Non-linear flow response and plane correlations

Li Yan

Department of Physics and Astronomy

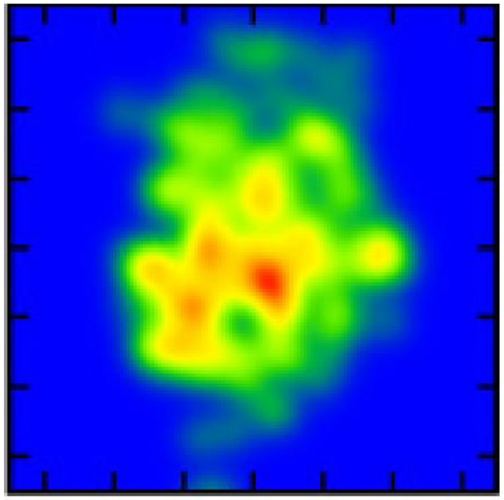
Stony Brook University

Quark Matter 2012
August 14, 2012

In collaboration with Derek Teaney

- e-Print: arXiv:1206.1905 [nucl-th].
- Phys. Rev. C83 (2011) 064904, e-Print: arXiv:1010.1876 [nucl-th].

Outline



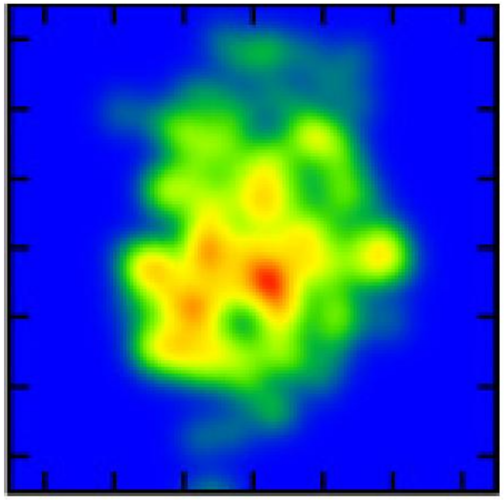
Initial state fluctuations + Event-By-Event hydro give

- Harmonic flow: v_2, v_3, etc.
- Correlations of reaction plane in final state.

Can we understand E-B-E hydro.?

Stony Brook University

Outline



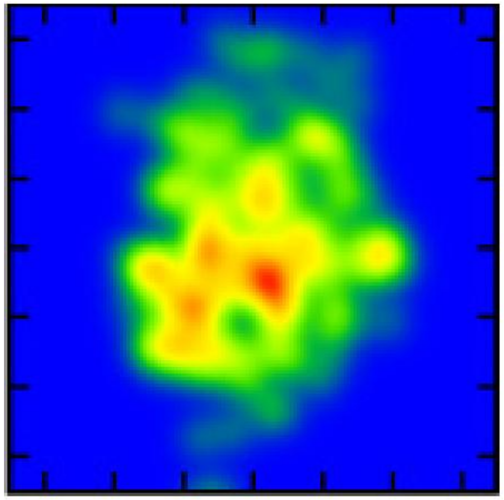
Initial state fluctuations + Event-By-Event hydro give

- Harmonic flow: v_2, v_3, etc.
- Correlations of reaction plane in final state.

Can we understand E-B-E hydro.?

1. Cumulant formalism for initial geometry with fluctuations.
2. Non-linear response formalism for (single-shot) hydrodynamics.
3. Reaction-plane correlations of final state (ATLAS results).

Outline



Initial state fluctuations + Event-By-Event hydro give

- Harmonic flow: v_2, v_3, etc.
- Correlations of reaction plane in final state.

Can we understand E-B-E hydro.?

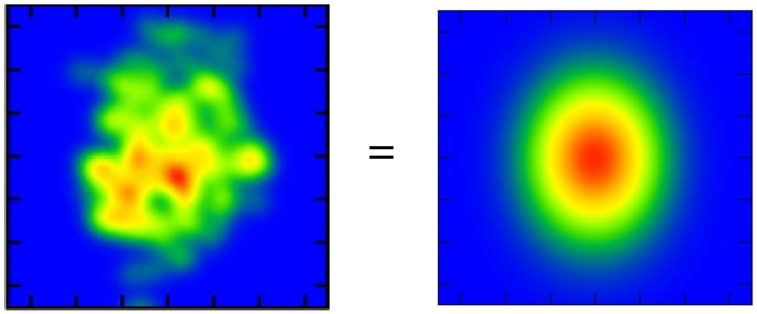
1. Cumulant formalism for initial geometry with fluctuations.
2. Non-linear response formalism for (single-shot) hydrodynamics.
3. Reaction-plane correlations of final state (ATLAS results).

$$\underbrace{\text{Initial correlations}}_{\text{Initial state(cumulants)}} + \underbrace{\text{Flow response}}_{\text{Linear \& Non-linear}} = \underbrace{\text{Reaction place correlations}}_{\text{Final state}}$$

Stony Brook University

Cumulants for initial state: (not moments!)

Fluctuations in initial state as corrections:



- Cumulant expansion:

$$\rho(x,y) = \text{Gaussian} + \underbrace{\text{1st cumulant}}_{\varepsilon_1} + \underbrace{\text{3rd cumulant}}_{\varepsilon_3} + \underbrace{\text{4th cumulant}}_{\mathcal{C}_4} + \ldots$$

- 4th Cumulant determines **eccentricity** $\mathcal{C}_4$ and **participant angle** Φ_4.

$$\underbrace{\mathcal{C}_4 e^{4i\Phi_4}}_{\text{4th cumulant}} = -\frac{1}{\langle r^4 \rangle}\Big[\underbrace{\langle r^4 e^{i4\phi_r} \rangle}_{\varepsilon_4:\ \text{moments def.}} - \underbrace{3\langle r^2 e^{i2\phi_2} \rangle^2}_{\text{subtract } \varepsilon_2^2} \Big]$$

e.g. Gaussian with ε_2 has $\mathcal{C}_4 = 0$, but $\varepsilon_4 \propto \varepsilon_2^2 \neq 0$.

- Why we use cumulants: avoid double counting in initial conditions.

We define all geometric deformations, *i.e.* $(\mathcal{C}_n, \Phi_n)$, with cumulants.

Non-linear response formalism (n=5 for example)

Flow generation in hydro:

$$\underbrace{v_5 e^{-i5\Psi_5}}_{\text{final state}} = \underbrace{\underbrace{\frac{w_5}{\mathcal{C}_5}}_{\text{linear resp.}} \times \underbrace{\mathcal{C}_5 e^{-i5\Phi_5}}_{\text{initial state}}}$$

[1] U. Heinz and Z. Qiu, and Gardim *et al.*

Stony Brook University

Non-linear response formalism (n=5 for example)

Flow generation in hydro:

$$\underbrace{v_5 e^{-i5\Psi_5}}_{\text{final state}} = \overbrace{\underbrace{\frac{w_5}{\mathcal{C}_5}}_{\text{linear resp.}} \times \underbrace{\mathcal{C}_5 e^{-i5\Phi_5}}_{\text{initial state}}}$$

$$\underbrace{\hphantom{\frac{w_5}{\mathcal{C}_5} \times \mathcal{C}_5 e^{-i5\Phi_5}}}_{^1\text{linear resp. fails for } n\geq 4}$$

[1] U. Heinz and Z. Qiu, and Gardim *et al.*

Stony Brook University

Non-linear response formalism (n=5 for example)

Flow generation in hydro:

$$\underbrace{v_5 e^{-i5\Psi_5}}_{\text{final state}} = \underbrace{\underbrace{\frac{w_5}{\mathcal{C}_5}}_{\text{linear resp.}} \times \underbrace{\mathcal{C}_5 e^{-i5\Phi_5}}_{\text{initial state}}}_{^{1}\text{linear resp. fails for } n\geq 4} + \underbrace{\frac{w_{5(23)}}{\varepsilon_2\varepsilon_3}}_{\text{non-linear resp.}} \times \underbrace{\varepsilon_2\varepsilon_3 e^{-i(3\Phi_3+2\Phi_2)}}_{\text{initial state}}$$

- Assume non-linear flow response to $\underline{\varepsilon_2\varepsilon_3}$.

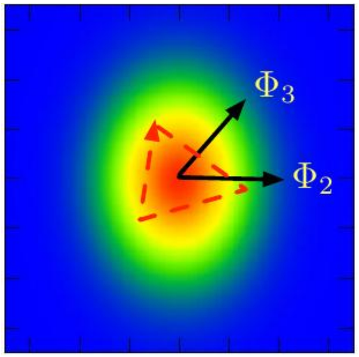
[1] U. Heinz and Z. Qiu, and Gardim *et al.*

Non-linear response formalism (n=5 for example)

Flow generation in hydro:

$$\underbrace{v_5 e^{-i5\Psi_5}}_{\text{final state}} = \underbrace{\underbrace{\frac{w_5}{\mathcal{C}_5}}_{\text{linear resp.}} \times \underbrace{\mathcal{C}_5 e^{-i5\Phi_5}}_{\text{initial state}}}_{^{1}\text{linear resp. fails for } n\geq 4} + \underbrace{\frac{w_{5(23)}}{\varepsilon_2\varepsilon_3}}_{\text{non-linear resp.}} \times \underbrace{\varepsilon_2\varepsilon_3 e^{-i(3\Phi_3+2\Phi_2)}}_{\text{initial state}}$$

- Assume non-linear flow response to $\underline{\varepsilon_2\varepsilon_3}$.



