The NLO inclusive forward hadron production in pA collisions

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- G. Chirilli, BX and F. Yuan, Phys. Rev. Lett. 108, 122301 (2012).
- G. Chirilli, BX and F. Yuan, arXiv:1203.6139.

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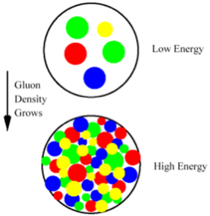
Saturation physics

Saturation physics describes the high density parton distributions in the high energy limit.



- Initial condition: McLerran-Venugopalan Model plus small-x evolution $\Rightarrow$ dense gluon distributions.
- In a physical process, in order to probe the dense nuclear matter precisely, the proper factorization is required.
- Factorization is about separation of short distant physics(perturbatively calculable hard factor) from large distant physics (parton distributions and fragmentation functions). Hard factor should always be finite and free of divergence of any kind.

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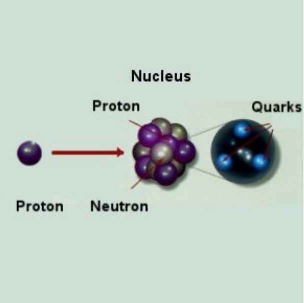
Forward observables at pA collisions



Why pA collisions?

- For pA (dilute-dense system) collisions, there is an effective k_t factorization.

$$\frac{d\sigma^{pA\to qfX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p q(x_p,\mu^2) x f(x, q_\perp^2) \frac{1}{\pi}\frac{d\hat{\sigma}}{d\hat{t}}.$$

- For dijet processes in pp, AA collisions, there is no k_t factorization[Collins, Qiu, 08],[Rogers, Mulders; 10].

Why forward?

- At forward rapidity y, $x_p \propto e^y$ is large, while $x_A \propto e^{-y}$ is small.
- Ideal place to find gluon saturation in the target nucleus.

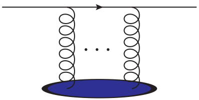
Forward hadron production in pA collisions

Consider the inclusive production of inclusive forward hadrons in pA collisions, i.e., in the process: [Dumitru, Jalilian-Marian, 02]

$$p + A \to H + X.$$

The leading order result for producing a hadron with transverse momentum $p_\perp$ at rapidity y_h

$$\frac{d\sigma_{\mathrm{LO}}^{pA\to hX}}{d^2 p_\perp d y_h} = \int_\tau^1 \frac{dz}{z^2}\left[\sum_f x_p q_f(x_p)\mathcal{F}(k_\perp)D_{h/q}(z) + x_p g(x_p)\tilde{\mathcal{F}}(k_\perp)D_{h/g}(z)\right].$$



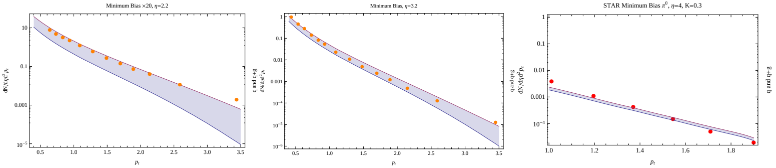
$$\Rightarrow \qquad U(x_\perp) = \mathcal{P}\exp\left\{ig_S\int_{-\infty}^{+\infty}\mathrm{d}x^+\, T^c A_c^-(x^+, x_\perp)\right\},$$

$$\mathcal{F}(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} S_Y^{(2)}(x_\perp, y_\perp).$$

- $p_\perp = zk_\perp$, $x_p = \frac{p_\perp}{z\sqrt{s}}e^{y_h}$ (large), $\tau = zx_p$ and $x_g = \frac{p_\perp}{z\sqrt{s}}e^{-y_h}$(small).
- $S_Y^{(2)}(x_\perp, y_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}U(x_\perp)U^\dagger(y_\perp)\right\rangle_Y$ with $Y \sim \ln 1/x_g$.
- The gluon channel with $\tilde{\mathcal{F}}(k_\perp)$ defined in the adjoint representation.
- Classical $p_\perp$ broadening calculation, no divergences, no evolution.

