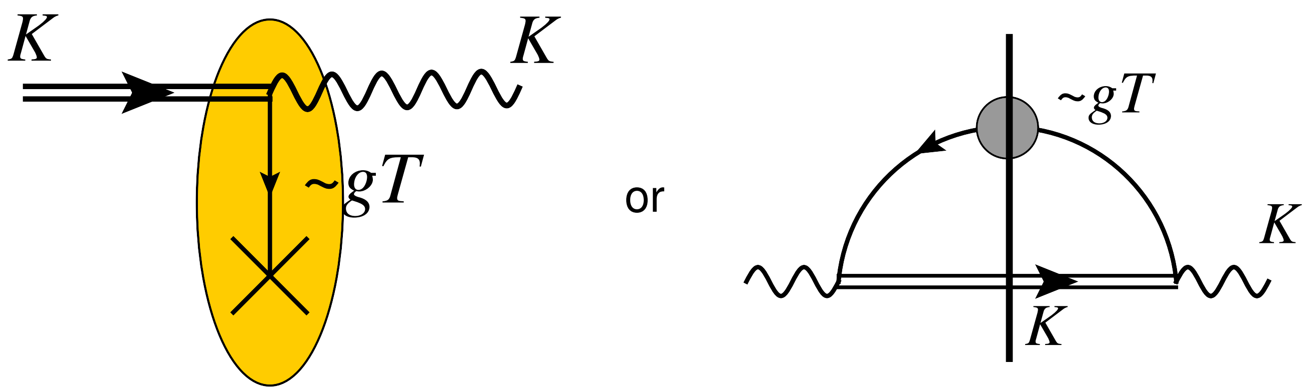
$$\sim e^2 \underbrace{m_\infty^2}_{g^2 C_F T^2/4} \times \underbrace{n_F(k)}_{\text{fermi dist.}} \times \left[\log(T/\mu) + C_{2to2}(k)\right]$$

2. Collinear Bremmstrahlung – a $1 \leftrightarrow 2$ processes



$$\sim e^2 m_\infty^2 n_F \left[\underbrace{C_{\rm bremm}(k)}_{\text{LPM + AMY and all that stuff!}} \right]$$

3. Quark Conversions – $1 \leftrightarrow 1$ processes (analogous to drag)



$$= \sim e^2 m_\infty^2 n_F \left[\log(\mu_\perp / m_\infty) + C_{\rm cnvrt}\right]$$

Full LO Rate is independent of scale $\mu_\perp$:

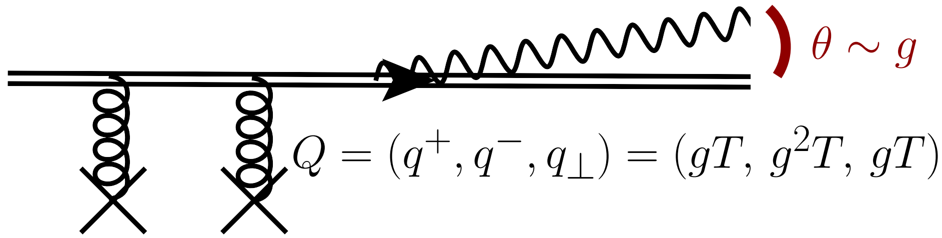
$$2k \frac{d\Gamma}{d^3k} \propto e^2 m_\infty^2 n_F \left[\log\left(T/m_\infty\right) + \underbrace{C_{\rm cnvrt} + C_{\rm bremm}(k) + C_{\rm cnvrt}}_{\equiv C_{LO}(k)} \right]$$

$O(g)$ Corrections to Hard Collisions, Bremm, Conversions:

1. No corrections to Hard Collisions:

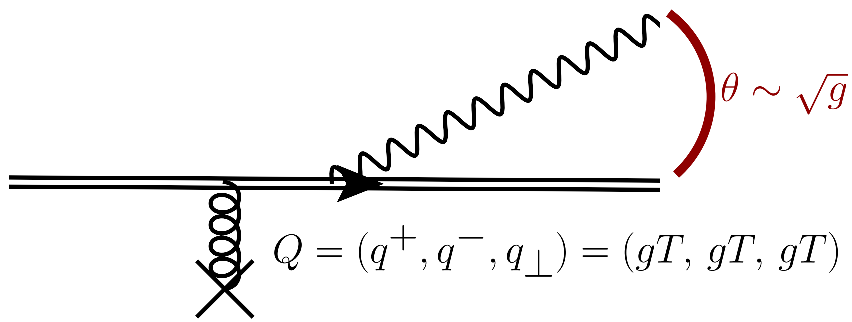
2. Corrections to Bremm:

   (a) Small angle bremm. Corrections to AMY coll. kernel. (Caron-Huot)

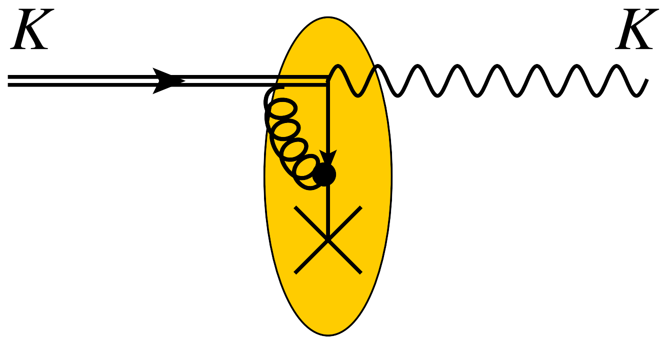


$$C_{LO}[q_\perp] = \frac{Tg^2 m_D^2}{q_\perp^2(q_\perp^2 + m_D^2)} \rightarrow \text{A complicated but analytic formula}$$

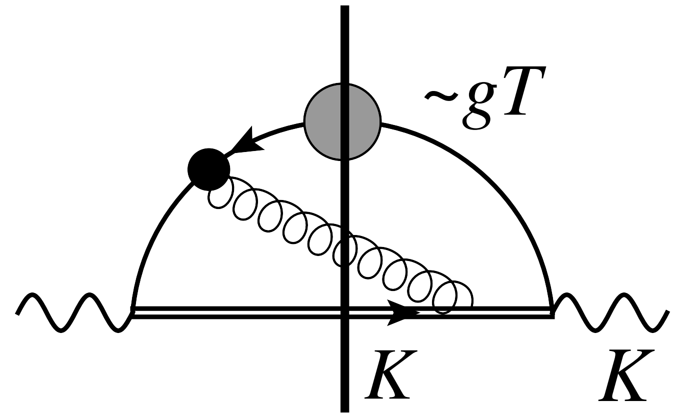
   (b) Larger angle bremm. Include collisions with energy exchange, $q^- \sim gT$.