[1] U. Heinz and Z. Qiu, and Gardim *et al.*

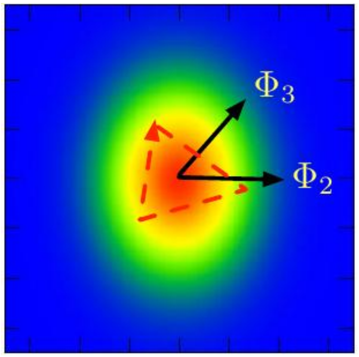
Stony Brook University

Non-linear response formalism (n=5 for example)

Flow generation in hydro:

$$\underbrace{v_5 e^{-i5\Psi_5}}_{\text{final state}} = \underbrace{\underbrace{\frac{w_5}{\mathcal{C}_5}}_{\text{linear resp.}} \times \underbrace{\mathcal{C}_5 e^{-i5\Phi_5}}_{\text{initial state}}}_{^1\text{linear resp. fails for } n\geq 4} + \underbrace{\frac{w_{5(23)}}{\varepsilon_2\varepsilon_3}}_{\text{non-linear resp.}} \times \underbrace{\varepsilon_2\varepsilon_3 e^{-i(3\Phi_3+2\Phi_2)}}_{\text{initial state}}$$

- Assume non-linear flow response to $\underline{\varepsilon_2\varepsilon_3}$.



- Calculations of $v_n\{2\}$: flow from two-particle correlation.

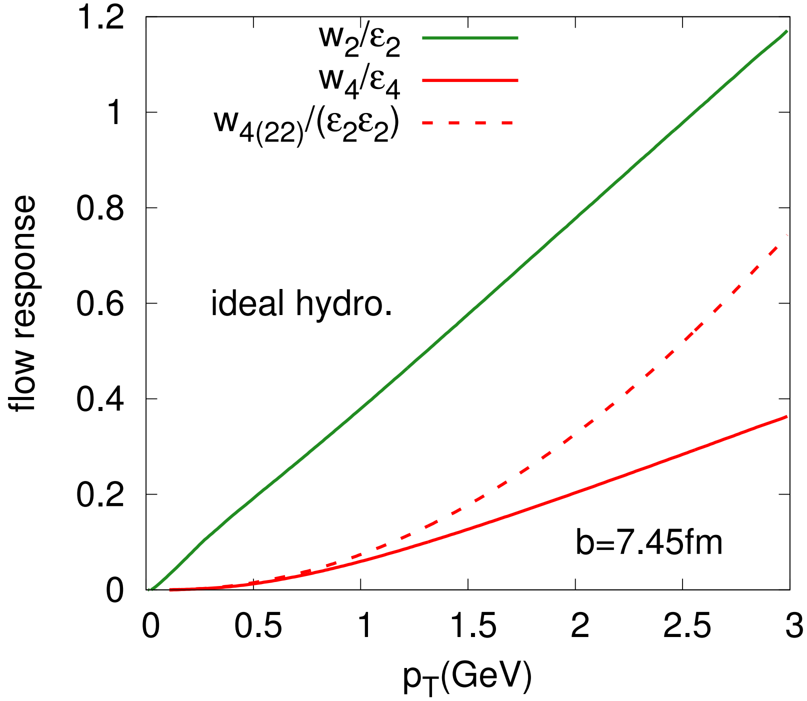
$$v_5\{2\} = \langle\langle |w_5 e^{-i5\Phi_5} + w_{5(23)} e^{-i(3\Phi_3+2\Phi_2)}|^2\rangle\rangle^{1/2}$$

$$v_5 \sim (\text{linear}) + (\text{non-linear}) + (\text{interference} \propto \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)).$$

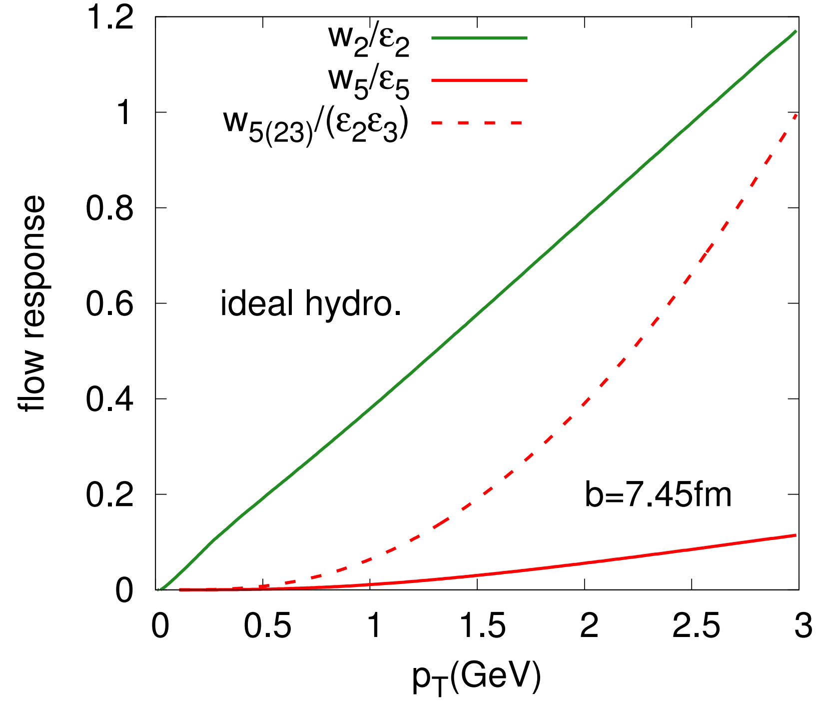
[1] U. Heinz and Z. Qiu, and Gardim *et al.*

Non-linear response dependence on p_T

w_4 and $w_{4(22)}$



w_5 and $w_{5(23)}$



- Small p_T: non-linear response is not distinguishable from linear response.
- Large p_T: linear response $\propto p_T$, non-linear response $\propto p_T^2$.[2]

So, non-linear response becomes more significant for larger p_T.

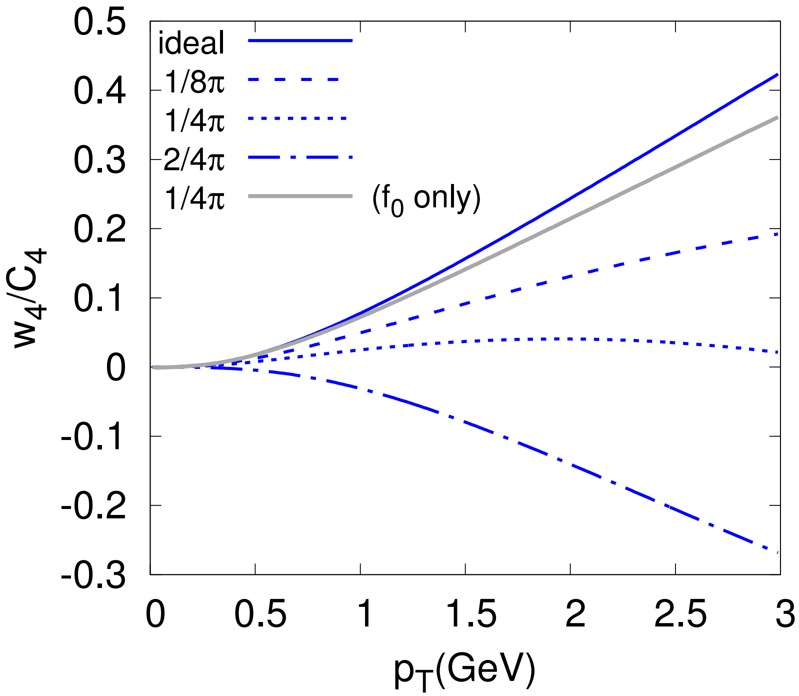
[2] (N. Borghini and J. Ollitrault)

Stony Brook University

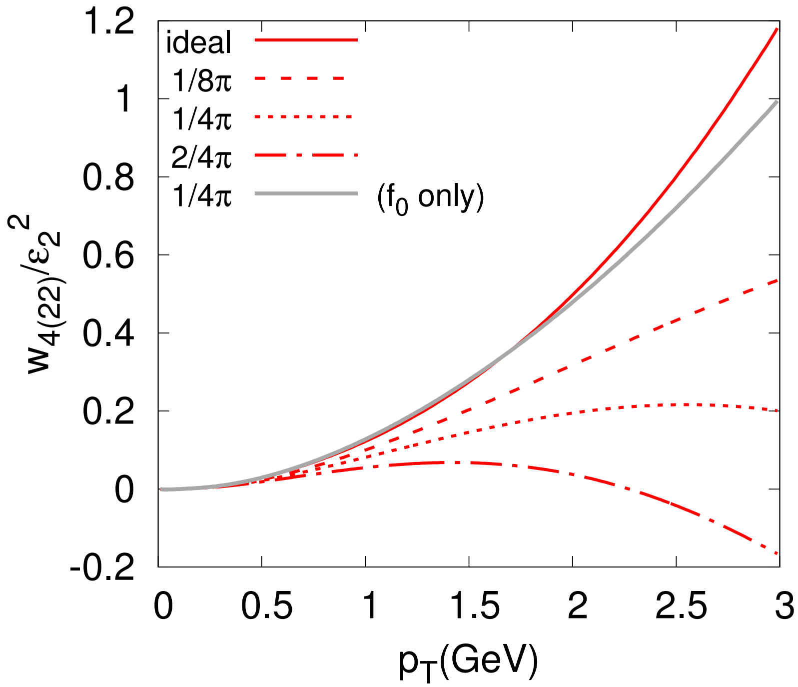
Non-linear response dependence on η/s

[3]Damping rate $\propto$ (harmonic order)^2 $\times$ η/s:

linear v_4



non-linear v_4



- Damping of $w_{4(22)}$ < damping of w_4, which can be generalized to $n \geq 4$.
- δf on freeze out may be questionable for higher order flow response.

For $(n \geq 4)$, non-linear response becomes more important for larger η/s.