Issues with the leading order calculation

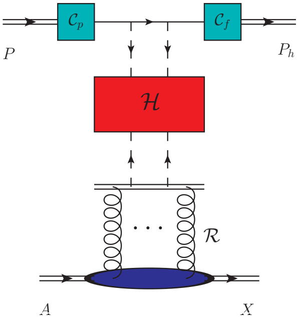
The comparison between the leading order calculation and the RHIC data:



Comments: Why do we need NLO calculations?

- LO calculation is order of magnitude estimate. Normally, we need to introduce the artificial K factor to fix the normalization. Fails to describe large $p_\perp$ data.
- There are large theoretical uncertainties due to renormalization/factorization scale dependence in $xf(x)$ and $D(z)$. Choice of the scale at LO requires information at NLO.
- In general, higher order in the perturbative series in α_s helps to increase the reliability of QCD predictions.
- NLO results reduce the scale dependence and may distort the shape of the cross section. $K = \frac{\sigma_{\rm LO}+\sigma_{\rm NLO}}{\sigma_{\rm LO}}$ is not a good approximation.
- NLO is vital in terms of establishing the QCD factorization in saturation physics.

The overall picture



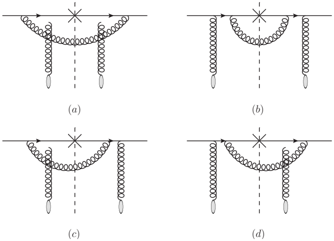
The QCD factorization formalism for this process reads as,

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\xi x f_a(x,\mu)D_{h/c}(z,\mu)\int [dx_\perp]S^Y_{a,c}([x_\perp])\mathcal{H}_{a\to c}(\alpha_s,\xi,[x_\perp]\mu)\ .$$

- For UGD, the rapidity divergence cannot be canceled between real and virtual gluon emission due to different restrictions on $k_\perp$.
- Removing the divergences via renormalization$\Rightarrow$ Finite results for hard factors.

Real diagrams

The real contributions in the coordinate space: Computing the real diagrams with a quark ($b_\perp$) and a gluon ($x_\perp$) in the final state in the dipole model in the coordinate space:



$$\frac{d\sigma^{qA\to qgX}}{d^3k_1 d^3k_2} = \alpha_S C_F \delta(p^+ - k_1^+ - k_2^+) \int \frac{\mathrm{d}^2 x_\perp}{(2\pi)^2} \frac{\mathrm{d}^2 x'_\perp}{(2\pi)^2} \frac{\mathrm{d}^2 b_\perp}{(2\pi)^2} \frac{\mathrm{d}^2 b'_\perp}{(2\pi)^2}$$

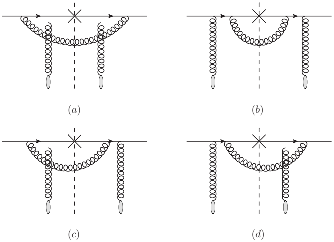
$$\times e^{-ik_{1\perp}\cdot(x_\perp - x'_\perp)} e^{-ik_{2\perp}\cdot(b_\perp - b'_\perp)} \sum_{\lambda\alpha\beta} \psi^{\lambda *}_{\alpha\beta}(u'_\perp)\psi^{\lambda}_{\alpha\beta}(u_\perp)$$

$$\times \Big[S_Y^{(6)}(b_\perp, x_\perp, b'_\perp, x'_\perp) + S_Y^{(2)}(v_\perp, v'_\perp) - S_Y^{(3)}(b_\perp, x_\perp, v'_\perp) - S_Y^{(3)}(v_\perp, x'_\perp, b'_\perp) \Big],$$

with $u_\perp = x_\perp - b_\perp$ and $v_\perp = (1-\xi)x_\perp + \xi b_\perp$.

