Non-linear flow response and plane correlations

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Outline

Initial state fluctuations + Event-By-Event hydro give

- Harmonic flow: $v_2$, $v_3$, etc.
- Correlations of reaction plane in final state.

Can we understand E-B-E hydro?
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1. Cumulant formalism for initial geometry with fluctuations.
3. Reaction-plane correlations of final state (ATLAS results).
Outline

Initial state fluctuations + Event-By-Event hydro give

- Harmonic flow: $v_2$, $v_3$, etc.
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Can we understand E-B-E hydro.?

1. Cumulant formalism for initial geometry with fluctuations.
3. Reaction-plane correlations of final state (ATLAS results).

\[ \text{Initial correlations} + \text{Flow response} = \text{Reaction place correlations} \]

\[ \text{Initial state(cumulants)} + \text{Linear & Non-linear} = \text{Final state} \]
Cumulants for initial state: (not moments!)

Fluctuations in initial state as corrections:

$\rho(x, y) = \text{Gaussian} + 1\text{st cumulant} + 3\text{rd cumulant} + 4\text{th cumulant} + \ldots$

- Cumulant expansion:

$$\rho(x, y) = \text{Gaussian} + 1\text{st cumulant} + 3\text{rd cumulant} + 4\text{th cumulant} + \ldots$$

- 4th Cumulant determines eccentricity $C_4$ and participant angle $\Phi_4$.

$$C_4 e^{4i\Phi_4} = -\left\langle \frac{1}{r^4} \right\rangle \left\langle r^4 e^{i4\phi_r} \right\rangle - 3\left\langle r^2 e^{i2\phi_2} \right\rangle^2$$

4th cumulant $\epsilon_4$: moments def. subtract $\epsilon_2^2$

e.g. Gaussian with $\epsilon_2$ has $C_4 = 0$, but $\epsilon_4 \propto \epsilon_2^2 \neq 0$.

- Why we use cumulants: avoid double counting in initial conditions.

We define all geometric deformations, i.e. $(C_n, \Phi_n)$, with cumulants.
Non-linear response formalism (n=5 for example)

Flow generation in hydro:

\[ v_5 e^{-i5\Psi_5} = \frac{w_5}{C_5} \times C_5 e^{-i5\Phi_5} \]

final state

linear resp.

initial state

\[ v_5 \sim (\text{linear}) + (\text{non-linear}) + (\text{interference} \propto \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)) . \]

\[^1\text{U. Heinz and Z. Qiu, and Gardim et al.}\]
Non-linear response formalism (n=5 for example)

Flow generation in hydro:

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final state  \quad \text{linear resp.}  \quad \text{initial state}

\[1\text{linear resp. fails for } n \geq 4\]

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Non-linear response formalism (n=5 for example)

Flow generation in hydro:

\[
\nu_5 e^{-i5\psi_5} = \frac{w_5}{C_5} \times C_5 e^{-i5\Phi_5} + \frac{w_5(23)}{\varepsilon_2\varepsilon_3} \times \varepsilon_2\varepsilon_3 e^{-i(3\Phi_3+2\Phi_2)}
\]

1 linear resp. fails for \(n \geq 4\)

- Assume non-linear flow response to \(\varepsilon_2\varepsilon_3\).

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- \( w_5 \) \( C_5 \) \( \varepsilon_2 \) \( \varepsilon_3 \)

\[ \varepsilon_2 \varepsilon_3 \]

\[ \Phi_3 \]

\[ \Phi_2 \]

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Non-linear response formalism (n=5 for example)

Flow generation in hydro:

\[ v_5 e^{-i5\Psi_5} = \left( w_5 \frac{C_5}{c_1} \right) e^{-i5\Phi_5} + \left( \frac{w_5(23)}{\varepsilon_2\varepsilon_3} \right) e^{-i(3\Phi_3 + 2\Phi_2)} \]

1. Linear response fails for \( n \geq 4 \)

- Assume non-linear flow response to \( \varepsilon_2\varepsilon_3 \).

- Calculations of \( v_n \{2\} \): flow from two-particle correlation.

\[ v_5 \{2\} = \left\langle \left| w_5 e^{-i5\Phi_5} + w_5(23) e^{-i(3\Phi_3 + 2\Phi_2)} \right|^2 \right\rangle^{1/2} \]

\[ v_5 \sim (\text{linear}) + (\text{non-linear}) + (\text{interference } \propto \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)) \]

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Non-linear response dependence on $p_T$

$w_4$ and $w_{4(22)}$

$w_5$ and $w_{5(23)}$

- Small $p_T$: non-linear response is not distinguishable from linear response.
- Large $p_T$: linear response $\propto p_T$, non-linear response $\propto p_T^2$.

So, non-linear response becomes more significant for larger $p_T$.

\footnote{(N. Borghini and J. Ollitrault)}
Damping rate \( \propto (\text{harmonic order})^2 \times \eta/s: \)

- Damping of \( w_{4(22)} \) < damping of \( w_4 \), which can be generalized to \( n \geq 4 \).
- \( \delta f \) on freeze out may be questionable for higher order flow response.