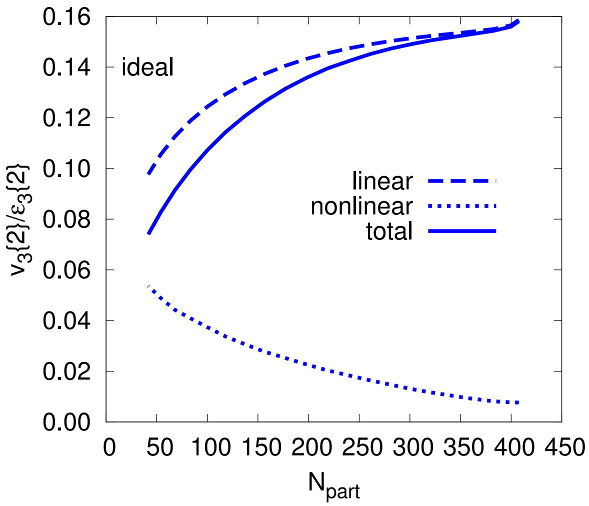
[3](S. Gubser and A. Yarom)

Stony Brook University

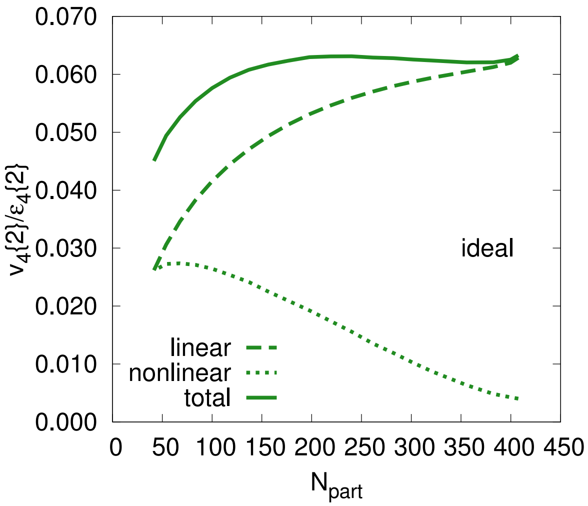
Non-linear response dependence on centrality: integrated $v_n\{2\}$

- $v_n\{2\}^2 = \underbrace{\text{linear response}}_{\langle\varepsilon^2\rangle} + \underbrace{\text{crossing terms}}_{\langle\varepsilon^2\cos(\ldots)\rangle} + \underbrace{\text{non-linear response}}_{\langle\varepsilon^4\rangle}$

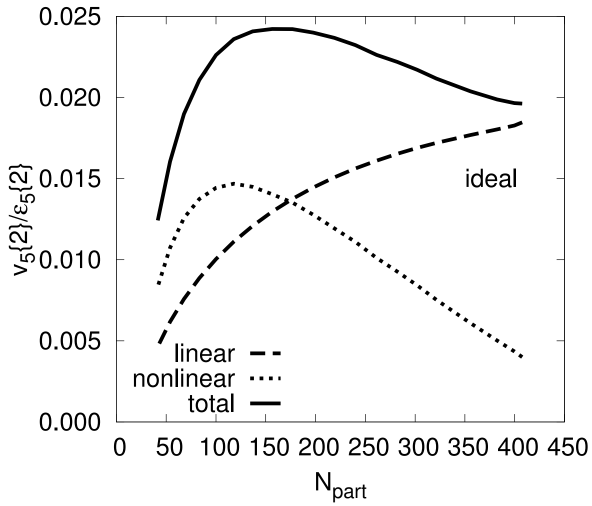
$v_3\{2\} : w_3, w_{3(12)}$



$v_4\{2\} : w_4, w_{4(22)}$



$v_5\{2\} : w_5, w_{5(23)}$



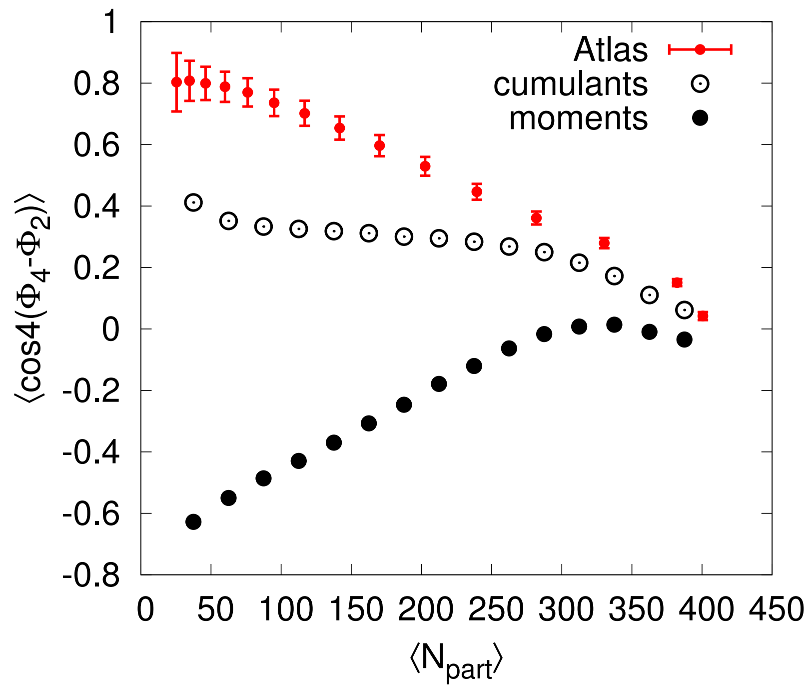
(LHC PbPb, ideal hydro, $T_{fo} = 150\text{MeV}$, PHOBOS MC-GLb.)

- Non-linear response is not important for v_3, but crucial for v_4 and v_5.
- Linear response dominates at central bins.
- Non-linear response dominates at peripheral bins.

Non-linear response becomes more important for larger centrality.

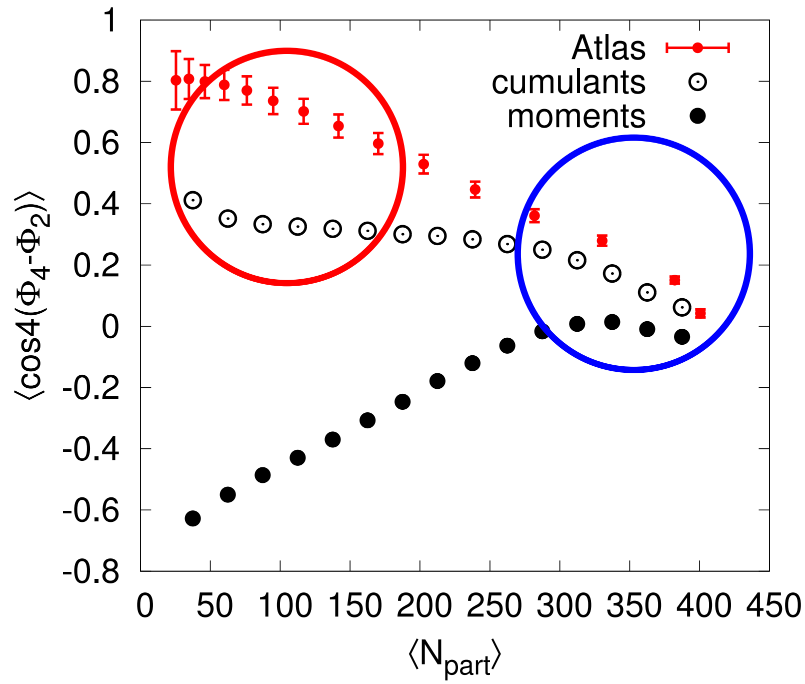
Reaction-plane correlations: linear response $(\Phi_n, \ldots) \Leftrightarrow (\Psi_n, \ldots)$

(Ψ_4, Ψ_2) correlations:



- Initial correlations from cumulants and moments are different.
- Deviations (cumulants def.) from experiment data imply NL response.
    1. At central bins(linear dominant): smaller deviations.
    2. At peripheral bins(non-linear dominant): larger deviations.

(Ψ_4, Ψ_2) correlations:



- Initial correlations from cumulants and moments are different.
- Deviations (cumulants def.) from experiment data imply NL response.
  1. At central bins(linear dominant): smaller deviations.
  2. At peripheral bins(non-linear dominant): larger deviations.

Reaction-plane correlations: non-linear response $(\Phi_n, \ldots) \not\Leftrightarrow (\Psi_n, \ldots)$

- $\langle\langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5)\rangle\rangle$: $\Psi_2 = \Phi_2$ and to the lowest order $\Psi_3 = \Phi_3$.

$$\langle\langle \cos(2\Psi_2+3\Psi_3-5\Psi_5)\rangle\rangle = \left\langle\left\langle \frac{\overbrace{\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)w_5}^{\text{linear}} + \overbrace{w_{5(23)}}^{\text{Non-linear}}}{|w_5 e^{-i5\Phi_5} + w_{5(23)} e^{-i(2\Phi_2+3\Phi_3)}|} \right\rangle\right\rangle$$

- Understand RP plane correlation with non-linear response formalism.

$$\text{RP correlations} = \langle\langle \underbrace{\text{PP correlations}}_{\color{blue}\text{Linear response limit}} + \underbrace{\text{NL correlations}}_{\color{red}\text{NL response limit}} \rangle\rangle$$

Stony Brook University

Reaction-plane correlations: non-linear response $(\Phi_n, \ldots) \not\Leftrightarrow (\Psi_n, \ldots)$

- $\langle\langle\cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5)\rangle\rangle$: $\Psi_2 = \Phi_2$ and to the lowest order $\Psi_3 = \Phi_3$.

$$\langle\langle\cos(2\Psi_2+3\Psi_3-5\Psi_5)\rangle\rangle = \left\langle\left\langle \frac{\overbrace{\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)w_5 w_3}^{\text{linear}} + \overbrace{w_{5(23)} w_3 + \ldots}^{\text{Non-linear}}}{|w_5 e^{-i5\Phi_5} + w_{5(23)} e^{-i(2\Phi_2+3\Phi_3)}||v_3|} \right\rangle\right\rangle$$

- Understand RP plane correlation with non-linear response formalism.

$$\text{RP correlations} = \langle\langle \underbrace{\text{PP correlations}}_{\color{blue}{\text{Linear response limit}}} + \underbrace{\text{NL correlations}}_{\color{red}{\text{NL response limit}}} \rangle\rangle$$