The real contributions in the coordinate space

Computing the real diagrams with a quark ($b_\perp$) and a gluon ($x_\perp$) in the final state in the dipole model in the coordinate space: [G. Chirilli, BX and F. Yuan, 11;12]



$$S_Y^{(6)}(b_\perp, x_\perp, b'_\perp, x'_\perp) = \frac{1}{C_F N_c} \left\langle \mathrm{Tr}\left(U(b_\perp)U^\dagger(b'_\perp)T^d T^c\right)\left[W(x_\perp)W^\dagger(x'_\perp)\right]^{cd}\right\rangle_Y,$$

$$S_Y^{(3)}(b_\perp, x_\perp, v'_\perp) = \frac{1}{C_F N_c} \left\langle \mathrm{Tr}\left(U(b_\perp)T^d U^\dagger(v'_\perp)T^c\right) W^{cd}(x_\perp)\right\rangle_Y.$$

- By integrating over the gluon momentum, we identify $x_\perp$ to $x'_\perp$ which simplifies $S_Y^{(6)}(b_\perp, x_\perp, b'_\perp, x'_\perp)$ to $S^{(2)}(b_\perp, b'_\perp)$.
- $S_Y^{(3)}(b_\perp, x_\perp, v'_\perp) = \frac{N_c}{2C_F}\left[S_Y^{(4)}(b_\perp, x_\perp, v'_\perp) - \frac{1}{N_c^2} S_Y^{(2)}(b_\perp, v'_\perp)\right]$

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The real contributions in the momentum space

By integrating over the gluon $(k_1^+, k_{1\perp})$, we can cast the real contribution into

$$\frac{\alpha_s}{2\pi^2}\int\frac{dz}{z^2}D_{h/q}(z)\int_{\tau/z}^1 d\xi\frac{1+\xi^2}{1-\xi}xq(x)\left\{C_F\int d^2k_{g\perp}\mathcal{I}(k_\perp,k_{g\perp})\right.$$
$$\left.+\frac{N_c}{2}\int d^2k_{g\perp}d^2k_{g1\perp}\mathcal{J}(k_\perp,k_{g\perp},k_{g1\perp})\right\},$$

where $x=\tau/z\xi$ and $\mathcal{I}$ and $\mathcal{J}$ are defined as

$$\mathcal{I}(k_\perp,k_{g\perp}) = \mathcal{F}(k_{g\perp})\left[\frac{k_\perp-k_{g\perp}}{(k_\perp-k_{g\perp})^2}-\frac{k_\perp-\xi k_{g\perp}}{(k_\perp-\xi k_{g\perp})^2}\right]^2,$$
$$\mathcal{J}(k_\perp,k_{g\perp},k_{g1\perp}) = \left[\mathcal{F}(k_{g\perp})\delta^{(2)}(k_{g1\perp}-k_{g\perp})-\mathcal{G}(k_{g\perp},k_{g1\perp})\right]\frac{2(k_\perp-\xi k_{g\perp})\cdot(k_\perp-k_{g1\perp})}{(k_\perp-\xi k_{g\perp})^2(k_\perp-k_{g1\perp})^2}$$
$$\text{with}\quad \mathcal{G}(k_\perp,l_\perp) = \int\frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^4}e^{-ik_\perp\cdot(x_\perp-b_\perp)-il_\perp\cdot(b_\perp-y_\perp)}S_Y^{(4)}(x_\perp,b_\perp,y_\perp).$$

Three types of divergences:

- $\xi\to 1 \Rightarrow$ Rapidity divergence. $y=\frac{1}{2}\ln\frac{l^+}{l^-}\to-\infty$
- $k_{g\perp}\to k_\perp \Rightarrow$ Collinear divergence associated with parton distributions.
- $k_{g\perp}\to k_\perp/\xi \Rightarrow$ Collinear divergence associated with fragmentation functions.