3. Corrections to Conversions:



or



Simon Caron-Huot

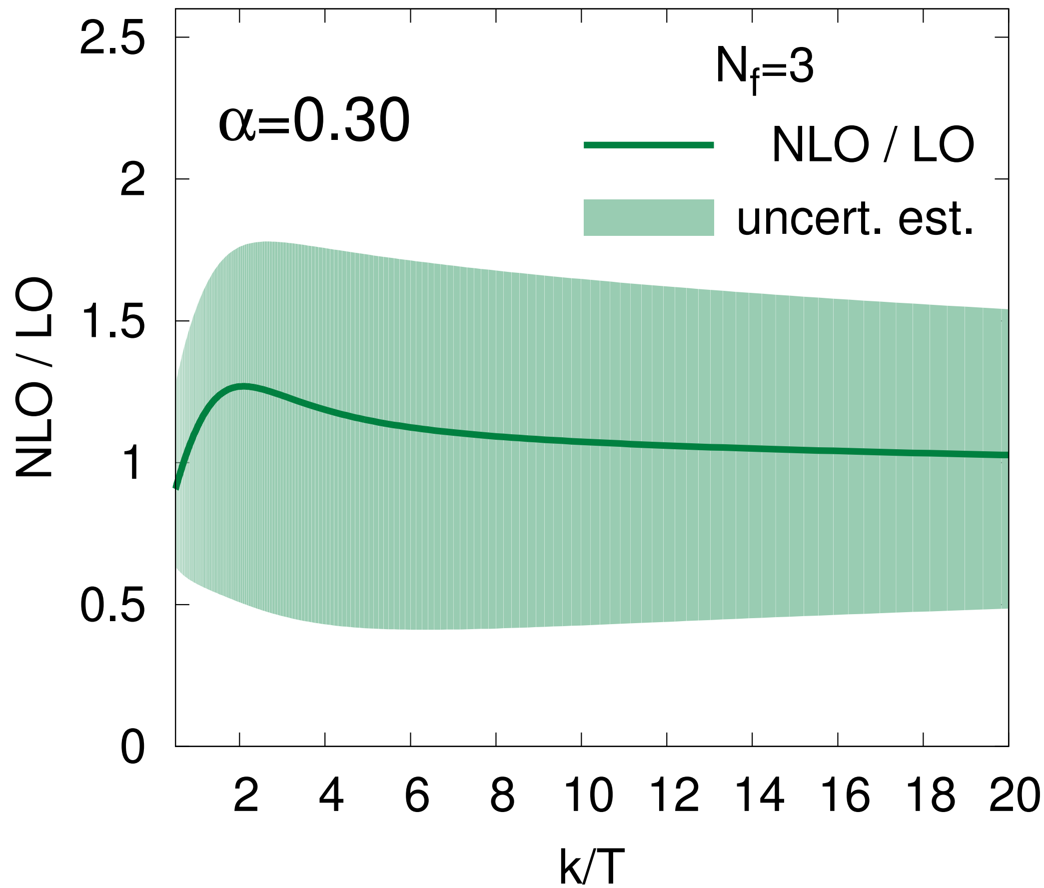
- Doable because of HTL sum rules (light cone causality)
- Gives a numerically small and momentum indep. contribution to the NLO rate

Full results depend on all these corrections.

These rates smoothly match onto each other as the kinematics change.

NLO Results: $\sim g^3 \log(1/g) + g^3$

$$2k\frac{\mathrm{d}\delta\Gamma_{LO}}{\mathrm{d}^3k} \propto e^2 m_\infty^2 n_F(k)\Big[\overbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\left(\frac{\sqrt{2Tm_D}}{m_\infty}\right)}^{\text{conversions}} + \overbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\mathrm{large}-\theta}(k)}^{\text{large-}\theta\text{-bremm}} + \overbrace{\frac{g^2 C_A T}{m_D}C_{\mathrm{small}-\theta}(k)}^{\text{small-}\theta\text{-bremm}}\Big]$$