For \( (n \geq 4) \), non-linear response becomes more important for larger \( \eta/s \).

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\(^3\)Damping rate \( \propto (\text{harmonic order})^2 \times \eta/s: \)

\[^3\text{(S. Gubser and A. Yarom)}\]
Non-linear response dependence on centrality: integrated $v_n\{2\}$

$\bullet$  $v_n\{2\}^2 = \text{linear response} + \text{crossing terms} + \text{non-linear response}$

$\langle \varepsilon^2 \rangle$  $\langle \varepsilon^2 \cos(...) \rangle$  $\langle \varepsilon^4 \rangle$

$v_3\{2\} : w_3, w_3(12)$  $v_4\{2\} : w_4, w_4(22)$  $v_5\{2\} : w_5, w_5(23)$

(LHC PbPb, ideal hydro, $T_{fo} = 150\text{MeV}$, PHOBOS MC-GLb.)

- Non-linear response is not important for $v_3$, but crucial for $v_4$ and $v_5$.
- Linear response dominates at central bins.
- Non-linear response dominates at peripheral bins.

Non-linear response becomes more important for larger centrality.
Reaction-plane correlations: linear response \((\Phi_n, \ldots) \Leftrightarrow (\Psi_n, \ldots)\)

\((\Psi_4, \Psi_2)\) correlations:

- Initial correlations from cumulants and moments are different.
- Deviations (cumulants def.) from experiment data imply NL response.
  1. At central bins (linear dominant): smaller deviations.
  2. At peripheral bins (non-linear dominant): larger deviations.
Reactivity-plane correlations: linear response \((\Phi_n, \ldots) \Leftrightarrow (\Psi_n, \ldots)\)

\((\Psi_4, \Psi_2)\) correlations:

- Initial correlations from cumulants and moments are different.
- Deviations (cumulants def.) from experiment data imply NL response.
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Reaction-plane correlations: non-linear response \((\Phi_n, \ldots) \Leftrightarrow (\Psi_n, \ldots)\)

- \(\langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle\): \(\Psi_2 = \Phi_2\) and to the lowest order \(\Psi_3 = \Phi_3\).

\[
\langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle = \langle \frac{\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)w_5}{|w_5 e^{-i5\Phi_5} + w_5(23)e^{-i(2\Phi_2 + 3\Phi_3)}|} + w_5(23) \rangle
\]

- Understand RP plane correlation with non-linear response formalism.

\[
\text{RP correlations} = \langle \text{PP correlations} + \text{NL correlations} \rangle
\]

\(\text{Linear response limit}\)

\(\text{NL response limit}\)
Reaction-plane correlations: non-linear response \((\Phi_n, \ldots) \leftrightarrow (\Psi_n, \ldots)\)

\[\langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle\]: \(\Psi_2 = \Phi_2\) and to the lowest order \(\Psi_3 = \Phi_3\).

\[
\langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle = \left\langle \frac{\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)w_5w_3 + w_5(23)w_3 + \cdots}{w_5e^{-i5\Phi_5} + w_5(23)e^{-i(2\Phi_2 + 3\Phi_3)}} \right\rangle
\]

Understand RP plane correlation with non-linear response formalism.

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\text{RP correlations} = \langle \underbrace{\text{PP correlations}}_{\text{Linear response limit}} + \underbrace{\text{NL correlations}}_{\text{NL response limit}} \rangle
\]
Reaction-plane correlations: non-linear response ($\Phi_n, \ldots$) ⇐ ($\Psi_n, \ldots$)

- $\langle\cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5)\rangle$: $\Psi_2 = \Phi_2$ and to the lowest order $\Psi_3 = \Phi_3$.

$$\langle\cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5)\rangle = \left\langle \frac{\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)w_5w_3 + w_5(23)w_3 + \ldots}{|w_5e^{-i5\Phi_5} + w_5(23)e^{-i(2\Phi_2 + 3\Phi_3)}||v_3|} \right\rangle$$

- Understand RP plane correlation with non-linear response formalism.

$$\text{RP correlations} = \left\langle \underbrace{\text{PP correlations}}_{\text{Linear response limit}} + \underbrace{\text{NL correlations}}_{\text{NL response limit}} \right\rangle$$

Non-linear limit

2−3−5 correlation
Reaction-plane correlations: non-linear response ($\Phi_n, \ldots \rightleftharpoons \Psi_n, \ldots$)

1. $\langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle$: $\Psi_2 = \Phi_2$ and to the lowest order $\Psi_3 = \Phi_3$.

$$\langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle = \langle \left\langle \frac{\cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) w_5 w_3 + w_5(23) w_3 + \ldots}{w_5 e^{-i5\Phi_5} + w_5(23) e^{-i(2\Phi_2 + 3\Phi_3) \|v_3\|}} \right\rangle \rangle$$