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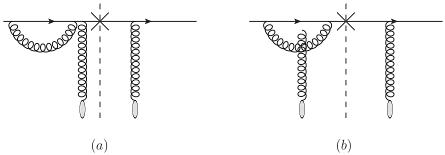
The virtual contributions in the momentum space

Now consider the virtual contribution



$$-2\alpha_s C_F \int \frac{d^2 v_\perp}{(2\pi)^2}\frac{d^2 v'_\perp}{(2\pi)^2}\frac{d^2 u_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(v_\perp - v'_\perp)} \sum_{\lambda\alpha\beta} \psi^{\lambda *}_{\alpha\beta}(u_\perp)\psi^{\lambda}_{\alpha\beta}(u_\perp)$$

$$\times \left[S^{(2)}_Y(v_\perp, v'_\perp) - S^{(3)}_Y(b_\perp, x_\perp, v'_\perp)\right]$$

$$\Rightarrow \quad -\frac{\alpha_s}{2\pi^2}\int \frac{dz}{z^2} D_{h/q}(z) x_p q(x_p) \int_0^1 d\xi \frac{1+\xi^2}{1-\xi}$$

$$\times \left\{ C_F \int d^2 q_\perp \mathcal{I}(q_\perp, k_\perp) + \frac{N_c}{2}\int d^2 q_\perp d^2 k_{g1\perp} \mathcal{J}(q_\perp, k_\perp, k_{g1\perp}) \right\}.$$

Three types of divergences:

- $\xi \to 1 \Rightarrow$ Rapidity divergence.
- Collinear divergence associated with parton distributions and fragmentation functions.

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The subtraction of the rapidity divergence

We remove the rapidity divergence from the real and virtual diagrams by the following subtraction:

$$\mathcal{F}(k_\perp) = \mathcal{F}^{(0)}(k_\perp) - \frac{\alpha_s N_c}{2\pi^2}\int_0^1 \frac{d\xi}{1-\xi}\int \frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp-y_\perp)} \times \frac{(x_\perp-y_\perp)^2}{(x_\perp-b_\perp)^2(y_\perp-b_\perp)^2}\left[S^{(2)}(x_\perp,y_\perp) - S^{(4)}(x_\perp,b_\perp,y_\perp)\right].$$

Comments:

- The rapidity divergence is an artifact that both the projectile and targets are put on the light cone in the high energy limit ($s\to\infty$).
- This divergence removing procedure is similar to the renormalization of parton distribution and fragmentation function in collinear factorization.
- Splitting functions becomes $\frac{1+\xi^2}{(1-\xi)_+}$ after the subtraction.
- Rapidity divergence disappears when the $k_\perp$ is integrated. Unique feature of unintegrated gluon distributions.

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The subtraction of the rapidity divergence

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This is equivalent to the Balitsky-Kovchegov equation:

$$\frac{\partial}{\partial Y} S_Y^{(2)}(x_\perp, y_\perp) = -\frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2b_\perp\,(x_\perp - y_\perp)^2}{(x_\perp - b_\perp)^2(y_\perp - b_\perp)^2}\left[S_Y^{(2)}(x_\perp, y_\perp) - S_Y^{(4)}(x_\perp, b_\perp, y_\perp)\right].$$

- Recall that $\mathcal{F}(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} S^{(2)}(x_\perp, y_\perp)$.
- The soft gluon is emitted from the projectile proton with momentum $(1-\xi)p^+$, and it is easy to see that the rapidity of this soft gluon goes to $-\infty$ when $\xi \to 1$ since the radiated gluon is now in the region $k_g^- \gg k_g^+$. As a matter of fact, this soft gluon can be regarded as collinear to the target nucleus which is moving on the backward light cone with the rapidity close to $-\infty$ and $P_A^- \gg P_A^+$.
- Renormalize the soft gluon into the gluon distribution function of the target nucleus through the BK evolution equation.

The subtraction of the collinear divergence

Let us take the following integral as an example:

$$\begin{aligned} I_1(k_\perp) &= \int \frac{d^2k_{g\perp}}{(2\pi)^2}\mathcal{F}(k_{g\perp})\frac{1}{(k_\perp - k_{g\perp})^2}\,, \\ &= \frac{1}{4\pi}\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}S_Y^{(2)}(x_\perp, y_\perp)\left(-\frac{1}{\hat{\epsilon}} + \ln\frac{c_0^2}{\mu^2 r_\perp^2}\right), \end{aligned}$$

where $c_0 = 2e^{-\gamma_E}$, γ_E is the Euler constant and $r_\perp = x_\perp - y_\perp$.