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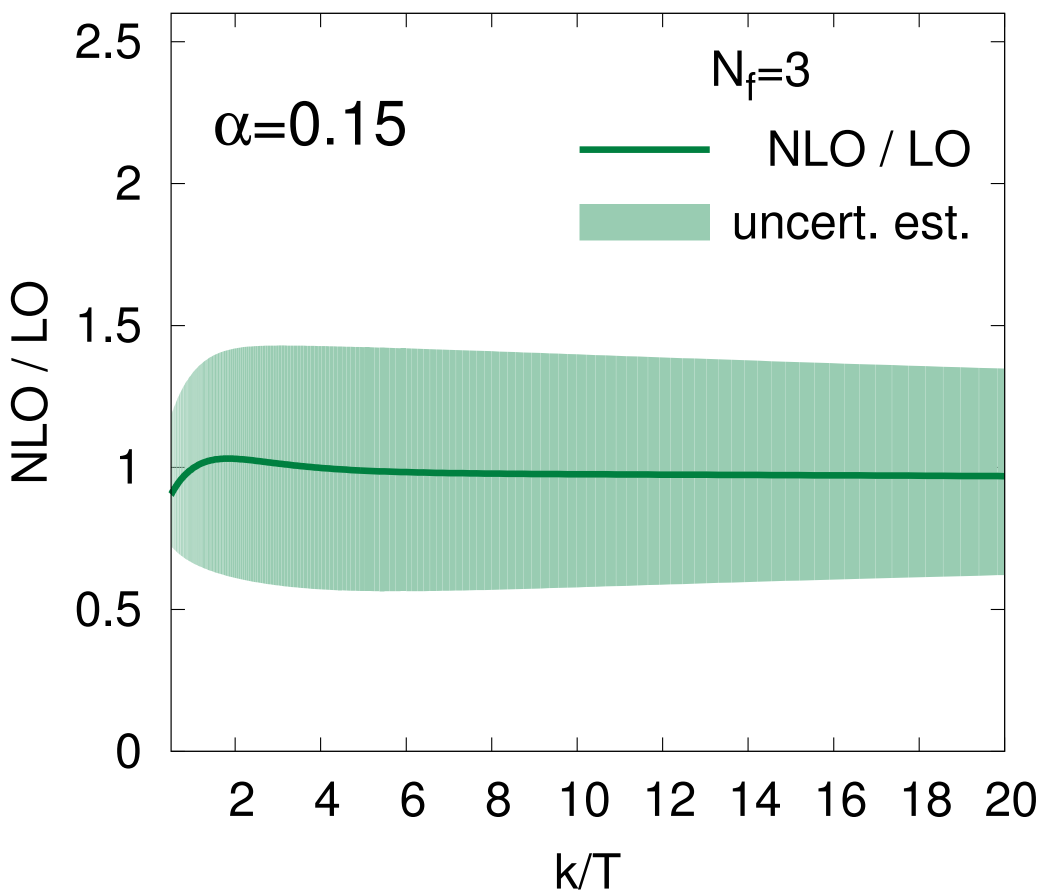