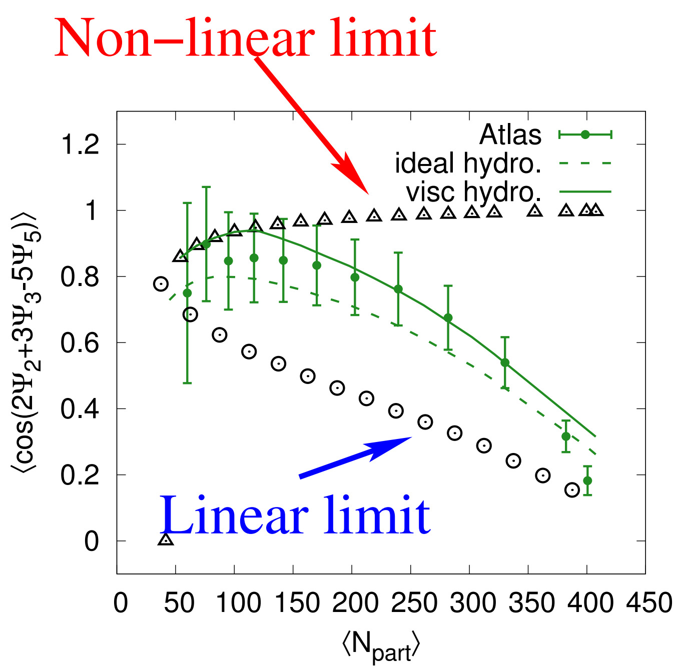
Stony Brook University

- $\langle\langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle\rangle$: $\Psi_2 = \Phi_2$ and to the lowest order $\Psi_3 = \Phi_3$.

$$\langle\langle \cos(2\Psi_2+3\Psi_3-5\Psi_5) \rangle\rangle = \left\langle\left\langle \frac{\overbrace{\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) w_5 w_3}^{\text{linear}} + \overbrace{w_{5(23)} w_3}^{\text{Non-linear}} + \ldots}{|w_5 e^{-i5\Phi_5} + w_{5(23)} e^{-i(2\Phi_2+3\Phi_3)}||v_3|} \right\rangle\right\rangle$$

- Understand RP plane correlation with non-linear response formalism.

$$\text{RP correlations} = \langle\langle \underbrace{\text{PP correlations}}_{\text{Linear response limit}} + \underbrace{\text{NL correlations}}_{\text{NL response limit}} \rangle\rangle$$



2–3–5 correlation

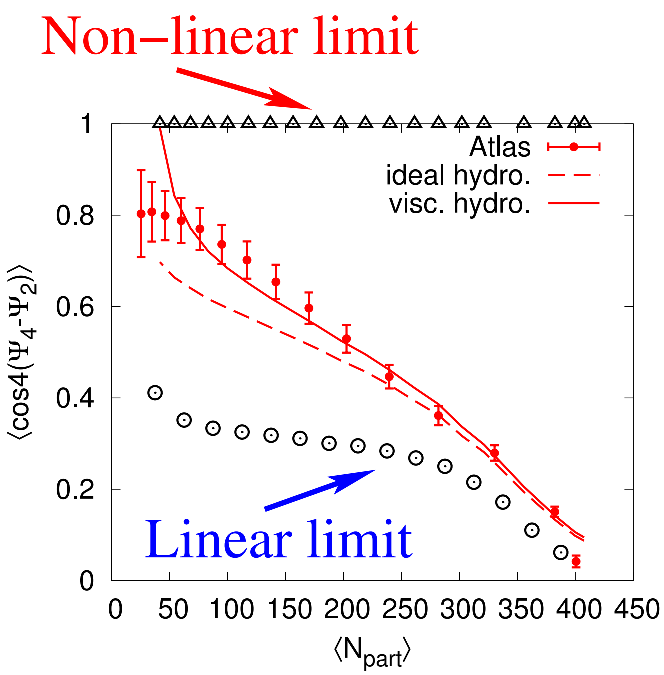
Reaction-plane correlations: non-linear response $(\Phi_n, \ldots) \not\Leftrightarrow (\Psi_n, \ldots)$

- $\langle\langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle\rangle$: $\Psi_2 = \Phi_2$ and to the lowest order $\Psi_3 = \Phi_3$.

$$\langle\langle \cos(2\Psi_2+3\Psi_3-5\Psi_5) \rangle\rangle = \left\langle\left\langle \frac{\overbrace{\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) w_5 w_3}^{\text{linear}} + \overbrace{w_{5(23)} w_3 + \ldots}^{\text{Non-linear}}}{|w_5 e^{-i5\Phi_5} + w_{5(23)} e^{-i(2\Phi_2+3\Phi_3)}||v_3|} \right\rangle\right\rangle$$

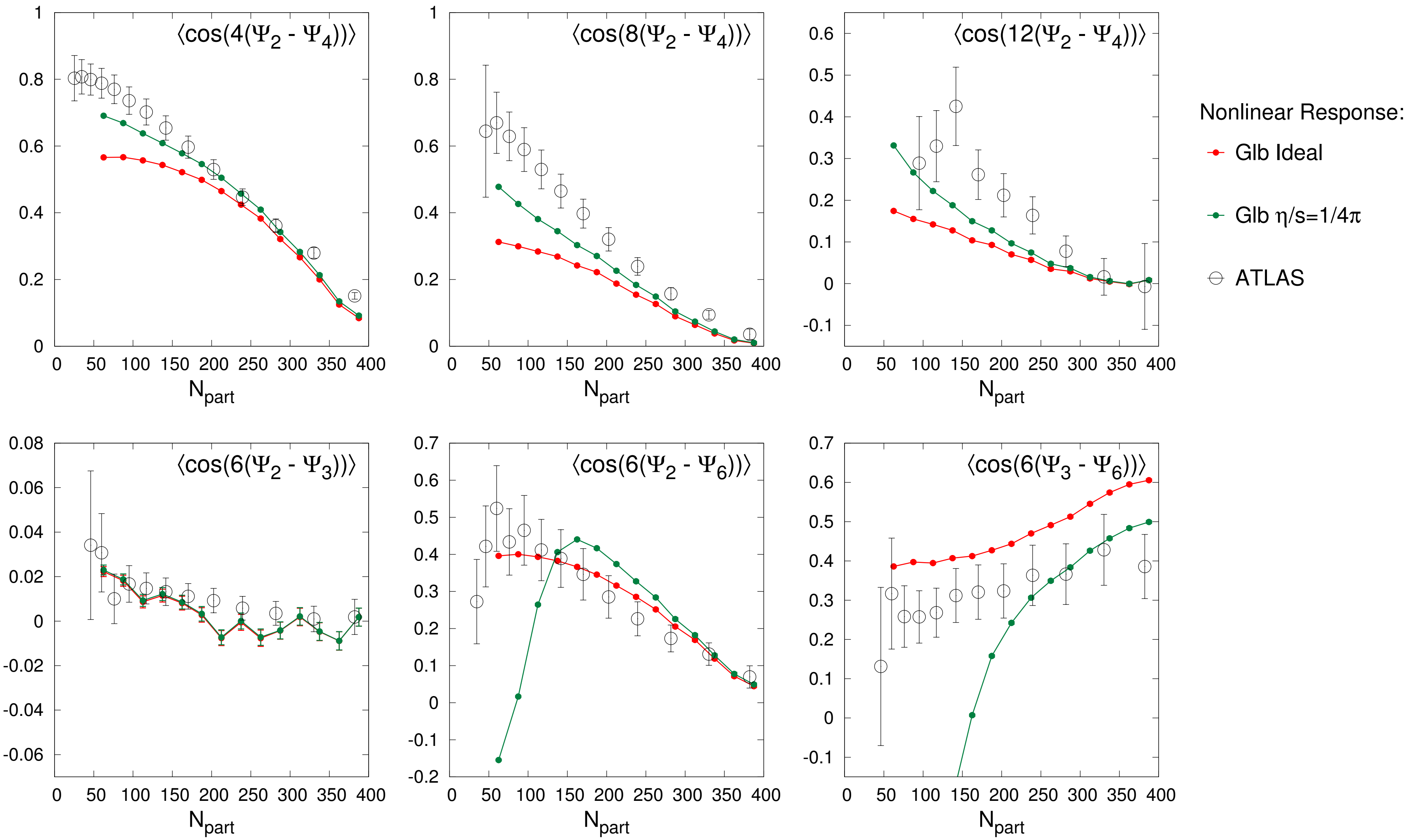
- Understand RP plane correlation with non-linear response formalism.

$$\text{RP correlations} = \langle\langle \underbrace{\text{PP correlations}}_{\text{Linear response limit}} + \underbrace{\text{NL correlations}}_{\text{NL response limit}} \rangle\rangle$$