- Use dimensional regularization ($D = 4 - 2\epsilon$) and the $\overline{\text{MS}}$ subtraction scheme ($\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$).
- $\int \frac{d^2k_{g\perp}}{(2\pi)^2} \Rightarrow \mu^{2\epsilon}\int \frac{d^{2-2\epsilon}k_{g\perp}}{(2\pi)^{2-2\epsilon}}$ where μ is the renormalization scale dependence coming from the strong coupling g.
- The terms proportional to the collinear divergence $\frac{1}{\hat{\epsilon}}$ should be factorized either into parton distribution functions or fragmentation functions.

The subtraction of the collinear divergence

Remove the collinear singularities by redefining the quark distribution and the quark fragmentation function as follows

$$
\begin{aligned}
q(x,\mu) &= q^{(0)}(x) - \frac{1}{\hat{\epsilon}}\frac{\alpha_s(\mu)}{2\pi}\int_x^1 \frac{d\xi}{\xi} C_F \mathcal{P}_{qq}(\xi) q\left(\frac{x}{\xi}\right), \\
D_{h/q}(z,\mu) &= D_{h/q}^{(0)}(z) - \frac{1}{\hat{\epsilon}}\frac{\alpha_s(\mu)}{2\pi}\int_z^1 \frac{d\xi}{\xi} C_F \mathcal{P}_{qq}(\xi) D_{h/q}\left(\frac{z}{\xi}\right),
\end{aligned}
$$

with

$$
\mathcal{P}_{qq}(\xi) = \underbrace{\frac{1+\xi^2}{(1-\xi)_+}}_{\text{Real Sub}} + \underbrace{\frac{3}{2}\delta(1-\xi)}_{\text{Virtual Sub}}.
$$

Comments:

- Reproducing the DGLAP equation for the quark channel. Other channels will complete the full equation.
- The emitted gluon is collinear to the initial state quark $\Rightarrow$ Renormalization of the parton distribution.
- The emitted gluon is collinear to the final state quark $\Rightarrow$ Renormalization of the fragmentation function.

Hard Factors

For the $q \to q$ channel, the factorization formula can be written as

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi xq(x,\mu)D_{h/q}(z,\mu)\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\left\{S_Y^{(2)}(x_\perp,y_\perp)\left[\mathcal{H}_{2qq}^{(0)}+\frac{\alpha_s}{2\pi}\mathcal{H}_{2qq}^{(1)}\right]\right.$$
$$\left. + \int \frac{d^2b_\perp}{(2\pi)^2} S_Y^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)}\right\}$$

with $\mathcal{H}_{2qq}^{(0)} = e^{-ik_\perp\cdot r_\perp}\delta(1-\xi)$ and

$$\mathcal{H}_{2qq}^{(1)} = C_F\mathcal{P}_{qq}(\xi)\ln\frac{c_0^2}{r_\perp^2\mu^2}\left(e^{-ik_\perp\cdot r_\perp}+\frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right) - 3C_F\delta(1-\xi)e^{-ik_\perp\cdot r_\perp}\ln\frac{c_0^2}{r_\perp^2 k_\perp^2}$$
$$-(2C_F-N_c)\,e^{-ik_\perp\cdot r_\perp}\left[\frac{1+\xi^2}{(1-\xi)_+}\widetilde{I}_{21} - \left(\frac{\left(1+\xi^2\right)\ln(1-\xi)^2}{1-\xi}\right)_+\right]$$