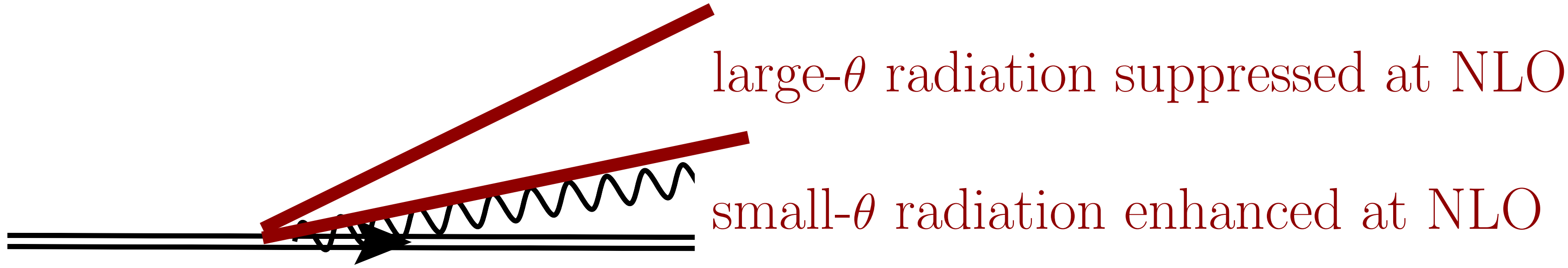
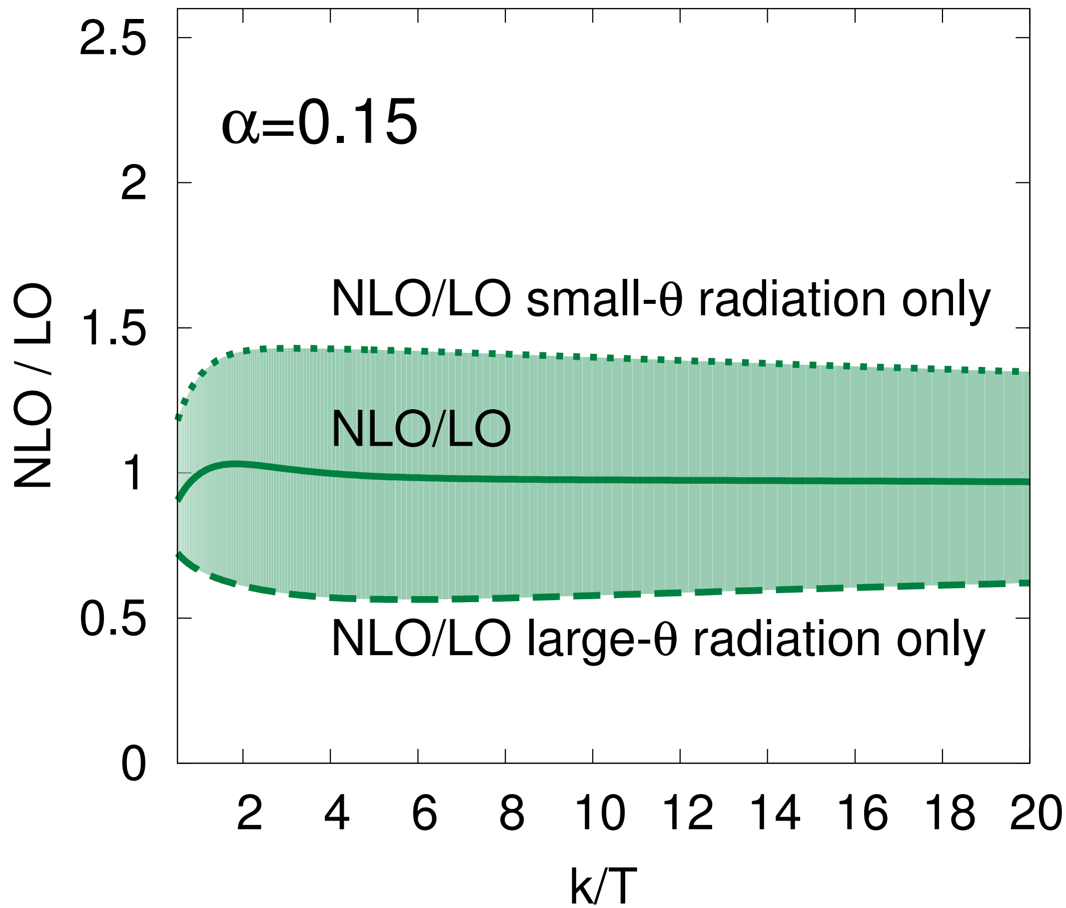