2–4 correlation

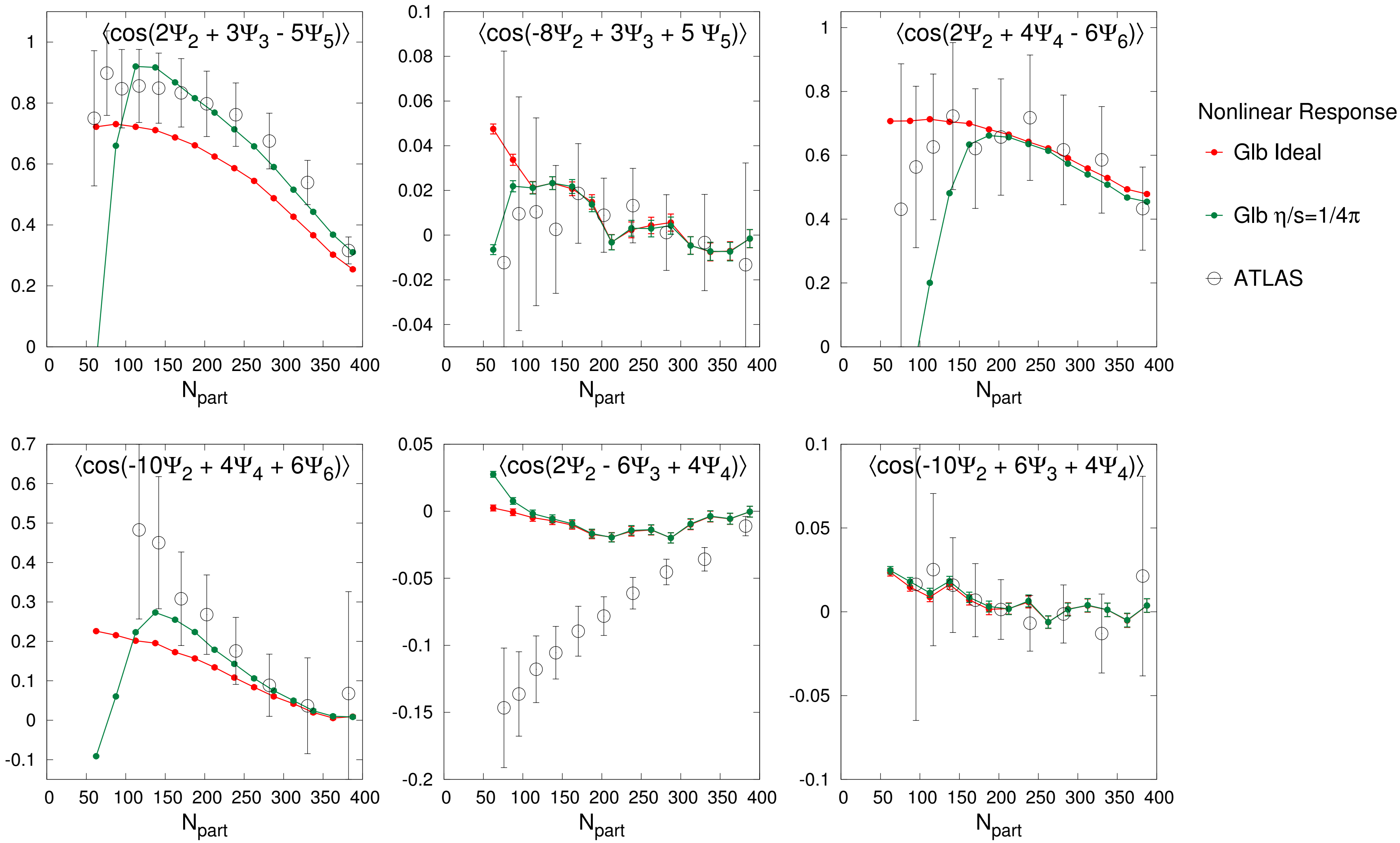
Two-plane correlations



- $v_3(3,12), v_4(4,22), v_5(5,23), v_6(33,24,222)$
- (LHC PbPb, $\eta/s = 1/4\pi$, $T_{fo} = 150\text{MeV}$, PHOBOS MC-GLb.)

Stony Brook University

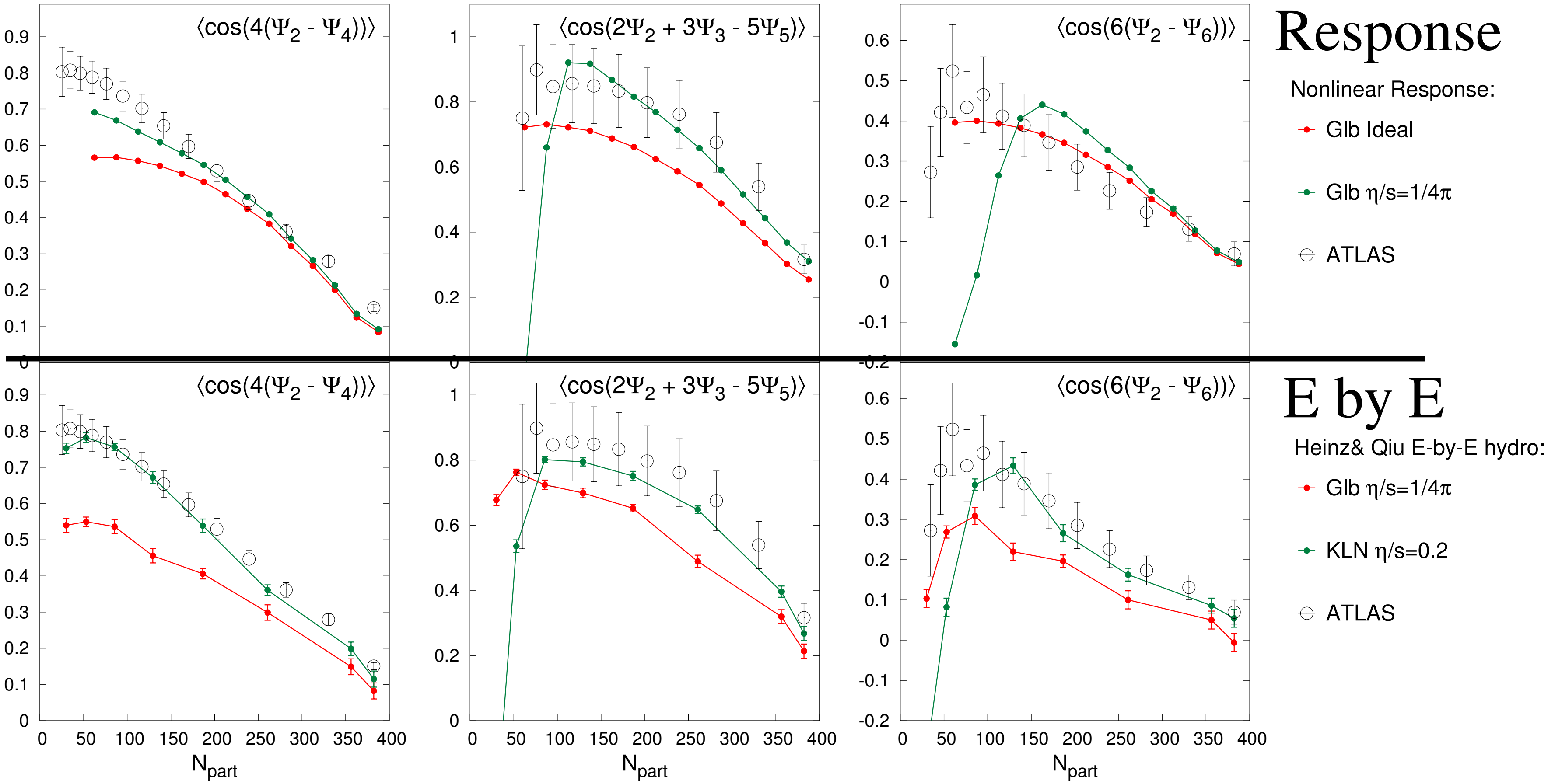
Three-plane correlations



- $v_3(3, 12), v_4(4, 22), v_5(5, 23), v_6(33, 24, 222)$
- (LHC PbPb, $\eta/s = 1/4\pi$, $T_{fo} = 150$MeV, PHOBOS MC-GLb.)

Stony Brook University

Not fair comparison with E-By-E hydro



Stony Brook University

Summary and conclusions

We have developed a non-linear response formalism for (single-show) hydro:

$$\underbrace{\text{Initial correlations}}_{\text{cumulants}} + \underbrace{\text{Flow response}}_{\text{Linear \& \underline{Non-linear}}} = \underbrace{\text{Reaction plane correlaions}}_{\text{Final state}}$$

- Ingredients:
  1. We use cumulant formalism to classify initial fluctuations.
  2. We take linear *and* non-linear response in hydro calculations, non-linear response *vs.* (p_T, η/s, centrality).
  3. Predictions are consistent to final state observables (E-B-E hydro).

Stony Brook University

Summary and conclusions

We have developed a non-linear response formalism for (single-show) hydro:

$$\underbrace{\text{Initial correlations}}_{\text{cumulants}} + \underbrace{\text{Flow response}}_{\text{Linear \& \underline{Non-linear}}} = \underbrace{\text{Reaction plane correlaions}}_{\text{Final state}}$$

- Ingredients:
  1. We use cumulant formalism to classify initial fluctuations.
  2. We take linear *and* non-linear response in hydro calculations, non-linear response *vs.* (p_T, η/s, centrality).
  3. Predictions are consistent to final state observables (E-B-E hydro).

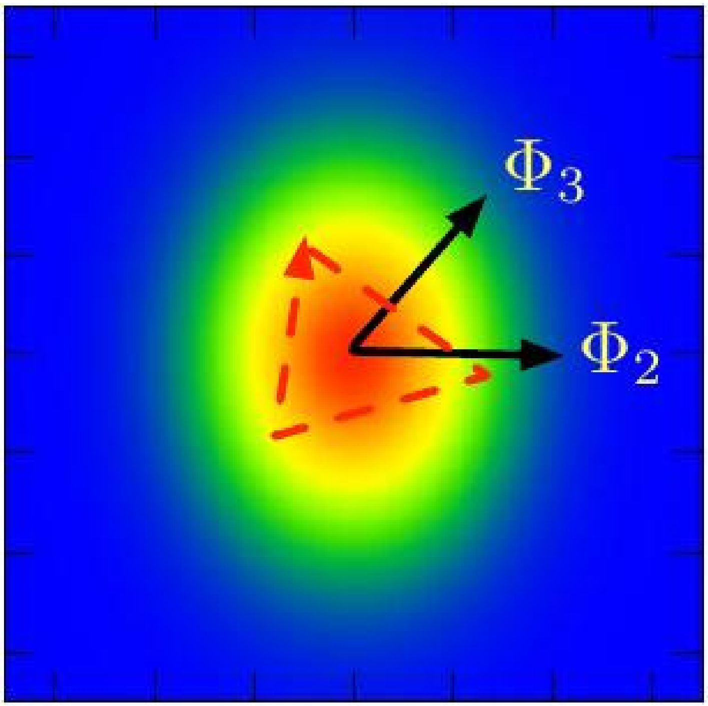
Thank you.