$$\mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_\perp\cdot r_\perp}\left\{e^{-i\frac{1-\xi}{\xi}k_\perp\cdot(x_\perp-b_\perp)}\frac{1+\xi^2}{(1-\xi)_+}\frac{1}{\xi}\frac{x_\perp-b_\perp}{(x_\perp-b_\perp)^2}\cdot\frac{y_\perp-b_\perp}{(y_\perp-b_\perp)^2}\right.$$
$$\left.-\delta(1-\xi)\int_0^1 d\xi'\frac{1+\xi'^2}{(1-\xi')_+}\left[\frac{e^{-i(1-\xi')k_\perp\cdot(y_\perp-b_\perp)}}{(b_\perp-y_\perp)^2} - \delta^{(2)}(b_\perp-y_\perp)\int d^2r'_\perp\frac{e^{ik_\perp\cdot r'_\perp}}{r'^2_\perp}\right]\right\}$$

where $$\widetilde{I}_{21} = \int\frac{d^2b_\perp}{\pi}\left\{e^{-i(1-\xi)k_\perp\cdot b_\perp}\left[\frac{b_\perp\cdot(\xi b_\perp - r_\perp)}{b_\perp^2\,(\xi b_\perp-r_\perp)^2} - \frac{1}{b_\perp^2}\right] + e^{-ik_\perp\cdot b_\perp}\frac{1}{b_\perp^2}\right\}.$$

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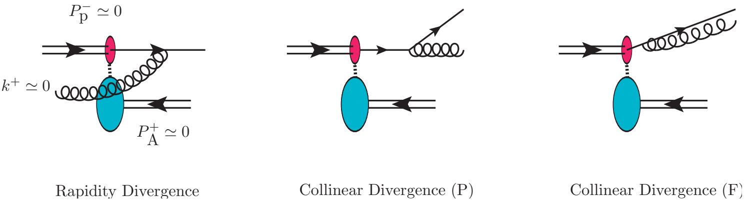
Physical interpretation

- Achieve a systematic factorization for the $p + A \to H + X$ process by systematically remove all the divergences!
- Gluons in different kinematical region give different divergences. 1.soft, collinear to the target nucleus; 2. collinear to the initial quark; 3. collinear to the final quark.



- All the rapidity divergence is absorbed into the UGD $\mathcal{F}(k_\perp)$ while collinear divergences are either factorized into collinear parton distributions or fragmentation functions.
- Large N_c limit is vital for the factorization in terms of getting rid of higher point functions.
- Consistent check: take the dilute limit, $k_\perp^2 \gg Q_s^2$, the result is consistent with the leading order collinear factorization formula.
- In terms of resummation, we will be able to resum up to $\alpha_s(\alpha_s \ln k_\perp^2)^n$ and $\alpha_s(\alpha_s \ln 1/x)^n$ terms.
- The other three channels $g \to g$, $g \to q$ and $q \to g$ follow accordingly.

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Numerical implementation of the NLO result

Consistent implementation should include all the NLO α_s corrections.

- NLO parton distributions. (Choose your favorite one, CTEQ or MSTW)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors.
- Use the one-loop approximation for the running coupling which is sufficient in this calculation.
- NLO BK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. (Hard)
  Alternate solution: Treat the dipole amplitude as an input, use GBW model or your favorite parametrization of dipole amplitudes with appropriate energy dependence, and then find the best fit by comparing with all the available data. Then make prediction for the LHC data.
- Looking at about $20 - 30$ percent uncertainty. Large N_c limit gives about 10 percent.
- Working in progress.

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Conclusion

- We calculate inclusive hadron productions in pA collisions in the small-x saturation formalism at one-loop order, and demonstrate the factorization in a rigorous and accurate way. Many more NLO calculations are now under study. (Higgs and WW, see F. Yuan's talk)
- The rapidity divergence with small-x dipole gluon distribution of the nucleus is factorized into the BK evolution of the dipole gluon distribution function.
- The collinear divergences associated with the incoming parton distribution of the nucleon and the outgoing fragmentation function of the final state hadron are factorized into the well-known DGLAP equation.
- The hard coefficient function, which is finite and free of divergence of any kind, is evaluated at one-loop order.
- Now we have a systematic NLO description of inclusive forward hadron productions in pA collisions which is ready for making reliable predictions and conducting precision test. Phenomenological applications are promising for both RHIC and LHC (upcoming pA run) experiments.

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