The different contributions at NLO (conversions are not numerically important)

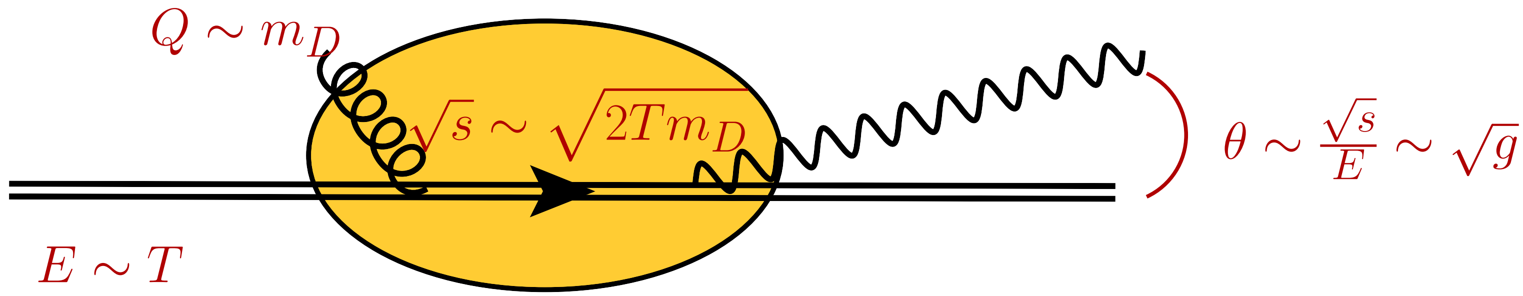




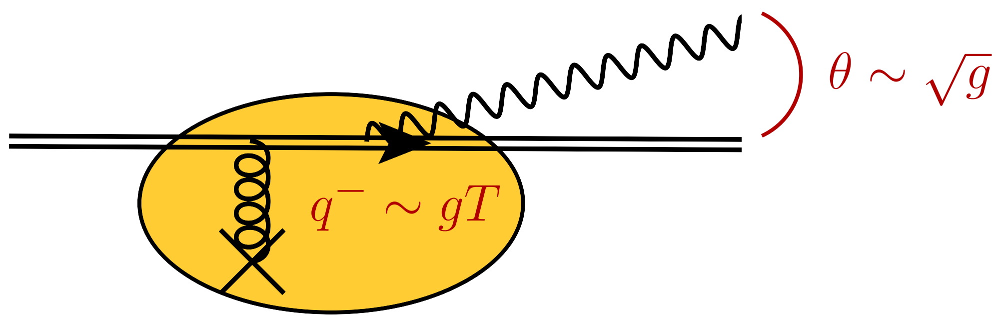
The calculation

Matching collisions to bremm

- When the gluon becomes soft (a plasmon), the $2 \leftrightarrow 2$ collision:



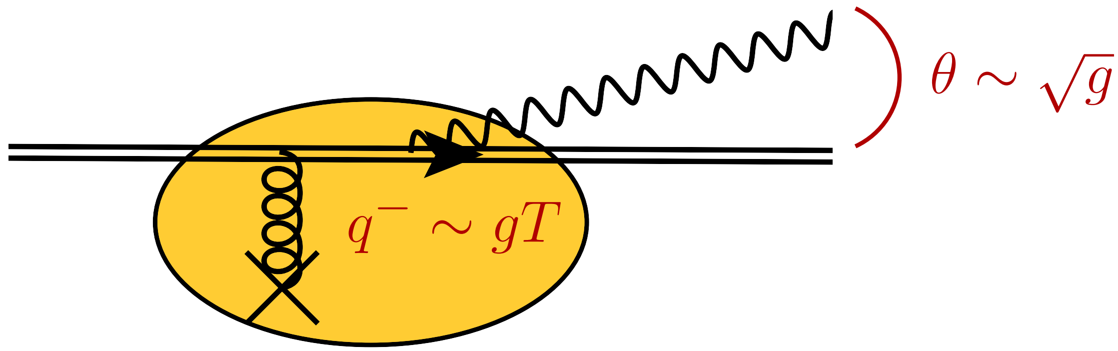
is not physically distinct from the wide angle bremm



Need both processes

- For harder gluons, $q^- \to T$, this becomes a normal $2 \to 2$ process.
- For softer gluons, $q^- \to g^2T$, this smoothly matches onto AMY.

Finite energy transfer sum-rule



- The AMY collision kernel $C[q_\perp]$ involves

Aurenche, Gelis, Zakarat

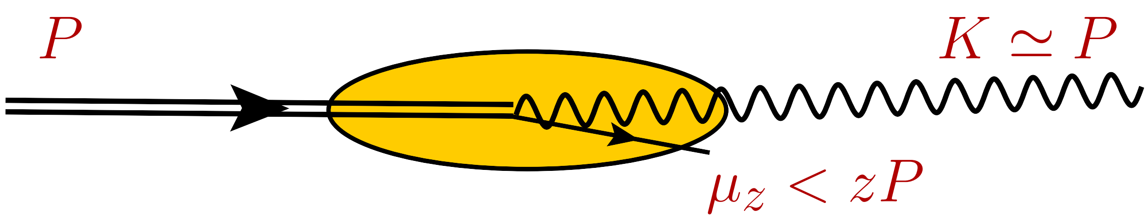
$$q_\perp^2 C[q_\perp] = \int_{-\infty}^{\infty} \frac{\mathrm{d}q^z}{2\pi} \, \langle F_{i+} F_{i+}(Q) \rangle|_{q^-=0} = \frac{T m_D^2}{q_T^2 + m_D^2} \, .$$