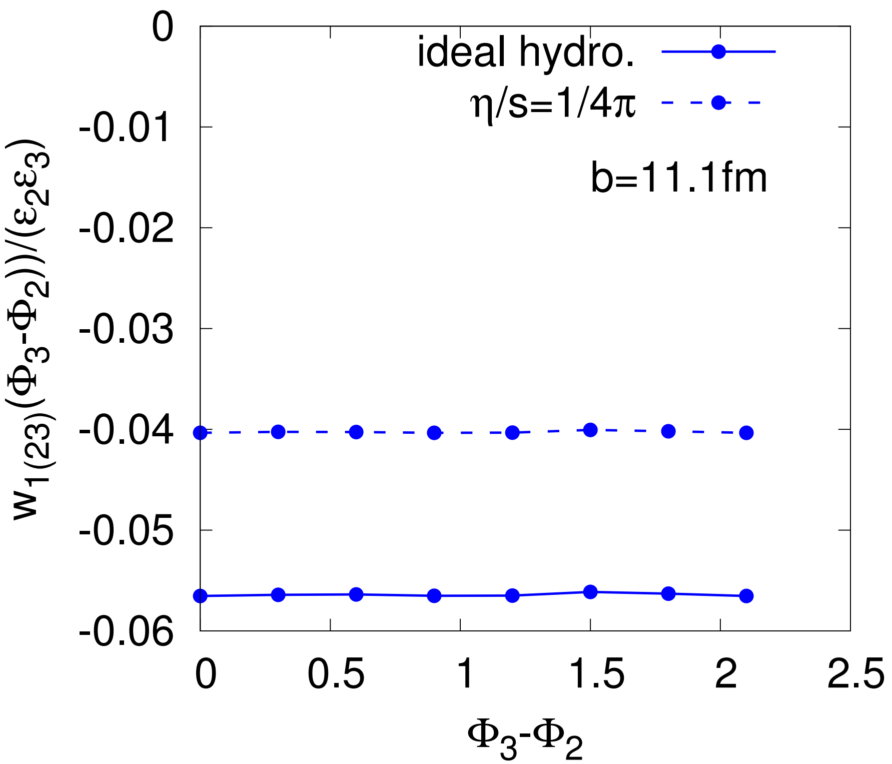
Stony Brook University

Back-up slides

Non-linear response dependence on angle

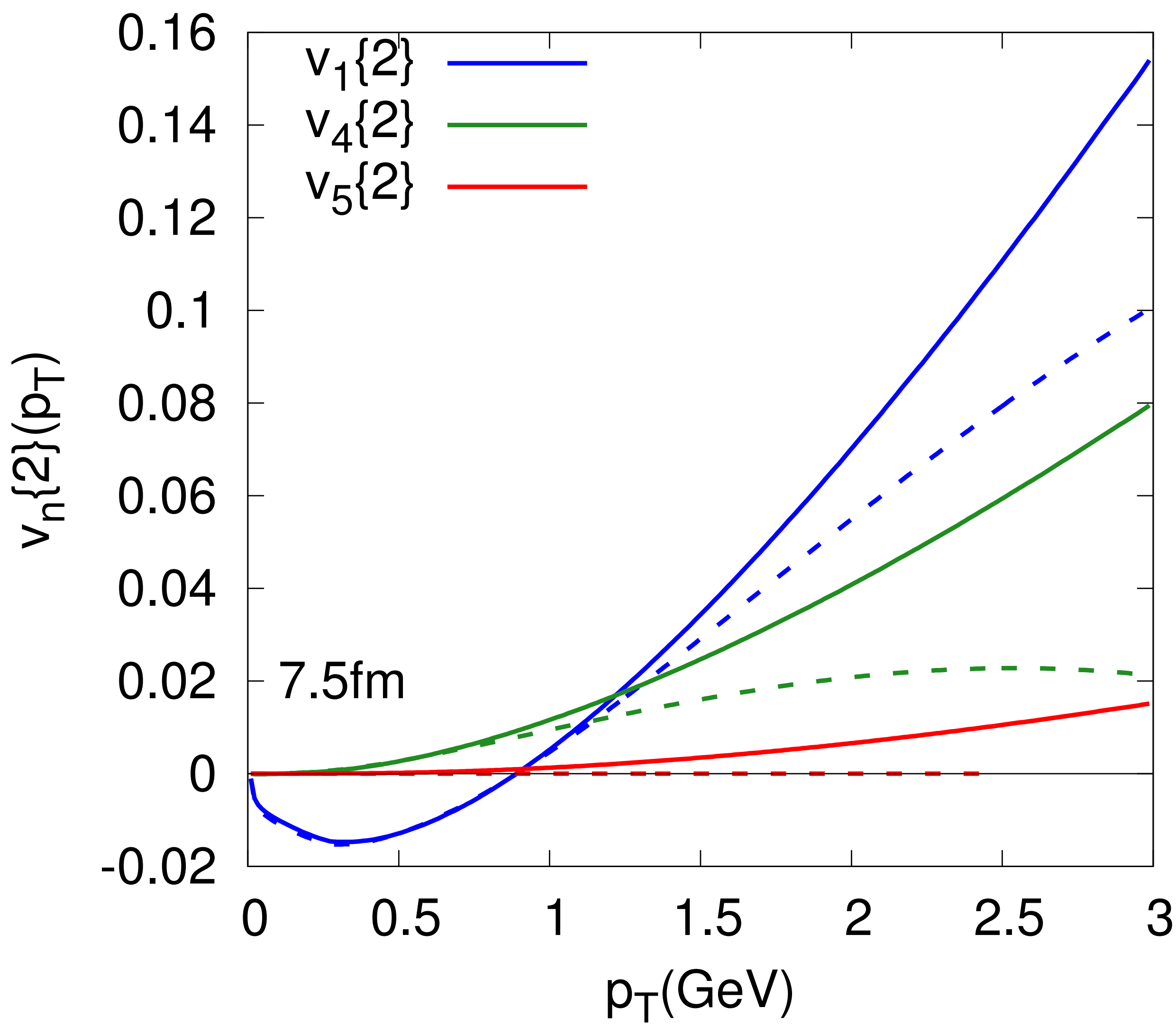
- For $w_{4(22)}$ the angle dependence is trivial.
- $\Phi_2 = \Phi_R$ fixed, while Φ_3 rotates $\to w_{1(23)}$, ($w_{5(23)}$ similar!).





Stony Brook University

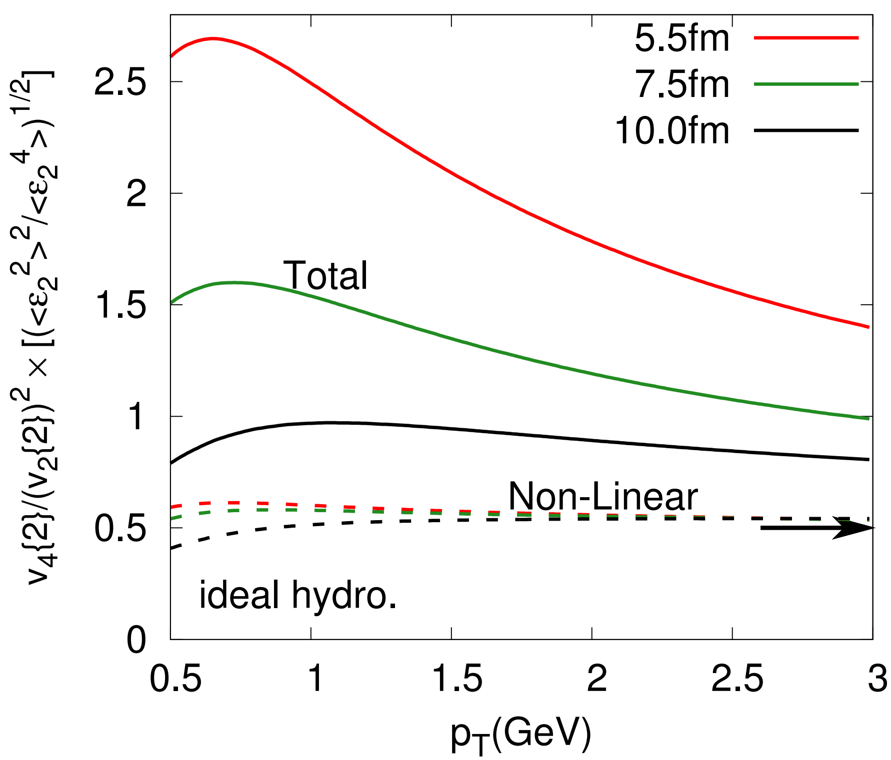
Differential flow at mid-central bin

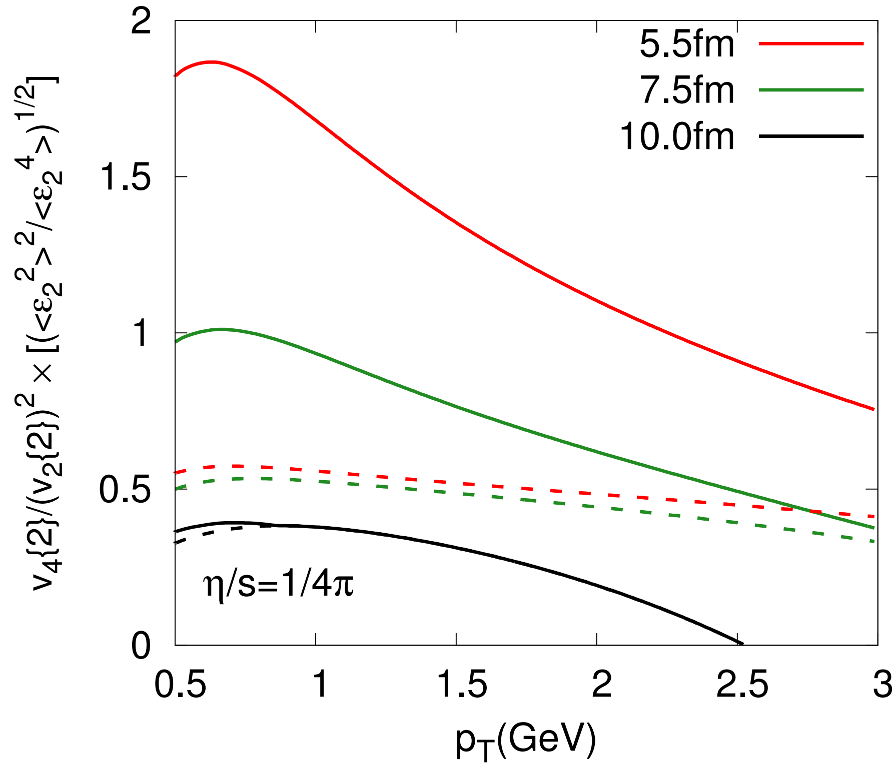


- Non-linear correction in v_1 not significant, compare to v_4 and v_5.
- Viscous damping at large p_T region.