2. Understand RP plane correlation with non-linear response formalism.

$$\text{RP correlations} = \langle \left\langle \text{PP correlations} \right\rangle + \left\langle \text{NL correlations} \right\rangle \rangle$$

- Linear response limit
- NL response limit

2–4 correlation

Non-linear limit

Linear limit
Two-plane correlations

\[
\langle \cos(4(\Psi_2 - \Psi_4)) \rangle
\]

\[
\langle \cos(8(\Psi_2 - \Psi_4)) \rangle
\]

\[
\langle \cos(12(\Psi_2 - \Psi_4)) \rangle
\]

Nonlinear Response:

- Glb Ideal
- Glb $\eta/s = 1/4\pi$
- $\sim$ ATLAS

\[
\langle \cos(6(\Psi_2 - \Psi_3)) \rangle
\]

\[
\langle \cos(6(\Psi_2 - \Psi_6)) \rangle
\]

\[
\langle \cos(6(\Psi_3 - \Psi_6)) \rangle
\]

$\rightarrow v_3(3, 12), v_4(4, 22), v_5(5, 23), v_6(33, 24, 222)$

$\rightarrow$ (LHC PbPb, $\eta/s = 1/4\pi$, $T_{fo} = 150$MeV, PHOBOS MC-GLb.)
Three-plane correlations

- $v_3(3, 12), v_4(4, 22), v_5(5, 23), v_6(33, 24, 222)$
- (LHC PbPb, $\eta/s = 1/4\pi, T_{fo} = 150\text{MeV}$, PHOBOS MC-GLb.)
Not fair comparison with E-By-E hydro

Response
Nonlinear Response:
- Gibb Ideal
- Gibb $\eta/s=1/4\pi$
- KLN $\eta/s=0.2$
- ATLAS

E by E
Heinz & Qiu E-by-E hydro:
- Gibb $\eta/s=1/4\pi$
- KLN $\eta/s=0.2$
- ATLAS
Summary and conclusions

We have developed a non-linear response formalism for (single-show) hydro:

\[
\text{Initial correlations} + \text{Flow response} = \text{Reaction plane correlations} \\
\text{cumulants} \quad \text{Linear & Non-linear} \quad \text{Final state}
\]

- Ingredients:

1. We use cumulant formalism to classify initial fluctuations.
2. We take linear and non-linear response in hydro calculations, non-linear response vs. \((p_T, \eta/s, \text{centrality})\).
3. Predictions are consistent to final state observables (E-B-E hydro).
Summary and conclusions

We have developed a non-linear response formalism for (single-show) hydro:

\[ \text{Initial correlations} + \text{Flow response} = \text{Reaction plane correlations} \]

- **Ingredients:**
  1. We use cumulant formalism to classify initial fluctuations.
  2. We take linear and non-linear response in hydro calculations, non-linear response vs. \((p_T, \eta/s, \text{centrality})\).
  3. Predictions are consistent to final state observables (E-B-E hydro).

Thank you.
Back-up slides
For $w_{4(22)}$ the angle dependence is trivial.

- $\Phi_2 = \Phi_R$ fixed, while $\Phi_3$ rotates $\rightarrow w_{1(23)}$, ($w_{5(23)}$ similar!).
Non-linear correction in $v_1$ not significant, compare to $v_4$ and $v_5$.

Viscous damping at large $p_T$ region.
\[ v_4\{2\}(p_T)/v_2\{2\}(p_T)^2 \]

- Scaling behavior reproduced for ideal hydro. large \( p_T \) limit.
- Linear contribution affects the scaling.
- The real observables are dressed with quantities from events average.
\[ v_5\{2\}(p_T)/(v_2\{2\}(p_T)v_3\{2\}(p_T)) \]

- Scaling behavior reproduced for ideal hydro. large \( p_T \) limit.
- Linear contribution affects the scaling.
- The real observables are dressed with quantities from events average.
Shape switch at certain time – $\Delta \Phi_n = |\pi/n|

Each of the deformations evolves independently.

Different angular deformations do NOT interact during expansion.
Damping rate as a function of $\eta/s$, linear response

Damping rate: $\Gamma_{n,m}\tau_{\text{final}} \rightarrow -\frac{\Delta w_n}{w_n^i} \propto (n+m)^2 \times \frac{\eta}{s}$

$n, m$ are cumulant indices.

- Viscous damping qualitatively follows the rule.
- For $n=4$ and $n=5$, viscous hydro. may have negative response.
- damping of $w_4^{(22)} \simeq 2 \times$ damping of $w_2 <$ damping of $w_4$.
- damping of $w_5^{(23)} \simeq$ damping of $w_2 +$ damping of $w_3 <$ damping of $w_5$. 
The magnitude of non-linear flow response (vs. centrality)

LHC PbPb: ideal hydro. and visc. hydro. ($\eta/s = 1/4\pi$), $T_\text{fo} = 150$ MeV.