- We need a finite $q^- = \delta E$ generalization of the sum rule

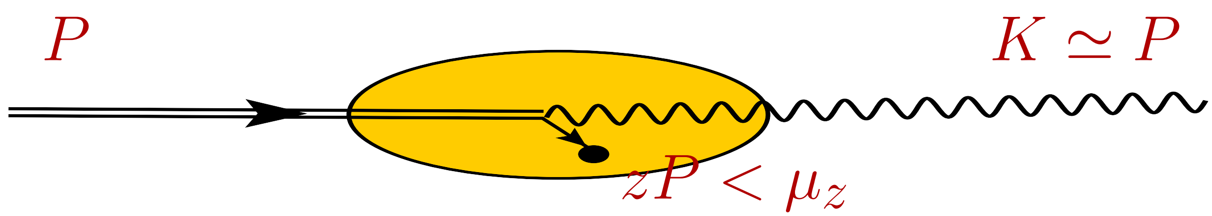
$$\int_{-\infty}^{\infty} \frac{\mathrm{d}q^z}{2\pi} \, \langle F_{i+} F_{i+}(Q) \rangle|_{q^-=\delta E} = T \left[\frac{2(\delta E)^2(\delta E^2 + q_\perp^2 + m_D^2) + m_D^2 q_\perp^2}{(\delta E^2 + q_\perp^2 + m_D^2)(\delta E^2 + q_\perp^2)} \right] .$$

Matching between bremm and conversions

- When the final quark line becomes soft, the bremm process :



is not physically distinct from the conversion process



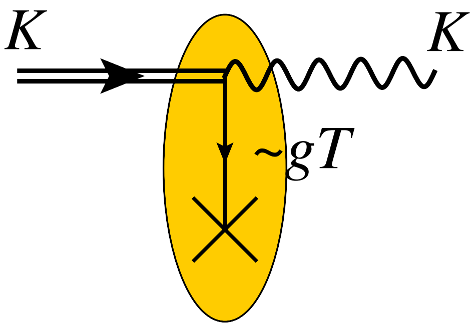
Separately both depend on the separation scale, μ, but the μ dep. cancels in sum

- The LO small-θ and large-θ bremm rates depend linearly and logarithmically on an infrared separation scale, μ.

The NLO conversion rate will depend on a UV cutoff μ and cancels this dependence

Computing the conversion rate with sum-rules (LO):

(see also Bodeker)



$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma_{\mathrm{cnvrt}}}{\mathrm{d}^3 k} \propto e^2 n_F(k)\, \hat{q}_{\mathrm{cnvrt}}(\mu)$$

- $\hat{q}_{\mathrm{cnvrt}}$ is the quark version of $\hat{q}$

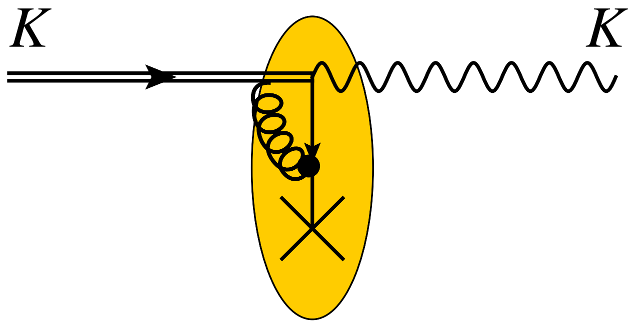
$$\hat{q}_{\mathrm{cnvrt}}(\mu_\perp) = \int^{\sim\mu} \frac{\mathrm{d}^2\mathbf{p}_T}{(2\pi)^2} \underbrace{\int_{-\mu}^{\mu} \frac{\mathrm{d}p^z}{2\pi} \mathrm{Tr}\left[\gamma_+ S^<(\omega,\mathbf{p})\right]_{\omega=p^z}}_{\text{evaluate with sum rule}}$$

$$= \int^{\mu} \frac{d^2\mathbf{p}_T}{(2\pi)^2} \frac{m_\infty^2}{p_T^2 + m_\infty^2}$$

where

$$S_R(X) = \left\langle \psi(X) e^{ig\int_0^X dx^\mu A_\mu} \bar{\psi}(0) \right\rangle$$