Stony Brook University

$v_4\{2\}(p_T)/v_2\{2\}(p_T)^2$

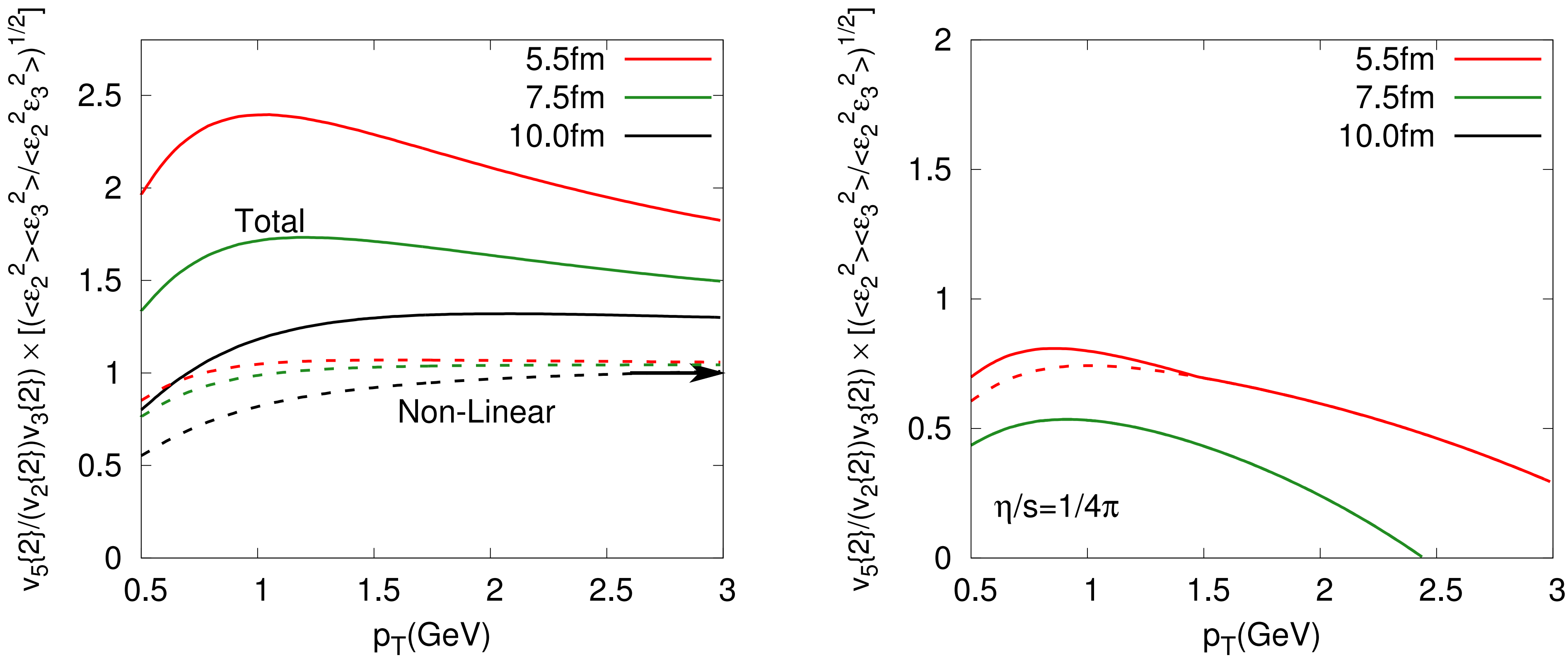




- Scaling behavior reproduced for ideal hydro. large p_T limit.
- Linear contribution affects the scaling.
- The real observables are dressed with quantities from events average.

Stony Brook University

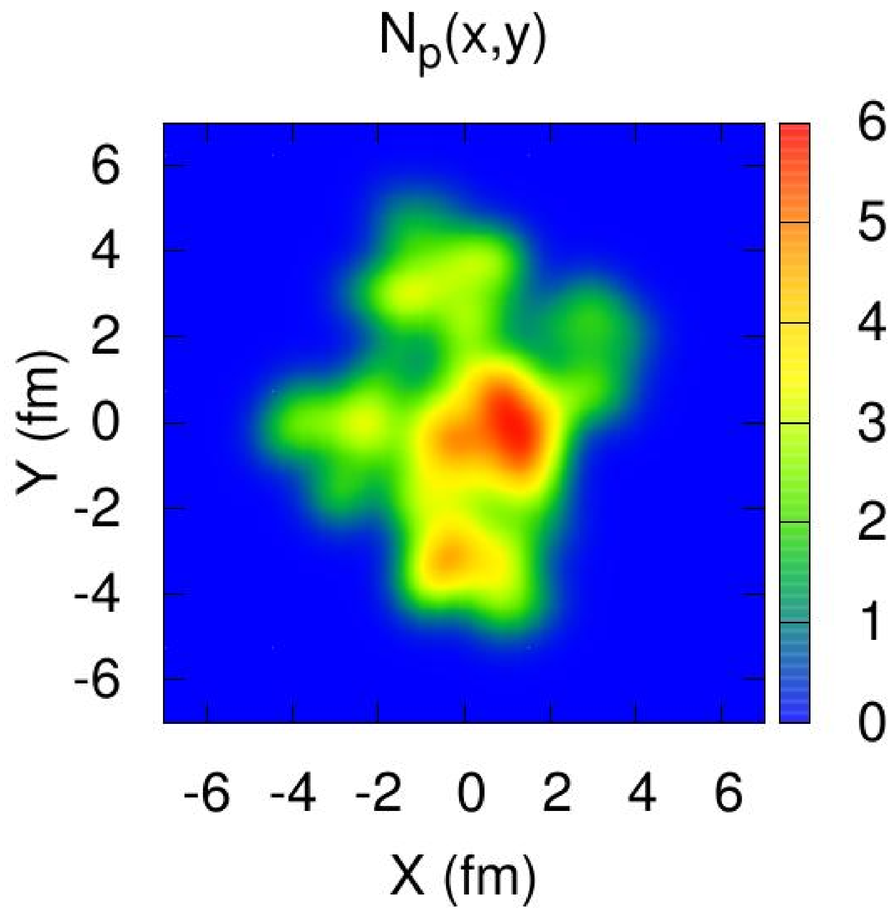
$v_5\{2\}(p_T)/(v_2\{2\}(p_T)v_3\{2\}(p_T))$

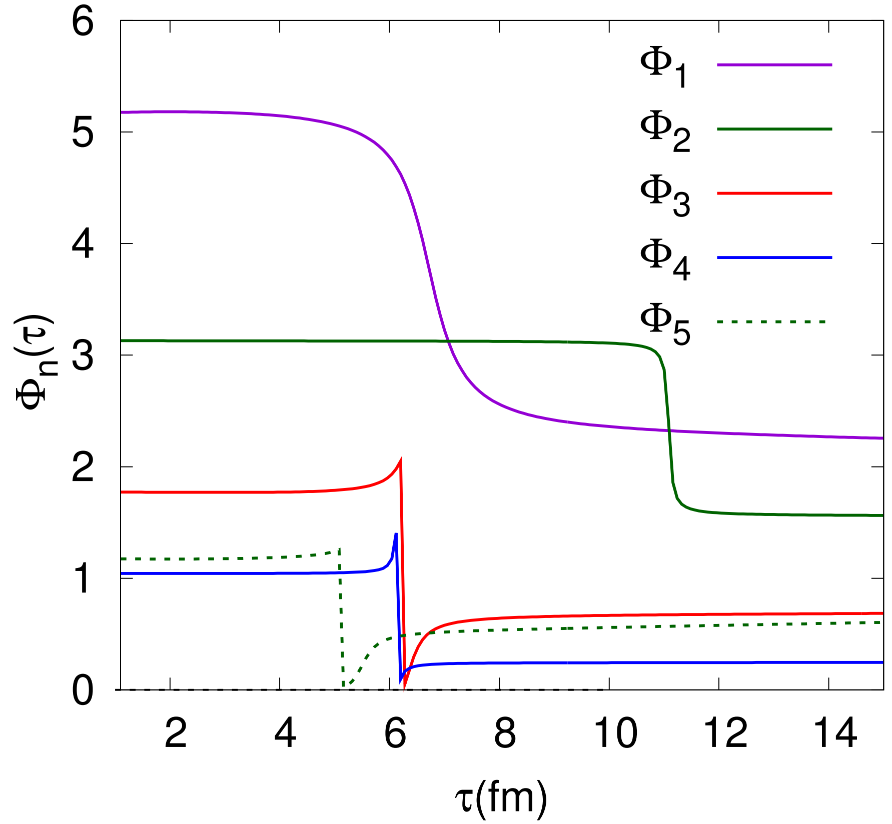


- Scaling behavior reproduced for ideal hydro. large p_T limit.
- Linear contribution affects the scaling.
- The real observables are dressed with quantities from events average.

Stony Brook University

Origins of non-linearity – medium expansion





- Shape switch at certain time – $\Delta\Phi_n = |\pi/n|$
- Each of the deformations evolves independently.

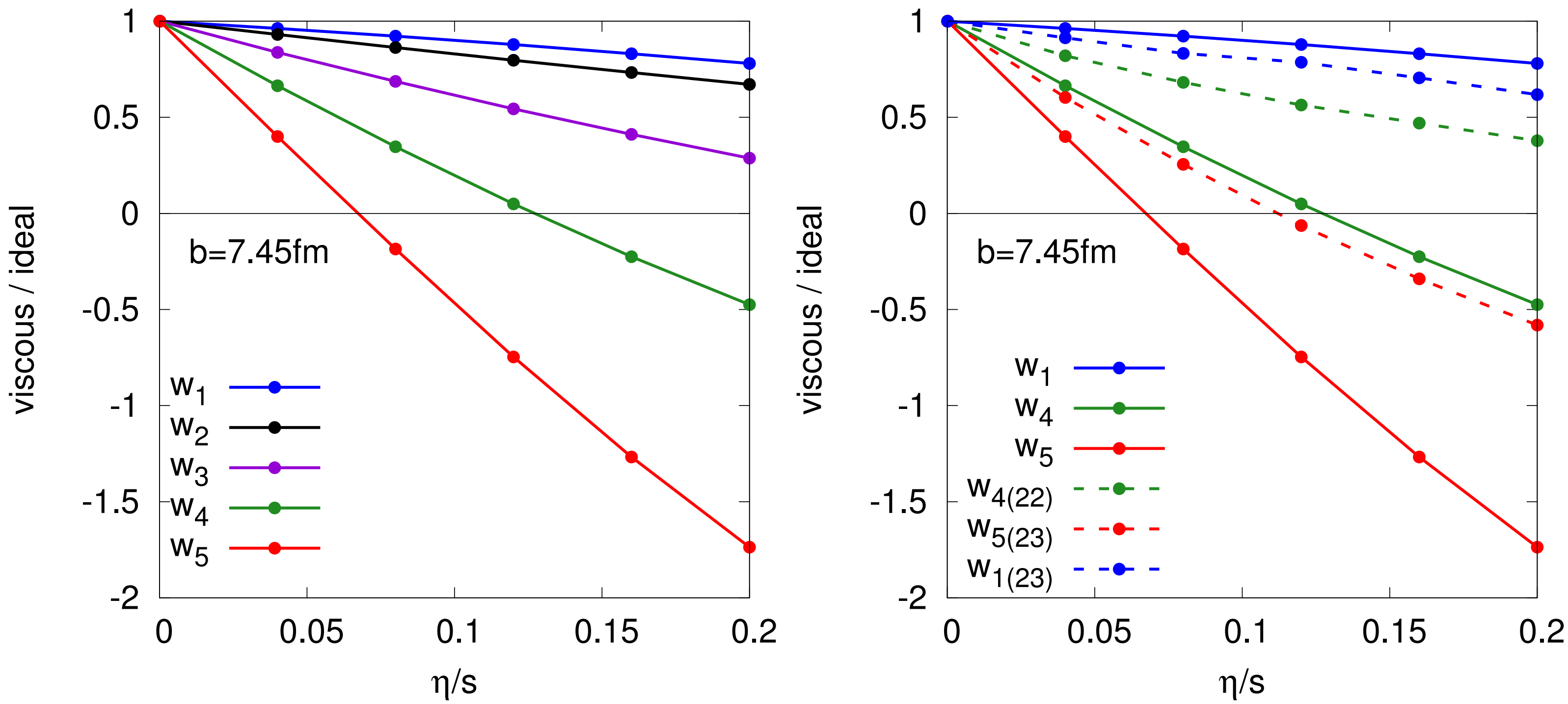
Different angular deformations do NOT interact during expansion.

Stony Brook University

Damping rate as a function of η/s, linear response

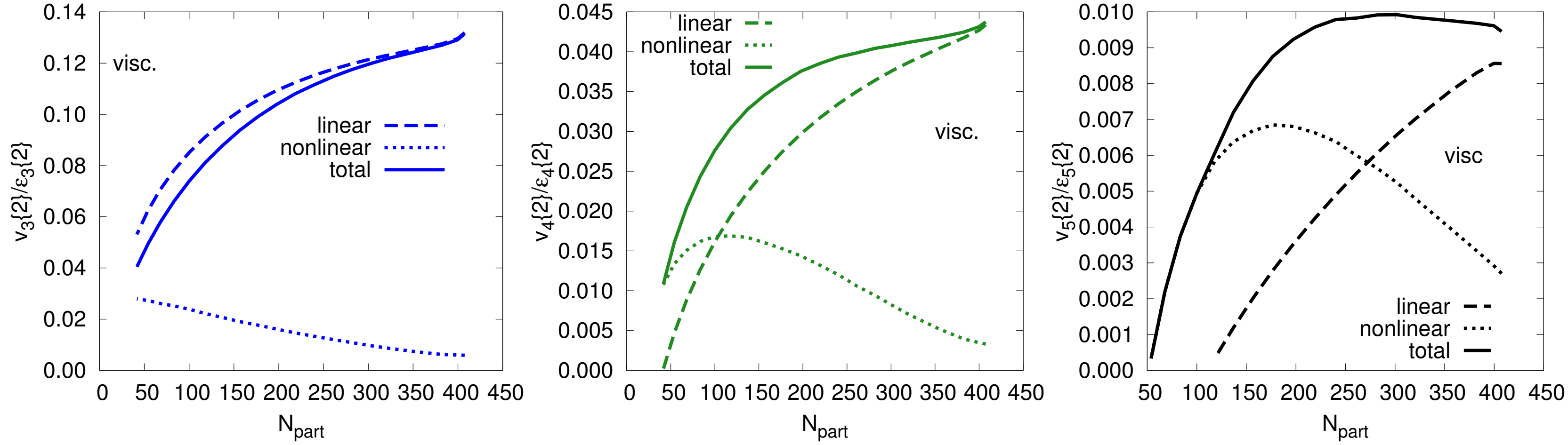
$$\text{Damping rate: } \Gamma_{n,m}\tau_{\text{final}} \rightarrow -\frac{\Delta w_n}{w_n^i} \propto \underline{(n+m)^2} \times \frac{\eta}{s}$$

n, m are cumulant indices.

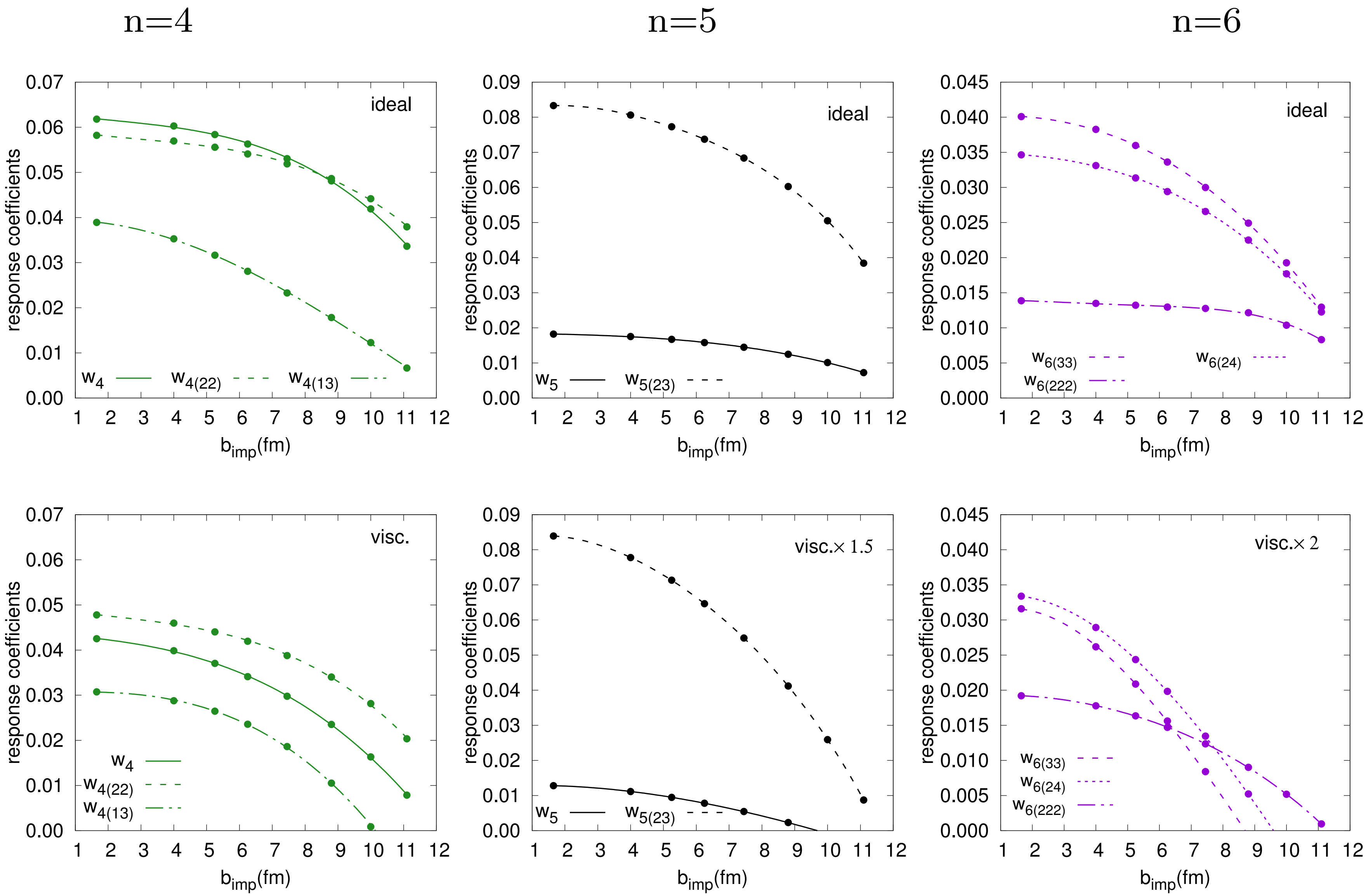


- Viscous damping qualitatively follows the rule.
- For n=4 and n=5, viscous hydro. may have negative response.
- damping of $w_{4(22)} \simeq 2 \times$ damping of $w_2 <$ damping of w_4.
- damping of $w_{5(23)} \simeq$ damping of w_2 + damping of $w_3 <$ damping of w_5.

Stony Brook University


visc.
linear
nonlinear
total
$v_3\{2\}/\varepsilon_3\{2\}$
N_{part}
linear
nonlinear
total
visc.
$v_4\{2\}/\varepsilon_4\{2\}$
N_{part}
visc
linear
nonlinear
total
$v_5\{2\}/\varepsilon_5\{2\}$
N_{part}

The magenitude of non-linear flow response (vs. centrality)



LHC PbPb: ideal hydro. and visc. hydro. ($\eta/s = 1/4\pi$), $T_{\mathrm{fo}} = 150$ MeV.