Computing the conversion rate at NLO with sum-rules:



$$2k(2\pi)^3 \frac{\mathrm{d}\Gamma_{\rm cnvrt}}{\mathrm{d}^3 k} \propto e^2\, n_F(k)\, \hat{q}_{\rm cnvrt}(\mu)$$

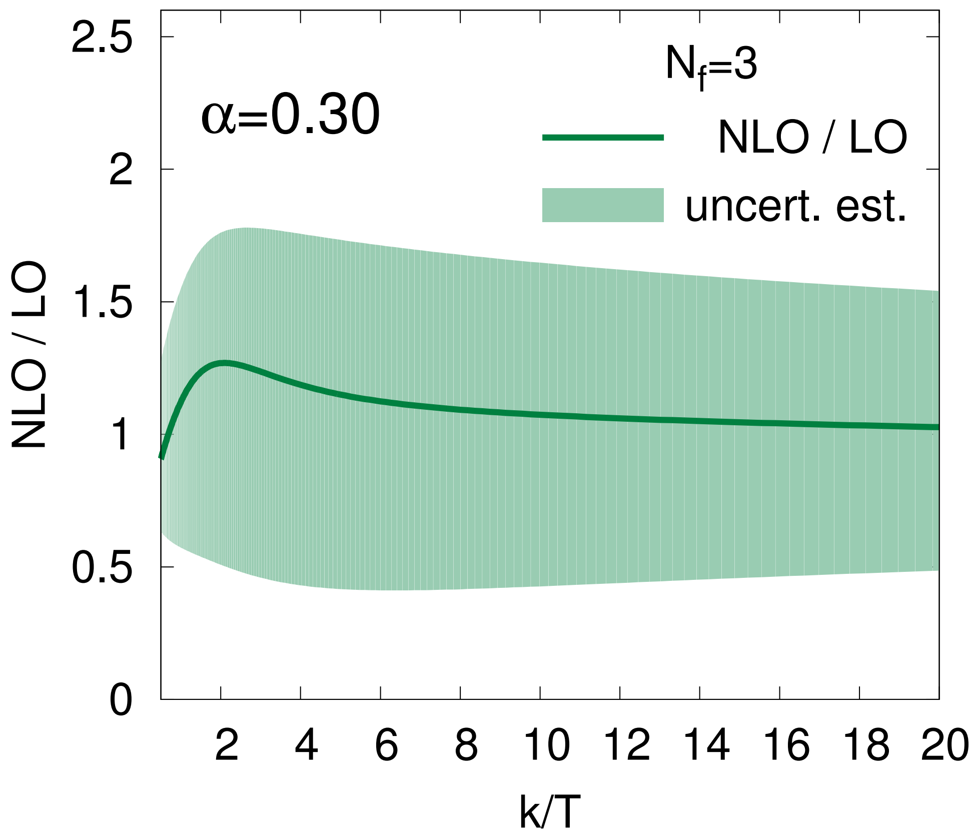
- At NLO we have only to replace $m_\infty^2 \to m_\infty^2 + \delta m_\infty^2$

$$\hat{q}_{\rm cnvrt} = \underbrace{\int^\mu \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \frac{m_\infty^2 + \delta m_\infty^2}{p_T^2 + m_\infty^2 + \delta m_\infty^2}}_{\text{finite + UV logarithmic divergence in } \mu} \quad + \text{UV linearly divergent in } \mu$$

The UV divergences of conversion rate match with the IR divergences of large and small angle bremm giving a finite answer

Conclusion

- The result again



Many things can be computed next (e.g. shear viscosity and e-loss)